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## AN INVESTIGATION OF AIRCRAFT HEATERS

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WASHINGTON

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# AN INVESTIGATION OF AIRCRAFT HEATERS

# XXI - MEASURED AND PREDICTED PERFORMANCE OF A FLATTENED-TUBE TYPE CROSSFLOW EXHAUST GAS AND AIR HEAT EXCHANGER

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## SUMMARY

Data on the thermal performance and the static pressure drop characteristics of a "flattened-tube type" crossflow exhaust gas and air heat exchanger are presented. A full-crossflow type air shroud was used for the tests on this exchanger.

The weight rates of exhaust gas which were used in the tests varied from 1700 lb/hr to 4200 lb/hr and the weight rates of ventilating air ranged from 1000 lb/hr to 4200 lb/hr. The inlet temperature of the exhaust gas was kept at approximately 1600° F. Static pressure drop measurements were made across the exhaust gas and ventilating air sides of the heater under isothermal and non-isothermal conditions.

The measured thermal outputs and static pressure drops are compared with predicted magnitudes.

## INTRODUCTION

Investigation of the performance characteristics of this flattened-tube type crossflow heat exchanger, designed for use in the exhaust gas streams of aircraft engines for cabin heating systems and for wing and tail surface anti-icing systems, was carried out on the large test stand of the Mechanical Engineering Laboratories of the University of California. (See description of this test stand in reference 1.)

The following data were obtained:

1. Weight rates of the exhaust gas and ventilating air through the respective sides of the heat exchanger

2. Temperatures of the exhaust gas and ventilating air at inlet and outlet of the exchanger

3. Static pressure drops across the exhaust gas and ventilating air sides of the heat exchanger for isothermal and non-isothermal conditions

This investigation, part of a research program conducted on aircraft heat exchangers at the University of California, was sponsored by and conducted with the financial assistance of the National Advisory Committee for Aeronautics.

## SYMBOLS

$a$ transverse spacing of tubes in banks, measured in multiples of the diameter of the cylinders

$A$ area of heat transfer, ft$^2$

$A_{cs}$ minimum cross-sectional area of flow for either fluid, ft$^2$

$A_h$ cross-sectional area of flow for either fluid measured within the heater, ft$^2$

$A_1$ cross-sectional area of flow for either fluid taken at the inlet pressure measuring station, ft$^2$

$A_2$ cross-sectional area of flow for either fluid taken at the outlet pressure measuring station, ft$^2$

$c_p$ heat capacity of the fluid at constant pressure, Btu/lb °F

$D$ hydraulic diameter, ft

$f_c$ unit thermal convective conductance (average with length), Btu/hr ft$^2$ °F

$f_{cc}$ unit thermal convective conductance for flow of exhaust gas over cylinders with a diameter equivalent to that of the circular leading and trailing edges of the flattened tubes (average for $N$ rows of cylinders), Btu/hr ft$^2$ °F

$f_{cf}$ unit thermal convective conductance, determined from equation (8), for flow of exhaust gas over flat plate portion of the flattened tubes (average with length), Btu/hr ft$^2$ °F

$f_{cx}$ point unit thermal convective conductance for flow over flat plates, Btu/hr ft$^2$ °F

$(f_cA)$ thermal conductance of either fluid, Btu/hr °F

$(f_cA)_c$ thermal conductance of exhaust gas for flow over rows of cylinders with diameters equal to that of the circular leading and trailing edges of the flattened tubes, Btu/hr °F

$(f_cA)_f$ thermal conductance of exhaust gas for flow over the flat plate portion of the flattened tubes, Btu/hr °F

$F_a$ tube arrangement modulus for flow of fluids over banks of staggered tubes

$g$ gravitational force per unit of mass, lb/(lb sec$^2$/ft)

$G$ weight rate of fluid per unit of area, lb/hr ft$^2$

$G_o$ weight rate of fluid per unit of area, based on minimum area, lb/hr ft$^2$

$K$ isothermal pressure drop factor defined by equation

$$\frac{\Delta P}{\gamma} = K \frac{u_m^2}{2g}$$

$l$ length of a duct measured from the entrance, ft

$n$ ratio of cross-sectional area of flow before expansion of the fluid passage to that after expansion of the fluid passage

$N$ number of rows in a tube bank

$q$ measured rate of enthalpy change of either fluid, Btu/hr or kBtu/hr (kBtu designates kilo Btu, or 1000 Btu/hr)

$t_w$ temperature of the tube wall, °F

$T_{av}$ arithmetic average mixed-mean absolute temperature of either fluid $\frac{T_1 + T_2}{2}$, °R

$T_f$ arithmetic average of tube wall absolute temperature and mixed-mean absolute temperature of fluid, °R

$T_{iso}$ mixed-mean absolute temperature of either fluid for the isothermal pressure drop tests, °R

$T$ mixed-mean absolute temperature of either fluid, °R

$u_m$ mean velocity of the fluid at the minimum cross-sectional area of the fluid passage, ft/sec

$UA$ over-all thermal conductance, Btu/hr °F

$W$ weight rate of either fluid, lb/hr

$Re$ Reynolds number, $\frac{GD}{3600\mu g}$

$x$ distance along a flat plate measured from the point of stagnation, ft

$x_1$ equivalent flat-plate distance measured along the circumference of cylindrical leading edge of a flattened tube from forward point of stagnation to beginning of flat plate section, ft

$x_2$ equivalent flat-plate distance measured along the periphery of a flattened tube from forward point of stagnation to end of flat plate section, ft

$\gamma$ weight density of fluid, lb/ft$^3$

$\Delta P$ static pressure drop, lb/ft$^2$

$\Delta P''$ static pressure drop (heater plus ducts) on either side, in. $H_2O$

$\Delta\tau_{mx}$ mean temperature difference for crossflow of fluids when the exhaust gas is mixed and the ventilating air is not mixed while passing through the heater, °F

$\mu$ viscosity of either fluid, lb sec/ft$^2$

$\zeta$ isothermal friction factor defined by the equation

$$\frac{\Delta P_{iso}}{\gamma} = \zeta \frac{l}{D} \frac{u_m^2}{2g}$$

$\zeta'$ friction factor for flow of fluids past tube banks (defined by equation (10))

$\tau$ mixed-mean temperature of either fluid, °F

$\varphi_x$ heater effectiveness for crossflow of fluids when exhaust gas is mixed and ventilating air is not mixed while passing through heater. This effectiveness is defined by equation
$q_a = W_a c_{p_a} (\tau_{g_1} - \tau_{a_1})\varphi_x$

Subscripts

a ventilating air side

c convective conductance ($f_c$ and so forth) and also sudden contraction ($K_c$)

oc convective conductance along a cylinder equivalent to leading and trailing edges of a flattened tube

cf convective conductance along flat-plate section of a flattened tube

e sudden expansion

f fluid property ($T_f$) and also flat plate $[(f_cA)_f]$

g exhaust gas side

h, htr heater

m mean values at any section of heater ($u_m$)

o maximum values ($G_o$)

x crossflow of fluids

av arithmetic average

contr sudden contraction

ducts ducts on either side of heat exchanger exp sudden expansion

fric friction

iso isothermal conditions

non-iso non-isothermal conditions

1 point 1, entrance section of heater and also beginning of flat-plate section of a flattened tube

2 point 2, exit section of heater and also end of flat plate section of a flattened tube

## DESCRIPTION OF HEATER AND TESTING PROCEDURE

The flattened-tube type crossflow heat exchanger is a unit consisting of 263 flattened tubes which convey the ventilating air, arranged as chords of the circular exhaust gas passage. The bundle of staggered tubes consists of 21 rows of flattened tubes, with alternate rows containing 12 and 13 tubes per row. The tubes, depending upon their chordwise location, vary in length from approximately 4 to 8 inches, the latter being approximately the inside diameter of the circular heater shell. (See the diagrammatic sketch, fig. 1, for the exact dimensions.)

The air shroud used, designated as UC-3 in this report, is designed to give full crossflow characteristics. Photographs of the heater and shroud are shown in figures 2 to 4.

