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AN INVESTIGATION OF AIRCRAFT HEATERS

XVI - DETERMINATION OF THE VISCOSITY OF EXHAUST GASES FROM A GASOLINE ENGINE

By L. M. K. Boelter and W. H. Sharp
University of California

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XVI - DETERMINATION OF THE VISCOSITY OF EXHAUST GASES FROM A GASOLINE ENGINE

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SUMMARY

The absolute viscosity of exhaust gases from a gasoline engine was measured at temperatures from 75° to 890° F and at compositions corresponding to fuel-air ratios ranging from 0.0625 to 0.167. The viscosity was found to be nearly independent of the fuel-air ratio and within 6 percent of the value for dry air at the same temperature. During the calibration tests measurements of the viscosities of air, N_2, O_2, and CO_2 were also obtained.

INTRODUCTION

Measurements of the viscosity of high-temperature exhaust gases from an internal-combustion engine were undertaken as part of a program to establish values of certain physical properties of these gases. These properties - viscosity, thermal conductivity, heat capacity, and density - are necessary in order to evaluate the heat-transfer moduli used in the predictions of the performance of exhaust gas and air heat exchangers.

The heat capacity and density of gaseous mixtures can be closely approximated from calculations based on measurements of these properties of the pure components. There exists no experimental evidence, however, to prove that any of the equations proposed for the calculation of viscosity or thermal conductivity yield the correct results for an exhaust gas mixture at high temperatures.

Measurement of the viscosity of exhaust gases was first undertaken. With the viscosity known, the thermal conductivity can be approximately computed by means of a relation derived from the kinetic theory of gases (reference 1).

The necessity of determining the viscosity at conditions of elevated temperatures eliminates the use of many of the precise methods already devised for use at room temperature. The presence of water vapor in the gases further reduces the number of available methods because of the difficulty of retaining this component in making volumetric flow measurements. The method of measuring the time of viscous flow of a gas through a capillary placed between two reservoirs, one at a continuously decreasing pressure and the other at a lower and constant pressure, was selected as being most likely to meet all the requirements.

A preliminary viscosimeter was constructed in order to test the accuracy of the method. Tests at room temperature on air, O_2, and CO_2 indicated that the method was satisfactorily accurate. A second piece of apparatus was then made to obtain measurements of viscosity at elevated temperatures. The apparatus constant K was determined by calibration tests on air, O_2, and N_2, and viscosity determinations were made on exhaust gases for several fuel-air ratios at temperatures up to 890° F.

The present work is not considered complete, but the results indicate that the method is satisfactory and that the preliminary data may be useful.

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SYMBOLS

P_0	atmospheric static pressure (lb)/(sq ft)
P_1	static pressure, in excess of P_0, of gas in viscosimeter at time θ_1 (lb)/(sq ft)
P_2	static pressure, in excess of P_0, of gas in viscosimeter at time θ_2 (lb)/(sq ft)

P_f	measured static pressure, in excess of P_0, of gas in viscosimeter at conclusion of run (lb)/(sq ft)
l	length of capillary tube (ft)
r	inside radius of capillary (ft)
μ	absolute viscosity of gas (lb)(sec)/(sq ft)
V_1	volume of tubing between zero level of manometer and stopcock 2 (cu ft)
V_2	viscosimeter-reservoir volume (cu ft)
V_3	volume of tubing between capillary and outlet stopcock 3 (cu ft)
θ_1	time at which gas pressure in viscosimeter is P_1 (sec)
θ_2	time at which gas pressure in viscosimeter is P_2 (sec)
$K = \frac{\pi r^4}{8 l V_2}$	calibration constant for viscosimeter
Re	Reynolds number $= \frac{ud\rho}{\mu}$
Re_{max}	Reynolds number for flow through capillary (at time θ_1) $= \frac{u_{max}\, d\rho}{\mu}$
ρ	mass density of gas (lb)(sec^2)/(ft^4)
d	inside diameter of capillary tubing (ft)
u_{max}	maximum cross-sectional mean velocity of gas between θ_1 and θ_2 (ft)/(sec)
u	mean cross-sectional velocity of gas (ft)/(sec)
B	constant in Sutherland equation, $\mu = \frac{BT^{1/2}}{1 + \frac{C}{T}} \left(\frac{\text{lb sec}}{\text{ft}^2\ {}^{\circ}\text{R}^{1/2}}\right)$

C constant in Sutherland equation, $\mu = \dfrac{BT^{1/2}}{1 + \dfrac{C}{T}}$ (°R)

ANALYSIS

In the case of the experiments reported herein, the fluid flows from a reservoir in which the pressure is continuously decreasing through the capillary tube into a region of constant pressure (atmospheric). The pressure in the upstream reservoir decreases only because of the flow out through the capillary.

Based on the Navier-Stokes differential equation of motion for viscous flow and neglecting acceleration factors, which are negligible for the condition of small pressure gradient along the capillary tube, the expression for the variation of pressure in the reservoir with time as derived in reference 2 is:

$$\frac{1}{2P_0} \log_e \left[\left(\frac{P_2 + 2P_0}{P_1 + 2P_0} \right) \frac{P_1}{P_2} \right] = \frac{\pi r^4}{16 \mu l V_2} (\theta_2 - \theta_1) \qquad (1)$$

Solving equation (1) for the gas viscosity μ yields:

$$\mu = -\left(\frac{\pi r^4}{8 l V_2} \right) \frac{P_0 (\theta_2 - \theta_1)}{\log_e \left[\left(\frac{P_2 + 2P_0}{P_1 + 2P_0} \right) \frac{P_1}{P_2} \right]} \qquad (2)$$

where

$$\frac{\pi r^4}{8 l V_2} = K \qquad (3)$$

Solving equation (2) for K yields:

$$K = \frac{\mu}{P_0 (\theta_2 - \theta_1)} \log_e \left[\left(\frac{P_2 + 2P_0}{P_1 + 2P_0} \right) \frac{P_1}{P_2} \right] \qquad (4)$$

The term K is a constant for the particular piece of apparatus used for the determination of the viscosity of the gas. The magnitude of K may be established in two ways: (1) By directly measuring r, l, and V_2 and computing K from equation (3); and (2) by calibrating the apparatus with tests on a gas of known viscosity and computing K from equation (4).

