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EFFECT OF HUMIDITY ON SEVERAL SUPERCHARGER PARAMETERS

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INTRODUCTION

The general practice in the calculation of results from supercharger tests is to use constant values of the specific heats, gas constant, and ratio of the specific heats of the air passing through the supercharger. The assumption is made that the effects of humidity and temperature on the gas properties will have a negligible effect on the parameters used to describe supercharger performance and that for all practical purposes the gas properties at some constant temperature and humidity can be used.

The object of the present report is to determine if the change of c_p, c_v, γ, and R of the air with extreme changes of humidity and test-bench temperature will have an appreciable effect on several parameters used to determine supercharger performance. The effects of temperature and humidity on the gas properties of normal air are first determined and then the effects of changes in the gas properties on the supercharger parameters are calculated. Normal air is defined as air of 29.92 inches of mercury pressure, 68° F temperature, and 36 percent relative humidity. The properties of normal air are given in reference 1 and are used by the NACA Subcommittee on Superchargers.

This work was done at the LMAL during February 1941.

ANALYSIS

Effect of Humidity and Temperature on Gas Properties

The specific heat of a mixture of dry air and water vapor may be written as follows:

$$c_p = \frac{c_{p_a} w_a + c_{p_m} w_m}{w_a + w_m}$$

A complete list of definitions of symbols used is given at the end of this report.

From the foregoing equation

$$c_p = c_{p_a}\left[1 + \left(\frac{c_{p_m}}{c_{p_a}} - 1\right)\left(\frac{w_m/w_a}{1 + w_m/w_a}\right)\right]$$

Now

$$r = \frac{w_m/w_a}{1 + w_m/w_a}$$

Then

$$c_p = c_{p_a}\left[1 + \left(\frac{c_{p_m}}{c_{p_a}} - 1\right) r\right] \qquad (1)$$

Solving equation (1) for c_{p_a}, the following equation is obtained:

$$c_{p_a} = c_p\left[1 - \left(\frac{r}{1 - r}\right)\left(\frac{c_{p_m}}{c_p} - 1\right)\right] \qquad (2)$$

When $r = r_n$, then $c_p = c_{p_n}$. Substituting these values in equation (2)

$$c_{p_a} = c_{p_n}\left[1 - \left(\frac{r_n}{1 - r_n}\right)\left(\frac{c_{p_m}}{c_{p_n}} - 1\right)\right] \qquad (3)$$

Substituting this value of c_{p_a} in equation (1)

$$c_p = c_{p_n}\left[1 + \left(\frac{c_{p_m}}{c_{p_n}} - 1\right)\left(\frac{\Delta r}{1 - r_n}\right)\right] \qquad (4)$$

where $\Delta r = r - r_n$. Equation (4) gives the specific heat at constant pressure with any specific humidity r in terms of the specific humidity and specific heat of normal air. In like manner, an equation can be derived for c_v, or

$$c_v = c_{v_n} \left[1 + \left(\frac{c_{v_m}}{c_{v_n}} - 1 \right) \left(\frac{\Delta r}{1 - r_n} \right) \right] \tag{5}$$

By dividing equation (4) by equation (5), the ratio of specific heats at any specific humidity r is obtained in terms of the ratio of specific heats with a specific humidity r_n, or

$$\gamma = \gamma_n \left\{ 1 - \left(\frac{\Delta r}{1 - r_n} \right) \frac{\left(\frac{c_{v_m}}{c_{v_n}} - \frac{c_{p_m}}{c_{p_n}} \right)}{\left[1 + \left(\frac{c_{v_m}}{c_{v_n}} - 1 \right) \left(\frac{\Delta r}{1 - r_n} \right) \right]} \right\} \tag{6}$$

