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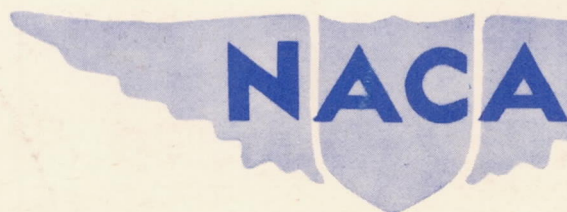
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AN INVESTIGATION OF AIRCRAFT HEATERS
XXV - USE OF THE THERMOPILE RADIOMETER

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

AN INVESTIGATION OF AIRCRAFT HEATERS
XXV - USE OF THE THERMOPILE RADIOMETER

By L. M. K. Boelter, R. Bromberg,
J. T. Gier, and E. R. Dempster

SUMMARY

An analysis of the thermopile radiometer as used in the measurement of irradiation and radiant power interchange is given. The theory of the instrument when used to measure irradiation and net radiant power interchange is developed, and the precautions to be observed when using it for these measurements are presented. Descriptions of the types used are given.

It was found that accurate measurements of irradiation or net radiant power interchange could be obtained by always allowing sufficient time for the readings to become constant and by arranging the system so that the front of the housing is not brought closer than twice the housing length to any reflecting surface. By observing these precautions when using the radiometers, the absolute error of any measurement of irradiation may be made very small. If these precautions are observed, the procedure in making a measurement of irradiation is to point the radiometer in a given direction,* obtain the thermopile electromotive force and the housing temperature by means of a potentiometer or other voltage-measuring device, and evaluate the results obtained in the following manner:

The irradiation is given by the product of the millivolts generated by the thermopile and a constant of the thermopile plus the product of the absolute temperature of the radiometer housing raised to the fourth power and a constant of the

*In this report a measurement of irradiation (or net power exchange by radiation) in a given direction refers to a measurement of power incident (or exchanged) in a solid angle (fraction of space) equal to that seen by the radiometer receiver ($F_0 \leftarrow R$ of half space).

radiometer. This sum will indicate the actual irradiation of the thermopile receiver by the source "viewed" including any gaseous radiation that may be present.

INTRODUCTION

Suitable instruments to evaluate the magnitudes of the several modes of heat transfer will aid in affecting a more precise thermal analysis on heat transfer systems.

A complete heat balance on an airplane or any of its parts requires a knowledge of the heat absorbed and lost by radiation. Besides data on the emissivities of the surfaces involved (reference 1), it is necessary to know the magnitude of the radiation incident upon any surface (irradiation). The thermopile radiometer provides a simple, rugged instrument to measure this irradiation.

Several thermopile radiometers of the plated junction type have been constructed and used at the University of California (reference 2). Recently, because of the requirements of certain problems which have been encountered, it was decided to investigate the possibilities of improving the design of the sensitive elements. As a result, it is now possible to build thermopile radiometers having much smaller physical dimensions and the same or greater sensitivities than the former units referred to above. For example, one of the older units, having an admittance angle of approximately 16° was $4\frac{1}{2}$ inches in diameter and $11\frac{1}{2}$ inches long. A new unit, having the same admittance angle and approximately the same sensitivity, is 1 inch in diameter and approximately $4\frac{1}{2}$ inches long. (See photographs, figures 3 to 6.)

This report presents an analysis of the operation of the compensated type thermopile radiometer and a description of some of its uses, including the measurement of irradiation and the measurement of net radiant power interchange.

The program of research upon which this report is based, conducted in the Spectro-Radiometric Laboratory of the Department of Mechanical Engineering of the University of California, was sponsored by and conducted with the financial assistance of the National Advisory Committee for Aeronautics.

The authors wish to express their appreciation to Mr. E. H. Morrin for his assistance in preparing this report.

DISCUSSION

Description

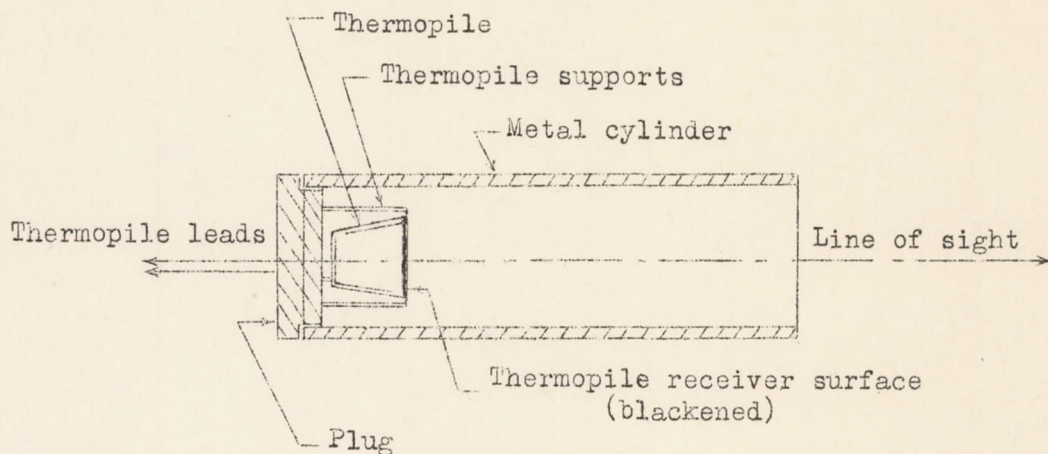
The thermopile radiometers are constructed as follows:

A metal cylinder is made with a removable plug or cover plate at one end, as shown in figure 1. The thermopile element is placed in the end of the cylinder, either as part of the removable plug or attached directly to the cylinder, so that the receiving surface of the thermopile faces the open end of the cylinder. One of the smaller models is equipped with a handle so that it may be used as a portable field instrument. The larger models are mounted on heavy bases and though not as portable, they are often more useful for laboratory purposes; the heavy housing provides a more stable datum and tends to reduce drifts. The exterior surfaces of the radiometer housings are chromium plated to provide a durable surface which has and maintains a high reflectivity. A thermocouple is peened into the housing for use in determination of the housing temperature.

For a more detailed discussion of the construction of plated-junction-type thermopiles, see reference 2.

