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GENERALIZED EQUATIONS FOR SELECTION CHARTS FOR HEAT EXCHANGERS IN AIRCRAFT

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SUMMARY

The equations that describe the operation of a heat exchanger – the equations for pressure drop, rate of heat transfer, and power expenditure – have been put into nondimensional, generalized form. These generalized equations can be used for constructing selection charts for various types of aircraft heat exchangers. Such charts would facilitate choice of heat-exchanger dimensions for any given set of operating conditions. A typical selection chart is presented.

INTRODUCTION

Until very recently, little detailed mathematical analysis of the design of heat exchangers used in aircraft has been published. Basic physical phenomena involved have been described, general trends in good design have been investigated, and selection charts for specific operating conditions have been developed in several papers such as references 1 through 6. Three papers (references 7, 8, and 9), which present new mathematical approaches to the problem of selecting efficient installations of specific types of heat exchanger, have recently been written. Reference 7 shows how to obtain the dimensions and the operating characteristics of cross-flow intercoolers that have small power expenditures. In reference 8 there is derived a formula that defines the volume of radiators using a minimum of power as a function of the airplane and air characteristics. A useful design chart with generalized coordinates is also presented in the same paper. In a third paper (reference 9) generalized coordinates are used to plot a surface representing all radiator designs.

This paper presents a method of deriving generalized coordinates for selection charts for any type of heat exchanger - cross-flow, counterflow, or parallel-flow - dissipating heat from any type of fluid and installed in an airplane.

SYMBOLS

The units given are the ones used in this paper. Any consistent system of units may be used.

c_1, c_2, c_3, c_4	empirical numerical constants
C_D	drag coefficient of airplane
C_L	lift coefficient of airplane
D	outside tube diameter, feet
D_h	hydraulic diameter of passage, feet
K	ratio of open area to total frontal area
g	acceleration due to gravity, 32.2 feet per second per second
h	surface heat-transfer coefficient, Btu per second per square foot per $^{\circ}F$
H	heat dissipation, Btu per second
k	thermal conductivity of fluid, Btu per second per square foot per $^{\circ}F$ per foot
K_1, K_2, K_3	constants
L	length of fluid passage, feet
L_n	length of heat exchanger in no-flow direction, feet
M	mass flow of air, pounds per second
n	number of tubes per square foot of face of heat exchanger with open ends of tubes
p	tube pitch, feet
Δp	pressure drop, pounds per square foot
P	power, foot-pounds per second

s	effective cooling surface per unit length of tube, square feet per foot
S	effective cooling surface, square feet
ΔT_i	initial temperature difference available for cooling, $^{\circ}F$
u	empirical exponent determined by type of flow
v	empirical exponent determined by type of flow
V	average velocity of fluid inside heat exchanger, feet per second
V_o	velocity of airplane, feet per second
W	weight of heat exchanger, pounds
x	empirical exponent determined by type of flow
y	empirical exponent determined by type of flow
ϵ	multiplying factor to take care of additional weight of heat exchanger mounting
ζ	mean temperature difference between hot fluid and cold fluid divided by ΔT_i
η	duct efficiency
μ	coefficient of viscosity of air, slugs per foot-second
ρ	mass density, slugs per cubic foot
ρ_W	weight density of heat exchanger, pounds per cubic foot
$\phi, \phi_1 \ldots \phi_n, \theta_b$	constants

