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THE STRENGTH AND STIFFNESS OF SHEAR WEBS

WITH AND WITHOUT LIGHTENING HOLES

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SUMMARY

Nearly 200 tests were made on the strength of shear webs of 24S-T aluminum alloy, with and without lightening holes. The tests were made in a jig of the single-specimen type, in which the specimen is free to collapse completely without developing diagonal tension. The lightening holes were circular and had either flanged edges or beaded edges, the specimens with flanged edges constituting by far the largest test group. The following equations were found for the shear stresses τ causing collapse, all stresses being given in kips per square inch:

(a) Solid webs: $\tau_{coll} = (37 - 0.283\, h/t)$ if $h/t < 60$ and $\tau_{coll} = 1200\, t/h$ if $h/t > 60$. The second formula applies only to sheet 0.036 inch thick; for other thicknesses, the collapsing stress may be obtained from a graph

(b) Webs with flanged holes:

$$\tau_{coll}(net) = k\left[\tau_{cr} + (\tau_{ult} - \tau_{cr})\, D/b\right]$$

where the shear stress is based on the net section

(c) Webs with beaded holes: $\tau_{coll} = 440\,(t/h)^{3/4}$ where the shear stress is based on the gross section. Within the rather narrow test range, the size and the spacing of the holes has a practically negligible effect on the strength of webs with beaded holes.

In these equations, h is the width of the sheet; t, the thickness; D, the hole diameter; b, the hole spacing; k, a correction factor (not differing greatly from unity), which depends

on the sheet thickness; τ_{cr}, the buckling stress; and τ_{ult}, the ultimate shear strength of the material.

Simple empirical formulas are given for the shear stiffness appropriate to various groups of specimens. For webs with flanged holes, design charts are presented; these charts make it possible to determine by inspection the proportions of the lightest web for a given set of design conditions.

INTRODUCTION

The shear webs employed in aircraft structures are frequently perforated with regularly spaced holes to lighten the web or to provide access to the interior of the structure. Round holes with flanged edges were used in airship girders before the metal monocoque structure came into general use for airplanes, and they continue to be the most common type of lightening hole.

The problem of computing the strength of a web with lightening holes by theoretical means offers formidable mathematical difficulties. There appears to be no published record of any attempt at a purely theoretical solution, the nearest approach being a general, but extremely laborious, method of computing the stresses in a web with plain holes. It has been necessary, therefore, to rely on tests for proving the strength of perforated webs. Individual tests are sufficient for the immediate purpose of proving the strength of a given design, but they furnish no information on the optimum design proportions. A sufficiently extensive series of systematic tests would furnish information on the optimum design proportions and would eliminate the need for many individual tests. Unfortunately, so many parameters are involved that a very large number of specimens would be necessary to cover the range of proportions; this obvious fact has acted as an effective deterrent for many years.

A fairly extensive series of tests was published by Schüssler (reference 1), but his results have not been fully accepted by aeronautical engineers. A number of aircraft manufacturers have been interested for some time in obtaining additional data; it was finally agreed that these manufacturers would furnish the test specimens and the NACA would do the testing. Each manufacturer was to use his standard dies for flanging but to provide a sufficient number of specimens to cover the range of variables as far as practicable. The specimens tested in the present investigation were furnished by the Bell Aircraft Corporation. Special acknowledgment is due this company for their willingness to cooperate by making a

large number of test specimens at a time when unprecedented demands are being made on all production facilities.

The extensive test work involved was performed by Mr. S. H. Diskin of the NACA staff.

TEST PROCEDURE

In its most general form, the problem of shear webs with lightening holes involves the following variables:

(1) Material of sheet

(2) Thickness of sheet, t

(3) Width of sheet, h

(4) Type of edge support of sheet

(5) Size of holes

(6) Shape of holes

(7) Spacing of holes, b

(8) Shape of flanges or beads around holes

It is obvious that systematic tests covering the entire range of all variables would require a prohibitive number of specimens. Any given investigation, then, can cover only a limited range of designs and, if it becomes apparent that a different range of designs offers promise of being better in some respect, a new series of tests will become necessary. The fact that additional tests are certain to be required makes it desirable to discuss in some detail the test procedure used and the difficulties encountered in these tests, in order that later investigations may benefit from the experience gained.

Test specimens.- The specimens furnished by the Bell Aircraft Corporation consisted of the following: 125 specimens with flanged holes, including 52 duplicates; 27 specimens with beaded holes, including 4 duplicates; 8 specimens with plain holes, including 4 duplicates; and 4 specimens without holes. Typical cross sections of the flanges and of the beads are shown in figure 1. All specimens were made of 24S-T aluminum alloy, as were 28 specimens without holes prepared by the NACA.

The perforated specimens ranged in thickness from 0.032 to 0.064 inch. Three standard widths of specimens with holes were furnished: 6, 5, and 4½ inches, measured between center lines of bolt rows. The nominal hole diameters (clear diameters) were 0.8, 1.1, and 1.6 inches. All specimens were about 33 inches long; the exact length L was determined in each case by the hole spacing, the end being taken halfway between holes. The free ends of the specimens were reinforced by 90° flanges having a width of 1 inch.

The specimens without holes ranged in thickness from 0.015 to 0.065 inch. They were about 33 inches long, with the exception of one specimen (t = 0.065 in., h/t = 210) that was 77.5 inches long. The widths of specimens without holes ranged from 1 to 13 inches.

Inspection of the specimens before the tests disclosed that a number of the flanged specimens had cracks in the flanges, sometimes radial and sometimes circumferential. Even in an extreme case, however, where every flange in the specimen was cracked circumferentially, the static strength of the specimen was evidently unimpaired.

Test jig.- Shear tests on sheets with or without holes have commonly been made in the type of jig shown schematically in figure 2(a). (See references 1, 2, and 3, p. 603.) This type of jig is very suitable for tests concerned with buckling loads; for tests concerned with ultimate loads, however, the jig is objectionable because the rigid fixation of the outer bars enables the shear webs to develop diagonal tension and, consequently, to develop higher loads than they could develop in the actual structure.

For the present investigation, the single test jig shown schematically in figure 2(b) was chosen. In this type of jig, the specimen is free to collapse completely when the buckles become deep enough to cause yielding of the material at the crests. Figure 3 is a scale drawing of the actual jig, and figure 4 shows the jig in use.

For a few tests, the jig was modified by joining the fixed bar and the movable bar by links to produce a parallelogram; in such a parallelogram jig, the conditions are between those in a single jig and those in a double jig. The tests, which are not included in the paper, indicated an increase in strength of about 10 percent over the single-jig results.

Very heavy bars were used to hold the specimen along the outer, or free, edge in order to insure as uniform as possible a distribution of the shear stress along the length of the specimen. The

importance of this consideration was first pointed out by Mathar. (See reference 2.) The bars that receive the concentrated test load (bar C in fig. 2(a); bar B in fig. 2(b)) are subjected to longitudinal stresses and strains; as a result, the displacement of the loading bars - and with it the shear strain in the specimens - is a maximum at the point of load application and decreases from there toward the end, or ends, of the bars. The introduction of the load at the middle of the bar (fig. 2(b)) instead of at the end (fig. 2(a)) offers two advantages: The maximum amount of nonuniformity of shear strain is reduced to one-fourth; and the maximum shear strain occurs in the middle of the specimen instead of at the ends, where conditions are already uncertain. The size of the bars was chosen such that, theoretically, the maximum shear strain in the specimen exceeded the average shear strain by less than 2 percent in the worst case when perforated specimens were being tested.

As shown in figures 3 and 4, two dial gages reading to 1/10000 inch were used to measure the shear deformation of the specimens.

The load was applied by a portable hydraulic testing machine; the accuracy of load measurement was one-half of 1 percent.

Attachment of specimens.- The large thickness of the loading bars made it impossible to use rivets for attaching the specimens to them; half-inch bolts were used for this purpose. The bolt holes were at first drilled through the specimen with a special lip-cutting drill. The shear deformation measured on the first specimen without lightening holes agreed with the calculated value within the accuracy of measurement, and the first tests with perforated specimens gave very smooth load-deformation curves. It was therefore believed that the method of drilling the holes was sufficiently accurate, particularly since the emphasis in these tests was on strength, not on stiffness. After two groups of specimens had been tested, however, it was found that, under average conditions in continued testing, the original accuracy of the holes could not be maintained; in all the rest of the specimens the holes were therefore drilled undersize and line-reamed. The reamed holes gave better results than the drilled holes at the expense of doubling the time required for testing; with drilled holes it had been possible to make four tests a day; with reamed holes the average dropped to two tests a day. For extensive test series, it would be desirable to use tapered holes in the test jig to provide for taking up the wear caused by repeated reaming operations.

Edge support.- The specimens were at first clamped directly between the loading bars (fig. 5(a)). A comparable degree of edge restraint is not likely to exist in an actual structure. A number of tests were therefore made with a practical substitute for

supported edges. The conversion into the second type of support was made as shown in figure 5(b). The loading bars were separated and drill rods were placed between the bars and the specimens along the inner edges of the bolt holes. The first type of support will be referred to as "bar support," the second type as "rod support." For the largest values of h/t tested, the bar support may be considered to give clamped edges, the rod support to give supported edges. At small values of h/t, the clamping effect of the bar support is apparently not sufficient to produce the equivalent of rigidly clamped edges. The rod support, on the other hand, has some restraining effect that becomes more noticeable at lower values of h/t; it is caused by the restraining action of the bolts on the parts of the specimen that overhang the rods.