Calibration

The radiometer may be calibrated with the use of a tungsten filament lamp which has been aged and calibrated against a radiation standard, or with an ideal radiator ("hohlraum"), or with any source of radiation the emissive power of which is known and for which the irradiation at the thermopile may be computed. For the thermopiles constructed it has been found that the electromotive force generated is directly proportional to the irradiation over a large range of values. Three or four points obtained with the calibrating source will yield an accurate calibration. This method of calibration depends on the fact (experimentally determined) that the thermopile receiver element absorbs independently of the various wavelengths emitted by the source.



Note: Inside of cylinder is blackened with a mixture of lampblack and turpentine and then sooted.

Fig. 1

Measurement of Irradiation

A fundamental form of the equation to be used with the compensated type thermopile radiometer is

$$G_o = K (mv) + F_o \leftarrow R \sigma T_h^4$$

or

$$G_o - F_o \leftarrow R \sigma T_h^4 = K (mv)$$

where

G_o irradiation of thermopile due to radiant energy entering opening in housing

K a constant of thermopile

mv electromotive force generated by thermopile (for zero current flow)

$F_o \leftarrow R$ fraction of power leaving front surface of receiver element which goes out through opening (shape modulus)

σ Stefan-Boltzmann radiation constant

T_h absolute temperature of radiometer housing (postulated to be uniform)

This equation also applies to the non-compensated* type of thermopile if the cold junctions are kept at the housing temperature and if the housing remains at air temperature. (See appendix A for a derivation of this equation.)

When the radiometer is directed toward a source or a surface, the radiant energy coming into the instrument is composed of the following:

- (1) The energy which would be present if the radiometer were removed
- (2) The energy which originates at the radiometer and is reflected back into the same instrument minus that energy which would have been reflected into the radiometer if it had not been shielded from the surface by the housing

It is actually desired to measure term (1) only (referred to in appendix B as G'), but the radiometer measures term (1) plus term (2) (referred to in this report as G_o). In other words, term (2) must be made very small; then G_o may be used as if it were the desired quantity. Term (2) may be made very small if the radiometer is located at a distance which is greater than twice its length from the source or the surface. Under these conditions, the absolute error due to the reflected energy from a perfectly reflecting diffuse surface will be less than $F_o \leftarrow R$ ($F_o \leftarrow R \sigma T_h^4$). (See appendix B for the proof.)

*In a compensated-type thermopile both sets of junctions are so constructed and disposed as to be similarly affected by changes in the air temperature. In the non-compensated type the cold junctions are kept at a constant temperature, and the emf generated is dependent on the air temperature.

If the temperature distribution along the housing is not uniform, an electromotive force may be generated even though $G_o - F_o \leftarrow R \sigma T_h^4$ is equal to zero because the irradiation of the hot and cold junctions will not be equal. It is because of this requirement that the radiometer housing is made of a material having a high thermal conductivity and a rather large mass. When use is made of the radiometer, many difficulties will be eliminated if sufficient time is allowed for the housing to come to equilibrium with the surroundings. Usually, only a few minutes are necessary to attain this condition. Factors that might cause an uneven temperature distribution along the housing should be avoided: for example, a hand placed on the housing while data are being taken.

Measurement of Net Exchange of Power by Radiation

The net exchange of power by radiation between a surface and the surroundings (the heat loss by radiation of a surface) may be obtained with the radiometer if the surroundings may be considered as having uniform emissive power. The question whether the surroundings are uniform enough for this measurement may be determined by sighting the radiometer at the surroundings and noting any high intensity sources. If there are any such sources, the procedure outlined in the following paragraph should be followed. For the case where the surroundings are uniform, equation (19) in appendix B shows that if the radiometer is pointed at the surface in question and "sees" only this surface and then is pointed at the surroundings in the opposite direction, the difference in readings will give

$$\left(\frac{q}{A}\right)_{\text{net}} = K (mv_{\text{surface}} - mv_{\text{surroundings}})$$

which is the net exchange by radiation between a surface and the surroundings if the surroundings absorb and radiate as an ideal radiator. (See reference 3, ch. XVIII, p. 20.) This equation applies only if the radiometer is used as specified under Measurement of Irradiation and also satisfies the condition of "seeing" only the surface in question when pointed at it. In other words, the surface sighted at must have an area at least large enough to cover the circle which would be seen by the radiometer receiver element. (See appendix B for derivation.)

For the case where the surroundings do not radiate uniformly, other methods must be used. If data are available on the emissivity of the surface in question, it is possible to obtain its radiant power loss by measuring its temperature. The power loss in any direction for any surface is its apparent emissive power in this direction times the shape modulus for the desired direction minus the absorbed power, or

$$\left(\frac{q}{A}\right)_{\text{net}} = \epsilon \sigma T^4 F_{0 \leftarrow R} - \alpha G_0$$

where G_0 is the value obtained with the radiometer.

This equation refers to an exchange of radiant power in that portion of space defined by the shape modulus $F_{0 \leftarrow R}$ only. Therefore, the power referred to is the emissive power of the body $\epsilon \sigma T^4$ times $F_{0 \leftarrow R}$. It should be noted that ϵ and α in this equation are not necessarily equal.*

In all measurements of irradiation, the result obtained is referred to a surface normal to the axis of the radiometer. If the radiometer axis makes an angle ϕ with the normal to the surface, on which the magnitude of the irradiation is desired, the incident power per unit area is obtained by multiplying the result by $\cos \phi$.

CONCLUSIONS

1. To make a measurement of irradiation:
 - (a) Point the radiometer at the object, keeping the front surface of the housing at least two radiometer lengths away from the nearest surface.
 - (b) After allowing time for the readings to become constant, take the housing temperature and the thermopile electromotive force readings.

*The emissivity (and consequently the absorptivity) is a function of wavelength, and, since the energy distribution corresponding to the incident and emitted energy is not necessarily the same, it may be necessary to use different factors: ϵ for the over-all emissivity corresponding to T , and α for the absorptivity corresponding to the distribution of G_0 .