Subscripts:

a	cold fluid side
b	hot fluid side
t	total
W	weight

Constants known in installation:

$$K_1 = \frac{n\ \Delta\ T_i \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}}}{\phi_3\ \phi_a\ H}$$

$$K_2 = \left[\frac{\phi_b}{\phi_a}\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}}\right]^{\frac{1}{v}}$$

$$K_3 = \frac{\left(\frac{\phi_3}{\phi_1}\right)^{\frac{1}{3-x}}}{\phi_3}$$

$$\phi_1 = \frac{2c_1\ \mu_a{}^x}{g^{3-x}\ \rho_a{}^2\ \eta_a\ p\ D^x (K_a)^{2-x}}$$

$$\phi_2 = \frac{2c_2\ \mu_b{}^y}{g^{3-y}\ \rho_b{}^2\ \eta_b\ D^{1+y}(K_b)^{2-y}}$$

$$\phi_3 = \epsilon\ \frac{C_D}{C_L}\ V_o\ \rho_W$$

$$\phi_a = \frac{(K_a)^u\ D^{1-u}\ \mu_a{}^u\ g^u}{k_a\ c_3\ s_a}$$

$$\phi_b = \frac{(K_b)^v\ D^{1-v}\ \mu_b{}^v\ g^v}{k_b\ c_4\ s_b}$$

$$\theta_b = \frac{2c_2\ \mu_b{}^y}{D^{1+y}\ \rho_b\ (gK_b)^{2-y}}$$

ANALYSIS

Three types of heat exchangers - ethylene-glycol radiators, oil coolers, and intercoolers - are used in aircraft today. In all three heat exchangers, the fluids either flow frictionally through passages or flow across banks of tubes. The physical phenomena occurring are described by pressure-drop and heat-transfer data that make possible the selection of heat exchangers for given conditions.

Data for the pressure drop across tube banks have been correlated (reference 10) by means of equations of the following type:

$$\frac{\Delta p D}{4 \frac{\rho V^2}{2} L} = c_1 \left(\frac{\mu}{\rho V D}\right)^x \tag{1}$$

Data for the pressure drop when fluid flows through passages are generally correlated (reference 11) by means of equations similar to the following:

$$\frac{\Delta p D_h}{4 \frac{\rho V^2}{2} L} = c_2 \left(\frac{\mu}{\rho V D_h}\right)^y \tag{2}$$

The general form of the equation correlating heat-transfer data for flow across tubes (reference 10) is

$$\frac{hD}{k} = c_3 \left(\frac{\rho V D}{\mu}\right)^u \tag{3}$$

The general form of the equation for frictional flow through passages (reference 11) is represented by

$$\frac{hD_h}{k} = c_4 \left(\frac{\rho V D_h}{\mu}\right)^v \tag{4}$$

The constants c_1, c_2, c_3, and c_4 and the exponents x, y, u, and v are determined by experiment.

Generalized equation for power expenditure.- The total power chargeable to a heat exchanger in an airplane is composed of three parts: the power required to force the hot fluid through the heat exchanger and the associated duct system, the power required to force the cold fluid through the heat exchanger and the associated duct system, and the power required to carry the weight of the heat exchanger and its supports. It is useful to note that the power used on the hot-fluid side is negligible in almost all heat exchangers. In intercoolers, the one type of heat exchanger in which this power sometimes is not negligible, the power is limited to a small value by the restriction on the pressure drop on the engine-air side. The power chargeable to heat exchangers can therefore be taken, for design purposes, to be the sum of the power required to force the cooling air through the heat exchanger and the duct system and the power required to carry the heat exchanger and its supports:

$$P = \frac{P_a}{\eta_a} + P_W = \frac{\Delta p_a\ M_a}{g\ \rho_a\ \eta_a} + P_W$$

A generalized equation for the power expended in forcing the air through the heat exchanger is obtained by solving for the pressure drop on the cooling-air side in equation (1), when the air flows across a bank of tubes, or in equation (2), when the air flows frictionally through passages, and by substituting for the velocity of the air in terms of the mass flow and the free area. For example, in the case of a cross-flow heat exchanger with the cooling air flowing across the tubes, the generalized equation for the cooling air power is

$$\frac{P_a}{\eta_a} = \frac{\Delta p_a\ M_a}{g\ \rho_a\ \eta_a} = \frac{2c_1\ \mu_a{}^x}{g^{3-x}\ \rho_a{}^2\ \eta_a\ D^x\ p\ K_a{}^{2-x}}\ \frac{M_a{}^{3-x}\ L_a}{(L_n\ L_b)^{2-x}}$$