The specimens with flanged holes were divided into two inter-locking groups; one group was tested with bar supports and the other group, with rod supports. The test points obtained with rod supports appeared to show less scatter than the test points obtained with bar supports, and the edge restraint provided by the rod support was more nearly representative of actual conditions. Rod supports were therefore used for most of the specimens with beaded holes and for the specimens with plain holes. Both types of support were used for specimens without holes.

Loading procedure.- In the main group of tests, each specimen was preloaded once or several times to about 20 percent of the maximum load and was adjusted until the two dial gages gave approximately equal readings. The load was then applied in increments of 500 or 1000 pounds until the specimen completely collapsed and the load dropped off. Dial-gage readings were taken at each load increment.

After the strength tests had been completed, a small number of duplicate specimens were tested in the following manner: Each specimen was preloaded and adjusted to give approximately equal readings on the two dial gages. The load was then increased by the usual increments to two-thirds of the estimated maximum value and decreased again to zero. A second run to two-thirds load and back to zero load was then made, and finally the specimen was loaded to destruction. These tests were intended chiefly to obtain some data on permanent set; incidentally, they served the usual purposes of repeat tests.

TEST RESULTS

The Strength of Shear Webs

The strength of solid shear webs.- The dimensions of the shear webs without holes and the maximum loads carried by them are given in table 1. The experimental shear stresses causing the webs to collapse τ_{coll} were calculated, from the test load causing the specimen to collapse P_{coll}, by the formula

$$\tau_{coll} = \frac{P_{coll}}{L_e \, t} \tag{1}$$

the effective length L_e being taken as (see fig. 5)

$$L_e = L - \frac{1}{2} h_1 \tag{2}$$

for bar supports as well as for rod supports. This correction for ineffectiveness at the free ends was also used by Schüssler (reference 1) and is based on photoelastic tests reported in reference 3 (p. 605). Strain measurements made on the upper half of one specimen with bar supports showed stresses equal to 79 and 99 percent of the calculated stress at distances of $0.2h_1$ and $0.4h_1$, respectively, from the end; the measured stress at the middle of the specimen was 105 percent of the calculated stress. This excess at the middle is explained qualitatively by the fact that the load is applied in concentrated form, as mentioned in the discussion of the test jig. The fact that a 5-percent excess was measured instead of a 2-percent excess, as estimated, may be due to experimental error, inadequacy of the simple formula used for making the estimate, local overstressing due to oversized holes, and finally to the high load carried in the solid specimen.

The experimental values of τ_{coll} are shown in figure 6. The evidence is not so complete as might be desired but appears to warrant the conclusion that the method of edge support does not affect the collapsing load. For values of $h/t < 60$, the data can be represented by the empirical formula

$$\tau_{coll} = (37 - 0.283 \, h/t) \text{ kips per square inch} \tag{3}$$

At values of $h/t > 60$, the curves separate for different thick-
nesses, the thinner sheets develop higher stresses than the thicker
sheets. For a thickness of 0.036 inch, the experimental curve for
$h/t > 60$ can be expressed by the empirical formula

$$\tau_{coll} = 1,200 \; t/h \quad \text{kips per square inch} \tag{4}$$

No attempt was made to express the curves for other thicknesses in
analytical form.

For comparison, figure 6 also shows the well-known theoretical
curves for the critical shear stresses τ_{cr}. These curves are valid
only as long as the stress in the material has not passed the limit
of proportionality; beyond this point, corrections must be made
analogous to the case of column curves at low slenderness ratios.
There is no established method of making such corrections in the
case of critical shear stress, but an upper limit for τ_{cr} may
obviously be obtained by using τ_{coll} whenever it is lower than τ_{cr}.

The strength of shear webs with flanged holes.- Because webs
with round flanged lightening holes are widely used, an effort was
made to develop an empirical strength formula of such a form that it
could be used for extrapolation beyond the test range[1] with a
reasonable degree of accuracy. The formula developed is

$$\tau_{coll} \; (\text{net}) = k \left[\tau_{cr} + (\tau_{ult} - \tau_{cr}) D/b \right] \tag{5}$$

where

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[1] Formula (5) for the strength of shear webs with flanged lightening
holes, as given in this report, was based on a fairly large
number of tests (119 tests). The range of some of the variables
was, however, quite limited; in particular, there were practically
no tests with a diameter-to-depth ratio greater than 0.5. Addi-
tional tests have been started to extend the range of variables;
only a few of these tests have now been completed (Sept. 1942),
but they appear to indicate definitely that the formula becomes
unconservative outside the test range. Pending the completion
of these tests, it is recommended that the application of
formula (5) be strictly confined to webs falling within the
test range, which may be defined as follows:

$$D/h < 0.5; \quad h < 5.5 \text{ inches}; \quad t > 0.32 \text{ inch}$$

τ_{coll} (net)   shear stress that causes collapse, based on the net
section. The net section per inch run is taken
as $t(1 - D/b)$

τ_{cr} critical stress at which the sheet would buckle if it
had no holes

τ_{ult} ultimate shear stress of material

D   clear diameter of holes

b   center-to-center spacing of holes

$$k = 0.675 + 7.5 \ t \ (t \leqq 0.050 \text{ in.})$$

$$k = 1.050 \ (t \geqq 0.050 \text{ in.})$$

$$(6)$$

It will be seen that formula (5) involves the properties of the
material; namely, τ_{ult} and E (in τ_{cr}). The formula gives either
approximately correct values or conservative values for all possible
limiting cases as follows:

When the holes are so closely spaced that the flanges of
adjacent holes touch each other $(D/b \longrightarrow 1)$, the shear stress
developed over the net section may be expected to equal the ultimate
shear stress of the material as long as the sheet is thick enough to
prevent buckling of the narrow net section. Formula (5) reduces for
the case of $D/b \longrightarrow 1$ to τ_{coll} (net) $= k\tau_{ult}$, which indicates a net
shear stress lower than τ_{ult} for thin sheet, increasing to a net
shear stress somewhat larger than τ_{ult} for thick sheet. This excess,
which has a maximum value of 5 percent according to formula (6), can
probably be explained by the fact that the value of τ_{ult} as obtained
from reference 4 is somewhat conservative.

When the holes become vanishingly small but a finite spacing is
still maintained or when the spacing becomes very large for any
arbitrary size of holes $(D/b \longrightarrow 0)$, formula (5) reduces to
τ_{coll} (net) $= k\tau_{cr}$. This value is conservative for large ratios of
h/t and approximately correct for low ratios of h/t provided that
the τ_{coll} curve is used as a cut-off curve for τ_{cr}, as suggested
in the discussion of the strength of solid webs.

The linear dependence of τ_{coll} on D/b was established
empirically; a sample test plot is shown in figure 7. It was first

believed that the change of τ_{coll} should depend on a function of
D, b, and h, the most obvious one being D^2/bh that expresses
the amount of lightening (ratio of area removed to original area)
except for inessential constants. It was found, however, that
much closer correlation could be obtained with the parameter D/b
than with D^2/bh.

Tables 2, 3, and 4 give the dimensions of the test specimens,
the test loads, the experimental values of τ_{coll}, and the calculated
values of τ_{coll} for the shear webs with flanged lightening holes.
The experimental values of τ_{coll} were calculated by the formula

$$\tau_{coll}\,(net) = P_{coll}/A_e \tag{7}$$

where the effective net cross-sectional area A_e was taken as

$$A_e = (n - 1)(b - D)t \tag{8}$$

n being the number of holes in the specimen.

The correction for ineffectiveness at the ends included in
formula (8) is based on the assumption that the material outboard
of the last hole on each end carries no stress. Qualitatively, this
correction seems more appropriate for perforated specimens than the
correction used for solid specimens, and it does not differ greatly
from the correction for solid specimens within the test range.
Quantitatively, however, the correction is not verified and
constitutes the largest item of uncertainty in the evaluation of the
test data. The error due to this uncertainty is estimated to be,
in most cases, less than 5 percent.

The calculated values of τ_{coll} were obtained by using formulas (5)
and (6). The values of τ_{cr} needed for use with formula (5) were
taken from the curves shown in figure 8. These curves were obtained
by drawing tentative straight lines on all test plots, analogous
to the plot shown in figure 7. The tentative values for τ_{cr} obtained
in this manner were then plotted against h/t and faired. The
modulus E was taken as 10,600 kips per square inch and the ultimate
strength as $\tau_{ult} = 37$ kips per square inch, according to reference 4.

It will be noted that, for the two main groups of tests (with
reamed bolt holes), formula (5) represents the test data quite well.