Heat transfer and static pressure drop data for the exchanger using this shroud are presented in tables I to III. Plots of these data as functions of the weight rates of the ventilating air and exhaust gas are presented in figures 5 to 8.

## METHOD OF ANALYSIS

### Heat Transfer

The thermal output of the exchanger was determined from the enthalpy change of the ventilating air:

$$q_a = W_a \, c_{p_a} \, (\tau_{a_2} - \tau_{a_1}) \tag{1}$$

in which $c_{p_a}$ was evaluated at the arithmetic average ventilating air temperature. A plot of $q_a$ against $W_a$ at constant values of the exhaust gas rate $(W_g)$ is presented in figure 5.

For the exhaust gas side of the heater

$$q_g = W_g \, c_{p_g} \, (\tau_{g_1} - \tau_{g_2}) \tag{2}$$

where $c_{p_g}$ is taken as that of air at the average exhaust gas temperature. For the case of a heat exchanger thermally insulated from its surroundings, $q_a$ would equal $q_g$. Because experience has shown $q_g$ to be the more reliable value, it is used in determining the over-all thermal conductance UA. (See equation (3).) The heat balance ratios $q_g/q_a$ are given in table I.

The over-all thermal conductance UA was evaluated from the equation:

$$q_a = (UA) \, \Delta\tau_{mx} \tag{3}$$

where $\Delta\tau_{mx}$ is the mean effective temperature difference for crossflow of fluids when the fluid on the exhaust gas side is mixed and that on the ventilating air side is not mixed while passing through the heater. This term is shown graphically in figure 31b of reference 2 as a function of the terminal temperatures of the exhaust gas and ventilating air. The variation of UA as a function of $W_a$ at various values of $W_g$ is shown in figure 6.

The thermal output of the exchanger for values of $\Delta\tau_{mx}$, $W_a$ and $W_g$ other than those used here may be predicted by determining UA at the desired weight rates from figure 6 and using those magnitudes in equation (3)*.

*This method is an approximation because it takes into consideration only the variation of UA with the weight rates of fluid, the effect of the different temperatures of the fluids being neglected. For a discussion of this effect, see appendix A of reference 3.

Predictions of the over-all thermal conductance UA were attempted. The expression

$$UA = \frac{1}{\left(\frac{1}{f_c A}\right)_a + \left(\frac{1}{f_c A}\right)_g} \tag{4}$$

was used (reference 2, equation (46)).

The thermal conductances for the tubular passages on the ventilating air side and for the tube bank on the exhaust gas side $(f_c A)_a$ and $(f_c A)_g$, are determined by use of the following equations:

1. Ventilating air side.- Because the flow passages for the ventilating air consisted of flattened tubes, the unit thermal conductance on the air side was determined from the equation for turbulent flow in ducts (see reference 2, equation (25)):

$$f_{ca} = 5.4 \times 10^{-4}\ T_{av}^{0.3}\ \frac{G^{0.8}}{D^{0.2}} \left[1 + 1.1\ \frac{D}{l}\right] \tag{5}$$

where

$f_{ca}$ unit thermal conductance for turbulent flow in ducts

$T_{av}$ average absolute temperature of ventilating air

$G$ weight rate of ventilating air per unit of cross-sectional area

$D$ hydraulic diameter of flattened tubes

$l$ length of flattened tubes

The thermal conductance of the air side is then

$$(f_c A)_a = f_{ca} \times A$$

where A is the area of heat transfer.

2. Exhaust gas side.- The thermal conductance of the exhaust gas side was obtained by combination of the thermal conductance of the circular portions of the flattened tube (leading and trailing surfaces) and the thermal conductance of the flattened portion of the tubes.

(a) The average unit thermal conductance over the cylindrical parts of the flattened tubes was taken as being equivalent to that over the cylinders of a bank of staggered tubes (see reference 2, equation (29a)), which is given by the equation:

$$f_{cc} = 14.5 \times 10^{-4} F_A T_f^{0.43} \frac{G_0^{0.6}}{D^{0.4}} \tag{6}$$

where

$f_{cc}$ average unit thermal conductance for any number of rows of cylinders

$F_a$ tube arrangement modulus for banks of staggered tubes

$T_f$ arithmetic average of absolute temperature of tube wall and of exhaust gas

$G_o$ maximum weight rate of exhaust gas per unit of area

D outer diameter of tube (in this case D was taken as the maximum width of the flattened tube)

(b) The unit thermal conductance over the flattened portion of the tubes was found by considering this portion of the tube as a flat plate and measuring its length from the forward point of stagnation. The average unit thermal conductance of this section was found by use of the equation for the point unit thermal conductance over a flat plate (see reference 2, equation (19)):

$$f_{cx} = 0.51 T_f^{0.3} \frac{(u_m\gamma)^{0.8}}{x^{0.2}} = 0.51 T_f^{0.3} \frac{(G/3600)^{0.8}}{x^{0.2}} \tag{7}$$

where

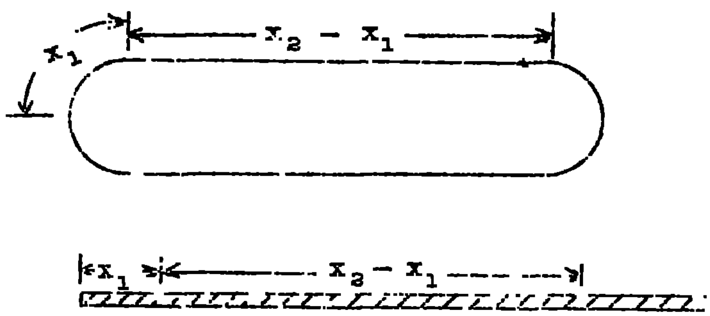
$f_{cx}$ point unit thermal conductance over a flat plate

$T_f$ arithmetic average absolute temperature of tube wall and of fluid

G weight rate of fluid per unit of cross-sectional area

x distance along flat plate measured from leading edge

To obtain the average (with length) unit thermal conductance, equation (7) must be integrated over the length of the flattened portion of the tube and the integral divided by the length over which it was taken.




Equivalent Flat Plate

$$f_{cx}\ (\text{average with length}) = \frac{\int_{x_1}^{x_2} f_{cx}\, dx}{x_2 - x_1}$$

$$= \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \left(0.51\ T_f^{0.3}\right) (G/3600)^{0.8}\, x^{-0.2}\, dx$$

$$= \frac{0.51\ T_f^{0.3}\ (G/3600)^{0.8}}{x_2 - x_1} \int_{x_1}^{x_2} x^{-0.2}\, dx$$

$$= \frac{0.51\ T_f^{0.3}\ (G/3600)^{0.8}}{x_2 - x_1} \left.\frac{x^{0.8}}{0.8}\right]_{x_1}^{x_2}$$

$$= \frac{0.51\ T_f^{0.3} (G/3600)^{0.8}\ (x_2^{0.8} - x_1^{0.8})}{0.8(x_2 - x_1)} \qquad (8)$$

where $x_1$ is the peripheral distance from the forward point of stagnation to the beginning of the flattened portion of the tube and $x_2$ is that to the end of the flattened portion of the tube.

The thermal conductance on the exhaust gas side of the tubes is the sum of both that over the cylindrical portion and that of the flattened portion of the tubes and therefore:

$$(f_cA)_g = (f_cA)_c + (f_cA)_f \tag{9}$$

where the first term on the right-hand side of the equation is the thermal conductance of the cylindrical portion of the tubes and the second term is the thermal conductance of the flattened portion of the tube.