- (c) Substitute these values in the equation

$$G_o = K (mv) + F_o \leftarrow R \sigma T_h^4$$

2. To make a measurement of the gain or loss of radiant power in a given direction by a body which receives an equal amount of radiation from all directions in half space:

- (a) Check by sighting in several directions (as outlined in 1(a) and 1(b)) to determine whether this condition is approximately true. If it is not, refer to paragraph 3, which follows.
- (b) Holding the radiometer near the body under consideration, sight at the surroundings in the desired direction and follow the procedure of 1(a), 1(b), and 1(c). Turn the radiometer through 180° , sighting at the body, and repeat the procedure of 1(a), 1(b), and 1(c).
- (c) The value of G_o obtained when sighted at the body minus the value of G_o obtained when sighting at the surroundings is the radiant power loss by the body in the direction referred to. The equation is

$$\left(\frac{q}{A}\right)_{\text{net}} = K \left[(mv)_{\text{surface}} - (mv)_{\text{surroundings}} \right]$$

- (d) This method applies only if, when sighting the radiometer at the surface in question, the radiometer sees only the surface and yet satisfies condition 1(a).

3. To make a measurement of the gain or loss of radiant power in a given direction by a body under any circumstances (including that of paragraph 2), the following procedure may be used:

- (a) Obtain the irradiation of the body as outlined in 1(a), 1(b), and 1(c). Call this G_o .
- (b) Measure the temperature of the surface of the body t_b by means of a thermocouple or other device.

(c) On the basis of other data, or on the basis of experience, approximate the emissivity of the surface ϵ .

(d) The radiant loss in a given direction is then

$$\left(\frac{q}{A}\right)_{\text{net}} = \left[\epsilon \sigma (t_b + 460)^4 \times F_{o \leftarrow R} - \alpha G_o \right]$$

University of California,
Berkeley, Calif., October 19, 1944.

APPENDIX A

EXAMPLES OF THE USE OF THE THERMOPILE RADIOMETER

In the following examples the case of a pilot in an airplane cabin is considered. The examples are dependent on each other, the final result being a radiant heat balance on the pilot.

In all irradiation measurements referred to in this and the following section of this appendix, it is assumed that the radiometer is held normal to the surface. If it is held at an angle ϕ to the normal of the surface, then the results should be multiplied by $\cos \phi$ in order to obtain the power per unit area of surface.

Example 1. Measurement of Irradiation

A pilot is seated behind a windshield receiving the direct radiation (normal to the window) of the sun. It is desired to measure the irradiation of the pilot due to the energy of the sun which is transmitted through the window. (Actually, the irradiation is by the sun and the small amount of sky seen by the radiometer.)

(a) The radiometer, which has been in the airplane cabin for about 5 minutes or more (during which time the

cabin conditions have not changed), is placed in front of the man and sighted in the direction of the sun, keeping the radiometer at least two radiometer lengths away from the window. After sufficient time is allowed for all emf's to become constant, the temperature of the radiometer housing and the emf generated by the thermopile are obtained by means of a portable potentiometer.*

(b) The irradiation of the man then is equal to

$$G_o = K(mv) + F_o \leftarrow R \sigma T_{\text{housing}}^4$$

A typical case would be as follows:

$$K \text{ (constant of the radiometer)} = 7.00 \frac{\text{Btu}}{\text{hr ft}^2 \text{ mv}}$$

$$F_o \leftarrow R \text{ (shape modulus of radiometer)} = 0.02$$

$$\sigma \text{ (Stefan-Boltzmann constant)} = 0.173 \times 10^{-8} \frac{\text{Btu}}{\text{hr ft}^2 (\text{°R})^4}$$

If the emf generated in this case were 42.0 mv and the housing temperature 20° F, the irradiation intercepted by the radiometer would be

$$\begin{aligned} G_o &= 7.00 \times 42.0 + 0.02 \times 0.173 \times 10^{-8} (20 + 460)^4 \\ &= 294.0 + 1.83 = 295.8 \frac{\text{Btu}}{\text{hr ft}^2} \end{aligned}$$

*If a portable potentiometer is not available, a sensitive millivoltmeter may be used. The emf generated

(mv) then would be equal to $E_m \frac{R_m + R_R}{R_m}$, where E_m is

the millivolt reading of the meter, R_m resistance of the meter, and R_R resistance of the radiometer thermopile.

Example 2

Measurement of Radiant Power Loss, Surroundings Uniform

It is desired to measure the power loss by radiation of a man in an airplane. The airplane is on the ground, sun behind it.

(a) The radiometer, after having been in the cabin for several minutes, is sighted about the cabin in the various directions that the man sees on the side desired. (That is, the man is considered to consist of two flat sides, and the portion of space seen by one side is half space.) If the readings obtained (allowing sufficient time for each reading to become constant) are nearly uniform, the average of the readings is noted. The radiometer is then sighted at the man, and the reading is noted. In all measurements the radiometer is kept at least two lengths away from the nearest surface. When sighting at the man, though, it must be close enough to see only the man, but still be at least two lengths away from him.

(b) The net loss by the man in the given direction is then

$$\left(\frac{q}{A}\right)_{\text{net}} = K \left[(mv)_{\text{man}} - (mv)_{\text{surroundings}} \right]$$

A typical case would be as follows:

(If the cabin wall temperature = 120° F)

$$K = 7.00 \frac{\text{Btu}}{\text{hr ft}^2 \text{ mv}}$$

$$F_o \leftarrow R = 0.02$$

Emf when sighting at surroundings = 0.0

Emf when sighting at man = -0.171 mv

Therefore, the radiant heat loss is $-7 \times 0.171 = -1.20 \frac{\text{Btu}}{\text{hr ft}^2}$

This value refers to the power lost to that portion of space defined by $F_{O \leftarrow R}$. Therefore the total loss to one-half space is this value divided by $F_{O \leftarrow R}$.

If similar readings are obtained in all directions when sighting at the man, his total heat loss to one side by radiation is then

$$\frac{-1.20}{F_{O \leftarrow R}} = \frac{-1.20}{0.02} = -60 \quad \frac{\text{Btu}}{\text{hr ft}^2}$$

or, in other words, he receives a net amount of radiation of 60 Btu/hr ft².

Example 3. Radiant Power Gain in Nonuniform Surroundings

It is desired to obtain the total radiant power exchange of a pilot seated behind a windshield, facing in- to the sun.