APPARATUS AND METHODS

Description of apparatus.- The final viscosimeter (fig. 1) was constructed entirely of Pyrex to fit it for medium-temperature measurement. The volume of the cylindrical reservoir was 50.1 cubic inches; the capillary was 35 inches long and 0.0183 inch in diameter before coiling. The capillary was wound into a 2-inch diameter helix so that the entire viscosimeter could be placed inside an insulated cylindrical hot-air furnace. Because it was impracticable to measure directly the gas temperatures in the reservoir and capillary, these temperatures were approximated by means of three (chromel-alumel) thermocouples which were inserted into wells in the reservoir, two more thermocouples in the hot-air stream around the capillary, and one thermocouple in each of the ducts that carried the hot air in and out of the furnace. Two small-bore tubes with stopcocks were added to provide a separate evacuation line and inlet for the gas samples. Both the volume V_1 of the tubing between the manometer and the reservoir, and the volume V_3 between the capillary and outlet stopcock were held to minimum values by the use of small tubing in order to reduce the corrections to the final measured pressure; the corrections depend on the magnitudes of these volumes and on the corresponding temperatures of the gas in these volumes. (See the appendix.)

The values of capillary radius r and length l, and reservoir volume V_2 necessary for determination of the viscosimeter constant K by the direct method involving equation (3) were obtained by measuring the length before coiling and by calculating the reservoir volume from the weight of water and the capillary radius from the weight of mercury contained. The value of r obtained in this manner for a tube with bore irregularities is not exactly the value which should be used in the capillary-flow equation since the equation involves r^4 and the method of measuring r^2. The value of K obtained by this method, however, was used only as a check on the value obtained by the indirect method (equation (4)) which consisted of calibration of the apparatus with tests on a gas of known viscosity: namely, air.

The viscosimeter was fastened to a wooden stand for convenient handling and thoroughly cleaned with alcoholic potassium hydroxide, hot nitric acid, and distilled water. The reservoir was packed in an insulating material (ground asbestos) and the entire apparatus was set up in a constant-temperature room. The air used in the calibration tests was dried and purified before it entered the reservoir. The samples of O_2 and N_2 used in other runs were introduced directly without purification.

Preliminary runs.- The first two series of runs, one on air and one on CO_2, were made at room temperature, at initial pressures of 2 to 7.5 inches of mercury (measured above atmospheric pressure with a mercury manometer), and with times of efflux of 105 to 210 seconds. The results showed a decrease in the apparent value of K with an increase in pressure. The value approached a constant, however, at the low pressures and concomitant low efflux rates.

In order to obtain more accurate measurements at low pressures, the mercury manometer was replaced with one of similar design but with a longer column and using Ellison* fluid. A series of runs was made at room temperature on air at a range of pressures equivalent to 3 to 25 inches of water. These runs showed that there was no apparent deviation in K over this initial pressure range. Tests on CO_2 were not carried out because of the slight solubility of CO_2 in Ellison fluid.

Medium-temperature runs on exhaust gases.- A series of runs on exhaust gases then was made at a temperature range of 78° to 890° F, at a range of fuel-air ratios from 0.0602 to 0.158, and at pressures equivalent to 3 to 28 inches of water. The composition of the exhaust gases at various fuel-air ratios was determined by Orsat analyses.

Calibration runs at room temperature.- A series of calibration runs then was carried out to determine if the medium-temperature runs had affected the apparatus constant K. These runs were made on air, O_2, and N_2 at room temperature, at a pressure range equivalent to 3 to 28 inches of water, and at times of efflux from 60 to 315 seconds.

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*A mineral oil plus red dye, sp. gr. = 0.835 at 75° F.

Check runs at medium temperatures.- A series of runs also was made to check the value of K at medium temperatures. The runs were made on air, O_2, and N_2 at a temperature range of 75° to 868° F and with pressure conditions and times of efflux similar to those of the calibration runs.

RESULTS AND DISCUSSION

Preliminary runs.- The results of the preliminary runs with the mercury manometer indicated variation in the apparent value of K with change in pressure. This variation may be attributed to Reynolds number effect for flow through curved passages. When K is plotted as a function of Re_{max} (fig. 2) the results of the runs on CO_2 coincide with the results of the two series of tests on air. This relation between K and Re_{max} shown in figure 2 agrees qualitatively with the work of C. M. White (reference 3) on steady flow of fluids through curved pipes. From a study of the data on the flow of three different fluids (air, oil, and water) White concluded that the static pressure drop along curved pipes could be calculated from the laminar-flow equation for straight pipes if the value of the dimensionless ratio $Re\left(\frac{d}{D}\right)^{1/2}$ were less than 11.6 where Re is Reynolds number, d is the inside diameter of the pipe, and D is the diameter of the coil. From this relation the maximum value of Reynolds number for which K should be constant was about 120, although for the case of flow under decreasing pressure it might be expected that the value of Re_{max} at time θ_1 could be somewhat higher than 120 before the effect on K was apparent. The experimental data show that K does decrease at values of Re_{max} only slightly above 120.