The over-all thermal conductance of the heater is then obtained from the equation:

$$UA = \frac{1}{\left(\frac{1}{f_cA}\right)_a + \left(\frac{1}{f_cA}\right)_g} = \frac{1}{\left(\frac{1}{f_cA}\right)_a + \frac{1}{(f_cA)_c + (f_cA)_f}} \tag{4}$$

Sample Calculations

(Based on run 24, table I)

$$W_a = 1430 \text{ lb/hr}$$

$$\tau_{a_1} = 108^\circ \text{ F}$$

$$W_g = 3240 \text{ lb/hr}$$

$$\tau_{g_1} = 1590^\circ \text{ F}$$

Postulate: Outlet ventilating air temperature, $\tau_{a_2} = 800^\circ$ F

Outlet exhaust gas temperature, $\tau_{g_2} = 1300^\circ$ F

therefore, $\tau_a$ (av) $\cong 450^\circ$ F, $\tau_g$ (av) $\cong 1450^\circ$ F

1. Thermal conductance on exhaust gas side:

Considering the thermal conductance of the cylindrical portion of flattened tubes and that of the flat portion of these tubes, then the thermal conductance of the exhaust gas side is given by the equation

$$(f_cA)_g = (f_cA)_f + (f_cA)_c$$

Evaluate the average thermal conductance over the cylindrical portion:

$$f_{cc} = 14.5 \times 10^{-4}\ F_a\ T_f^{0.43}\ \frac{G_0^{0.6}}{D^{0.4}} \qquad (6)$$

$F_a$ tube arrangement modulus = 1.55

(see table II of reference 2)

$T_f$ (assuming thermal conductances on the two sides of the heat exchanger to be approximately equal)*

$$= \left[\frac{t_w + T_g(av)}{2} + 460\right]\ {}^{o}R = \left[\frac{450 + 1450}{2} + 1450\right] \times \frac{1}{2} + 460$$

$$= \frac{950 + 1450}{2} + 460 = 1660^{o}\ R$$

$$T_f^{0.43} = (1660)^{0.43} = 24.2$$

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*This assumption is not valid as seen from the calculations of the thermal conductances, but if these thermal conductances are used to obtain a better approximation of $t_w$, the wall temperature, it is found that an error of about 10 percent in the value of $T_f$ was obtained in the previous calculations. However, since $T_f$ enters into the equation for the unit thermal conductance only to the 0.43 power, the error committed using the first approximation was actually only about 4 percent.

$G_o$ weight rate per unit area

$$G_o = \frac{W_g}{A_{cs}}\ \frac{3240}{0.1945} = 16{,}700\ \text{lb/hr ft}^2$$

$$G_o^{0.8} = 343$$

(The area $A_{cs}$ is the minimum cross-sectional area of flow in the tube bank, measured along the diagonals in this case.) (See fig. 1.)

Outer diameter of equivalent cylinder

$$D = \frac{0.188}{12} = 0.0157\ \text{ft}$$

$$D^{0.4} = 0.189$$

$$f_{cc} = 14.5 \times 10^{-4} \times 1.55 \times 24.2 \times \frac{343}{0.189} = 98.6\ \text{Btu/hr ft}^2\ {}^{\circ}\text{F}$$

$$(f_cA)_c = (98.6 \times 7.20) = 709\ \text{Btu/hr}\ {}^{\circ}\text{F}$$

Evaluate the average thermal conductance over the flat plate portion:

$$f_{cf} = \frac{0.51\ Tf^{0.3}\ (G/3600)^{0.8}}{0.8\ (x_2 - x_1)}\ (x_2^{0.8} - x_1^{0.8}) \qquad (8)$$

$$\left(\frac{x_2^{0.8} - x_1^{0.8}}{x_2 - x_1}\right) = \left\{\frac{(0.0539)^{0.8} - (0.0123)^{0.8}}{0.0539 - 0.0123}\right\} = \frac{0.0665}{0.0416} = 1.60$$

$$f_{cf} = \frac{0.51 \times 1.60}{0.8}\ T_f^{0.3}\ (G/3600)^{0.8} = 1.02\ T_f^{0.3}\left(\frac{G}{3600}\right)^{0.8}$$

$$= \frac{1.02}{700}\ T_f^{0.3}\ G^{0.8} = 1.46 \times 10^{-3} \times T_f^{0.3}\ G^{0.8}$$

$$T_f = 1660^{\circ}\ \text{R}$$

$$T_f^{0.3} = (1660)^{0.3} = 9.25$$

Weight rate per unit area $G = \frac{W_g}{A_{cs}} = \frac{3240}{0.233} = 13,900$ lb/hr ft$^2$

($A_{cs}$ evaluated in the flat plate section of the tube rows)

$G^{0.8} = 2080$

$f_{cf} = 1.46 \times 10^{-3} \times 9.25 \times 2080 = 28.0$ Btu/hr ft$^2$ $^\circ$F

$(f_cA)_f = (28.0 \times 12.2) = 341$ Btu/hr $^\circ$F

Thermal conductance on exhaust gas side is then

$$(f_cA)_g = (f_cA)_c + (f_cA)_f = 709 + 341 = 1050 \text{ Btu/hr } ^\circ\text{F} \quad (9)$$

$$\left(\frac{1}{f_cA}\right)_g = 0.000953$$

<u>2. Thermal conductance on ventilating air side:</u>

$$f_{ca} = 5.4 \times 10^{-4}\ T_{av}^{0.3}\ \frac{G^{0.8}}{D^{0.2}} \left[1 + 1.1\ \frac{D}{l}\right] \quad (5)$$

$T_{av}$ (arithmetic mean absolute temperature of the ventilating air) = $(454 + 460) = 914^\circ$ R

$T_{av}^{0.3} = 7.72$

Weight rate per unit area $G = \frac{W_a}{A_{cs}} = \frac{1430}{0.145} = 9860$ lb/hr ft$^2$

$G^{0.8} = (9860)^{0.8} = 1570$

Hydraulic diameter $D = \frac{4\ A_{cs}}{\text{Wetted perimeter}} = \frac{4 \times 0.145}{31.0} = 0.0188$ ft

$D^{0.2} = 0.451$

$\left(1 + 1.1\ \frac{D}{l}\right) = 1.04$

$$f_{ca} = 5.4 \times 10^{-4} \times 7.72 \times \frac{1570}{0.451}\left[1 + 1.1 \times \frac{0.0188}{0.556}\right]$$

$$= 15.1 \text{ Btu/hr ft}^2 \ {}^{\circ}\text{F}$$

$$(f_cA)_a = (15.1 \times 17.3) = 260 \text{ Btu/hr } {}^{\circ}\text{F}$$

$$(f_cA)_a^{-1} = 0.00384$$

3. Over-all thermal conductance:

$$UA = \frac{1}{\left(\frac{1}{f_cA}\right)_a + \left(\frac{1}{f_cA}\right)_g} = \frac{1}{0.00384 + 0.00095} = \frac{1}{0.00479}$$

$$= 211 \text{ Btu/hr } {}^{\circ}\text{F}$$

Check the outlet temperatures postulated.

$$q_a = W_a c_{pa}(\tau_{g_1} - \tau_{a_1})\varphi_x = W_a c_{pa}(\tau_{a_2} - \tau_{a_1}) = W_g c_{pg}(\tau_{g_1} - \tau_{g_2})$$

$$\tau_{a_2} - \tau_{a_1} = (\tau_{g_1} - \tau_{a_1})\varphi_x, \quad \tau_{a_2} = (\tau_{g_1} - \tau_{a_1})\varphi_x + \tau_{a_1}$$

$$\tau_{g_1} - \tau_{g_2} = \frac{W_a c_{pa}(\tau_{g_1} - \tau_{a_1})}{W_g c_{pg}}, \quad \tau_{g_2} = \tau_{g_1} - \frac{W_a c_{pa}}{W_g c_{pg}}(\tau_{g_1} - \tau_{a_1})\varphi_x$$

Evaluate $\varphi$ from figure 34 of reference 2.

$$\varphi = 0.422$$

$$\tau_{a_2} = (1480)\ 0.422 + 108 = 733^{\circ}\text{ F}$$

$$\tau_{g_2} = 1590 - \frac{1430 \times 0.242}{3240 \times 0.276}(1480) \times 0.422 = 1590 - 240 = 1350^{\circ}\text{ F}$$

The check temperatures are close to the values postulated: 800° F for the ventilating air and 1300° F for the exhaust gas. If a much closer prediction of UA is desired, one can use these corrected temperatures to obtain new values of the unit thermal conductances and, consequently, of the over-all thermal conductance UA. Carrying out this procedure, however, changes UA by only 1 percent; the corrected value of UA being 209 Btu/hr °F.