Errors in excess of 10 percent are shown for 11 percent of all tests, and the maximum errors are 22 percent on the conservative side and 10 percent are shown for 11 percent of all tests, and the maximum errors are 22 percent on the conservative side and 10 percent are shown by 16 percent of all tests; the maximum error on the conservative side is 24 percent and the maximum error on the unconservative side, 20 percent. Compared with the formulas of reference 1, formula (5) has, therefore, the twofold advantage of somewhat better accuracy and of much greater usefulness for extrapolating beyond the test range. The test group with drilled bolt holes averages 10 percent low, presumably reflecting the influence of uneven load distribution caused by irregular oversized holes.

The strength of shear webs with beaded holes.- The results of the tests on webs with beaded holes are given in table 5. Application of the formula developed for webs with flanged holes showed large irregular scatter, indicating that the behavior of the webs with beaded holes differs considerably from the behavior of the webs with flanged holes. The beads stiffen a fairly large portion of the sheet and, as a result, the webs with beaded holes appear to act more nearly as uniformly stiffened sheets. The collapsing stress of webs with beaded holes is therefore based on the gross, not on the net, section and is calculated by the formulas used for webs without holes, namely, formulas (1) and (2). In order to emphasize this point, the shear stress thus calculated will be designated τ_{coll} (gross).

The experimental values of τ_{coll} (gross) are plotted in figure 9 against the ratio h/t. Curve A is plotted from the equation

$$\tau_{coll} = 440 \ (t/h)^{3/4} \text{ kips per square inch} \tag{9}$$

This formula represents all the test data for beaded holes with about the same degree of accuracy as formula (5) represents the test data on webs with flanged holes. On the webs having a hole diameter $D = 1.05$ inches, the influence of hole spacing is sufficiently definite to justify the fairing of individual curves for different hole spacings b. Curve B in figure 9 is faired through the test points for webs with $b = 4$ inches, curve C through the test points for webs with $b = 3$ inches. The curve for $b = 3.5$ inches was omitted to simplify the figure. For the webs having a hole diameter $D = 1.60$ inches, the tests indicate no relation between the allowable stress and the hole spacing. The number of tests is not sufficient to draw more definite conclusions on the influence of hole size and hole spacing.

Three beaded-hole specimens were tested with bar supports.
It will be noted that the test points fall practically on the same
curves as points for tests with rod supports. The conclusion
that the method of edge support does not influence the strength of
webs with beaded holes is in agreement with the conclusion first
stated that this type of web fails in the same general manner as a
uniform sheet, because the tests on solid webs indicated no
influence of the method of edge support on the strength.

The strength of webs with plain holes.- Since only four
different sizes of webs with plain holes (without flanges) were tested,
it is impossible to draw any general conclusions. The test results
are given in table 6.

The Stiffness of Shear Webs

The shear displacement δ of a solid web is given by the
elementary formula

$$\delta = \frac{T}{G} h_1 = \frac{P\, h_1}{L_e tG} \tag{10}$$

as long as the sheet does not buckle and the limit of proportionality
of the material is not exceeded. The depth h_1 of the web between
the center lines of the bolt rows is used in all cases when deforma-
tions are being calculated.

The displacement of a perforated web may be calculated by
the same formula if the product tG in formula (10) is multiplied
by an efficiency factor η. This factor will be denoted by η_0
when it applies to the initial straight-line part of the load
deformation curve. For many webs, this initial straight-line part
is so short as to be of little practical significance. The factor η
(without subscript) recommended for general use is, therefore, based
on the measured displacement δ at two-thirds of the collapsing load;
this load was chosen because, under present design requirements, the
limit load is two-thirds of the ultimate design load.

A simple formula for the shear-stiffness factor may be obtained
by assuming that the material between the holes and the edges is
entirely ineffective, leaving as effective material rectangular
strips having a length $(b-D)$; the formula is evidently

$$\eta = 1 - D/b \tag{11}$$

If this formula is modified by introducing an exponent m

$$\eta = 1 - (D/b)^m \qquad (11a)$$

it may be adjusted to fit individual groups of test data as well as the scatter of the data will permit.

The experimental displacement curves often exhibited marked irregularities; some of these irregularities were probably caused by loose fit of the bolts, some by buckling between the lightening holes. No attempt was made, therefore, to derive formulas of general validity to represent the experimental shear-stiffness factors.

Only the results for webs with rod supports are given. It is believed that the restraining influence exerted by the bar supports on the shear displacements is never approached in a practical structure, and the results obtained with bar supports are, consequently, of no practical interest.

The stiffness of solid webs.- By definition, the shear-stiffness factor η_0 equals unity for solid webs.

If buckling begins at a load less than $2/3$ P_{coll}, the value of η will depend on the amount of buckling. The condition is similar to that in diagonal-tension fields but is complicated by the fact that a web free to collapse is more sensitive to initial buckles than a diagonal-tension web. There were additional experimental difficulties in some cases, such as the small magnitude of the displacements caused by h_1 being very small, and uncertainties concerning the fit of the bolts. As a result, the usable data obtained are too isolated to warrant publication.

The stiffness of webs with flanged holes.- The basic formula (11) was found to represent quite well the experimental values of η_0 obtained for webs with flanged holes having thicknesses from 0.040 to 0.064 inch (fig. 10). For webs having a thickness of 0.032 inch, the values of η_0 were appreciably lower (fig. 11). The factor η for the stiffness at $2/3$ P_{coll} is shown in figure 12; all thicknesses of sheet are included in this plot because there was no discernible influence of the thickness on the stiffness factor. Figure 13 shows the factors η obtained on the specimens used for permanent-set tests. These specimens had been loaded twice to $2/3$ P_{coll}; it may be assumed, therefore, that the play in the bolt holes was fairly well eliminated, and the results average correspondingly higher than the results shown in figure 12.

It may be concluded from figures 12 and 13 that the stiffness factor may be taken as

$$\eta = 1 - (D/b)^{2}/_{3} \tag{12}$$

for webs with flanged holes when the joint along the loaded edge has no play; since a well-riveted joint has no play, the formula should be applicable to webs with riveted joints.

<u>The stiffness of webs with beaded holes</u>.- The basic formula (11) represents fairly well the experimental values of η_0 for webs with beaded holes having a thickness of 0.064 inch (fig. 14). For smaller thicknesses, the values of η_0 are lower (fig. 15).

The shear-stiffness factor η of webs with beaded holes at high loads exhibits the same characteristic as the strength of these webs; namely, that the influence of hole size and hole spacing is negligible within the test range (fig. 16). The thickness, however, has some influence and the experimental averages can be expressed by the empirical formula

$$\eta = 0.1 + 4.5t \quad (t \leqq 0.064 \text{ in.}) \tag{13}$$

Permanent-Set Tests

The permanent set of shear web may be thought of as caused by two distinct phenomena: (1) permanent set of the specimen itself and (2) permanent set in the joints - riveted or bolted - along the edges.

The magnitude of the permanent set suffered by the specimen itself depends on the magnitude of the maximum stress and on the extent of the region experiencing high stresses. In perforated webs, the maximum stress covers only a very narrow band in the net section. There may be some concentration of stress, but this concentration would be too localized to affect appreciably the permanent set of the entire specimen. There may exist a buckle over the net section, adding local bending stresses to the basic shear stresses; in the range covered by the tests, however, these buckles were always very small if at all perceptible, and they disappeared completely upon removal of the load.

At the two-thirds load chosen as standard for defining the permanent set, the maximum stress in a perforated shear web may,

therefore, be taken as approximately equal to $2/3 \tau_{coll}$. Since
τ_{coll} is always less than τ_{ult}, the maximum stress is always less
then $2/3 \tau_{ult}$. In 24S-T aluminum alloy, the yield stress is roughly
equal to $2/3 \tau_{ult}$. Consequently, there is little likelihood of
an appreciable amount of permanent set occurring in the net section
of a perforated web loaded to $2/3 P_{coll}$.

Permanent set in riveted joints is caused by bearing failures
of the sheet or the rivets and by deformation of the rivets. This
subject forms a separate field of study and need not be considered
here. Permanent set in bolted joints is caused chiefly by bearing
failures of the sheet and by slippage in oversized holes.

The results of the permanent-set tests are given in table 7.
It will be seen that the permanent sets of specimens with flanged
holes tested with bar supports range roughly from 5 to 10 percent of
the displacement under load. The net shear stresses are below the
yield stress of the material, and the sets recorded are, therefore,
believed to be mostly caused by slippage in the bolt holes.

The permanent set recorded for webs with flanged holes tested
with rod supports are about ten times as large as those with bar
supports. Since the net shear stresses are of the same order of
magnitude for both groups of tests, it must be concluded that the
slippage in the bolt holes and the bearing failures of the sheet were
much more pronounced in the tests with rod supports than in those
with bar supports. The difference presumably arises from the fact
that the bar supports transmit an appreciable part of the load by
friction, thus relieving the bearing pressures and delaying the
occurrence of slip. In addition, the bolts are subjected to a certain
amount of bending when the loading bars are separated by the rods.

The belief that the recorded set is largely caused by slippage
is supported by a study of the load-displacement curves discussed in
the appendix. These curves suggest strongly that large amounts of
slippage take place at loads between 4 and 8 kips when the rod
supports are used. The possibility of large amounts of slippage despite
the use of reamed holes is explained by wear in the test jig. An
index to the relative amount of wear in the jig is furnished by the
test numbers, which are given in tables 1 to 4; it may be noted that
the set tests on specimens with flanged holes carry test numbers 161
to 175. The irregular shape of the worn holes and the large thickness
of the loading bars made it impossible to measure the actual amount
of wear in the holes; it is estimated, however, that the wear in
many holes amounted to at least 0.002 to 0.004 inch when the set
tests were being made.