$$= \phi_1\ L_n\ L_b\ L_a \left(\frac{M_a}{L_n\ L_b}\right)^{3-x} \qquad (5)$$

The power required to carry and support the heat exchanger is given simply by the equation

$$P_W = \epsilon\ \frac{C_D}{C_L}\ V_o\ W = \epsilon\ \frac{C_D}{C_L}\ V_o\ \rho_W\ L_n\ L_b\ L_a = \phi_3\ L_n\ L_b\ L_a \qquad (6)$$

By a combination of equations (5) and (6), the generalized equation for the power expenditure chargeable to the installation is obtained:

$$P = L_n L_b L_a \left[\phi_1 \left(\frac{M_a}{L_n L_b} \right)^{3-x} + \phi_3 \right] \qquad (7)$$

Generalized heat-balance equation.- In order to dissipate the required amount of heat H, the following heat-balance equation must be satisfied:

$$\frac{H}{\Delta T_i} = h_t S_t \zeta \qquad (8)$$

The total resistance to the flow of heat is the sum of the thermal resistances on the two sides of the heat exchanger:

$$\frac{1}{h_t S_t} = \frac{1}{h_a S_a} + \frac{1}{h_b S_b} = \frac{1}{n L_n L_b L_a} \left(\frac{1}{h_a s_a} + \frac{1}{h_b s_b} \right) \qquad (9)$$

If equations (3) and (4) for the heat-transfer coefficients are solved, the velocities eliminated as before, and the result substituted into equation (9), the generalized equation for the total thermal resistance is found. In the typical case where the cooling air flows across the tubes and the hot fluid flows through the tubes, the equation for the total thermal resistance becomes

$$\frac{1}{h_t S_t} = \frac{1}{n L_n L_b L_a} \left[\phi_a \left(\frac{L_n L_b}{M_a} \right)^u + \phi_b \left(\frac{L_n L_a}{M_b} \right)^v \right] \qquad (10)$$

When equation (10) is combined with the heat-balance equation (8) and simplified, there is obtained:

$$\frac{\Delta T_i \zeta n L_n L_b L_a}{H} = \phi_a \left(\frac{L_n L_b}{M_a} \right)^u + \phi_b \left(\frac{L_n L_a}{M_b} \right)^v \qquad (11)$$

Equation (11) is the generalized form of the heat-balance equation for heat exchangers.

Derivation of generalized coordinates.- Equation (11), the generalized heat-balance equation, in combination with equation (7), the generalized equation for power expenditure, completely describes the characteristics of all cross-flow heat exchangers. Similar equations completely describe the characteristics of counterflow and parallel-flow heat exchangers. These equations must be used in obtaining the generalized coordinates for the selection charts of heat exchangers. The derivation of the generalized coordinates for the selection charts of cross-flow heat exchangers will now be given. Exactly analogous derivations are applicable to counterflow and parallel-flow heat exchangers.

Equation (7), the generalized equation for the power expenditure, relates four quantities: the power expenditure, $L_n L_b L_a$, $\phi_1\left(\frac{M_a}{L_n L_b}\right)^{3-x}$, and ϕ_3. More than three quantities interrelated by a single equation cannot be plotted on a single chart. Equation (7) can be expressed, however, as a relation among only three instead of four quantities if it is rewritten in the form

$$P = \phi_3 \; L_n \; L_b \; L_a \left[\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n \; L_b}\right)^{3-x} + 1\right] \qquad (12)$$

It is to be noted that, in order for the generalized equation for power expenditure to relate three quantities,

$$\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n \; L_b}\right)^{3-x}$$

must be taken to be one of the quantities. The other two quantities may be taken as P and $\phi_3 \; L_n \; L_b \; L_a$ or as P and $\phi_3 \; L_n \; L_b \; L_a$ multiplied by any function or group of constants whatsoever. A consideration of the heat-balance equation will determine the best form for these quantities.