(a) First, the total irradiation of the pilot will be obtained. It will be assumed that the cabin may be divided into two sections, the front and the rear. The front surface of the pilot is exposed to the front cabin section. Also, since the solid angle seen by the radiometer when viewing the sun takes up such a small section of the space seen by the front surface of the pilot, it is assumed that the whole front cabin section (including the window) is seen by the front surface of the pilot. The irradiated surfaces will be divided up as follows:

(Assume the total area subject to radiation as equal to 15.5 sq ft, see reference 4).

Receiving radiation from the sun 4 ft²

Receiving radiation from front wall sections 8 ft²

Receiving radiation from rear wall section . 4½ ft²

In contact with seat 3 ft²

(4 ft² receives radiation from sun and walls)

The total area receiving radiation is, therefore, 12.5 ft².

With the same radiometer as in example 1, the following readings are obtained:

When sighted at front wall section = -0.12 mv,
so that

$$G_o = -(0.12 \times 7.0) + 1.83 = 0.99 \text{ Btu/hr ft}^2$$

When sighted at rear wall section = -0.098 mv,
so that

$$G_o = -(0.098 \times 7.0) + 1.83 = 1.15 \text{ Btu/hr ft}^2$$

When sighted at sun = 42.0 mv
so that

$$G_o = (42.0 \times 7.0) + 1.83 = 295.8 \text{ Btu/hr ft}^2$$

(as in example 1)

The apparent emissive power of any surface sighted at is $G_o/F_o \leftarrow R$, and the total irradiation from uniformly radiating surroundings is equal to the emissive power of the surroundings.

Therefore, the total radiant power incident on the pilot is

$$\begin{aligned} 4 \times 295.8 + 8 \times \frac{0.99}{0.02} + 4.5 \frac{1.15}{0.02} &= 1184 + 396 + 259 \\ &= 1839 \text{ Btu/hr} \end{aligned}$$

This would be equivalent to an average irradiation of

$$\frac{1839}{12.5} = 147 \text{ Btu/hr ft}^2$$

The surface of the man will be considered to be a gray body (so that the emissivity is equal to the absorptivity). On the basis of other data (reference 1) an approximation of the emissivity of the pilot's clothing must be made, and the surface temperature obtained, by

use of a touch thermocouple. The value of ϵ , α , is taken as 0.90 and the pilot's clothes temperature as 80° F; the net loss by the pilot is then

$$0.90 \left[0.173 \left(\frac{540}{100} \right)^4 - 147 \right] 12.5 = (132 - 132) 12.5 = 0 \text{ Btu/hr}$$

That is, the net loss of radiant power by the pilot is 0 Btu/hr, or an average value of 0 Btu/hr ft².

In a typical, comfortably heated room, the radiant loss by a man is about 180 Btu/hr over an area of 15.5 square feet, or 12 Btu/hr ft², and the convective loss is about 120 Btu/hr over an area of 19.5 square feet, or 6 Btu/hr ft², so that the elimination of the radiant loss is an important factor. (See reference 4.)

If it is desired to evaluate the total irradiation on a surface in surroundings the emissive power of which varies greatly with angle, it is suggested that the procedure outlined in reference 3 (ch. XVIII, pp. 45-50) be followed.

APPENDIX B

THEORY GOVERNING THE USE OF THE THERMOPILE RADIOMETER

SYMBOLS

- A area of a surface, ft²
- E emissive power of an ideal radiator at the temperature of the surface under consideration
 = σT^4 , Btu/hr ft²
- $F_{a \leftarrow b}$ shape modulus, the fraction of energy originally leaving a perfectly diffusing surface b of uniform temperature which reaches a surface a before any reflections have taken place; the numbers of the various surfaces referred to are inserted in place of a and b .
- G_e irradiation of thermopile due to the radiant energy coming through the opening in the housing, Btu/hr ft²

- G' the portion of G_o which would be present if the radiometer were removed: the value desired, Btu/hr ft²
- K constant of radiometer thermopile, Btu/hr ft² mv
- l distance from front of radiometer to nearest reflecting surface in direct line of "sight" of radiometer, ft
- L distance from thermopile to front of radiometer housing, ft
- mv electromotive force generated by thermopile, mv
- n l/L
- q heat transferred, Btu/hr
- t temperature, °F
- T absolute temperature ($t + 460$), °R
- Z $(1 - \epsilon_1) (1 - \epsilon_2) F_{3 \leftarrow 1} \times F_{1 \leftarrow 3}$
- β error term in radiometer equation
- ϵ emissivity of a surface (all surfaces are treated as if ϵ were independent of wavelength)
- φ angle between normal to surface and axis of radiometer, deg
- σ Stefan-Boltzmann constant, $\frac{0.173 \times 10^{-8} \text{ Btu}}{\text{hr ft}^2 (\text{°R})^4}$ (See International Critical Tables.)

Subscripts

- o opening of radiometer housing
- R radiometer receiver element
- 1 same as R
- 2 surface 2 in fig. 1

3 surface 3 in fig. 1

4 surface 4 in fig. 1

Measurement of Irradiation

The equation $G_o = K(mv) + F_o \leftarrow R \sigma T_h^4$ may be derived as follows:

First, imagine the radiometer housing to be closed at the front, and the thermopile connected to a device (such as a potentiometer) which indicates the emf generated by the thermopile. Under this condition the output of the thermopile will be zero, since both junctions of the thermopile will be irradiated equally. (See reference 1 for a more detailed picture of the actual construction.) If the front opening is removed, the junctions facing in this direction will receive less radiation from the housing than the shielded junctions, the amount less being the radiation that would have been received from this hypothetical front cover. This amount would be $F_o \leftarrow R \sigma T_h^4$. This may be visualized as a negative irradiation of the junctions exposed to the housing opening equal to $-F_o \leftarrow R \sigma T_h^4$. Owing to irradiation from external sources G_o , the total irradiation falling on the exposed junctions is greater than that falling on the shielded junctions by the amount $G_o - F_o \leftarrow R \sigma T_h^4$. Consequently, the reading on the measuring device will be proportional to this quantity.

In the foregoing expression, the emissivity of the housing was taken as unity. This can be shown to be very nearly true because of the following:

(1) The back surface of a long cylinder (length to diameter ratio 1.9) the surface of which has an emissivity of 0.75 radiates and absorbs as if its emissivity were 0.99. (See reference 5.)