## Isothermal Pressure Drop

The isothermal pressure drop on the gas side of the heater was postulated to be that over rows of cylindrical tube banks. Each flattened tube was considered as an equivalent cylinder with a diameter identical to that of the circular leading section of the flattened tube. The pressure drop through these rows of tubes is based upon the relation (see reference 4)

$$\Delta P = 4\zeta' N \frac{(G_0/3600)^2}{2g\,\gamma} \tag{10}$$

where

$\Delta P$ pressure drop

$\zeta'$ friction factor for flow of fluids past tube banks

$G_0$ maximum weight rate of fluid per unit area

$N$ number of rows in direction of flow

$g$ gravitational force per unit of mass

$\gamma$ weight density of fluid

The friction factor $\zeta'$ is defined by the equation (see reference 5, equation (6)):

$$\zeta' = \left[0.25 + \frac{0.118}{(a-1)^{1.08}}\right] \mathrm{Re}^{-0.16} \tag{11}$$

where $a$ is the center-to-center spacing of the tubes in any one row, in terms of the tube diameter (center-to-center distance of adjacent tubes is equal to $aD$, $D$ being the diameter), and Re is the Reynolds number based on the maximum weight rate of fluid per unit cross-sectional area $G_0$ and on the outside diameter of the tubes D. For the flattened tube, the outside diameter D is taken as that of the cylindrical leading edge of the tubes.

The results obtained with these flattened tubes were compared with those of Joyner and Palmer (reference 9) on the pressure drop across banks of elliptical tubes. The results of those tests indicated a much lower pressure drop than that through the bank of flattened tubes. The difference is due, partially, to the fact that the banks tested by Joyner and Palmer were designed to give an approximately constant cross-sectional area of flow; whereas the flattened tube bank had a small gap between the rows of staggered tubes. A rough estimate of the pressure drop across the flattened-tube bank could be made by considering the pressure drop due to expansion and contraction between each row of tubes.

A trial calculation showed that the pressure drop due to skin friction over the flat portion of the flattened tubes was small compared to the pressure drop due to drag across the cylindrical leading and trailing edges of the tubes. This contribution to the static pressure drop was, therefore, not included in the predicted values.

The results of the measurement and the prediction of the pressure drop on the exhaust gas side of the heater are given in table II and figure 8.

The <u>isothermal pressure drop on the air side</u> of the heater was obtained from the measured isothermal pressure drop across both heater and ducts by subtracting the measured pressure drop across the ducts alone

$$\Delta P_{htr} = \Delta P_{iso} - \Delta P_{ducts} \qquad (12)$$

The value of $\Delta P_{ducts}$ at any weight rate is obtained from a curve based on experimentally determined points.

The predictions of the isothermal pressure drop on the air side of the heater are based upon the postulation of the following system; (1) A sudden contraction as the air leaves the ducts of the shroud and enters the heater passages, (2) a frictional pressure drop as the air flows through the passages of a uniform mean length l, and (3) a sudden expansion as the air leaves the heater passages and enters the outlet duct of the shroud. The equations which form the basis of the prediction are:

(1) Sudden contraction

$$\frac{\Delta P_{contr}}{\gamma} = K_c \frac{\mu_m^2}{2g} \tag{13}$$

where $u_m$ is the mean velocity within the passages ($K_c$ based on the velocity in the smaller area) and $K_c$ is a "head loss" coefficient obtained from reference 6 or 7.

(2) Friction loss

$$\frac{\Delta P_{fric}}{\gamma} = \zeta \frac{l}{D} \frac{u_m^2}{2g} \tag{14}$$

where

$\zeta$ friction factor (obtained from reference 6 or 7)

$l$ mean length

$D$ hydraulic diameter of the passage

$u_m$ mean velocity within the passages

(3) Sudden expansion

$$\frac{\Delta P_{exp}}{\gamma} = K_e \frac{u_m^2}{2g} \tag{15}$$

where $u_m$ is the mean velocity within the passages and $K_e$ is a head loss coefficient defined by $K_e = (1-m)^2$, where $m$ is the ratio of the cross-sectional area before expansion to that after expansion. (See reference 6 or 7.)

The over-all static pressure drop across the air side of the heater is then the sum of these terms:

$$\frac{\Delta P_{htr}}{\gamma} = \frac{\Delta P_{contr}}{\gamma} + \frac{\Delta P_{fric}}{\gamma} + \frac{\Delta P_{exp}}{\gamma} \tag{16}$$

The measured and predicted values of the isothermal static pressure drop on the ventilating air side of the heat exchanger are given in table II and are plotted in figure 7. The values plotted are those of $\Delta P''_{iso}$, the sum of $\Delta P''_{htr}$ and $\Delta P''_{ducts}$; the curve designated as the predicted isothermal pressure drop being a plot of values of $\Delta P''_{htr}$ (pred.) plus $\Delta P''_{ducts}$ (meas.).

## Non-Isothermal Pressure Drop

The non-isothermal static pressure drop across the air and gas sides of the heat exchanger was predicted from the isothermal measurements by means of equation (54), reference 2:

$$\Delta P_{non-iso} = \Delta P_{iso} \left(\frac{T_{av}}{T_{iso}}\right)^{1.13}$$

$$+ \left(\frac{W}{3600}\right)^2 \frac{1}{2g\gamma_1 A_h^2} \left[\left(\frac{A_h}{A_2}^2 + 1\right) \frac{T_2}{T_1} - \left(\frac{A_h}{A_1}^2 + 1\right)\right] \qquad (17)$$

where

$\Delta P_{iso}$ measured over-all isothermal static pressure drop at temperature $T_{iso}$

$T_1$, $T_2$ mixed-mean absolute temperatures of fluid at inlet and outlet of the heater, respectively

$T_{av}$ arithmetic average of $T_1$ and $T_2$

$W$ fluid weight rate

$\gamma_1$ weight density evaluated at temperature $T_1$ of fluid at inlet to heater

$A_h$ cross-sectional area of flow within heater

$A_1$ cross-sectional area at inlet pressure measuring station

$A_2$ cross-sectional area of flow at outlet pressure measuring station

A comparison of measured and predicted non-isothermal pressure drops across each side of the heater is presented in table III and is shown graphically in figures 7 and 8.

## DISCUSSION

The results of the tests on this flattened tube-type crossflow heat exchanger are shown graphically in figures 5 to 8. These graphs are based on data presented in tables I to III.

### Heat Transfer

Although some experimental work has been done on flow of fluids over banks of staggered tubes with shapes other than cylindrical (references 8 and 9), little has been done in the way of the establishment of definitive relations between such variables as spacing, tube dimensions, and weight rates of fluid. For this reason, the thermal performance of this heater was analyzed in the manner previously described.

The method of prediction yielded results which are, on the average, within 8 percent of the experimentally determined values. The slope of the experimentally determined values of the over-all thermal conductance UA, when plotted as a function of the ventilating air rate, is less than that of the predicted values and the deviation between measured and predicted values increases at the lower ventilating air rates. Inspection of the prediction equations for the unit thermal conductance reveals that the unit thermal conductance for flow over flat plates and for flow through ducts varies as the 0.8 power of the fluid weight rate and for flow over banks of cylinders varies as the 0.6 power of ventilating air rate. This analysis is only an approximation, because in the actual case the thermal conductance at the rear of the flattened tube will not be the same as that at the rear of a right circular cylinder of the same diameter as the trailing edge of the tube, nor will the unit thermal conductance at the beginning of the flattened portion of the tube be the same as that over a flat plate at a distance corresponding to that from the front stagnation point of the cylinder.

## Isothermal Static Pressure Drop

The predicted isothermal static pressure drop was within 20 percent of the measured value on the ventilating air side of the heater and was within 25 percent, on the average, of the measured values on the exhaust gas side of the heater. The magnitude of the prediction of the exhaust gas side isothermal pressure drop indicates that use of the equation for the pressure drop across banks of circular tubes is adequate for prediction of the pressure drop across banks of flattened tubes.