Calibration runs at room temperature.- The variations in pressure and times of efflux in these tests (see table 1) resulted in no discernible effect on the values of K obtained. The average deviations of the individual runs from the mean of all runs of the three series was 0.35 percent for 85 runs on air, 0.26 percent for 20 runs on N_2, and 1.0 percent for 19 runs on O_2. The mean value of K for each of the three series was the same; namely,

$K = 14.7 \times 10^{-13}$; while the value obtained by direct measurements was 15.3×10^{-13}. The value of K used in the viscosity calculations for other gases was that obtained by indirect measurement - that is, 14.7×10^{-13}.

Check runs at medium temperatures.- For purposes of comparison, the value of K determined from low-pressure (difference) measurements on air at room temperature was used in conjunction with the pressure-time measurements of the medium temperature runs to calculate the viscosities of N_2, O_2, and air at higher temperatures. (These values are compared in fig. 3 with curves drawn through the points obtained by previous investigators.) This value of K was used because it is based on the viscosity of air at room temperature, a subject that has been very carefully investigated, and because room-temperature measurements are subject to less error than higher-temperature measurements. The curves of the values obtained for the viscosities of O_2 and N_2 at room temperature coincide with the curves obtained by previous investigators, but at higher temperatures, the curves for O_2 and N_2 as well as for air, lie slightly above the curves of previous investigators. The maximum deviation of the experimental curves from the curves obtained by previous investigators are approximately 1 percent for air and O_2 and 2.5 percent for N_2. (See table 2 for summary of data.)

Exhaust gas measurements.- The results of the exhaust gas measurements are given in table 3 and figure 4. These results indicate that up to 890° F the viscosity of exhaust gases from gasoline engines is less than 6 percent below that of air at the same temperature and is only slightly affected by changes in fuel-air ratio. The purpose of the following discussion is to indicate: (1) Why the viscosity of exhaust gas mixtures should be approximately the same as the viscosity of air at the temperatures of these measurements and closer at higher temperatures, and (2) why changes in the fuel-air ratio produce only slight changes in the viscosity of the exhaust-gas mixtures.

Over the range 75° to 1600° F the viscosity curve for oxygen lies about 10 percent above that for air; while the curves for all the gases that replace oxygen in the combustion mixture lie below the air curve. (See fig. 5.) Although the exact viscosity of the mixture of gases cannot be accurately determined from a knowledge only of the composition of the mixture and the viscosities of its constituents, the data clearly indicate that the viscosity of exhaust gases should be close to and probably below that of air at the same temperature. Furthermore, the percentage deviation in the magnitudes of the viscosities of the principal combustion products, CO_2 and water vapor, from air at the same temperature decreases considerably with temperature while the viscosities of O_2, N_2, CO, and H_2 retain their relative positions with respect to air. This relation indicates that, at higher temperatures, the values of the viscosity of air and the exhaust-gas mixtures should tend to converge slightly.

Further evidence that at high temperatures the viscosities of the exhaust gas mixtures are only slightly below the viscosity of air at the same temperature is obtained by use of the Sutherland equation

$$\mu = \frac{BT^{1/2}}{1 + \frac{C}{T}} \tag{5}$$

where B and C are constants for each gas or mixture and T is the absolute temperature. This equation, which has been experimentally verified over a wide range of temperatures for pure gases by several investigators, also appears to hold well for mixtures (reference 4). Rearrangement of the equation to

$$T = B\left(\frac{T^{3/2}}{\mu}\right) - C \tag{6}$$

gives a straight line, if the equation is valid, when T is plotted against $\frac{T^{3/2}}{\mu}$. Plots of this type for air and the exhaust gas mixtures (fig. 6) give straight lines which, when extrapolated to 1500° F, indicate that the viscosity of the lean mixtures is only 2.0 percent below, and the rich mixtures 3.6 percent below, the viscosity of air at 1500° F.

The slight effect of the fuel-air ratio on the viscosity of the exhaust gases is made apparent by a consideration of the effect of the fuel-air ratio on the composition of the exhaust gases, as shown in the following table:

	Percentage composition by volume			
Fuel-air ratio	CO_2	CO	O_2	H_2O (calculated)
0.167-0.143	4.3	12	0.7	
.100- .091	6.7	7.7	1.4	
.063- .059	11.0	.1	2.8	12

If the fuel-air ratio is <u>decreased</u> beyond the optimum, the CO_2 content is increased at the expense of the CO. This change tends to reduce slightly the viscosity of the mixture, but the reduction is opposed by the increase in the O_2 content. If the fuel-air ratio is <u>increased</u> beyond the optimum, practically all the O_2 is removed from the air, and the steam and H_2 contents are slightly increased. This change might tend to reduce the viscosity, but it is opposed by the decrease in the CO_2-CO ratio. In general, although these changes in composition do not produce opposing effects of equal magnitude, all the changes in composition are small and their net effect on the viscosity are so negligible that the viscosity of the exhaust gases is practically equal to the viscosity of air at the same temperature.

CONCLUSIONS

1. Measurements of the viscosity of exhaust gases from a gasoline engine at temperatures from 75° to 890° F give values that are 3 to 6 percent below the viscosity of air at the same temperature.

2. Extrapolation of the results by Sutherland's equation indicates that at the temperature at which the exhaust gases leave the engine, about 1500° F, the viscosity of exhaust gas mixtures is only 2 to 4 percent below that of air at the same temperature.

3. Variation of the engine fuel-air ratio from 0.063 to 0.167 had very little effect on the viscosity of the exhaust gases.

University of California,
Berkeley, Calif., April 1944.