(2) The length to diameter ratio of the radiometers used was approximately 3.5.

(3) The emissivity of the surface was approximately 0.9.

It follows from the foregoing that the output of the radiometer is proportional to the exchange of radiant power between the sources seen by the thermopile through the housing opening and a hypothetical surface of the same emissive power as the radiometer housing placed in the position of the thermopile receiver element.

In the remainder of this appendix the term G_o and the net interchange term $G_o - F_o \leftarrow R \sigma T_h^4$ will be discussed: the surfaces in fig. 2 will be referred to by number. The configuration shown in fig. 2 is one commonly encountered: by considering surface 3 to be composed of several parts, none of which see the other, the system may be made to apply to many cases. By combining a number of surfaces in the direct line of sight of the radiometer into one equivalent surface, the analysis presented here may be made to apply to almost any system. Although such equivalent methods may not be exact, they are extremely useful in practice. Regardless of the distribution of radiation from the surroundings, it must be remembered that the radiometer always indicates the actual irradiation incident on the thermopile receiver element, including any gaseous radiation which may be present.

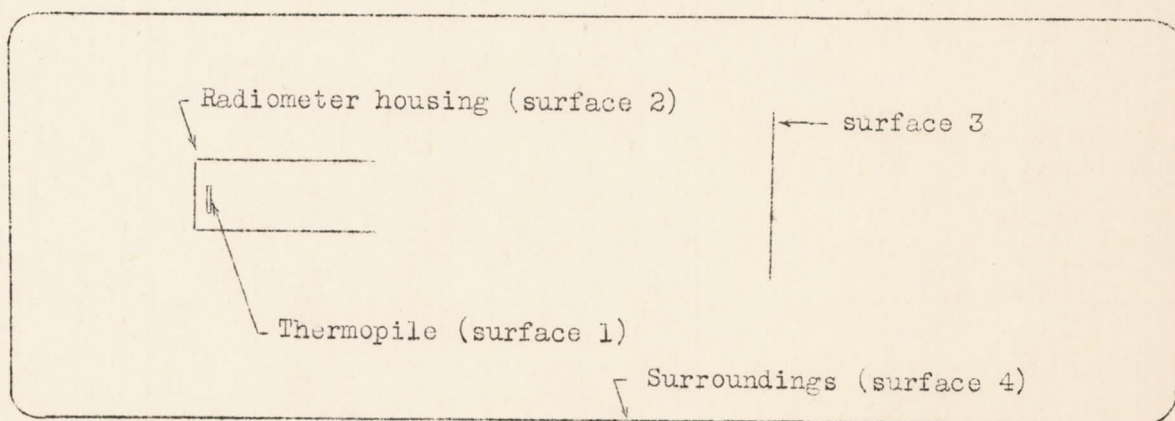


Figure 2.

The thermal system is based upon the following postulates:

1. All surfaces are perfectly diffusing.
2. All surfaces are opaque.
3. All surfaces are uniform in temperature.
4. Surfaces 1 and 3 are a sufficient distance apart that each point on surface 1 is essentially equidistant from each point on surface 3.
5. The exterior of the radiometer housing is perfectly reflecting.
6. The emissivity of each surface is uniform with temperature and wavelength.
7. The surroundings (in addition to the housing interior) have an emissivity of unity.
8. The intervening gaseous mediums are non-absorbing and non-radiating.

The above-listed conditions are very nearly satisfied in most cases.

In the analysis of G_0 , consider first what happens to any energy leaving the front surface of the thermopile element. This will be called $E'A_1$.

$E'A_1$ radiates in three directions:

1. To the radiometer housing (surface 2)
2. To the surroundings (surface 4)
3. To surface 3

Therefore:

$$E'A_1 = F_{2 \leftarrow 1} E'A_1 + F_{4 \leftarrow 1} E'A_1 + F_{3 \leftarrow 1} E'A_1$$

The energy $F_{2 \leftarrow 1} E'A_1$ is completely absorbed, as is also $F_{4 \leftarrow 1} E'A_1$. Consider now the term $F_{3 \leftarrow 1} E'A_1$. This is partially absorbed and partially reflected;

$\epsilon_3 F_{3 \leftarrow 1} E'A_1$ is absorbed, and $(1 - \epsilon_3) F_{3 \leftarrow 1} E'A_1$ reflected. Of this last term $(1 - \epsilon_3) F_{3 \leftarrow 1} F_{4 \leftarrow 3} E'A_1$ goes in the direction of the surroundings and is absorbed. Term $(1 - \epsilon_3) F_{3 \leftarrow 1} F_{2 \leftarrow 3} E'A_1$ goes in the direction of the housing interior and is absorbed, and $(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1$ is incident on the thermopile receiver element. Consequently,

$$\begin{aligned} E'A_1 = & F_{2 \leftarrow 1} E'A_1 + F_{4 \leftarrow 1} E'A_1 + \epsilon_3 F_{3 \leftarrow 1} E'A_1 \\ & + (1 - \epsilon_3) F_{3 \leftarrow 1} F_{4 \leftarrow 3} E'A_1 + (1 - \epsilon_3) F_{3 \leftarrow 1} F_{2 \leftarrow 3} E'A_1 \\ & + (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1 \end{aligned}$$

and of the original amount $E'A_1$ leaving surface 1,

$$(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1 \quad (a)$$

is returned. Of this, $\epsilon_1 (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1$ is reabsorbed, and $(1 - \epsilon_1) (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1$ is reflected. This new quantity may be treated as the original quantity $E'A_1$. Therefore, owing to this term, the quantity

$$(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} (1 - \epsilon_1) (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1 \quad (b)$$

is incident on the thermopile receiver strip. On each succeeding reflection, an additional amount of energy will be incident. It will be noted that each term is equal to the preceding term times the factor $(1 - \epsilon_1) (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3}$; and therefore summation of all such terms may be expressed as:

$$\begin{aligned} & (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1 [1 + Z + Z^2 + \dots] \\ & = \frac{(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} E'A_1}{1 - Z} \quad (2) \end{aligned}$$

where $Z = (1 - \epsilon_1) (1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3}$