The value of the "duct loss" obtained experimentally by measuring the pressure drop across the ducts alone is only an approximation to the value of the duct loss which must be subtracted from the isothermal pressure drop in order to obtain the pressure drop across the air side of the heater alone. This is substantiated by the fact that the pressure drops across the heater alone obtained in this manner are about 30 percent higher than experimental data given in reference 10 and also by the fact that in the present report the predictions of the pressure drop across the air side of the heater alone are within about 10 percent of the data in reference 10.

A head loss coefficient K was determined from the over-all isothermal pressure drop according to the equation:

$$\frac{\Delta P_{htr}}{\gamma} = K \frac{u_m^2}{2g} \tag{18}$$

The values of K obtained were approximately 3.0 on the ventilating air side and about 5.2 on the exhaust gas side. (On the exhaust gas side of the heater $\Delta P_{htr} = \Delta P_{iso}$ since there are no contracting or expanding sections in the system, exclusive of the heater section itself.) Comparison of these values with head-loss coefficients for other heaters (see previous reports of the series) will reveal that these values are of the usual magnitude on the air side, but the values on the gas side are much higher. This is due to the expansions and contractions which occur in the tube banks of the exhaust gas side of this heater.

## Non-Isothermal Static Pressure Drop

The prediction of the non-isothermal static pressure drop from the isothermal pressure drop, by means of equation (17), (reference 2, equation (54)), was, on the average, within 16 percent of the measured values on the ventilating air side, and was, on the average, within 21 percent of the measured values on the exhaust gas side.

The difference between the slopes of the predicted and measured pressure drop curves on the ventilating air side can be explained by the fact that the use of an arithmetic mean temperature in the first term on the right hand side of equation (17) is an approximation which is less valid at lower weight rates when the difference between the fluid temperatures at inlet and outlet is greatest.

As explained in reference 2, the non-isothermal static pressure drop includes losses due to the acceleration of the fluid (second term on the right-hand side of equation (17)). These losses are of thermal origin and hence cannot be recovered by mechanical means. Such losses can only be recovered by cooling the heated ventilating air (or heating the cooled exhaust gases, as the case may be). Such a process would defeat the purposes of a cabin heater, but occurs in the operation of thermal anti-icers for airplane wings.

## CONCLUSIONS

1. The over-all thermal conductance for a flattened-tube type crossflow heat exchanger was estimated, on the average, within 8 percent of the experimentally determined values.

2. The isothermal static pressure drop was estimated, on the average, within 15 percent of the measured values on the ventilating air side and within 25 percent of the measured values on the exhaust gas side.

3. Predictions of the non-isothermal static pressure drops from the isothermal static pressure drops were within 16 percent of the measured values on the ventilating air side and within 21 percent of the measured values on the exhaust gas side.

University of California,
Berkeley, Calif., July 1944.

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# TABLE I - EXPERIMENTAL RESULTS ON FLATTENED-TUBE HEATER

## USING UC-3 SHROUD

| | AIR SIDE | | | | | | GAS SIDE | | | | | | | | OVER-ALL PERF. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Run | $T_{a_1}$ | $T_{a_2}$ | $\Delta T_a$ | $W_a$ | $\Delta P_a$ | $q_a$ | $T_{g_1}$ | $T_{g_2}$ | $\Delta T_g$ | $W_g$ | $\Delta P_g$ | $q_g$ | $q_g/q_a$ | | $\Delta T_{mx}$ | UA | |
| No. | °F | °F | °F | Lb/hr. | Inches $H_2O$ | KBtu/hr | °F | °F | °F | Lb/hr | Inches $H_2O$ | KBtu/hr | | | °F | Btu/hr°F | |
| 17 | 102 | 812 | 710 | 1000 | 1.87 | 172 | 1573 | 1222 | 351 | 1690 | 3.34 | 161 | 0.94 | | 900 | 191 | |
| 18 | 104 | 619 | 515 | 1740 | 4.02 | 217 | 1543 | 1146 | 397 | 1700 | 3.24 | 183 | 0.84 | | 950 | 228 | |
| 19 | 104 | 508 | 404 | 2660 | 7.62 | 260 | 1534 | 1066 | 468 | 1700 | 3.04 | 216 | 0.83 | | 970 | 268 | |
| 23 | 104 | 905 | 801 | 1000 | 2.07 | 194 | 1591 | 1381 | 210 | 3240 | 13.6 | 188 | 0.97 | | 925 | 210 | |
| 24 | 108 | 795 | 687 | 1430 | 3.32 | 238 | 1594 | 1364 | 230 | 3240 | 13.6 | 206 | 0.87 | | 1000 | 238 | |
| 25 | 104 | 615 | 511 | 2640 | 8.20 | 326 | 1599 | 1299 | 300 | 3270 | 13.4 | 271 | 0.83 | | 1060 | 308 | |
| 26 | 107 | 675 | 568 | 2280 | 6.68 | 316 | 1582 | 1329 | 253 | 4160 | 23.2 | 290 | 0.92 | | 1040 | 302 | |
| 27 | 105 | 594 | 489 | 3170 | 10.8 | 376 | 1582 | 1299 | 283 | 4190 | 23.1 | 328 | 0.87 | | 1070 | 351 | |
| 28 | 105 | 541 | 436 | 4160 | 16.5 | 438 | 1578 | 1248 | 330 | 4190 | 22.8 | 382 | 0.87 | | 1060 | 410 | |

TABLE II

ISOTHERMAL STATIC PRESSURE DROP DATA

Flattened-Tube Bank Crossflow Heat Exchanger

| W (lb/hr) | G (lb/hr ft²) | $\Delta P''_{iso}$ (meas.) (in. $H_2O$) | $= \Delta P''_{ducts}$ (meas.) (in. $H_2O$) | $+ \Delta P''_{htr}$ (in. $H_2O$) | $\Delta P''_{htr}$ (pred.) (in. $H_2O$) | K[a] | Re |
|---|---|---|---|---|---|---|---|
| Air Side | | | | | | | |
| 1500 | 10,300 | 1.45 | 0.35 | 1.10 | 0.83 | 3.2 | [b]4,180 |
| 3000 | 20,700 | 5.50 | 1.30 | 4.20 | 3.12 | 3.0 | 8,380 |
| 6000 | 41,300 | 20.2 | 4.70 | 15.5 | 11.7 | 2.8 | 16,700 |
| Gas Side | | | | | | | |
| 2000 | 10,300 | 1.60 | 0.0 | 1.60 | 2.15 | 4.6 | [c]42,000 |
| 3500 | 18,000 | 5.00 | 0.0 | 5.00 | 6.10 | 4.7 | 73,000 |
| 6000 | 30,800 | 15.0 | 0.0 | 15.0 | 16.6 | 4.8 | 126,000 |

[a]K defined by $\frac{\Delta P_{htr}}{\gamma} = K \frac{u_m^2}{2g}$

[b]Reynolds number based on hydraulic diameter of a flattened tube $D_H$.

[c]Reynolds number based on the diameter of a cylinder equivalent to the leading and trailing edge flattened tube.

NOTE.- In figures 7 and 8, the curves labeled "predicted isothermal pressure drop" are obtained by plotting the sum of the $\Delta P''_{duct}$ (meas.) and $\Delta P''_{htr}$ (pred.) (columns 4 and 5 in this table).