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APPENDIX

ACCURACY AND CORRECTIONS

Accuracy of the Measurements of Viscosity

The accuracy of the viscosities obtained from the present measurements is determined by the following factors:

1. The accuracy of the viscosity value used to determine the apparatus constant K

2. The validity of the postulates made in the derivation of the flow equation (2)

3. The accuracy of measurement of the quantities P_0, P_1, P_2, θ_1, and θ_2

4. The magnitude of the temperature variation during the test runs

5. The temperature gradient along the capillary tube

An analysis of the magnitude of these estimated errors indicates that at room temperature the individual runs should be within ±0.3 percent of the mean of all the runs of the series and that the error in the mean value should be less than ±0.3 percent. At higher temperatures, where temperature gradients and fluctuations are larger, the maximum deviations from the mean value and the error of the mean value both may be of the order of ±2.5 percent, if there is only a small number of runs in the series. The experimental results (figs. 3 and 4) indicate that the deviations and the errors are of approximately the magnitude estimated, except that at medium temperatures the errors always appear to be positive. Comparison with other investigations indicates that the experimental values obtained herein are slightly higher than previously reported. On this basis, the viscosities of the exhaust gas mixtures are probably about 1 to 2 percent above the true value.

Corrections Applicable to Equation (2)

Equation (2) postulates that the static pressure drop at any point between the reservoir and the atmosphere is due only to the viscous drag at the wall of the capillary tube. Actually other mechanisms cause additional energy losses and contribute to the over-all pressure drop. Briefly, these additional mechanisms are manifested as:

(a) Initial acceleration of the gas in the capillary at time (θ_1) of opening of stopcock 3

(b) Inlet pressure drop which includes loss due to contraction and also pressure drop due to acceleration

(c) Pressure drop due to change in velocity distribution along entrance length

(d) Pressure drop due to acceleration caused by change in density of fluid along the tube

(e) Pressure drop due to expansion at end of capillary tube

An analysis of these pressure-drop components indicates that their magnitudes are small (less than the experimental error) and that equation (2) can be used without appreciable error.

Correction to Measurement of Final Pressure

The measured final pressure P_f had to be corrected to obtain the value of P_2 to be substituted into equation (2) for calculation of the fluid viscosity. This correction is necessary because it is impossible to measure the gas pressure P_2 existing in the reservoir at time θ_2. At time θ_2, as the outlet stopcock was closed and the watch simultaneously stopped, the gas in the bulb continued to flow through the capillary until the pressure in the small volume V_3 was raised from P_0 to P_f. When stopcock 2 in the manometer line was opened in order to measure the reservoir pressure at the end of the run, part of the gas in V_1 which was still under pressure P_1, escaped into the reservoir. This flow continued until the pressure P_f was established throughout V_1, V_2, and V_3.

The correction to be applied to P_f is obtained by application of the perfect gas law to the pertinent volumes, pressures, and weights of gas. Because the pressure differences and volumes V_1 and V_3 were small, the maximum correction of P_f to P_2 was only of the order of 0.1 inch of water.

TABLE 1.- RESULTS OF CALIBRATION RUNS

Number of runs	Gas used	Temperature ($^\circ$F)	Viscosity, μ $\left(\frac{\text{lb/sec}}{\text{sq ft}}\right)$	Range[a] of P_1 (in. water)	Range of θ	K (b)	Mean deviation (percent)
85	Air	74	3.83×10^{-7}	3.0-28	60-215	14.7×10^{-13}	0.35
19	O_2	79	4.29	6.3-25	120-180	14.7	1.0
20	N_2	75	3.67	5.4-25	120-315	14.7	.26

[a] Ellison fluid was used in the manometer for the measurements.

[b] The value of K calculated from direct measurement[c] of r, l, and V_2 was $K = 15.3 \times 10^{-13}$. The value of $K = 14.7 \times 10^{-13}$ was taken as the viscosimeter constant.

[c] The measurement of r was made before the capillary was coiled.

TABLE 2.- RESULTS OF CHECK RUNS AT MEDIUM TEMPERATURES

Gas	Temperature[a] (°F)	Number of runs	Viscosity, μ $\left(\frac{\text{lb/sec}}{\text{sq ft}}\right)$ (b)	Mean deviation (percent)
Air	256	9	4.78×10^{-13}	1.5
	431	6	5.58	1.6
	468	9	5.78	1.0
	598	4	6.24	3.0
	621	5	6.43	1.4
	868	9	7.30	1.5
				1.5 average
O_2	401	8	6.12	1.1
	474	7	6.60	.6
	594	9	7.14	1.2
	761	3	7.84	.8
				1.0 average
N_2	401	10	5.28	0.8
	502	7	5.76	1.4
	598	12	6.03	1.6
	755	10	6.68	.8
				1.2 average

[a] Temperature indicated by thermocouple number 4 in reservoir.

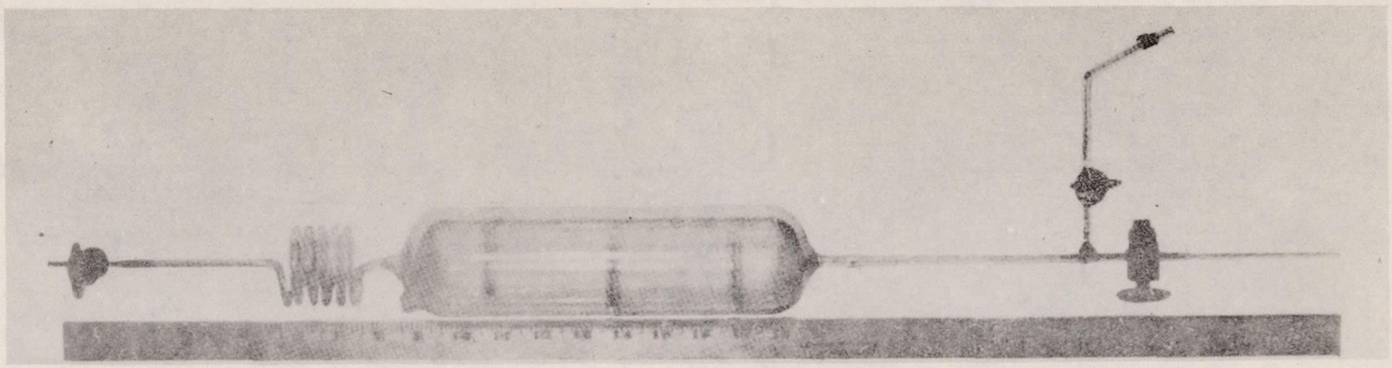
[b] These measured values were obtained by use of viscosimeter constant K based on air measurements at 76° F.