If the view is accepted that the set in the specimen itself is small compared with the set in the joint, then it must be concluded that the sets recorded in table 7 cannot be considered as representative of actual structures and, moreover, that it will be impossible to obtain representative data on permanent set by the method of testing employed in this investigation. It will be necessary to use a method of testing that permits reproduction of the joint used in the actual structure.

DESIGN FOR MAXIMUM STRENGTH-WEIGHT RATIO

The formulas and curves given in the preceding sections answer the problem of stress analysis; that is, the problem of predicting the strength of a web of given proportions. Attention will not be given to the design problem that is normally posed in the following form: Given the depth of a web and the load that it must carry, what proportions should be chosen to obtain the web with the minimum weight?

Solid webs.- In webs without holes, the thickness is the only variable that can be chosen. Inasmuch as the strength requirement must be met first, there is no possibility of fulfilling any additional requirements for minimum weight.

Webs with beaded holes.- Since the effect of hole size and hole spacing on the strength of webs with beaded holes was not established very definitely by the tests, it is not possible to give a definite procedure for designing such webs in order to obtain the best structural efficiency. Comparative calculations within the test range indicate that the efficiency of webs with 1.6-inch holes is very nearly the same as that of webs with 1.05-inch holes, the maximum differences being about 2 percent. Webs with 1.6-inch holes can be designed for strength on the basis of formula (9); this formula establishes the necessary thickness of the web, and the minimum weight is then obtained by using the minimum hole spacing for which the formula has been verified; namely, $b = 3.5$ inches.

Webs with flanged holes.- In webs with flanged holes, it is possible to meet the strength requirement with a series of webs differing in thickness, hole size, and hole spacing. It is not practicable to develop a direct analytical method of finding the lightest web among all the webs that meet the strength requirement; the problem was solved, therefore, by preparing design charts in the following manner: Fixed values were assumed for h and t, while D was varied systematically. Values of τ_{cr} were calculated by the standard formula

$$\tau_{cr} = 4.8E \ (t/h)^2 \tag{14}$$

assuming simply supported edges in order to be conservative.
Omitting factors that do not affect the maximum problem, the shear
force q per inch run is

$$q \propto \left[\tau_{cr} + \left(\tau_{ult} - \tau_{cr} \right) D/b \right] (1 - D/b) \tag{15}$$

Omitting again factors that do not affect the maximum problem,
the weight W of the web per inch run is

$$W \propto h \left(1 - \frac{\pi D^2}{4bh} \right) \tag{16}$$

Dividing expression (15) by (16) gave an expression for the strength-
weight ratio; when the derivative of this expression with respect to
the ratio D/b was equated to zero, a quadratic equation was obtained
that gave the optimum D/b ratio necessary to obtain the maximum
strength-weight ratio for a given combination of values of h, t,
and D. The solution of the quadratic equation is

$$R = A \pm \sqrt{A^2 - B} \tag{17}$$

where

R optimum value of D/b

A $\dfrac{4h}{\pi D}$

B $A + \dfrac{\tau_{cr}(1 - A)}{\tau_{ult} - \tau_{cr}}$

The physically possible values of R lie between zero and unity;
the applicable root of the equation, therefore, was found by inspection
to be the one using the minus sign before the square root. From the
value of R thus found, the best spacing b was obtained, and the
transverse shear strength S of the web was then calculated by the
formula

$$S = \tau_{coll}ht\,(1 - D/b) \tag{18}$$

The result was a series of points from which the strength curve for the assumed values of h and t could be plotted against D/h. In this manner, strength curves were calculated for various standard values of t and for two values of h delimiting the test range. The strength curves are shown in figures 17(a) and 17(b) as lines sloping down to the right. The number at each point gives the value of R determining the optimum hole spacing.

For each web calculated as described, the weight was then calculated. The weights obtained were used to construct curves of equal weight, shown as lines sloping up to the right. In order to facilitate comparisons, the equal weight curves are not numbered in terms of actual weights but in terms of the thickness t_s of the corresponding solid sheets.

There were no tests available with $D/h < 0.14$. The strength curves and the equal weight curves were therefore stopped at $D/h = 0.15$, and straight guide lines were drawn to the values of $D/h = 0$, which are based on the tests on sheets without holes. Individual judgment must be used should it be necessary to design webs falling within this region.

It will be noted that the strength curves, when extended to $D/h = 0$, pass near the points derived from tests on sheets without holes for a certain range but not over the entire range of the two charts. Theoretically, there is no reason why the strength curves should pass through these points, because the theoretical case of a web with vanishingly small holes is not identical with the case of a sheet without holes. The strength curves assume that the optimum hole spacing is used in each case, which means that there is a finite reduction of section along the center line of the web even when the holes became vanishingly small. On the other hand, the validity of equation (5) is assured only if there is a flange of a certain depth around each hole. In the case of very small holes, there must exist, then, a ridge of closely spaced flanges along the center line of the web, and this ridge would exert a stiffening influence. It should be realized, however, that this reasoning is theoretical and qualitative only. Caution should be used in designing perforated webs in the region where the strength of the solid sheet is appreciably lower than the strength of the perforated sheet until full experimental verification is obtained for this region.

For webs having a depth of either 4 or 8 inches, the answer to any design problem may be obtained from figure 17 by inspection.

For the design of webs with intermediate depths, figure 18 was prepared, using figure 17 as a basis. The ratio $D/h = 0.8$ is about the maximum value that can be used in practice without having undue interference between the flanges and the rivet rows; figure 18(b) will therefore be used to obtain the most efficient designs, because inspection of figure 17 indicates that the most efficient design is always obtained by using as large a hole as possible. If it should be necessary to use smaller holes, the allowable value of the running shear S/h may be obtained by interpolating between the curves of figures 18(a) and 18(b).

The design charts are based on the assumption that the optimum hole spacing is used. Larger hole spacing will increase the strength but will lower the strength-weight ratio. Smaller hole spacing will lower the strength as well as the strength-weight ratio. The influence of the hole spacing is illustrated by the three test groups shown in figures 19(a), 19(b), and 19(c). The figures illustrate the value of the formula for finding the optimum hole spacing when the optimum falls outside the test range.

Examples for use of design charts.- Example A: A web 4 inches deep is required to carry a transverse shear load of 1550 pounds. Find the design proportions giving the best strength-weight ratio, assuming that practical considerations limit the value of D/h to 0.8.

By inspection of figure 17(a), it is found that a web 0.040 inch thick will just carry the required load. The hole diameter is $0.8 \times 4 = 3.2$ inches. The chart gives $R = 0.57$; the optimum hole spacing is, therefore, $b = 3.2/0.57 = 5.6$ inches. The weight of this web is slightly more than that of a solid web 0.025 inch thick.

Example B: A web 6 inches deep is required to carry a transverse shear load of 2280 pounds. Find the design proportions giving the best strength-weight ratio, assuming that practical design considerations limit the value of D/h to 0.8.

The required running shear is $S/h = 2280/6 = 380$ pounds per inch. Figure 18(b) shows that to carry this running shear with a depth of 6 inches, a thickness of 0.040 inch is required. By interpolation, the value of R is 0.591. The hole diameter is $0.8 \times 6 = 4.8$ inches; the optimum hole spacing is therefore $4.8/0.591 = 8.12$ inches.

Comparison of three types of web.- Comparisons between solid webs with flanged holes may be made conveniently by inspection of figure 17. It will be seen that the perforated webs may be stronger or weaker than solid webs of the same thickness. For a given strength, however, the most efficient web is always a perforated web, never a solid web.

Comparative calculations for webs with flanged holes and webs with beaded holes are shown in table 8. The ratio D/h for flanged holes was limited to 0.8, because larger ratios may cause interference between the flanges and the rivets. The strength of the webs with beaded holes was based on formula (9). The hole diameter was taken as 1.6 inches, and the hole spacing as 3 inches, which is about the closest spacing possible. This close spacing, although beyond the test range, was chosen in order to make the comparison more favorable for the beaded holes. As table 8 shows, however, the webs with flanged holes require a smaller volume of material and, consequently, are more efficient than the webs with beaded holes unless the webs have a very low h/t ratio.

Comparisons not included here show that for the same thickness and hole diameter, the web with beaded holes will carry more load, or at least the same load, as the web with flanged holes. The web with flanged holes can be made more efficient, however, by using larger holes, while the size of the bead effectively limits the size of the hole.

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APPENDIX

LOAD-DISPLACEMENT CURVES OF SHEAR WEBS

While a discussion of load-displacement curves is only of slight
interest to the designer and to the stress analyst, it is of interest
to the engineer confronted with the task of devising a test procedure.
Increasing attention is being paid to questions of stiffness and,
consequently, there will be an increasing demand for information that
can be obtained only by tests. A discusion of points brought out in
the present investigation will therefore be in order in preparation
for future tests.