That is, if an amount of power $E_1 A_1$ leaves the surface of the thermopile receiver, the amount represented by equation (2) is returned by reflection from surface 3 and is again incident on the thermopile. The terms making up G_0 may now be calculated. The power incident on 1 (through the opening) before any reflections from 1 is from the surroundings (direct)

$$F_{1 \leftarrow 4} E_4 A_4 \quad (3)$$

from the surrounds (by reflection from surface 3)

$$(1 - \epsilon_3) F_{3 \leftarrow 4} F_{1 \leftarrow 3} E_4 A_4 \quad (4)$$

from surface 3

$$F_{1 \leftarrow 3} \epsilon_3 E_3 A_3 \quad (5)$$

from surface 2 (by reflection from surface 3)

$$(1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 2} E_2 A_2 \quad (6)$$

In addition there will be the amount of power emitted by the radiometer housing which is reflected from surface 1 to surface 3 and then reflected from surface 3 to surface 1:

$$(1 - \epsilon_1) F_{1 \leftarrow 2} A_2 E_2 (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} \quad (6a)$$

The sum of the powers represented by (3), (4), (5), (6), and (6a) are incident upon the receiver element, and of this sum the fraction $(1 - \epsilon_1)$ is reflected. This reflected term may be treated as the original term $E_1 A_1$ was treated. Therefore, utilizing expression (2), the energy per unit time incident on surface 1 due to power emitted from surfaces 1, 2, 3, and 4, including the inter-reflections is

$$\begin{aligned} & [(3) + (4) + (5) + (6) + (6a)] \\ & + \frac{(1 - \epsilon_3) (1 - \epsilon_1) F_{3 \leftarrow 1} F_{1 \leftarrow 3} [(3) + (4) + (5) + (6) + (6a)]}{1 - Z} \\ & + \frac{(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} \epsilon_1 [E_1 A_1]}{1 - Z} \end{aligned} \quad (7)$$

By making use of the "reciprocity" relation $F_{a \leftarrow b} A_b = F_{b \leftarrow a} A_a$ (reference 3, ch. XVIII, p. 12), all the terms may be expressed as a function of A_1 . Expression (7) then becomes

$$[F_{4 \leftarrow 1} E_4 A_1 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_4 A_1 + \epsilon_3 F_{3 \leftarrow 1} E_3 A_1 + (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2 A_1] \quad (8a)$$

$$+ \frac{(8a)[(1 - \epsilon_1)(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3}]}{1 - Z} \quad (8b)$$

$$+ \frac{(1 - \epsilon_3)(F_{3 \leftarrow 1}) F_{1 \leftarrow 3} \epsilon_1 E_1 A_1}{1 - Z} \quad (8c)$$

$$+ \frac{F_{2 \leftarrow 1} E_2 A_1 Z}{1 - Z} \quad (8d)$$

Expression (8) (or (7)) is equal to $G_0 A_1$. Equating $G_0 A_1$ and (8), and dividing through by A_1 , the equation

$$\begin{aligned} G_0 = & [(F_{4 \leftarrow 1} E_4 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_4 + F_{3 \leftarrow 1} \epsilon_3 E_3) \\ & + (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2] \\ & + \frac{[(F_{4 \leftarrow 1} E_4 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_4 + F_{3 \leftarrow 1} \epsilon_3 E_3) + (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2] Z}{1 - Z} \\ & + \frac{(1 - \epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} \epsilon_1 E_1}{1 - Z} + \frac{F_{2 \leftarrow 1} E_2 Z}{1 - Z} \end{aligned} \quad (9)$$

is obtained.

If the radiometer were not present, the irradiation of a plane at the position of the receiver element would be

$$G' = F_{4 \leftarrow 1} E_4 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_4 + F_{3 \leftarrow 1} \epsilon_3 E_3 \quad (10)$$

This is the desired quantity. Equation (9) may now be written

$$\begin{aligned} G_0 = G' + G' \frac{Z}{1 - Z} + \frac{(1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2}{1 - Z} \\ + (1 - \epsilon_3) \frac{F_{3 \leftarrow 1} F_{1 \leftarrow 3} \epsilon_1 E_1}{1 - Z} + \frac{F_{2 \leftarrow 1} E_2 Z}{1 - Z} \end{aligned} \quad (11)$$

Errors in Measuring Irradiation

The first term on the right-hand side represents the irradiation of the thermopile due to the various sources in the system which are "seen" by the thermopile through the housing opening - as if the radiometer were not there. The remaining terms are due to inter-reflections, between the radiometer and surface 3, of power emitted by the sources in question and emitted by the radiometer. Accurate measurements of G' make it necessary that the last four terms in equation (11) be very small. The magnitude of these terms may be evaluated. If it is supposed that surface 3 is large enough so that the receiver element sees, directly, only surface 3, the worst possible case will have been considered. For the type of construction used, the area of the receiver strip is about three-quarters of the area of the radiometer opening. If the distance from the front of the housing to surface 3 is called L , and the distance from the thermopile to the front of the housing is $\underline{L}$, then the distance from the element to surface 3 is $l + L$, or if $l = nL$, it is $(n+1)L$. The approximate area of the surface seen will be $(n+1)L/L$ times (area of opening).

If $A_1 = 3/4$ times (area of opening), then the area of surface 3 seen will be

$$\frac{(n+1)L}{L} \times 4/3 \quad A_1 = A_3$$

and

$$\frac{A_1}{A_3} = 3/4 \quad \frac{1}{n+1}$$

and

$$F_{3 \leftarrow 1} A_1 = F_{1 \leftarrow 3} A_3$$

or

$$\begin{aligned} F_{3 \leftarrow 1} \times F_{1 \leftarrow 3} &= (F_{3 \leftarrow 1})^2 \frac{A_1}{A_3} = \\ &= (F_{3 \leftarrow 1})^2 \times \frac{3}{4(n+1)} \end{aligned}$$

Taking the worst possible case of $\epsilon_3 = 0$, and assuming $\epsilon_1 = 0.9$ (which is probably close to the correct value),

$$\begin{aligned} Z &= (1-\epsilon_1)(1-\epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3} = (1) \times (0.1) F_{0 \leftarrow R} \\ &= (F_{0 \leftarrow R})^2 \times \frac{3 \times 10^{-1}}{4(n+1)} \end{aligned}$$