TABLE 3.- RESULTS OF MEASUREMENTS ON EXHAUST GASES

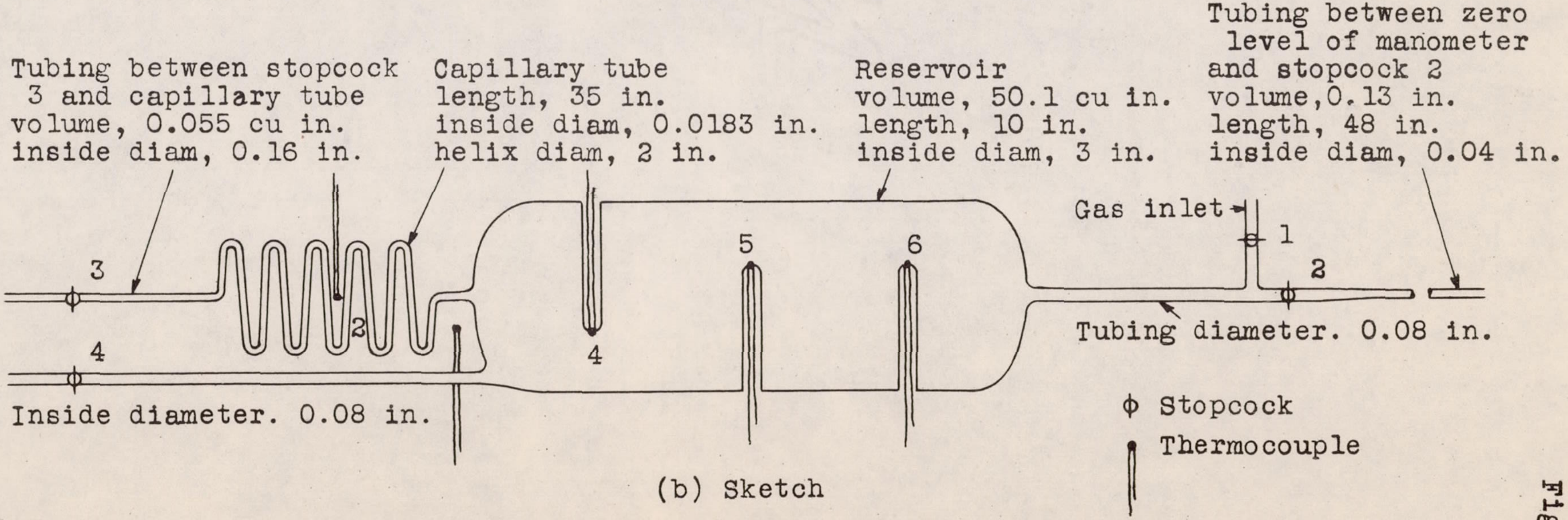
Temperature (°F)	Number of runs	Fuel-air ratio	Viscosity, μ $\left(\frac{\text{lb/sec}}{\text{sq ft}}\right)$	Mean deviation (percent)
[a]78	8	0.158	3.55×10^{-7}	0.8
[a]78	3	.075	3.65	.2
352	2	.152	5.01	.6
377	9	.100	5.10	1.0
378	6	.060	5.14	.8
488	4	.078	5.62	2.0
490	4	[b]R	5.47	1.3
490	2	.060	5.54	1.1
502	4	.060	5.66	2.0
590	11	.094	6.01	.6
598	4	.060	6.14	.9
599	9	.152	5.91	.7
672	4	.106	6.30	1.1
695	8	.060	6.47	1.0
800	10	[b]R	6.74	2.4
828	8	.106	6.83	1.4
890	9	[b]R	7.08	.9
				1.3 (average)

[a] The preliminary viscosimeter was used for the two room-temerature tests; all other measurements were made with the final viscosimeter.

[b] The three sets of measurements that have a fuel-air ratio denoted as R were very rich mixtures for which the fuel-air data were not recorded.



(a) Photograph



(b) Sketch

Figure 1. Medium temperature viscosimeter.