The formula for the gas constant of a mixture of dry air and water

$$R = \frac{R_a\ w_a + R_m\ w_m}{w_a + w_m}$$

or

$$R = R_a \left[1 + r \left(\frac{R_m}{R_a} - 1 \right) \right] \tag{7}$$

but

$$\frac{R_m}{R_a} = \frac{\mu_a}{\mu_m}$$

where μ is the molecular weight of a gas. Then

$$R = R_a \left[1 + \left(\frac{\mu_a}{\mu_m} - 1 \right) r \right] \tag{8}$$

Solving equation (8) for R_a, letting $r = r_n$ so that $R = R_n$, gives

$$R_a = \frac{R_n}{1 + \left(\frac{\mu_a}{\mu_m} - 1 \right) r_n} \tag{9}$$

Substituting equation (9) in equation (8), gives the following equation:

$$R = R_n \left[1 + \frac{\left(\frac{\mu_a}{\mu_m} - 1 \right) \Delta r}{1 + \left(\frac{\mu_a}{\mu_m} - 1 \right) r_n} \right] \tag{10}$$

The most convenient way to determine supercharger parameters is to base the calculations on normal air properties, and then apply corrections based upon the changes in the gas properties from normal air values. The changes in c_p, c_v, γ, and R in increments of their values for normal air are shown in the following equations which were obtained from equations (4), (5), (6), and (10):

$$\frac{\Delta c_p}{c_{p_n}} = \left(\frac{c_{p_m}}{c_{p_n}} - 1 \right) \frac{\Delta r}{1 - r_n} \tag{11}$$

$$\frac{\Delta c_v}{c_{v_n}} = \left(\frac{c_{v_m}}{c_{v_n}} - 1 \right) \frac{\Delta r}{1 - r_n} \tag{12}$$

$$\frac{\Delta\gamma}{\gamma_n} = -\frac{\left(\frac{c_{v_m}}{c_{v_n}} - \frac{c_{p_m}}{c_{p_n}}\right)}{\left[1 + \left(\frac{c_{v_m}}{c_{v_n}} - 1\right)\frac{\Delta r}{1 - r_n}\right]}\frac{\Delta r}{1 - r_n} \tag{13}$$

$$\frac{\Delta R}{R_n} = \frac{\left(\frac{\mu_a}{\mu_m} - 1\right)\Delta r}{1 + \left(\frac{\mu_a}{\mu_m} - 1\right)r_n} \tag{14}$$

Formulas for the variation with temperature of c_{p_a}, c_{p_m}, c_{v_a}, and c_{v_m} are given in reference 2. The specific heats γ and R should be obtained for the average supercharger air temperature, but calculations from the formulas in reference 2 showed little change in the values from 68° to 212° F, the latter being the average air temperature assumed. For normal test-bench conditions, then, the gas properties may be considered as constant with respect to temperature variations. The values will be taken at 68° F.

The Effect of Changes in Gas Properties on Supercharger Parameters

The effects of the foregoing changes in gas properties on various supercharger parameters will now be determined. The NACA measures the volume of air with a thinplate orifice, and it can be shown from the formula for flow through the orifice that the volume of air Q at the inlet of the supercharger varies with the gas constant R as follows:

$$Q = K\sqrt{R} \tag{15}$$

where K is a value not dependent on humidity.

$$\frac{dQ}{Q} = \frac{1}{Q}\,\frac{KdR}{2\sqrt{R}} \tag{16}$$

Substituting the value of Q in equation (16) gives

$$\frac{dQ}{Q} = \frac{1}{2}\,\frac{dR}{R} \tag{17}$$

or, the percent change in volume at the inlet of the supercharger is equal to half the percent change in R.

One parameter that is used to a great extent in supercharger plots is H_{ad}, the adiabatic work per pound of air. Now H_{ad} can be obtained from the formula

$$H_{ad} = c_p\, T_{1t} Y \tag{18}$$

where T_1 is total temperature at the inlet of the supercharger, °F absolute.

$$Y = \left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{Y-1}{Y}} - 1$$

p_{2t} total pressure at outlet of supercharger, inches mercury absolute

p_{1t} total pressure at inlet of supercharger, inches mercury absolute

The adiabatic efficiency is then equal to

$$\eta_{ad} = \frac{H_{ad}}{c_p\ (T_{2t} - T_{1t})} = \frac{T_{1t}\ Y}{T_{2t} - T_{1t}} \qquad (19)$$

where T_{2t} is the total temperature at the outlet of the supercharger.