It is impractical to present all the data; only samples are shown
for the most important test groups. In order to avoid personal bias
in the choice of the samples, the choice was made by arbitrarily
designating test numbers without referring to the curves.

The sample curves for solid webs are shown in figure 20. It
will be seen that the initial tangent agrees quite well with the
calculated straight line, but the initial straight-line part of the
curve may be quite short.

In references 1 and 2 it is stated that the typical load-
displacement diagram starts as a straight line, then bends through
a knee into a second straight line with smaller slope, and finally
rounds over into a curve approaching the horizontal. The knee between
the two straight-line parts was interpreted in these two references
as indicating the buckling load.

The curve shown in figure 20 for specimen 1 answers this general
description, and the knee of the curve lies in the region of the
critical load calculated on the assumption of supported edges. On
the curve for specimen 4D, however, there is obviously no relation
between the location of the knee of the curve and the critical load.

On the perforated webs with bar supports and drilled bolt holes
(fig. 21) the curves do not show a knee that might be considered as
indicating a buckling load. On the same type of specimens with reamed
holes, a knee might be identified on three of the four curves shown
(fig. 22).

On the perforated webs with rod supports (fig. 23), all the curves
show a more or less pronounced irregularity. The displacement curve
indicates a sudden reduction of shear stiffness, followed by a sudden

increase of a smaller amount. It would be very difficult to explain
this action as being due to buckling, when the specimen is free to
collapse. On the other hand, it is easy to explain this action on
the assumption that the bolt holes were oversize and that the sudden
apparent loss of stiffness is, in fact, caused by slippage.

If the displacement curves obtained with bar supports are re-
examined in the light of this conclusion, it will be seen that they
show similar tendencies, only much less pronounced. Since the bar
supports give a much larger contact area on the specimens than the
rod supports, slippage probably occurs more gradually and is thus
effectively masked.

It is stated in reference 1 that the knee of the load-displacement
curve was used as prime evidence of buckling but that corroborative
evidence was obtained by observing reflections on the surface of the
specimen between lightening holes. This method is quite sensitive
for detecting the instant at which a plane surface begins to curve
slightly, but it is difficult to detect changes of curvature by this
method. In the specimens used for the present investigation, it was
generally found that the flanging operation had left the sheet slightly
curved between the holes, so that it was difficult to detect buckles
at an early stage of development by observing reflections. In general,
clearly visible buckles began to appear at about 2/3 P_{coll}. Earlier
buckling was noted on some solid sheets and on a number of specimens
with bar supports and reamed holes, but the buckles were often so
shallow that their existence remained doubtful over a large range of
loading, sometimes over a range equal to one-third of the collapsing
load.

The observations made lead to the conclusion that the load-
displacement curves obtained in these tests are falsified by
slippage in the bolt holes, to a moderate extent when bar supports
were used and to a marked extent when rod supports were used. It
may also be concluded that whenever there is any possibility of such
slippage, a knee in the load-deformation curve cannot be regarded
as a reliable indication that buckling occurs in the specimen.

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TABLE 1
SOLID SHEAR WEBS

Specimen	Test	L_e (in.)	t (in.)	h (in.)	P_{coll} (kips)	Exp. τ_{coll} (kips/sq in.)	Calc. τ_{coll} (kips/sq in.)	$\dfrac{Exp.}{Calc.}$
				Bar supports, reamed bolt holes				
2B	37	32.28	0.0374	3.94	15.70	13.00	11.38	1.14
2C	44	32.38	.0394	3.94	15.90	12.46	12.00	1.04
2D	42	32.63	.0385	2.94	20.75	16.52	15.70	1.05
2E	43	32.78	.0396	2.94	21.30	16.41	16.18	1.01
4B	38	32.34	.0643	4.03	40.00	19.23	19.00	1.01
4C	39	32.02	.0642	3.97	41.50	20.14	19.30	1.04
4D	40	32.44	.0634	2.94	47.90	23.29	23.88	.98
4E	41	32.69	.0637	3.00	49.00	23.56	23.56	1.00
137A	176	27.64	.0420	9.97	5.50	4.74	4.90	.97
137B	177	27.52	.0419	9.97	6.50	5.64	4.88	1.16
138A	178	29.52	.0232	6.22	3.48	5.08	5.30	.96
138B	179	29.57	.0233	6.25	3.60	5.24	5.30	.99
139A	184	32.11	.0426	1.03	40.90	29.90	30.14	.99
139B	185	32.11	.0424	1.03	41.90	30.79	30.12	1.02
140	190	32.13	.0148	1.00	8.76	18.42	17.90	1.03
142	191	28.97	.0631	7.00	18.00	9.88	10.65	.93
143	192	69.94	.0619	13.00	21.20	4.90	5.00	.98

Average of ratios above unity (10 tests) = 1.05
Average of ratios below unity (7 tests) = .97
Average of all ratios   (17 tests) = 1.02

Specimen	Test	L_e (in.)	t (in.)	h (in.)	P_{coll} (kips)	Exp. τ_{coll} (kips/sq in.)	Calc. τ_{coll} (kips/sq in.)	$\dfrac{Exp.}{Calc.}$
				Rod supports, reamed bolt holes				
2F	85	31.94	0.0377	5.31	13.00	10.80	8.45	1.28
4F	86	32.06	.0641	5.31	30.30	14.80	14.50	1.02
133A	131	31.22	.0142	3.99	3.30	7.45	7.60	.98
133B	132	31.24	.0139	3.96	3.43	7.90	7.58	1.04
134A	129	28.08	.0401	10.02	5.30	4.71	4.64	1.02
134B	130	28.27	.0392	10.02	4.64	4.19	4.53	.92
135A	181	31.99	.0144	2.71	4.50	9.77	8.30	1.18
135B	182	31.95	.0144	2.71	3.75	8.15	8.30	.98
136A	146	29.59	.0393	7.18	8.96	7.70	6.46	1.19
136B	180	29.56	.0417	7.18	8.96	7.27	6.88	1.06
141	193	32.41	.0148	1.74	5.18	10.80	11.06	.98

Average of ratios above unity (7 tests) = 1.11
Average of ratios below unity (4 tests) = .97
Average of all ratios   (11 tests) = 1.06

Specimen	Test	L_e (in.)	t (in.)	h (in.)	P_{coll} (kips)	Exp. τ_{coll} (kips/sq in.)	Calc. τ_{coll} (kips/sq in.)	$\dfrac{Exp.}{Calc.}$
				Bar supports, drilled bolt holes[a]				
1	32	30.75	0.0315	4.00	9.66	9.97	9.50	1.05
2	17	30.72	.0406	4.00	14.15	11.35	12.22	.93
3	12	30.59	.0512	4.19	18.90	12.07	14.65	.82
4	1	30.72	.0656	4.06	36.00	17.87	19.20	.93

Average of all ratios (4 tests) = 0.93

[a]Specimens with drilled bolt holes
are not shown on plot.

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TABLE 2
SHEAR WEBS WITH FLANGED LIGHTENING HOLES
[Bar supports, reamed bolt holes]