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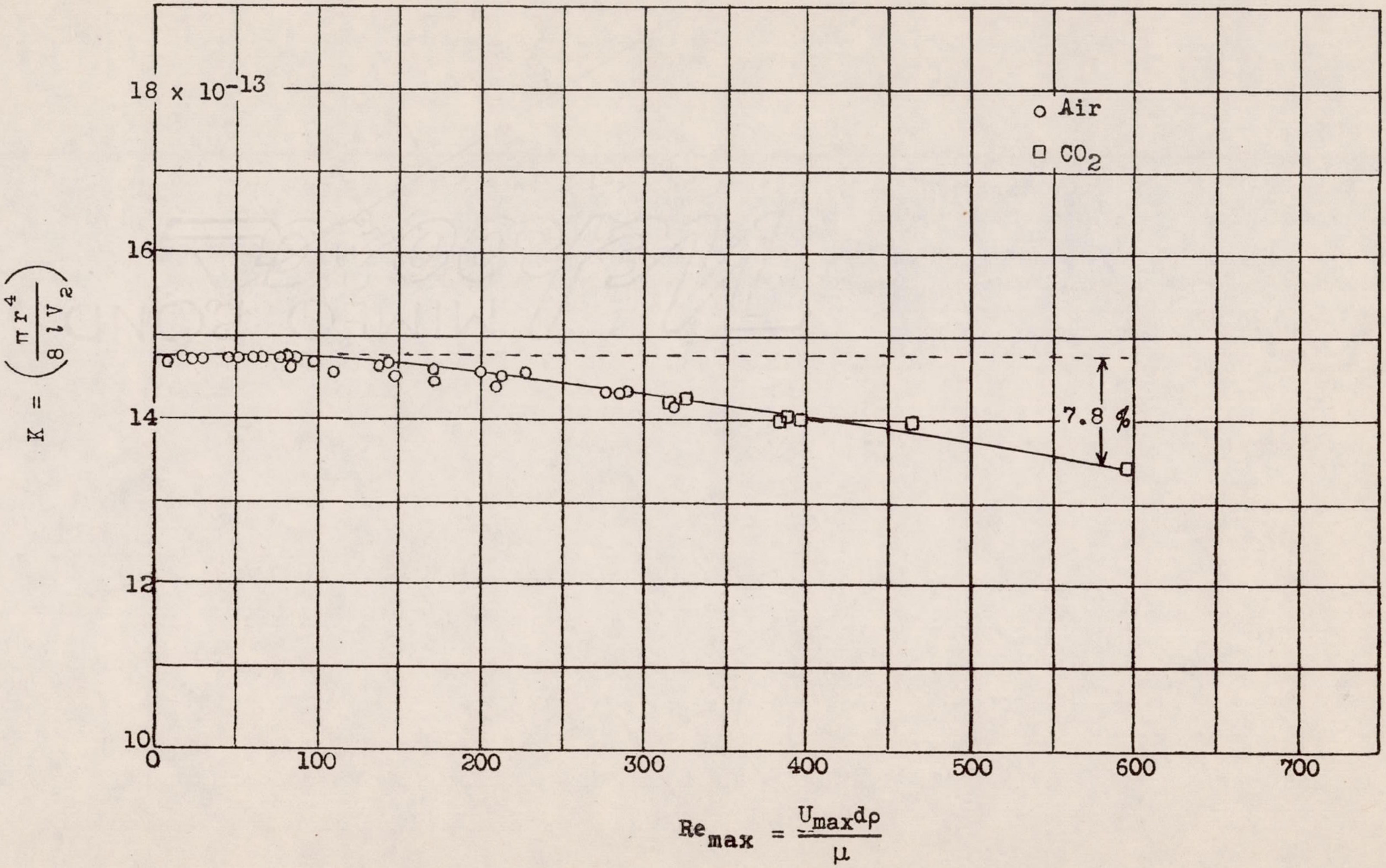

Figure 2.- Viscosimeter constant as a function of Reynolds number.

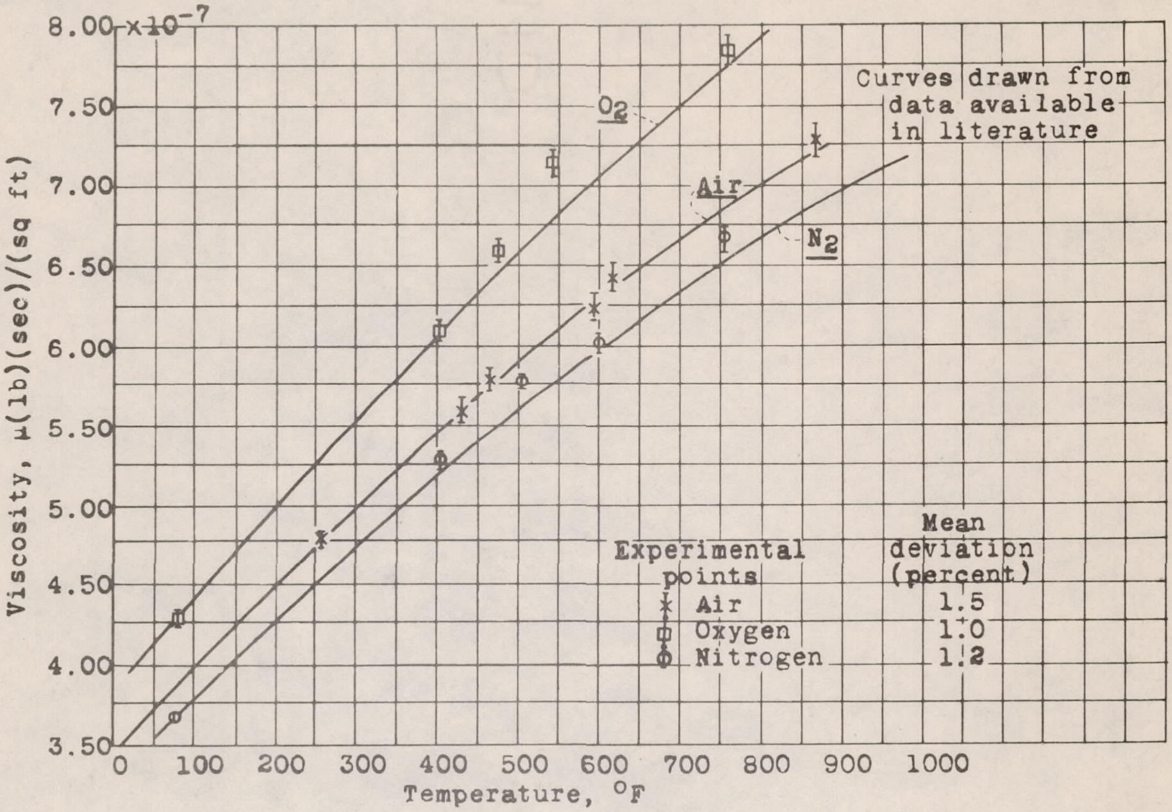

Figure 3.- Viscosity of air, oxygen, and nitrogen.