The generalized heat-balance equation (11) relates three quantities:

$$\frac{\Delta T_i \; \xi \; n \; L_n \; L_b \; L_a}{H}, \; \phi_a\left(\frac{L_n \; L_b}{M_a}\right)^u, \; \text{and} \; \phi_b\left(\frac{L_n \; L_a}{M_b}\right)^v$$

The quantity $\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n \; L_b}\right)^{3-x}$ is brought into the heat-balance relation by converting equation (11) into

$$\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n\ L_b}\right)^{3-x}$$

$$= \left[\frac{1}{\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\ \zeta\ n\ L_n\ L_b\ L_a}{H\ \phi_a} - \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\phi_b}{\phi_a}\left(\frac{L_n\ L_a}{M_b}\right)^v}\right]^{\frac{3-x}{u}} \qquad (13)$$

The heat-balance equation (13) contains a new quantity

$\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\ \zeta\ n\ L_n\ L_b\ L_a}{H\ \phi_a}$ that is easily introduced into

the equation for the power expenditure by multiplying both sides of equation (12) by

$$\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\ \zeta\ n}{H\ \phi_a\ \phi_3}:$$

$$\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i \zeta n P}{H\ \phi_a \phi_3} = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i \zeta n L_n L_b L_a}{H\ \phi_a}\left[\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n L_b}\right)^{3-x} + 1\right] \qquad (14)$$

The heat-balance and power expenditure relationships are expressed by equations (13) and (14) in terms of four quantities of which two occur in both equations. It is easily shown that these equations, as well as the four quantities involved in them, are nondimensional.*

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* From equation (14), $\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n\ L_b}\right)^{3-x}$ must have the same dimensions as dimensionless unity and, in order for equation (13) to be dimensionally satisfied, the quantities $L_n\ L_b\ L_a\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\ \zeta\ n}{H\ \phi_a}$ and $\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\phi_b}{\phi_a}\left(\frac{L_n\ L_a}{M_b}\right)^v$ must be non-

(Continued on p. 10)

Equations (13) and (14) can be rewritten as

$$\left[\frac{1}{L_n' L_b' L_a' - (L_n' L_a')^v}\right]^{\frac{3-x}{u}} = \left(\frac{1}{L_n' L_b'}\right)^{3-x} \tag{15}$$

$$P' = L_n' L_b' L_a' \left[\left(\frac{1}{L_n' L_b'}\right)^{3-x} + 1\right] \tag{16}$$

where L_n', L_b', L_a', and P' are nondimensional and are defined by

$$L_n' = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u-uv+v}{v(3-x)}} \left(\frac{\phi_b}{\phi_a}\right)^{\frac{1}{v}} \frac{H\,\phi_a}{M_b\,\Delta T_i\,\zeta\,n\,M_a} L_n = \frac{K_2\,K_3}{K_1\,\zeta\,M_a\,M_b} L_n \tag{17}$$

$$L_b' = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u(v-1)}{v(3-x)}} \frac{\Delta T_i\,\zeta\,n\,M_b}{H\,\phi_a\left(\frac{\phi_b}{\phi_a}\right)^{\frac{1}{v}}} L_b = \frac{K_1\,\phi_3\,\zeta\,M_b}{K_2} L_b \tag{18}$$

$$L_a' = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u-1}{3-x}} \frac{\Delta T_i\,\zeta\,n\,M_a}{H\,\phi_a} L_a = \frac{K_1\,\zeta\,M_a}{K_3} L_a \tag{19}$$

$$P' = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\,\zeta\,n}{H\,\phi_a\,\phi_3} P = K_1\,\zeta\,P \tag{20}$$

(Continued from p. 9)
dimensional if $\frac{\phi_1}{\phi_3}\left(\frac{M_a}{L_n\,L_b}\right)^{3-x}$ is nondimensional. If equation (14) is referred to again, it is seen that the fourth quantity $P\left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\,\zeta\,n}{H\,\phi_a\,\phi_3}$ must also be nondimensional.