Specimen	Test	L (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	A_e (sq in.)	P_{coll} (kips)	Exp. τ_{coll}(net) (kips/sq in)	Calc. τ_{coll}(net) (kips/sq in)	$\dfrac{Exp.}{Calc.}$
Strength tests
6 A	65	31.3	21	0.0311	3.97	0.77	1.50	0.454	9.35	20.59	19.51	1.06
7 A	66	33.0	19	.0314	3.97	.77	1.75	.554	10.50	18.96	17.49	1.08
8 A	67	33.7	17	.0313	3.97	.77	2.00	.616	10.65	17.29	15.86	1.09
10 A	45	31.2	21	.0409	3.97	.77	1.50	.597	15.80	26.46	22.63	1.17
11 A	46	33.0	19	.0411	4.00	.77	1.75	.725	16.40	22.62	20.56	1.10
12 A	47	33.8	17	.0400	3.94	.74	2.00	.806	14.10	17.49	18.32	.95
14 B	80	31.3	21	.0519	3.97	.75	1.50	.779	21.50	27.62	26.27	1.05
15 B	77	33.0	19	.0524	3.94	.75	1.75	.943	23.70	25.13	24.73	1.02
16 B	81	33.8	17	.0507	3.97	.75	2.00	1.014	24.25	23.92	22.78	1.05
19 B	79	33.0	19	.0654	4.03	.77	1.75	1.154	32.00	27.73	28.00	.99
21 A	69	35.3	19	.0311	4.00	1.14	1.88	.412	10.00	24.30	22.23	1.09
22 A	70	33.4	15	.0312	3.97	1.14	2.25	.485	9.75	20.11	19.36	1.04
23 A	71	33.8	13	.0311	4.00	1.14	2.63	.554	10.00	18.04	17.19	1.05
24	68	32.7	11	.0311	4.00	1.14	3.00	.582	9.90	17.00	15.68	1.08
25 A	48	35.3	19	.0391	4.00	1.13	1.88	.524	14.05	26.80	24.50	1.09
26 A	49	33.5	15	.0392	3.94	1.13	2.25	.615	14.35	23.34	21.76	1.07
27 A	51	33.8	13	.0422	3.97	1.14	2.63	.752	16.30	21.68	20.89	1.04
28	50	32.8	11	.0419	3.97	1.15	3.00	.775	15.70	20.25	19.33	1.05
29 B	82	35.4	19	.0519	3.94	1.15	1.88	.677	20.70	30.56	29.19	1.05
30 B	76	33.5	15	.0522	3.94	1.15	2.25	.804	23.70	29.48	26.72	1.10
31 B	83	33.9	13	.0520	3.97	1.15	2.63	.920	23.25	25.26	24.74	1.02
34 B	78	33.5	15	.0652	3.97	1.15	2.25	1.004	32.00	31.87	29.56	1.08
37 A	72	32.2	13	.0309	3.97	1.63	2.50	.323	6.96	21.57	23.48	.92
38	73	32.7	11	.0313	3.97	1.65	3.00	.423	7.63	18.05	20.65	.87
39	74	34.7	10	.0309	3.97	1.65	3.50	.515	9.40	18.27	18.25	1.00
40	75	35.7	9	.0310	3.97	1.65	4.00	.583	9.60	16.47	16.58	.99
41 A	52	32.2	13	.0420	3.94	1.62	2.50	.444	11.80	26.61	26.82	.99
42	53	32.7	11	.0421	4.00	1.65	3.00	.568	13.80	24.28	24.02	1.01
43	54	34.7	10	.0422	3.94	1.65	3.50	.703	15.40	21.92	21.99	1.00
45 B	84	33.2	13	.0528	3.97	1.60	2.50	.570	16.20	28.41	29.96	.95
57 A	56	32.3	13	.0408	2.97	1.65	2.50	.416	11.00	26.43	28.71	.92
58 A	55	32.8	11	.0407	2.97	1.65	3.00	.550	13.55	24.66	26.20	.94
59 A	57	34.7	10	.0396	2.97	1.65	3.50	.659	15.50	23.51	23.90	.98
60	58	35.8	9	.0404	2.97	1.65	4.00	.760	16.80	22.12	22.97	.96
71 A	63	34.7	10	.0319	2.50	1.62	3.50	.540	11.30	20.94	21.77	.96
72 A	64	35.8	9	.0312	2.47	1.62	4.00	.594	11.80	19.87	20.24	.98
73 A	59	32.3	13	.0410	2.47	1.65	2.50	.418	11.30	27.02	30.41	.89
74 A	60	32.7	11	.0393	2.47	1.65	3.00	.531	13.30	28.48	27.63	.91
75 A	61	34.7	10	.0395	2.47	1.65	3.50	.658	16.20	27.05	26.27	.94
76 A	62	35.8	9	.0391	2.44	1.65	4.00	.735	17.00	26.00	25.17	.92

Average of ratios above unity (22 tests) = 1.06
Average of ratios below unity (18 tests) = .95
Average of all ratios   (40 tests) = 1.01

Duplicate strength tests
7 B	172	32.9	19	0.0308	4.00	0.77	1.75	0.543	10.00	18.41	17.27	1.07
11 B	173	33.0	19	.0398	4.00	.77	1.75	.702	14.80	21.08	20.11	1.05
22 B	174	33.4	15	.0302	4.03	1.15	2.25	.465	9.85	21.18	19.14	1.11
26 B	175	33.5	15	.0403	4.00	1.15	2.25	.621	14.60	23.53	22.29	1.06
53 B	171	32.3	13	.0300	2.53	1.65	2.50	.306	6.89	22.52	25.30	.89
69 C	170	32.2	13	.0306	2.53	1.65	2.50	.312	7.67	24.58	25.57	.96

Average of ratios above unity (4 tests) = 1.07
Average of ratios below unity (2 tests) = .93
Average of all ratios   (6 tests) = 1.02

TABLE 3
SHEAR WEBS WITH FLANGED LIGHTENING HOLES
Rod supports, reamed bolt holes

Specimen	Test	L (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	A_e (sq in.)	P_{coll} (kips)	Exp. τ_{coll} (net) (kips/sq in.)	Calc. τ_{coll} (net) (kips/sq in.)	Exp. Calc.
						Strength tests						
6B	102	31.2	21	0.0310	5.27	0.77	1.50	0.453	8.72	19.27	18.02	1.07
8B	103	33.8	17	.0313	5.27	.77	2.00	.616	9.40	15.26	13.97	1.09
10B	87	31.3	21	.0404	5.31	.77	1.50	.590	12.20	20.68	19.99	1.03
12B	88	33.8	17	.0407	5.31	.77	2.00	.801	15.15	18.91	15.77	1.20
21B	104	35.3	19	.0314	5.27	1.15	1.88	.410	8.40	20.50	21.31	.96
23B	105	33.9	13	.0314	5.27	1.15	2.63	.556	8.92	16.05	15.69	1.02
25B	89	35.3	19	.0402	5.27	1.13	1.88	.539	12.95	24.02	22.94	1.05
27B	90	33.8	13	.0399	5.27	1.15	2.63	.706	12.15	17.20	17.39	.99
33B	100	35.4	19	.0652	5.27	1.15	1.88	.851	28.90	33.96	27.76	1.22
35B	101	33.8	13	.0652	5.27	1.15	2.63	1.154	29.90	25.91	22.74	1.14
37B	106	32.2	13	.0311	5.27	1.65	2.50	.317	6.90	21.75	22.72	.96
41B	91	32.3	13	.0394	5.31	1.60	2.50	.426	9.25	21.74	23.98	.91
57B	115	32.3	13	.0402	4.27	1.65	2.50	.410	10.00	24.39	25.46	.96
58B	116	32.8	11	.0403	4.24	1.65	3.00	.544	12.20	22.42	22.06	1.02
59B	117	34.8	10	.0412	4.27	1.65	3.50	.686	13.70	19.97	19.83	1.01
61A	118	32.3	13	.0504	4.18	1.60	2.50	.544	14.60	26.82	28.30	.95
62	119	32.7	11	.0525	4.24	1.65	3.00	.709	18.90	26.66	25.97	1.03
63	120	34.7	10	.0529	4.24	1.65	3.50	.881	21.80	24.75	23.80	1.04
64	121	35.7	9	.0529	4.27	1.65	4.00	.995	23.80	23.93	22.03	1.09
65B	122	32.3	13	.0665	4.27	1.65	2.50	.694	21.70	31.26	31.35	1.00
67	124	34.7	10	.0649	4.37	1.63	3.50	1.092	29.45	26.97	26.49	1.02
68	123	35.8	9	.0635	4.37	1.63	4.00	1.204	30.00	24.92	24.71	1.01
69B	107	32.3	13	.0308	3.77	1.65	2.50	.314	6.90	21.96	23.19	.95
70B	108	32.8	11	.0310	3.74	1.65	3.00	.419	8.20	19.59	19.94	.98
71B	109	34.8	10	.0305	3.74	1.65	3.50	.508	9.00	17.72	17.40	1.02
72B	110	35.8	9	.0308	3.77	1.63	4.00	.584	9.55	16.35	15.51	1.05
73B	92	32.2	13	.0405	3.81	1.65	2.50	.413	10.80	26.14	26.10	1.00
74B	93	32.7	11	.0417	3.81	1.65	3.00	.563	13.20	23.45	23.27	1.01
75B	94	34.7	10	.0410	3.81	1.65	3.50	.683	15.00	21.97	20.68	1.06
76B	95	35.7	9	.0409	3.81	1.65	4.00	.769	14.60	18.99	18.89	1.01
77A	111	32.2	13	.0498	3.74	1.60	2.50	.538	14.65	27.24	29.24	.93
78A	112	32.8	11	.0532	3.74	1.60	3.00	.745	19.20	25.78	27.37	.94
79A	113	34.8	10	.0524	3.74	1.65	3.50	.873	22.30	25.56	25.60	1.00
80A	114	35.8	9	.0496	3.77	1.65	4.00	.933	21.85	23.43	22.94	1.02
81B	96	32.2	13	.0649	3.77	1.65	2.50	.662	21.50	32.48	32.70	.99
82B	97	32.8	11	.0635	3.81	1.65	3.00	.857	24.60	28.69	30.23	.95
83B	98	34.8	10	.0652	3.77	1.60	3.50	1.115	30.70	27.53	29.11	.95
84B	99	35.8	9	.0651	3.77	1.65	4.00	1.224	33.30	27.21	28.28	.96
84A	183	35.7	9	.0651	4.38	1.65	4.00	1.224	30.70	25.08	25.24	.99

Average of ratios above unity (22 tests) = 1.05
Average of ratios below unity (17 tests) = .96
Average of all ratios (39 tests) = 1.01