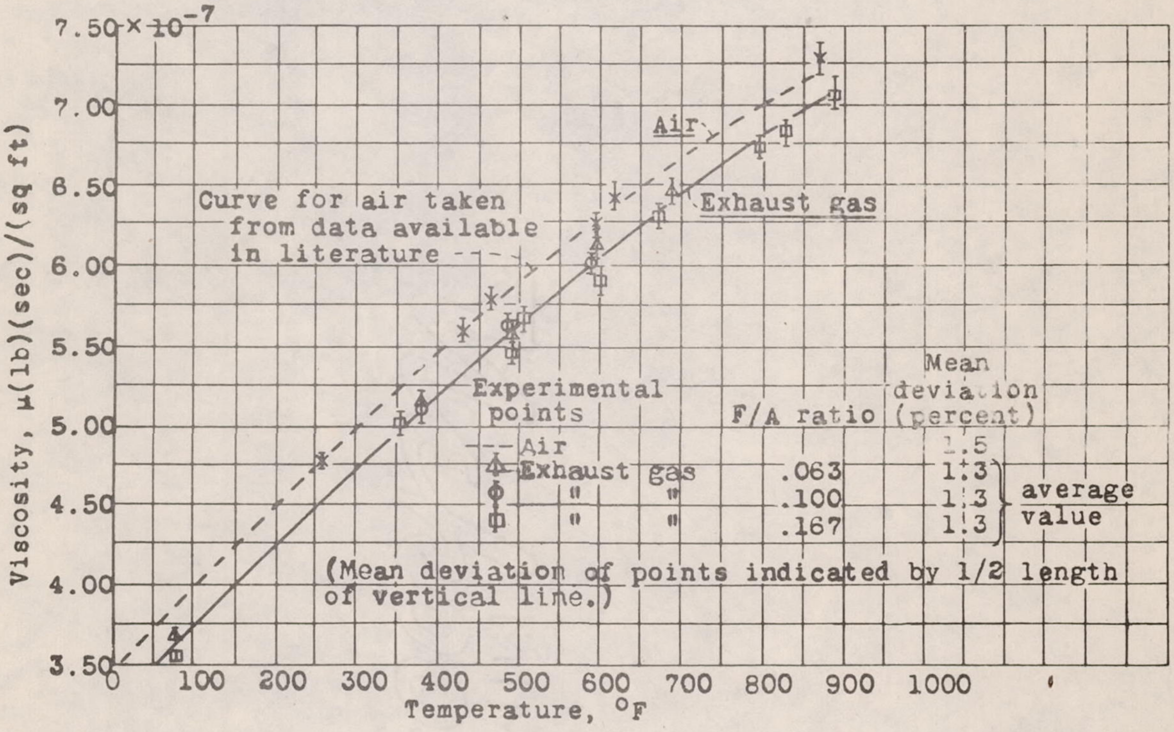

Figure 4.- Viscosity of air and exhaust gas at various temperatures and fuel-air ratios.