TABLE III

NON-ISOTHERMAL STATIC PRESSURE DROP DATA

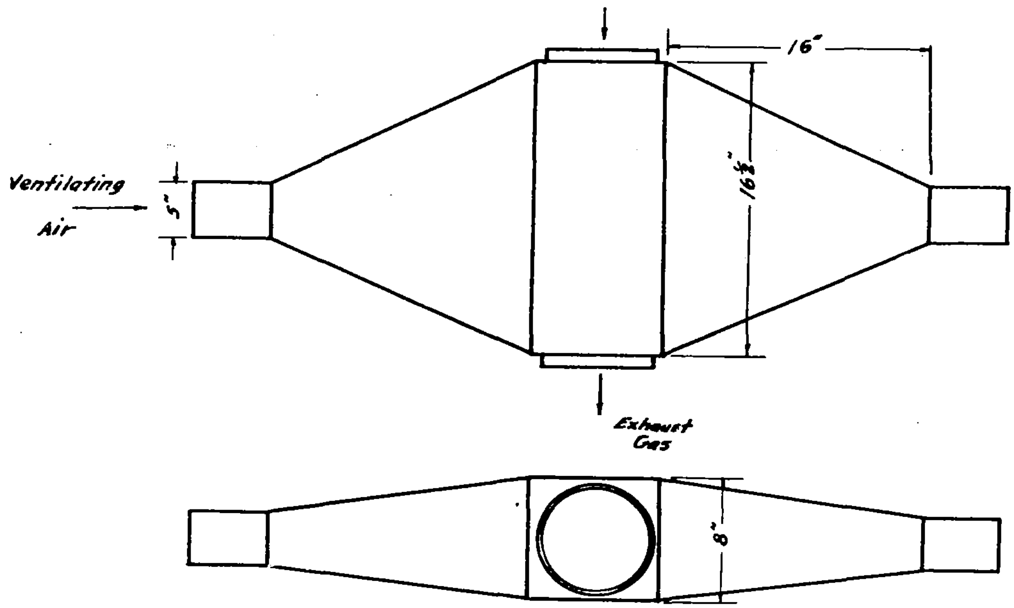
Flattened-Tube Bank Crossflow Heat Exchanger

| Run | W (lb/hr) | $T_1$ (°R) | $T_2$ (°R) | $T_{av}$ (°R) | $\Delta P_{iso}^{*}$ (in. $H_2O$) | $\Delta P^{*}$ non-iso (meas.) (in. $H_2O$) | $\Delta P^{*a}$ non-iso (pred.) (in. $H_2O$) |
|---|---|---|---|---|---|---|---|
| Exhaust Gas Side | | | | | | | |
| 18 | 1700 | 2003 | 1606 | 1805 | 1.15 | 3.24 | 4.10 |
| 25 | 3270 | 2059 | 1759 | 1909 | 4.37 | 13.4 | 16.9 |
| 28 | 4100 | 2042 | 1759 | 1900 | 6.90 | 22.8 | 26.5 |
| Ventilating Air Side | | | | | | | |
| 24 | 1430 | 568 | 1255 | 912 | 1.30 | 3.32 | 3.08 |
| 26 | 2280 | 567 | 1135 | 851 | 3.25 | 6.68 | 6.94 |
| 27 | 3170 | 565 | 1054 | 809 | 6.10 | 10.8 | 12.1 |

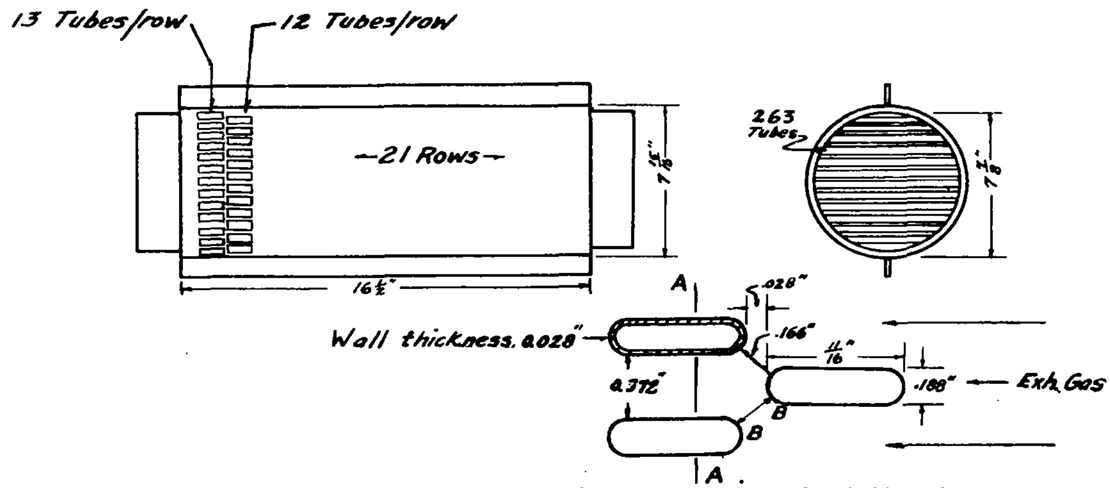
[a]Predicted values are based on equation (17):

$$\Delta P_{non-iso} = \Delta P_{iso} \left(\frac{T_{av}}{T_{iso}}\right)^{1.13}$$

$$+ \left(\frac{W}{3600}\right)^2 \frac{1}{2g\gamma_1 A_h^2} \left[\left(\frac{A_h^2}{A_2^2} + 1\right)\frac{T_2}{T_1} - \left(\frac{A_h^2}{A_1^2} + 1\right)\right] \quad (17)$$




UC-3-Shroud




Enlarged Section of Flattened Tubes

| | AIR SIDE | EXHAUST GAS SIDE Section A-A (Flat Plate) | Section B-B (Cylindrical End) |
|---|---|---|---|
| Cross Sectional Area Ft.² (Total) | 0.145 | 0.229 | 0.195 |
| Heat Transfer Area Ft² (Total) | 17.3 | 12.2 | 7.20 |
| Hydraulic Diameter Ft. | 0.0188 | — | — |

Weight of Heater - 38 Lbs.

Fig. 1 - Schematic Diagram of Flattened-Tube Type Heat Exchanger, and UC-3 Shroud.

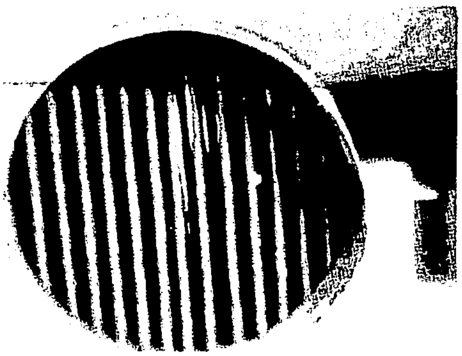


Figure 2.- Photograph of the flattened-tube crossflow type heat exchanger.

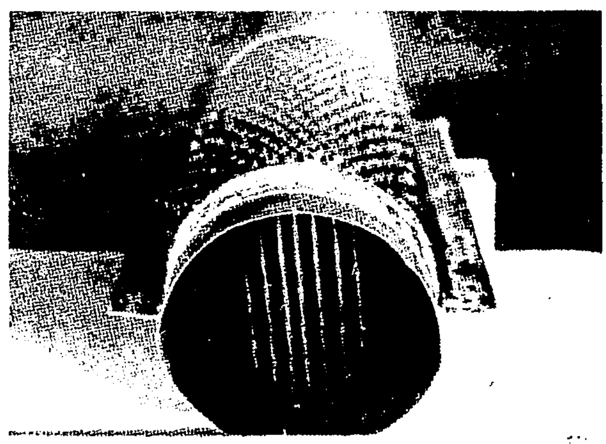


Figure 3.- Photograph of the flattened-tube crossflow type heat exchanger.

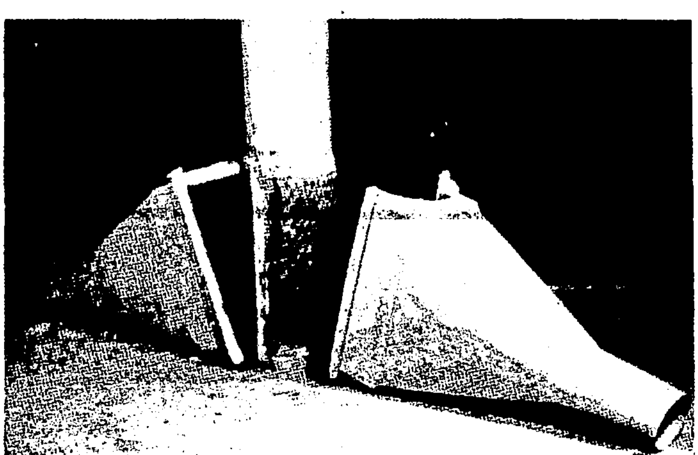


Figure 4.- Photograph of the flattened-tube cross-flow type heat exchanger and the UC-3 ventilating-air shroud.