Differentiating Y,

$$dY = \left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\gamma-1}{\gamma}} \log_e \frac{p_{2t}}{p_{1t}}\ d\ \left(1 - \frac{1}{\gamma}\right)$$

or

$$dY = (Y + 1)\ \log_e \frac{p_{2t}}{p_{1t}}\ \frac{d\gamma}{\gamma^2} \qquad (20)$$

Now

$$\log_e\ (Y + 1) = \frac{\gamma - 1}{\gamma}\ \log_e \left(\frac{p_{2t}}{p_{1t}}\right) \qquad (21)$$

Substituting for $\log_e \dfrac{p_{2t}}{p_{1t}}$ its value as determined from equation (21) in equation (20) and dividing by Y the following equation is obtained

$$\frac{dY}{Y} = \left(\frac{1}{\gamma - 1}\right)\left[\left(1 + \frac{1}{Y}\right) \log_e\ (1 + Y)\right]\ \frac{d\gamma}{\gamma} \qquad (22)$$

also

$$d(H_{ad}) = d(c_p\ T_{1t}\ Y) = T_{1t}\ Y\ dc_p + c_p\ T_{1t}\ dY$$

$$= c_p\ T_{1t}\ Y \left(\frac{dc_p}{c_p} + \frac{dY}{Y}\right)$$

Then

$$\frac{dH_{ad}}{H_{ad}} = \frac{d(c_p)}{c_p} + \frac{dY}{Y} \qquad (23)$$

or the percent change in H_{ad} is equal to the sum of the percentage changes in c_p and Y.

$$d\eta_{ad} = d\left(\frac{T_{1t}\ Y}{T_{2t} - T_{1t}}\right) = \frac{T_{1t}\ Y}{T_{2t} - T_{1t}}\ \frac{dY}{Y} = \eta_{ad}\ \frac{dY}{Y}$$

then

$$\frac{d\eta_{ad}}{\eta_{ad}} = \frac{dY}{Y} \qquad (24)$$

or the percentage change in η_{ad} equals the percentage change in Y.

The pressure ratio corrected to 60° F is given by

$$\left(\frac{p_{2t}}{p_{1t}}\right)_{60} = \left(1 + \frac{T_{1t}}{519.6}\ Y\right)^{\frac{\gamma_n}{\gamma_n - 1}} \qquad (25)$$

Differentiating equation (25) and dividing by $(p_{2t}/p_{1t})_{60}$ gives

$$\frac{d\left(\frac{p_{2t}}{p_{1t}}\right)_{60}}{\left(\frac{p_{2t}}{p_{1t}}\right)_{60}} = \frac{\gamma_n}{\gamma_n - 1}\left(\frac{\frac{T_{1t}\ Y}{519.6}}{1 + \frac{T_{1t}\ Y}{519.6}}\right)\frac{dY}{Y} \qquad (26)$$

Another parameter that is used is the pressure coefficient which is defined as

$$q_{ad} = \frac{H_{ad}}{V^2/g}$$

where V is the impeller tip speed. Then

$$\frac{dq_{ad}}{q_{ad}} = \frac{dH_{ad}}{H_{ad}} = \frac{dc_p}{c_p} + \frac{dY}{Y} \tag{27}$$

as shown in equation (23). In like manner, the shaft efficiency, η_s, sometimes used is defined as

$$\eta_s = \frac{WH_{ad}}{P}$$

where W is the rate of air flow and P is the net shaft power. Then

$$d\eta_s = dW\left(\frac{H_{ad}}{P}\right) + dH_{ad}\left(\frac{W}{P}\right)$$

(The power measurement does not depend upon calculations involving humidity.)