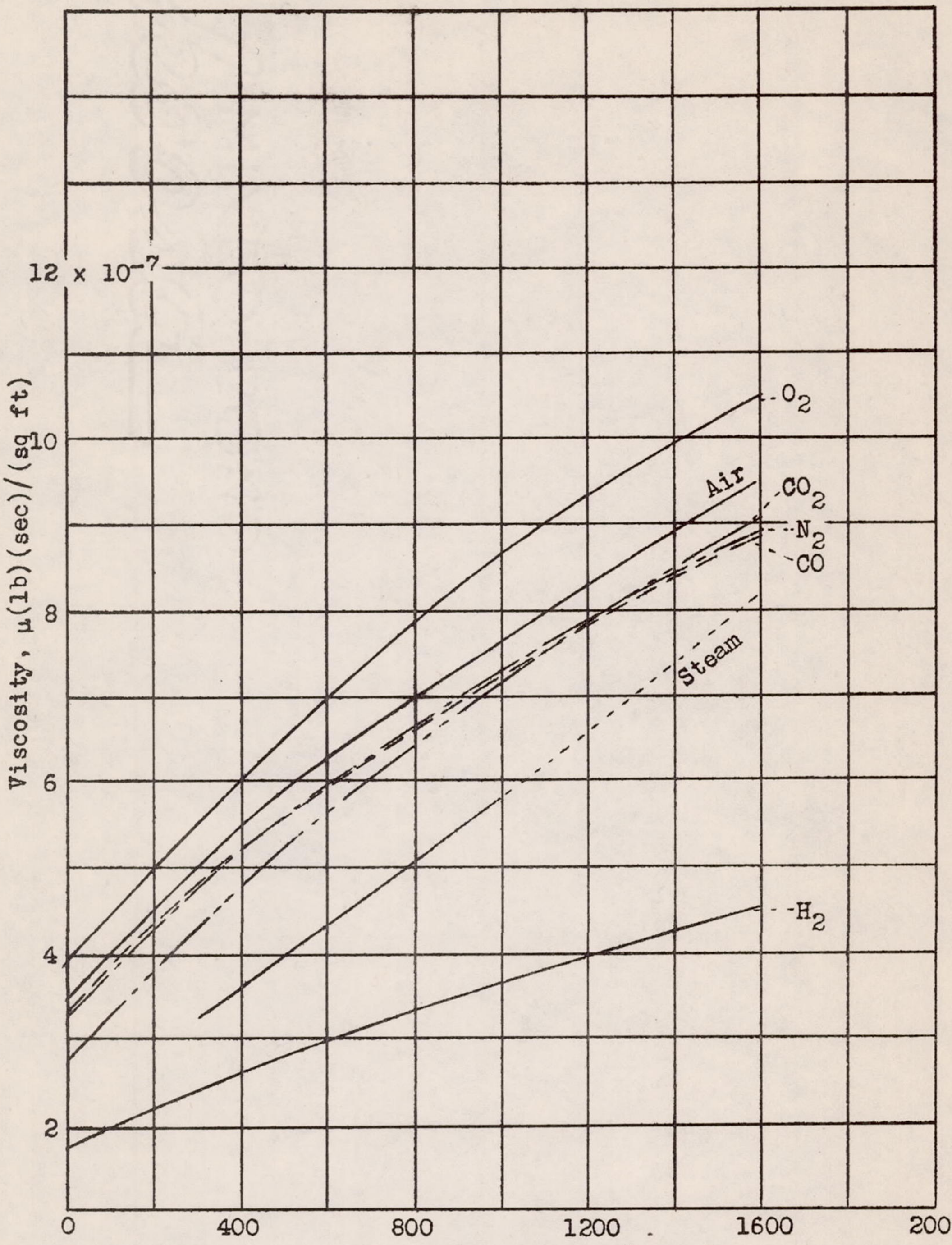

Figure 5.- Viscosity of principal exhaust-gas constituents. (Data from reference 1.)

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$B = 2.30 \times 10^{-8} \dfrac{\text{lb}\cdot\text{sec}}{\text{ft}^2\cdot{}^{\circ}\text{R}^{1/2}}$

$C = 260\ {}^{\circ}\text{R}$

$B = 2.32 \times 10^{-8} \dfrac{\text{lb}\cdot\text{sec}}{\text{ft}^2\cdot{}^{\circ}\text{R}^{1/2}}$

$C = 200\ {}^{\circ}\text{R}$

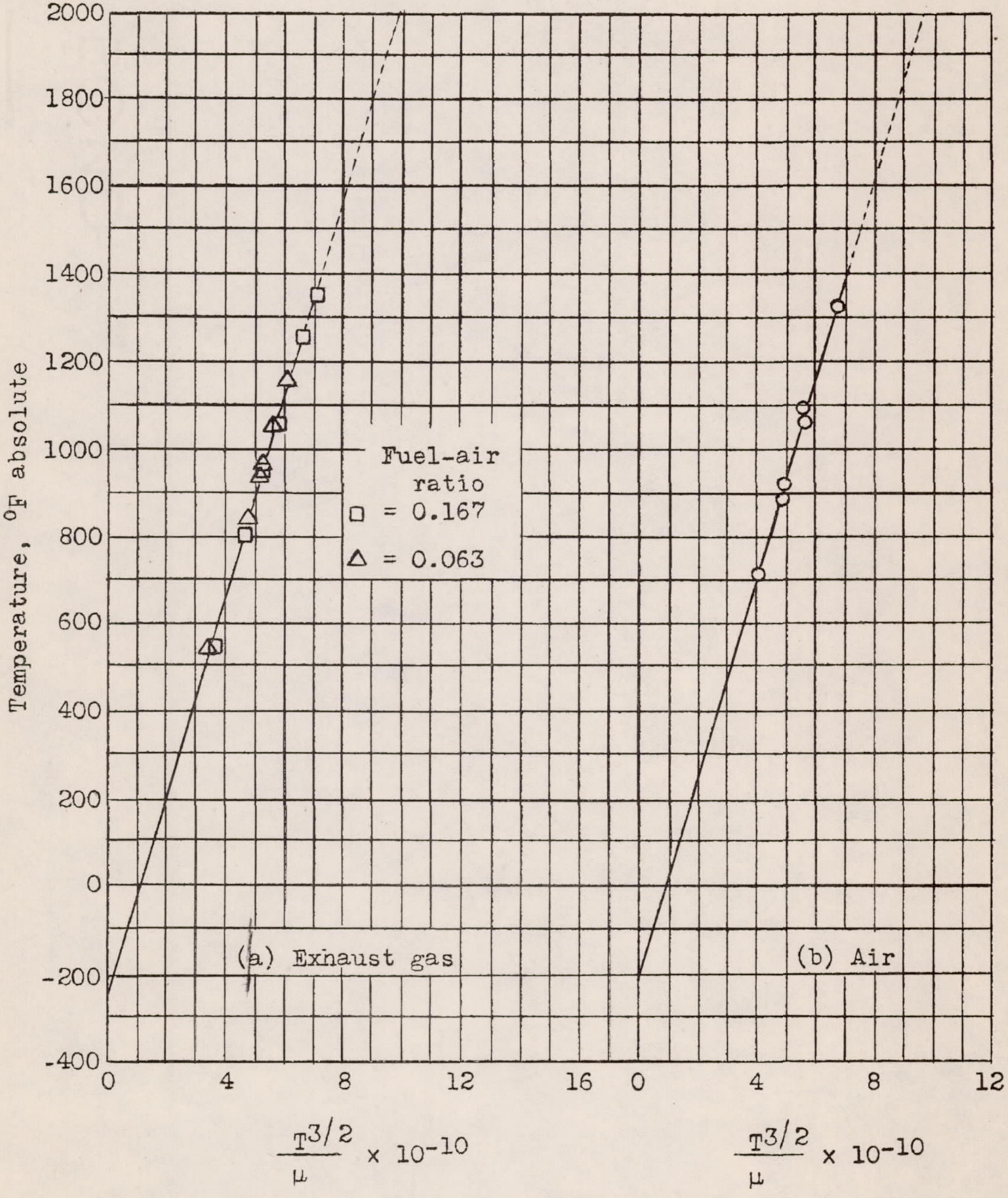

Figure 6.- Sutherland-equation plot, $T = \left(\frac{BT^{3/2}}{\mu}\right) - C$

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