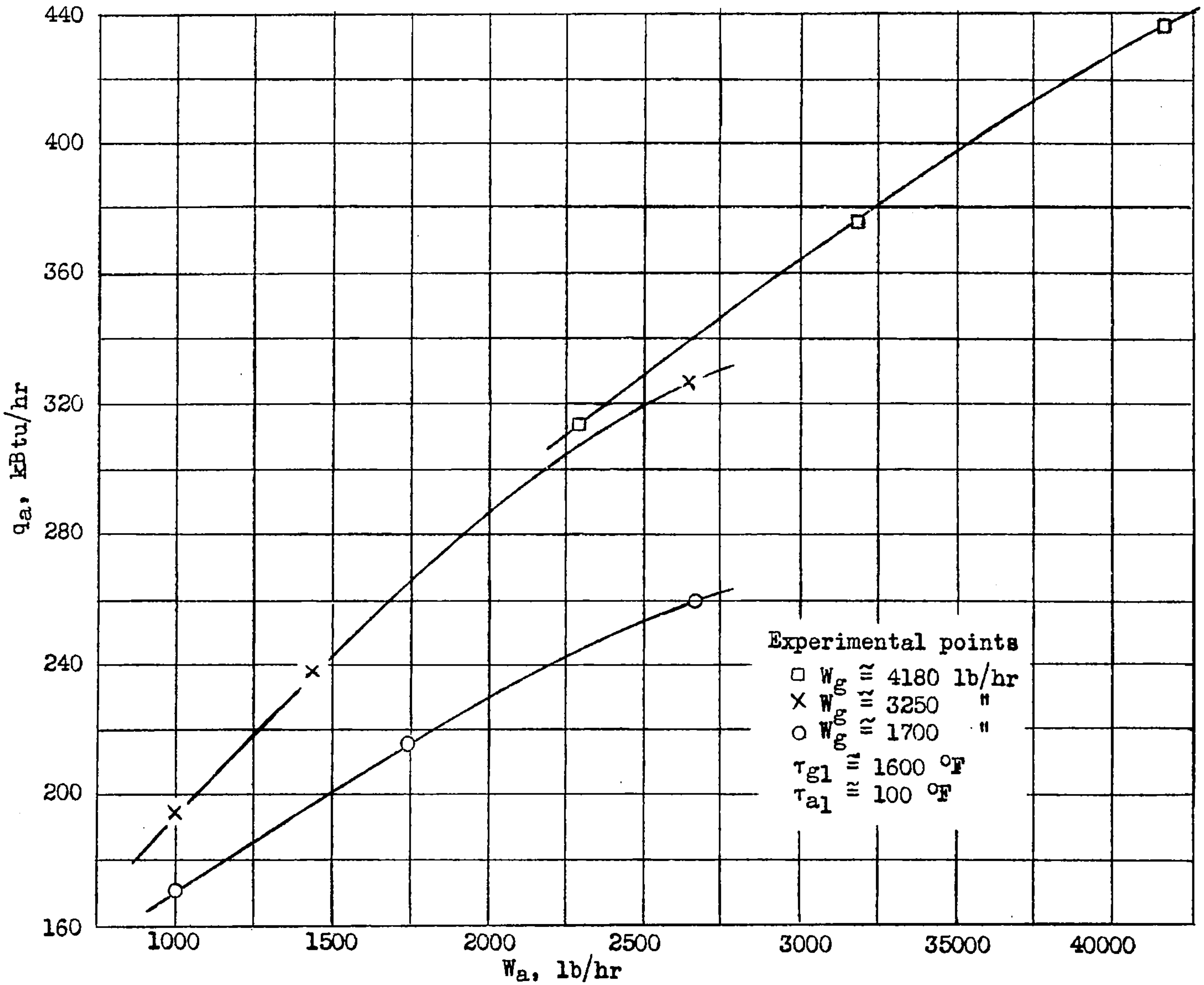



Figure 5.- Thermal output of the flattened-tube type heat exchanger, using UC-3 shroud, as a function of the ventilating air rate.

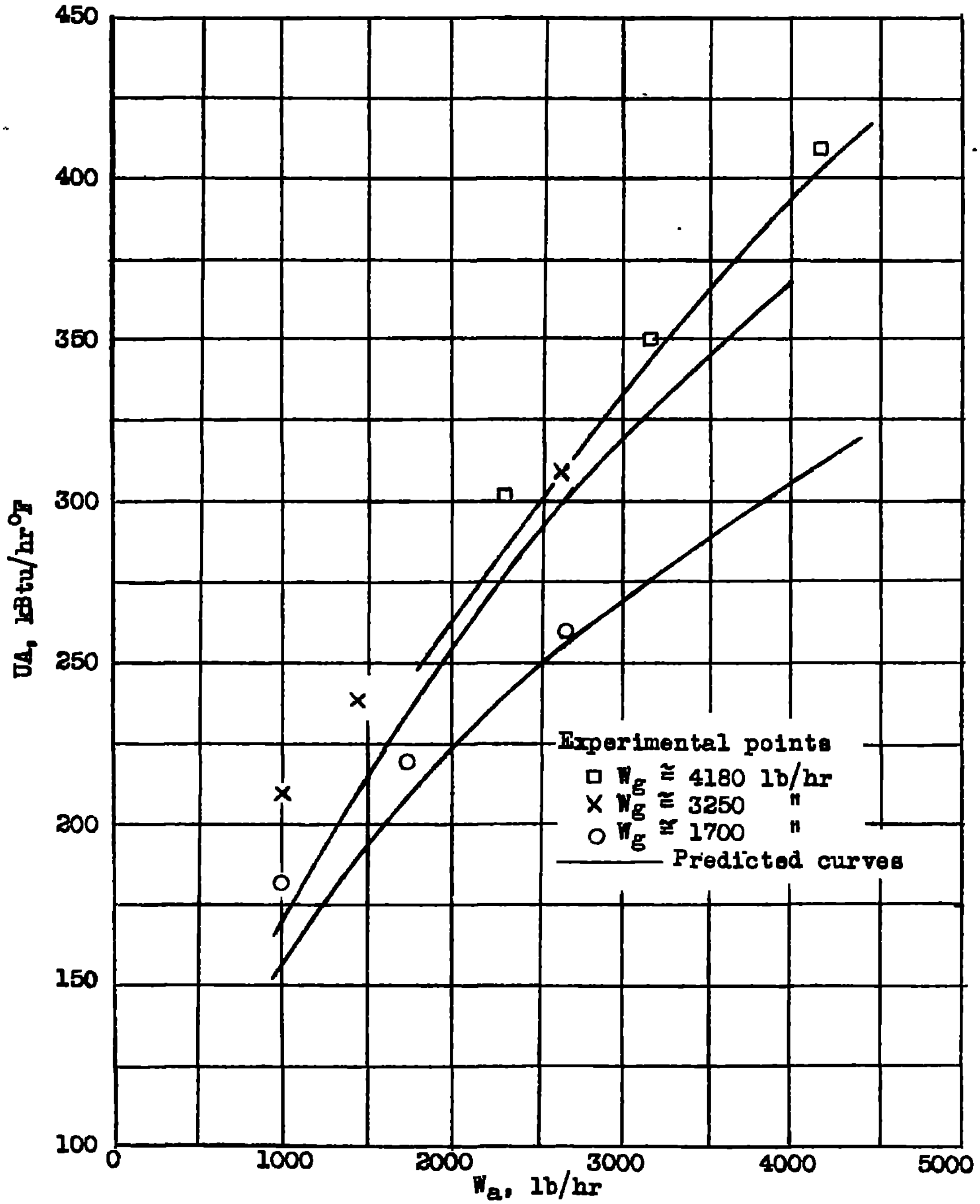



Figure 6.- Overall conductance of a flattened-tube type heat exchanger, using UC-3 shroud, as a function of the ventilating air rate.