(since $F_{3 \leftarrow 1} = F_{0 \leftarrow R}$). Assuming $F_{2 \leftarrow 3} = F_{1 \leftarrow 3}$ and $F_{2 \leftarrow 1} = 1$ does not introduce a large error, the last four terms in (11) may be written (taking $E_2 = E_1$) approximately

$$\left[\frac{3 \times 10^{-1}}{4(n+1)} (F_{0 \leftarrow R})^2 G' \right] + \left[\frac{6 (F_{0 \leftarrow R})^2}{4(n+1)} E_2 \right] + \frac{3 \times 10^{-1}}{4(n+1)} (F_{0 \leftarrow R})^2 \times E_2$$

(Since $1 - Z \cong 1$)

Since the last term in this expression is only 5 percent of the second term, it may be neglected. The error term now becomes

$$\beta \leq \left[\frac{3 \times 10^{-1}}{4(n+1)} (F_{0 \leftarrow R})^2 \right] G' + \left[\frac{6}{4(n+1)} F_{0 \leftarrow R} \right] F_{0 \leftarrow R} E_2$$

In most of the radiometers used, $F_{0 \leftarrow R}$ was approximately 0.02 (admittance angle of approximately 16°). Even if a radiometer having $F_{0 \leftarrow R}$ equal 0.5 (admittance angle of 90° were used (which is very unlikely)), the first term above would be approximately

$$\left(\frac{0.02}{n+1} \right) G'$$

If $n = 0$ (radiometer touching surface), this term is 2 percent of G' . In the much more likely case of $F_{0 \leftarrow R} = 0.02$, this term is 0.003 percent of G' . It follows that the first term in the foregoing expression may be neglected in most cases likely to be encountered. Consequently the error is

$$\beta \leq \left[\frac{6}{4(n+1)} F_{0 \leftarrow R} \right] F_{0 \leftarrow R} \sigma T_h^4$$

(since $\sigma T_h^4 = E_2$)

If $n = 1\frac{1}{2}$, β is less than $F_{0 \leftarrow R}$ times the term $F_{0 \leftarrow R} \sigma T_h^4$. If n is always made equal to or greater than 2, the error is proportionally less, and if the surface sighted at is partially absorbing, the error will again be proportionally less. This is also true if the intervening mediums are absorbing.

If desired, a curve of this error term (β) may be plotted for the particular shape modulus used in order to determine an appropriate value of n .

In cases where $F_{0 \leftarrow R} \sigma T_h^4$ is a small fraction of G' , the percentage error becomes negligible.

Measurement of Net Radiant Power Interchange

For some applications it may be desirable to obtain the net interchange of radiant power between the radiometer and the objects or surfaces seen through the opening by the element.

The net gain of power (referred to a surface in place of the receiver element having the emissive power of the housing) was shown to be equal to

$$G_0 - F_{0 \leftarrow 1} \sigma T_2^4 = K(mv)$$

For the purposes of this analysis it will be useful to treat the term $F_{0 \leftarrow R} \sigma T_2^4$ as if it represented the power radiated out of the radiometer by the hypothetical surface referred to previously. This power may be considered as going to surface 3 directly and being absorbed, to surface 4 directly and being absorbed, to surface 4 by reflection from surface 3, to surface 2 by reflection from surface 3, and back to surface 1 by reflection from surface 3. This result may be expressed as

$$\begin{aligned} F_{0 \leftarrow R} \sigma T_2^4 &= F_{0 \leftarrow 1} E_2 = \epsilon_3 F_{3 \leftarrow 1} E_2 + F_{4 \leftarrow 1} E_2 \\ &+ (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &+ (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &+ (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \end{aligned} \quad (12)$$

because $F_{0 \leftarrow 1} = F_{3 \leftarrow 1} + F_{4 \leftarrow 1} = F_{0 \leftarrow R}$

E_2 is used here since it is equal to σT_2^4 . Now, consider the term $(1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2$. Upon striking surface 1, part is absorbed and part reflected. Therefore, it is equal to

$$\epsilon_1 (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 + (1 - \epsilon_1) (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \quad (13)$$

The second term here may be considered in two parts:

$$\left. \begin{aligned} & (1 - \epsilon_1)(1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \times F_{2 \leftarrow 1} \\ \text{and} \\ & (1 - \epsilon_1)(1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \times F_{0 \leftarrow 1} = F_{2 \leftarrow 1} E_2 \times Z + F_{0 \leftarrow 1} E_2 \times Z \end{aligned} \right\} \quad (14)$$

The last term here, $F_{0 \leftarrow 1} E_2 \times Z$ may then be treated exactly as was $F_{0 \leftarrow 1} E_2$. Consequently, equation (12) becomes:

$$\begin{aligned} F_{0 \leftarrow 1} E_2 &= [\epsilon_3 F_{3 \leftarrow 1} E_2 + F_{4 \leftarrow 1} E_2 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &\quad + (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2 + \epsilon_1 (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &\quad + F_{2 \leftarrow 1} E_2 \times Z] + F_{0 \leftarrow 1} E_2 \times Z \\ &= [\epsilon_3 F_{3 \leftarrow 1} E_2 + F_{4 \leftarrow 1} E_2 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &\quad + (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2 + \epsilon_1 (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &\quad + F_{2 \leftarrow 1} E_2 \times Z] (1 + Z) + F_{0 \leftarrow 1} E_2 \times Z^2 \end{aligned} \quad (15)$$

Continuing this process, the expression

$$\begin{aligned} F_{0 \leftarrow 1} E_2 &= [\epsilon_3 F_{3 \leftarrow 1} E_2 + F_{4 \leftarrow 1} E_2 + (1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &\quad + (1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2 + \epsilon_1 (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2 \\ &\quad + F_{2 \leftarrow 1} E_2 \times Z] [1 + Z + Z^2 + \dots] \end{aligned} \quad (16)$$

is obtained. The remainder term, after repeating this n times would be $F_{0 \leftarrow 1} E_2 \times Z^{n+1}$, and as $n \rightarrow \infty$, $Z^{n+1} \rightarrow 0$ because Z is less than 1. Since $(1 + Z + Z^2 + Z^3 \dots) = 1/(1 - Z)$, (16) becomes

$$\begin{aligned} F_{0 \leftarrow 1} E_2 &= \frac{F_{4 \leftarrow 1} E_2}{1 - Z} + \frac{(1 - \epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1} E_2}{1 - Z} + \frac{\epsilon_3 F_{3 \leftarrow 1} E_2}{1 - Z} \\ &\quad + \frac{(1 - \epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1} E_2}{1 - Z} + \frac{\epsilon_1 (1 - \epsilon_3) F_{1 \leftarrow 3} F_{3 \leftarrow 1} E_2}{1 - Z} + \frac{F_{2 \leftarrow 1} E_2 Z}{1 - Z} \end{aligned} \quad (17)$$