Or

$$d\eta_s = \eta_s\left(\frac{dH_{ad}}{H_{ad}} + \frac{dW}{W}\right)$$

Then

$$\frac{d\eta_s}{\eta_s} = \frac{dH_{ad}}{H_{ad}} + \frac{dW}{W} \tag{28}$$

From equation (15) and the gas laws, it can be shown that

$$\frac{dW}{W} = -\frac{1}{2}\frac{dR}{R} \tag{29}$$

Substituting equations (23) and (29) in equation (28), the change of η_s with change in c_p, γ, and R is obtained. Or

$$\frac{d\eta_s}{\eta_s} = \frac{dc_p}{c_p} + \frac{d\gamma}{\gamma} - \frac{1}{2}\frac{dR}{R} \tag{30}$$

METHODS

The method of determining the change in gas properties from their values for normal air with change of humidity is to first substitute the values of c_p, c_v, γ, and r for normal air and the values of c_{p_m}, c_{v_m}, μ_a, and μ_m in equations (11), (12), (13), and (14). The values of the gas properties and specific humidity are obtained from reference 1 and the values of c_{p_m} and c_{v_m} from reference 2. Somewhat different values of c_{p_m} and c_{v_n} are given in the A.S.H.V.E. Guide, 1935 and in the International Critical Tables. Because of this disagreement, the values shown below were chosen arbitrarily. The values are summed up as follows:

p_n	29.92 in. Hg abs.
T_n	527.6° F abs.
r_n	0.00522 pound per pound
c_{p_n}	189.05 ft lb/lb/°F or 0.2430 Btu/lb/°F
c_{v_n}	135.55 ft lb/lb/°F or 0.1742 Btu/lb/°F
γ_n	1.3947
c_{p_m}	0.4466 Btu/lb/°F
c_{v_m}	0.3396 Btu/lb/°F

μ_a 28.95 lb/mol

μ_m 18.016 lb/mol

Substitution of the foregoing values in equations (11) to (14) results in the following equations:

$$\frac{\Delta c_p}{c_{p_n}} = 0.8423\ (r - 0.00522) \tag{31}$$

$$\frac{\Delta c_v}{c_{v_n}} = 0.9545\ (r - 0.00522) \tag{32}$$

$$\frac{\Delta \gamma}{\gamma_n} = -\frac{0.1123\ (r - 0.00522)}{1 + 0.9545\ (r - 0.00522)} \tag{33}$$

$$\frac{\Delta R}{R_n} = 0.6050\ (r - 0.00522) \tag{34}$$

The changes in the gas properties are then obtained from the foregoing formulas and a humidity chart as follows: From the relative humidity and the dry bulb reading (air temperature), or from the dry and wet bulb reading, the mixing ratio or the specific humidity r may be obtained, depending upon the chart used. Then from the value of r, the gas properties may be calculated.

If r is expressed in grains per pound, equations (31) to (34) will change to

$$\frac{\Delta c_p}{c_{p_n}} = 0.0001203\ (q - 36.5) \tag{35}$$

$$\frac{\Delta c_v}{c_{v_n}} = 0.0001364\ (q - 36.5) \tag{36}$$

$$\frac{\Delta\gamma}{\gamma_n} = - \frac{0.00001604\ (q - 36.5)}{1 + 0.0001364\ (q - 36.5)} \tag{37}$$

$$\frac{\Delta R}{R_n} = 0.00008643\ (q - 36.5) \tag{38}$$

where q is the specific humidity in grains per pound.