Equations (15) and (16) are the final nondimensional forms of the generalized equations for the heat balance and the power expenditure of heat exchangers. In the application of these equations to the plotting of selection charts, it is convenient to use P' as a dependent variable whose value is determined by two independent variables. These independent variables may be taken to be any two of the variables: L_n', L_b', L_a', and convenient groupings of L_n', L_b', and L_a', such as L_a' and $L_n'\,L_b'$. Two groupings of general interest are obtained by converting equations (1) and (2) for the pressure drops into forms involving L_n', L_b' and L_a'. In this manner the equation for the pressure drop on the cooling-air side is transformed into

$$\Delta p_a' = \frac{L_n'\,L_b'\,L_a'}{(L_n'\,L_b')^{3-x}} = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u}{3-x}} \frac{\Delta T_i\,\zeta\,n\,M_a}{\phi_3\,\phi_a\,H\,g\,\rho_a\,\eta_a}\,\Delta p_a$$

$$= \frac{K_1\,\zeta\,M_a}{g\,\rho_a\,\eta_a}\,\Delta p_a \qquad (21)$$

The equation for the pressure drop on the hot-fluid side becomes

$$\Delta p_b' = \frac{L_n'\,L_b'\,L_a'}{(L_n'\,L_a')^{3-y}} = \left(\frac{\phi_3}{\phi_1}\right)^{\frac{u(v-3+y)}{v(3-x)}} \frac{\Delta T_i\,\zeta\,n\,M_b}{\phi_a\,H\,\theta_b\left(\frac{\phi_b}{\phi_a}\right)^{\frac{3-y}{v}}}\,\Delta p_b$$

$$= \frac{K_1\,\zeta\,\phi_3\,M_b}{\theta_b\,(K_2)^{3-y}}\,\Delta p_b \qquad (22)$$

Equations (15) and (16), the generalized heat-balance and power-expenditure equations, can be reduced in many cases to a single equation for convenience in plotting selection charts. Some of the derived equations of general interest follow:

$$P' = L_n'\,L_b'\,L_a'\left\{1 + \left[\frac{1}{L_n'\,L_b'\,L_a' - (L_n'\,L_a')^{v}}\right]^{\frac{3-x}{u}}\right\} \qquad (23)$$

$$P' = L_n' L_b' L_a' \left\{ 1 + \left[\frac{1}{L_n' L_b' L_a' - \left(\frac{L_n' L_b' L_a'}{L_b'} \right)^v} \right]^{\frac{3-x}{u}} \right\} \tag{24}$$

$$P' = L_n' L_b' L_a' \left\{ \left[1 + \left(\frac{L_a'}{L_n' L_b' L_a'} \right)^{3-x} \right] \right\} \tag{25}$$

$$P' = L_n' L_b' L_a' + \Delta p_a' \tag{26}$$

$$P' = L_n' L_b' L_a' \left\{ 1 + \left[\frac{1}{L_n' L_b' L_a' - \left(\frac{L_n' L_b' L_a'}{\Delta p_b'} \right)^{\frac{v}{3-y}}} \right]^{\frac{3-x}{u}} \right\} \tag{27}$$

As an example of the application of these equations, the generalized intercooler selection charts of figure 1 were plotted from equations (24), (25), (26), and (27). These charts show the intercooler power expenditure represented by P' as a function of the intercooler dimensions and the pressure drops on the hot-air and cold-air sides represented by the primed quantities. The charts readily show the relative merits of various intercooler designs.