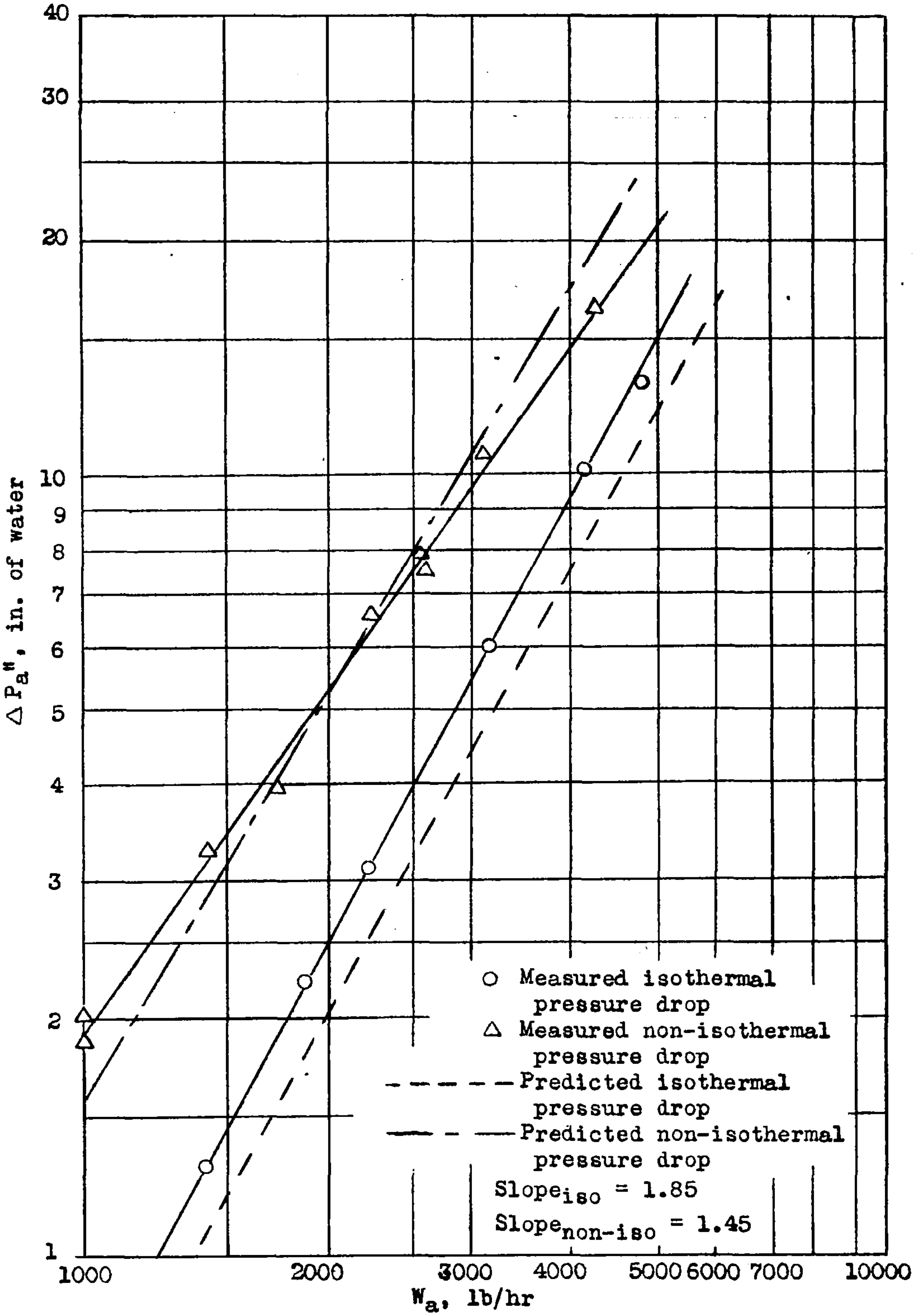



Figure 7.- Static pressure drop across air-side of flattened-tube type heat exchanger as a function of the ventilating air rate, using UC-3 shroud.

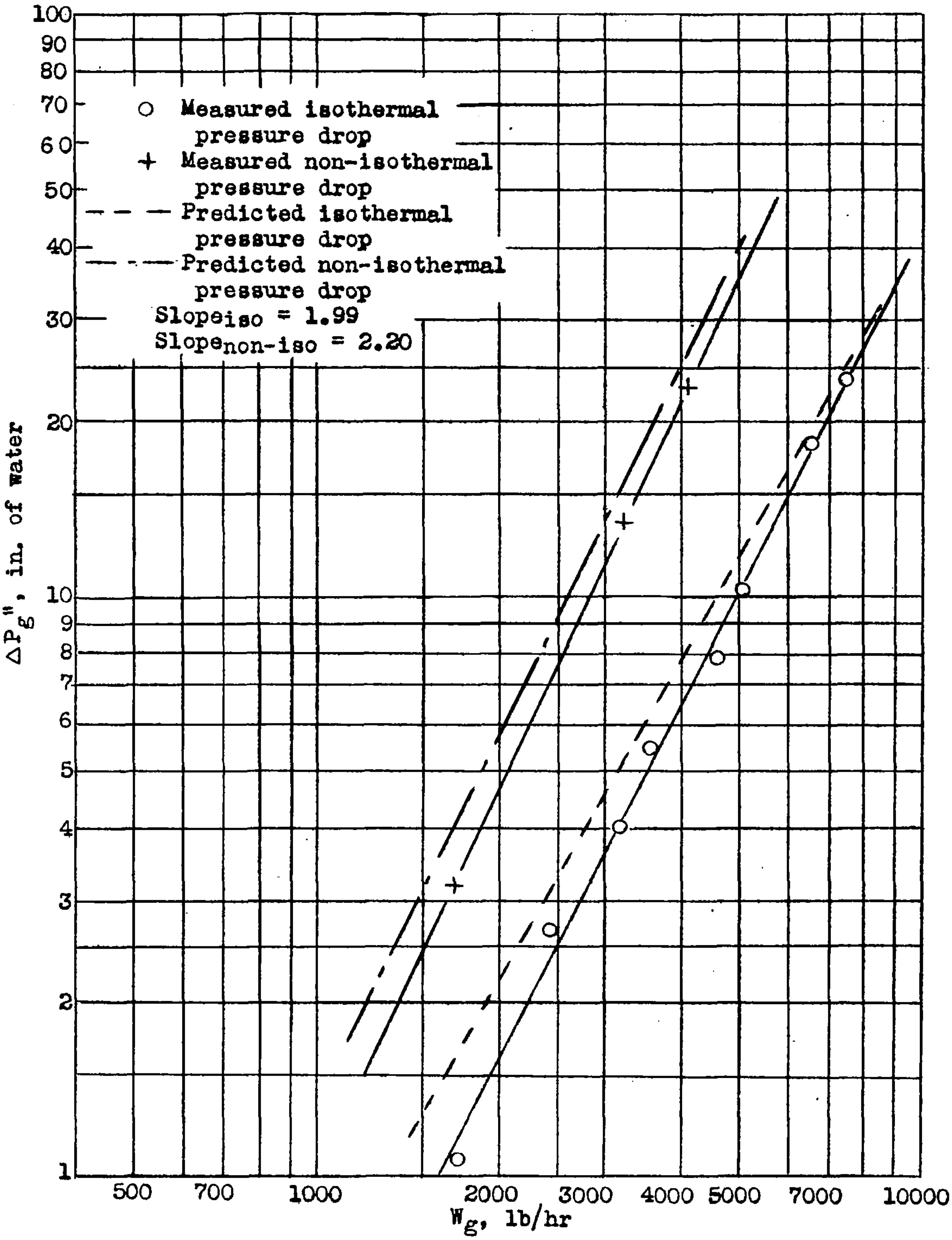



Figure 8.- Static pressure drop across exhaust gas side of flattened-tube type heat exchanger as a function of exhaust-gas rate.