The changes in supercharger parameters for changes in gas properties as caused by changes in humidity are obtained by substitution of equations (35), (37), and (38) where needed in equations (17), (22), (23), (24), (26), (27), and (30). The factor $\left(1 + \frac{1}{Y}\right) \log_e (1 + Y)$ in equation (22) in the range $1 \lesssim \frac{p_2}{p_1} \lesssim 2$ varies from 1.000 to 1.100. At the value $p_2/p_1 = 1.5$, $\left(1 + \frac{1}{Y}\right) \log_e (1 + Y) = 1.058$. The value 1.06 will be assumed as sufficiently accurate for the whole range of operation. The extreme value of the error in the factor in the use of 1.06 where $1 \lesssim \frac{p_2}{p_1} \lesssim 2$ is then 6 percent. Assuming a large value for $\frac{dY}{Y}$ such as 1 percent, then an error of 6 percent will make this value 1.06 percent. Then the error in Y is only 0.06 percent. In like manner, the error in $d\eta_{ad}/\eta_{ad}$ would be 6 percent and the error in η_{ad} only 0.06 percent. Therefore $\left(1 + \frac{1}{Y}\right) \log_e (1 + Y) = 1.06$ may be used with sufficient accuracy over the range of supercharger operation. The resulting equations for changes in supercharger parameters with changes of humidity are:

$$\frac{\Delta Q}{Q} = 0.00004321\ (q - 36.5) \tag{39}$$

$$\frac{\Delta Y}{Y} = - \frac{0.00004315\ (q - 36.5)}{1 + 0.0001364\ (q - 36.5)} \tag{40}$$

The approximation will be made that $1 + 0.0001364\ (q - 36.5) = 1$.

Then

$$\frac{\Delta H_{ad}}{H_{ad}} = (0.0001203 - 0.00004315)\ (q - 36.5)$$

$$= 0.00007715\ (q - 36.5) \qquad (41)$$

$$\frac{\Delta \eta_{ad}}{\eta_{ad}} = -\ 0.00004315\ (q - 36.5) \qquad (42)$$

$$\frac{\Delta\left(\frac{p_{at}}{p_{1t}}\right)_{60}}{\left(\frac{p_{at}}{p_{1t}}\right)_{60}} = -\ 0.0001524 \left(\frac{\frac{T_{1t}}{519.6}\gamma}{1 + \frac{T_{1t}}{519.6}\gamma}\right)(q - 36.5) \qquad (43)$$

$$\frac{\Delta q_{ad}}{q_{ad}} = 0.00007715\ (q - 36.5) \qquad (44)$$

$$\frac{\Delta \eta_s}{\eta_s} = 0.00003394\ (q - 36.5) \qquad (45)$$

Equations (39) to (45) give the increments to add to or subtract from the supercharger parameters for changes of humidity from that for normal air.

RESULTS

In order to give an idea of the variation to be expected under ordinary operating conditions, the values of c_p, c_v, γ, and R for wet air have been calculated for various temperatures to be expected to occur throughout the year, and for various relative humidities at those temperatures. The temperatures given are outside air temperatures, but the results are not affected by the artificial heating of air inside the building if no water vapor is added by an air-conditioning system. The specific humidity is not affected by the heating and it is upon this quantity which the gas properties depend. Table I shows the variation in percent of the gas properties of the wet air as compared with normal air calculated according to equations (31), (32), (33), and (34). The humidity values were obtained from the Bulkeley Psychrometric Chart from the A.S.H.V.E. Guide for 1935. A more detailed table for variation of γ may be found on page 10 of reference 1. The variation of γ with humidity is negligible but the change in c_p from a cold, dry day to a hot, humid day is more than 3 percent, the change in c_v about 4 percent, and the change in R a little less than $2\frac{1}{2}$ percent. At present, in supercharger calculations the gas properties are based on air at a temperature of 68° F and 36 percent relative humidity. Table I shows that the change in c_p from this condition to that for a hot, humid day is about 3 percent, the change in c_v about $3\frac{1}{2}$ percent, and the change in R a little more than 2 percent.