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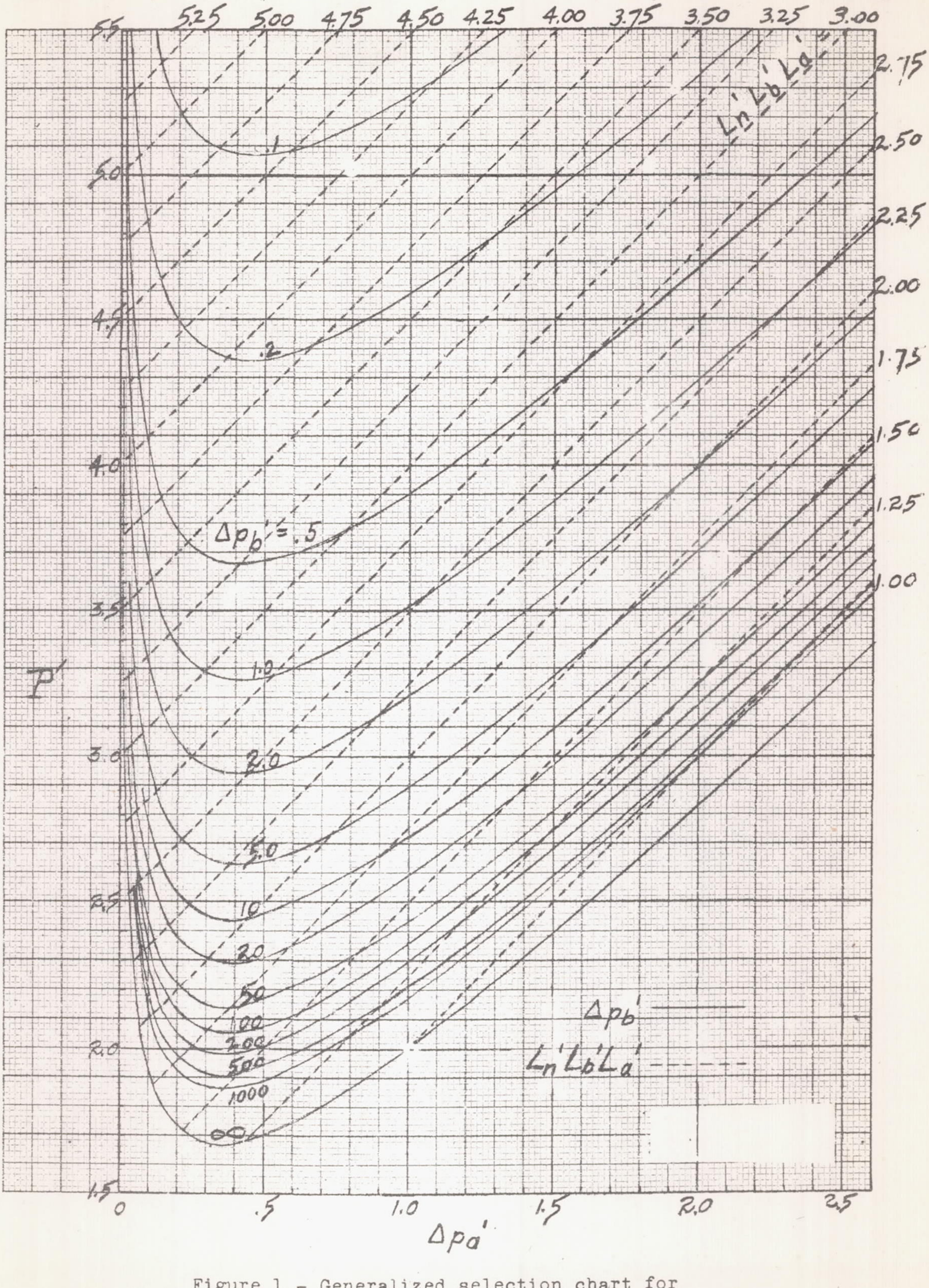

Figure 1.- Generalized selection chart for tubular intercoolers.