Combining equations (9) and (16),

$$\begin{aligned}
G_0 - F_{0 \leftarrow 1} E_2 &= \frac{F_{4 \leftarrow 1}}{1-Z} (E_4 - E_2) + \frac{(1-\epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1}}{1-Z} (E_4 - E_2) \\
&+ \frac{\epsilon_3 F_{3 \leftarrow 1} (E_3 - E_2)}{1-Z} + \frac{(1-\epsilon_3) F_{2 \leftarrow 3} F_{3 \leftarrow 1}}{1-Z} (E_2 - E_2) \\
&+ \epsilon_1 \frac{(1-\epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3}}{1-Z} (E_1 - E_2) \\
&+ \frac{F_{2 \leftarrow 1} Z}{1-Z} (E_2 - E_2)
\end{aligned}$$

or

$$\begin{aligned}
G_0 - F_{0 \leftarrow 1} E_2 &= \frac{F_{4 \leftarrow 1}}{1-Z} (E_4 - E_2) + \frac{(1-\epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1}}{1-Z} (E_4 - E_2) \\
&+ \frac{\epsilon_3 F_{3 \leftarrow 1}}{1-Z} (E_3 - E_2) + \frac{\epsilon_1 (1-\epsilon_3) F_{3 \leftarrow 1} F_{1 \leftarrow 3}}{1-Z} (E_1 - E_2) \quad (18)
\end{aligned}$$

Because T_1 is very nearly equal to T_2 , (it follows that the last term above is very nearly zero and may be neglected. Therefore, the net exchange of radiant power between the radiometer and the sources seen by the thermopile element (referred to a surface in place of the receiver element, having the emissive power of the housing) is

$$\begin{aligned}
\left(\frac{q}{A}\right)_{\text{net}} &= \frac{F_{4 \leftarrow 1}}{1-Z} (E_4 - E_2) + \frac{(1-\epsilon_3) F_{4 \leftarrow 3} F_{3 \leftarrow 1}}{1-Z} (E_4 - E_2) \\
&+ \frac{\epsilon_3 F_{3 \leftarrow 1}}{1-Z} (E_3 - E_2) = K(\text{mv}) \quad (19)
\end{aligned}$$

This equation is analogous to that given in reference 1. A particular use of this equation is in emissivity measurements. (See reference 1.) In this case $E_4 = E_2$, the resulting expression being

$$\frac{\epsilon_3 F_{3 \leftarrow 1}}{1-Z} (E_3 - E_2) = K(\text{mv}) \quad (20)$$

By use of this equation, the emissivities of the type described in reference 1 may be obtained.

If it is desired to find the net exchange of radiant power between surface 3 and the surroundings (surface 4), equation (19) reveals the following:

For the case where the surroundings radiate uniformly and the radiometer is at sufficient distance from the surface sighted at (and yet "sees" only the surface in question, surface 3) so that $E_{4 \leftarrow 3}$ is nearly 1,

$$(F_{4 \leftarrow 1} = 0, \text{ and } Z \rightarrow 0)$$

$$\begin{aligned} K(mv)_a &= (1 - \epsilon_3) F_{3 \leftarrow 1} (E_4 - E_2) + \epsilon_3 F_{3 \leftarrow 1} (E_3 - E_2) \\ &= F_{3 \leftarrow 1} E_4 - F_{3 \leftarrow 1} E_2 - \epsilon_3 F_{3 \leftarrow 1} E_4 + \epsilon_3 F_{3 \leftarrow 1} E_3 \end{aligned}$$

or

$$K(mv)_a = F_{3 \leftarrow 1} (E_4 - E_2) + \epsilon_3 F_{3 \leftarrow 1} (E_3 - E_4)$$

When the radiometer is sighted in the opposite direction, the radiometer would give

$$K(mv)_b = F_{3 \leftarrow 1} (E_4 - E_2)$$

Therefore, the difference is

$$K [(mv)_a - (mv)_b] = \epsilon_3 F_{3 \leftarrow 1} (E_3 - E_4)$$

which is the power lost by radiation to the surroundings in the direction of the radiometer by surface 3.

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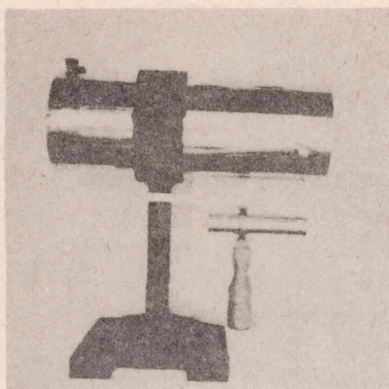


Figure 3.- This picture illustrates the reduction in size without reduction in voltage response to irradiation made possible by improved design of thermopile.

Figure 4.- Sensitive elements of thermopile radiometers shown in figure 3 are shown adjacent to the improved portable model.

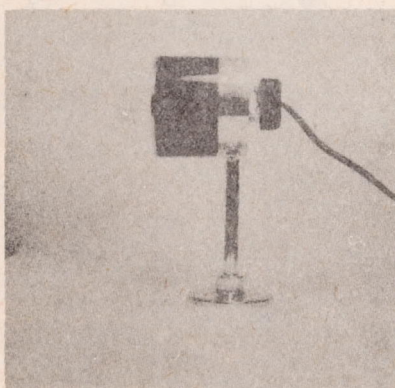


Figure 5.- Assembled portable radiometer employing thermopile similar to that used in larger model of figure 3.

Figure 6.- Unassembled portable radiometer as shown in figure 5.