For an extreme case of 100° F and 97-percent relative humidity, the changes in the supercharger parameters from their values for normal air will be:

$$\frac{\Delta Q}{Q} = 0.00004321\ (290 - 36.5) \times 100 = 1.1 \text{ percent}$$

$$\frac{\Delta H_{ad}}{H_{ad}} = 0.00007715\ (290 - 36.5) \times 100 = 2.0 \text{ percent}$$

$$\frac{\Delta \eta_{ad}}{\eta_{ad}} = -0.00004315\ (290 - 36.5) \times 100 = -1.1 \text{ percent}$$

$$\frac{\Delta q_{ad}}{q_{ad}} = 0.00007715\ (290 - 36.5) \times 100 = 2.0 \text{ percent}$$

$$\frac{\Delta \eta_s}{\eta_s} = 0.00003394\ (290 - 36.5) \times 100 = 0.86 \text{ percent}$$

In most supercharger test work, the weather changes are not as severe as those assumed in the above calculations, and constant values of the gas properties can be assumed. It is recommended that normal air gas properties be used in calculating supercharger parameters unless the specific humidity changes more than 55 grains per pound from its value for normal air. Corrections then should be applied to the supercharger parameters according to equations (39) to (45).

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SYMBOLS

c_p	specific heat of gas at constant pressure, Btu per °F per pound or foot-pounds per °F per pound
c_v	specific heat of gas at constant volume Btu per °F per pound or foot-pounds per °F per pound
H_{ad}	isentropic increase in total enthalpy per unit mass for a given pressure rise, foot-pounds per pound mass
p	pressure of gas, inches of Hg (abs.) obtained from corrected barometric pressure and pressure readings
q	specific humidity of gas, grains (H_2O) per pound (mixture)
q_{ad}	pressure coefficient
Q	volume flow at supercharger inlet, cubic feet per second
r	specific humidity of gas, pounds (H_2O) per pound (mixture)
R	gas constant for a particular gas, Btu per °F per pound or foot-pounds per °F per pound
T	temperature of gas, degrees Rankine
V	tip speed of impeller, feet per second
W	flow rate, pounds per second
w	specific weight of gas or vapor, pounds per cubic foot

$$Y \equiv \left(\frac{P_{2t}}{P_{1t}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{H_{ad}}{c_p\ T_{1t}}$$

$$\gamma \equiv \frac{c_p}{c_v}$$

$\Delta r \equiv r - r_n$

η_{ad}	adiabatic efficiency
η_s	adiabatic shaft efficiency
μ	molecular weight, pounds per mol

SUBSCRIPTS

a	refers to dry air
m	refers to moisture
n	refers to normal air
t	refers to total (or stagnation) value
1	refers to supercharger inlet condition
2	refers to supercharger outlet condition
60	refers to the pressure ratio corrected to inlet temperature of 60° F, on the assumption that $T_{1t}Y = (459.6 + 60)\,Y_{60}$

TABLE I

Outside-air temperature (°F)	Relative humidity (percent)	Specific humidity = r (lb/lb)	Δr $(r-r_n)$ (lb/lb)	$\frac{\Delta c_p}{c_{p_n}} \times 100$ (percent)	$\frac{\Delta c_v}{c_{v_n}} \times 100$ (percent)	$\frac{\Delta R}{R_n} \times 100$ (percent)	$\frac{\Delta \gamma}{\gamma_n} \times 100$ (percent)
100	97	0.04143	0.03621	3.05	3.46	2.19	-0.39
100	46	.01886	.01364	1.15	1.30	.83	-.15
100	10	.00406	-.00116	-.10	-.11	-.70	-.01
68	100	.01444	.00922	.78	.88	.56	-.10
68	36	.00522	.00000	.00	.00	.00	.00
68	10	.001433	-.00379	-.32	-.36	-.23	.04
30	100	.00344	-.00178	-.15	-.17	-.11	.02
30	50	.00171	-.00351	-.30	-.34	-.21	.04
30	10	.00034	-.00488	-.41	-.47	-.30	.06