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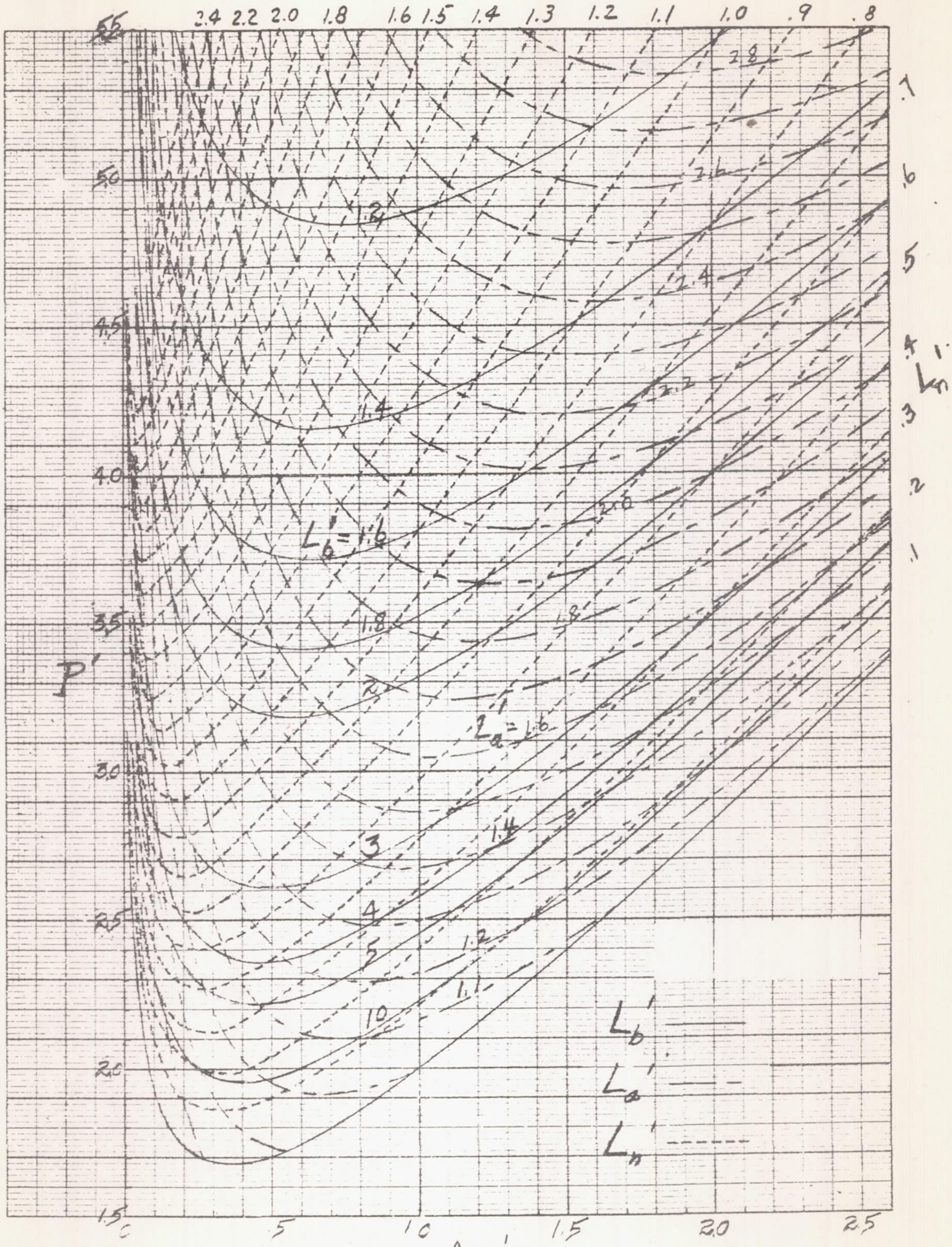

Figure 1.- Concluded.