Specimen	Test	L (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	A_e (sq in.)	P_{coll} (kips)	Exp. τ_{coll} (net) (kips/sq in.)	Calc. τ_{coll} (net) (kips/sq in.)	Exp. Calc.
						Duplicate strength tests						
61B	165	32.3	13	0.0523	4.31	1.60	2.50	0.565	14.70	26.03	28.33	0.92
71C	169	34.7	10	.0303	3.84	1.65	3.50	.505	8.70	17.24	17.26	1.00
77B	161	32.2	13	.0522	3.81	1.60	2.50	.564	15.35	27.23	29.59	.92
78B	162	32.7	11	.0500	3.81	1.60	3.00	.700	16.55	23.64	26.26	.90
79B	163	34.7	10	.0500	3.81	1.65	3.50	.833	20.50	24.62	24.59	1.00
80B	164	35.7	9	.0524	3.81	1.65	4.00	.985	23.50	23.86	23.83	1.00
81A	166	32.2	13	.0630	3.81	1.65	2.50	.643	20.20	31.43	32.25	.97
82A	167	32.7	11	.0642	3.73	1.60	3.00	.899	25.95	28.87	30.43	.95
83A	168	34.7	10	.0651	3.88	1.65	3.50	1.084	29.60	27.31	28.80	.95

Average of ratios above unity (2 tests) = 1.00
Average of ratios below unity (7 tests) = .94
Average of all ratios (9 tests) = .96

TABLE 4
SHEAR WEBS WITH FLANGED LIGHTENING HOLES
[Bar supports, drilled bolt holes]

Specimen	Test	L (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	A_e (sq in.)	P_{coll} (kips)	Exp. τ(net) (kips/sq in.)	Calc. τ(net) (kips/sq in.)	Exp. Calc.
14A	14	31.3	21	0.0495	4.06	0.75	1.50	0.743	14.70	19.80	25.33	0.78
15A	15	32.9	19	.0510	4.06	.75	1.75	.918	19.20	20.92	23.90	.88
16A	16	33.7	17	.0507	4.03	.75	2.00	1.014	18.50	18.24	22.53	.81
19A	3	32.9	19	.0658	4.00	.77	1.75	1.161	28.00	24.12	28.22	.85
29A	18	35.3	19	.0509	4.03	1.15	1.88	.664	20.25	30.49	28.79	1.06
30A	19	33.4	15	.0522	4.06	1.17	2.25	.789	18.40	23.31	26.57	.88
31A	20	33.7	13	.0527	4.06	1.15	2.63	.933	21.40	22.94	24.61	.93
32	21	32.7	11	.0527	4.06	1.15	3.00	.975	19.60	20.10	23.23	.87
33A	27	35.4	19	.0651	4.03	1.15	1.88	.850	26.20	30.84	31.31	.99
34A	28	33.5	15	.0649	4.06	1.15	2.25	1.000	28.45	28.46	29.18	.98
35A	13	33.8	13	.0651	4.03	1.15	2.63	1.152	27.80	24.13	27.90	.86
36	29	32.7	11	.0654	4.06	1.15	3.00	1.210	29.00	23.97	26.79	.89
45A	22	32.3	13	.0521	4.13	1.60	2.50	.563	14.95	26.57	29.48	.90
46	23	32.7	11	.0518	4.03	1.60	3.00	.725	17.40	23.99	26.91	.89
47	24	34.7	10	.0516	4.06	1.58	3.50	.892	20.10	22.54	24.65	.91
48	26	35.7	9	.0510	4.03	1.63	4.00	.967	21.95	22.70	23.47	.97
49A	6	32.2	13	.0654	4.06	1.63	2.50	.683	19.50	28.56	32.04	.89
50	9	32.7	11	.0648	4.06	1.65	3.00	.875	23.20	26.52	29.92	.89
51	10	34.7	10	.0657	4.03	1.60	3.50	1.123	27.90	24.84	28.42	.87
52	11	35.8	9	.0642	4.00	1.60	4.00	1.233	30.60	24.82	27.05	.92
53A	33	32.2	13	.0311	3.06	1.63	2.50	.325	7.09	21.84	24.46	.89
56	34	35.8	9	.0317	3.06	1.65	4.00	.596	10.25	17.20	18.43	.93
65A	30	32.2	13	.0655	3.19	1.60	2.50	.707	22.00	31.10	33.62	.93
66	31	32.8	11	.0662	3.06	1.63	3.00	.907	26.05	28.72	32.55	.88
70A	35	32.7	11	.0312	2.56	1.62	3.00	.431	8.40	19.51	22.93	.85

Average of ratios above unity (1 test) = 1.06
Average of ratios below unity (24 tests) = .89
Average of all ratios (25 tests) = .90

TABLE 6
SHEAR WEBS WITH PLAIN LIGHTENING HOLES
[Rod supports, reamed bolt holes]

Specimen	Test	L_e (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	P_{coll} (kips)	Exp. τ_{coll}(gross) (kips/sq in.)	A_e (sq in.)	Exp. τ_{coll}(net) (kips/sq in.)
					Strength tests						
NF1A	156	28.05	25	0.0311	5.26	1.00	1.25	3.61	4.14	0.187	19.35
NF2A	155	27.95	25	.0402	5.27	1.00	1.25	5.67	5.05	.241	23.51
NF3A	154	28.02	25	.0514	5.27	1.00	1.25	8.40	5.83	.308	27.24
NF4A	153	27.89	25	.0649	5.27	1.00	1.25	11.90	6.58	.389	30.56
					Duplicate strength tests						
NF1B	189	27.95	25	0.0307	5.34	1.00	1.25	3.60	4.20	0.184	19.54
NF2B	188	27.98	25	.0400	5.27	1.00	1.25	5.84	5.22	.240	24.33
NF3B	187	27.97	25	.0499	5.31	1.00	1.25	8.36	5.99	.299	27.92
NF4B	186	27.99	25	.0643	5.20	1.00	1.25	12.45	6.83	.386	32.27

TABLE 5
SHEAR WEBS WITH BEADED LIGHTENING HOLES

Specimen	Test	L_e (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	P_{coll} (kips)	Exp. τ_{coll} (gross) (kips/sq in)	Calc. τ_{coll} (net) (kips/sq in)	$\dfrac{Exp.}{Calc.}$
						Strength tests					
					Rod supports, reamed bolt holes						
101A	147	29.64	11	0.0316	5.27	1.05	3.00	9.00	9.61	9.46	1.02
102A	141	29.77	11	.0394	5.27	1.10	3.00	12.70	10.83	11.15	.97
103A	125	29.70	11	.0508	5.27	1.05	3.00	20.10	13.32	13.58	.98
104A	140	29.74	11	.0641	5.27	1.03	3.00	30.40	15.95	16.20	.98
105	150	32.23	11	.0304	4.29	1.05	3.00	10.45	10.67	10.72	.99
106	151	33.17	10	.0302	4.27	1.08	3.50	11.60	11.58	10.70	1.08
107	152	33.17	9	.0300	4.27	1.08	4.00	12.35	12.41	10.66	1.16
108	143	30.24	11	.0400	4.27	1.05	3.00	15.30	12.65	13.30	.95
109	144	32.24	10	.0400	4.27	1.07	3.50	18.20	14.12	13.30	1.06
110	145	33.24	9	.0394	4.27	1.07	4.00	19.75	15.08	13.12	1.15
111	128	30.20	11	.0502	4.27	1.03	3.00	22.00	14.51	15.80	.92
112	133	32.19	10	.0521	4.31	1.03	3.50	25.90	15.44	16.10	.96
113	134	33.20	9	.0505	4.27	1.03	4.00	28.50	16.96	15.88	1.07
114	135	30.19	11	.0640	4.31	1.03	3.00	34.00	17.60	18.76	.94
115	136	32.16	10	.0628	4.31	1.03	3.50	39.00	19.31	18.50	1.04
116	137	33.17	9	.0643	4.34	1.03	4.00	41.80	19.60	18.72	1.05
119	148	31.67	10	.0311	5.27	1.60	3.50	9.95	10.10	9.34	1.08
120	149	32.69	9	.0300	5.31	1.60	4.00	9.40	9.59	9.04	1.06
123	142	31.67	10	.0376	5.27	1.60	3.50	14.00	11.76	10.76	1.09
127	126	31.70	10	.0503	5.27	1.60	3.50	20.90	13.11	13.50	.97
128	127	32.70	9	.0506	5.27	1.60	4.00	22.25	13.45	13.56	.99
131	138	31.63	10	.0635	5.31	1.60	3.50	30.70	15.29	16.00	.96
132	139	32.70	9	.0637	5.27	1.60	4.00	31.90	15.31	16.10	.95

Average of ratios above unity (11 tests) = 1.08
Average of ratios below unity (12 tests) = .96
Average of all ratios   (23 tests) = 1.02

Specimen	Test	L_e (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	P_{coll} (kips)	Exp. τ_{coll} (gross) (kips/sq in)	Calc. τ_{coll} (net) (kips/sq in)	$\dfrac{Exp.}{Calc.}$
					Duplicate strength tests						
					Rod supports, reamed bolt holes						
104B	157	29.64	11	0.0641	5.27	1.03	3.00	30.70	16.16	16.16	1.00

Specimen	Test	L_e (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	P_{coll} (kips)	Exp. τ_{coll} (gross) (kips/sq in)	Calc. τ_{coll} (net) (kips/sq in)	$\dfrac{Exp.}{Calc.}$
					Strength tests						
					Bar supports, reamed bolt holes						
101B	160	29.74	11	0.0303	3.95	1.05	3.00	11.20	12.43	11.38	1.09
102B	159	29.70	11	.0390	3.97	1.04	3.00	15.80	13.64	13.78	.99
103B	158	29.67	11	.0511	3.97	1.03	3.00	23.60	15.57	16.90	.92

Average of all ratios (3 tests) = 1.00

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TABLE 7
PERMANENT-SET TESTS

Specimen	δ (in.)	Set after run 1 (in.)	Set after run 2 (in.)	P (kips)	τ (Kips/sq in.)
Webs with flanged holes and bar supports					
7B	179×10^{-4}	11×10^{-4}	0×10^{-4}	6.9	12.7[a]
11B	193	20	2	10.6	15.1
22B	174	18	2	6.4	13.8
26B	198	16	5	9.8	15.8
53B	195	11	0	4.5	14.7
69C	175	14	0	4.5	14.4
Webs with flanged holes and rod supports					
61B	296	111	10	10.0	17.7[a]
71C	225	67	13	6.0	11.9
77B	272	124	15	10.0	17.7
78B	254	136	12	12.0	14.3
79B	272	151	17	14.0	16.9
80B	210	104	9	15.0	15.8
81A	352	150	16	14.0	21.8
82A	278	152	13	16.5	18.3
83A	325	125	10	20.5	19.0
84A	282	134	3	20.0	16.4
Webs with beaded holes and bar supports					
101B	212	23	9	6.0	6.7[b]
102B	210	26	5	10.0	8.6
103B	288	99	13	13.0	8.6
Webs with beaded holes and rod supports					
104B	375	82	5	20.0	10.5[b]
Webs with plain holes and rod supports					
NF1B	181	12	0	2.4	13.0[a]
NF2B	155	10	6	3.75	15.6
NF3B	170	22	4	5.5	18.4
NF4B	208	11	5	7.8	20.2
Solid webs with bar supports					
137B	154	4	1	3.6	3.1[a]
138B	132	0	0	2.3	3.4
139B	242	194	7	27.0	19.8
Solid webs with rod supports					
135B	230	50	2	3.0	6.5[a]
136B	289	27	2	6.3	5.1

[a] τ(net).
[b] τ(gross).

TABLE 8

COMPARISONS BETWEEN WEBS WITH FLANGED HOLES AND WEBS WITH BEADED HOLES

Flanged holes : D/h = 0.8
Beaded holes : D = 1.6 in. , b = 3.0 in.

h (in.)	t (in.)	S (lb)	$V_{flanged}$ (in³/in.)	V_{beaded} (in³/in.)
4	0.064	3500	0.1811	0.1745
4	.025	790	.0621	.0745
8	.064	4840	.3216	.4170
8	.025	1520	.1225	.2150

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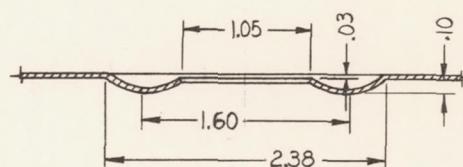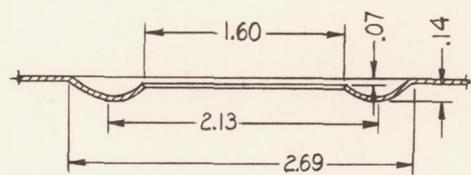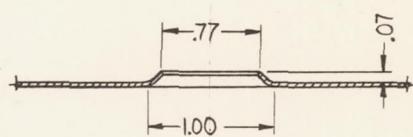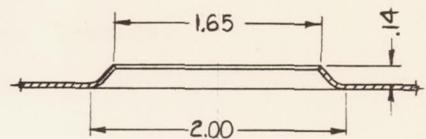
Figure 1.- Typical cross sections of flanges and beads.
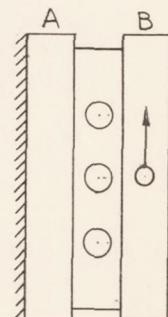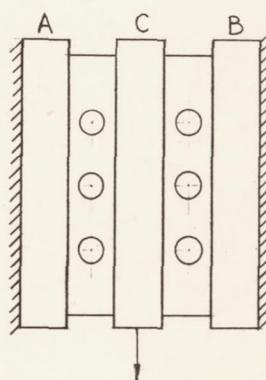
(a) Double jig.   (b) Single jig.

Figure 2.- Schematic arrangements of test jigs.
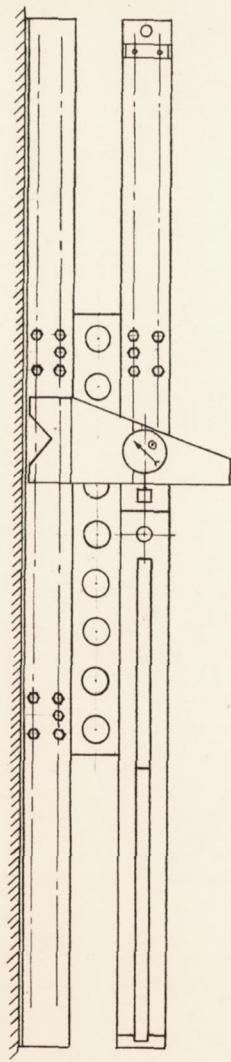
Side view
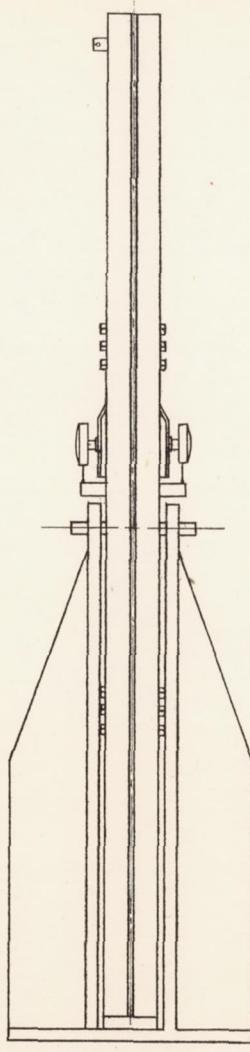
Front view

Figure 3.— Test jig.

(a) Bar supports.

Shim h_1 Shim

h

(b) Rod supports.

Shim h_1 Shim

h
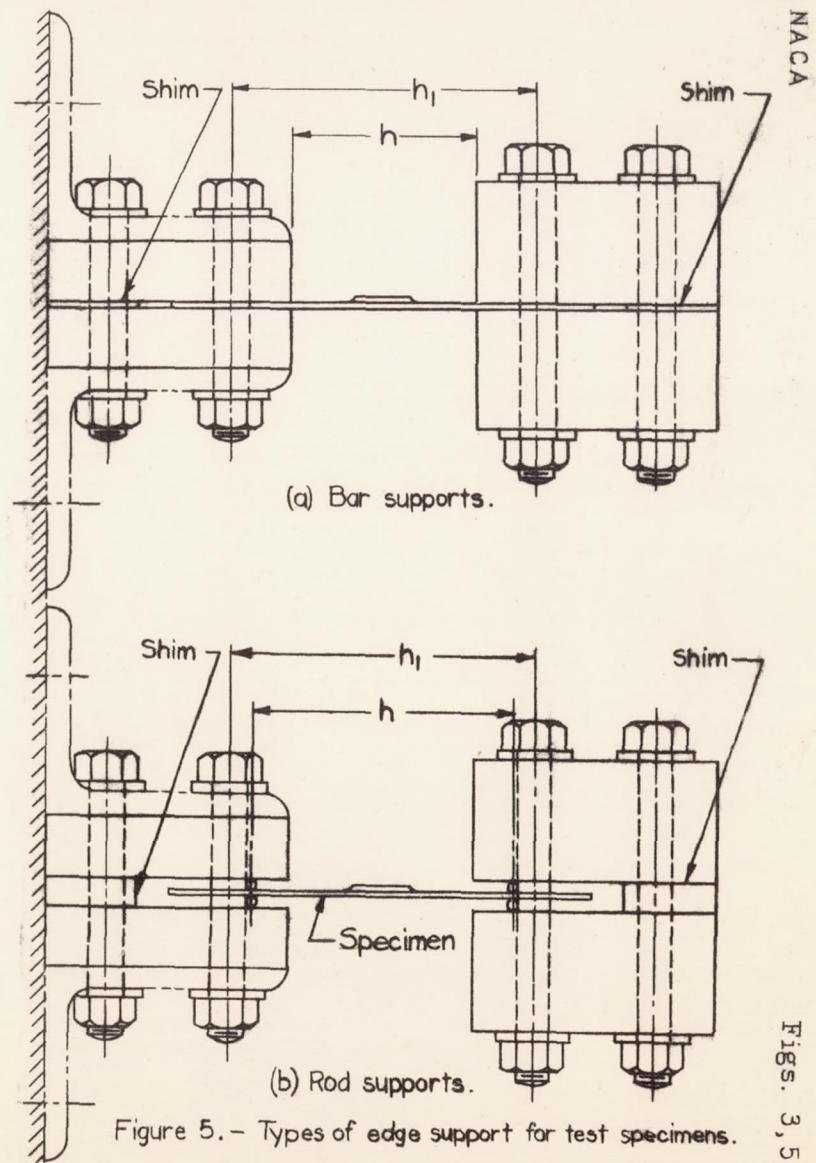
Specimen

Figure 5.— Types of edge support for test specimens.
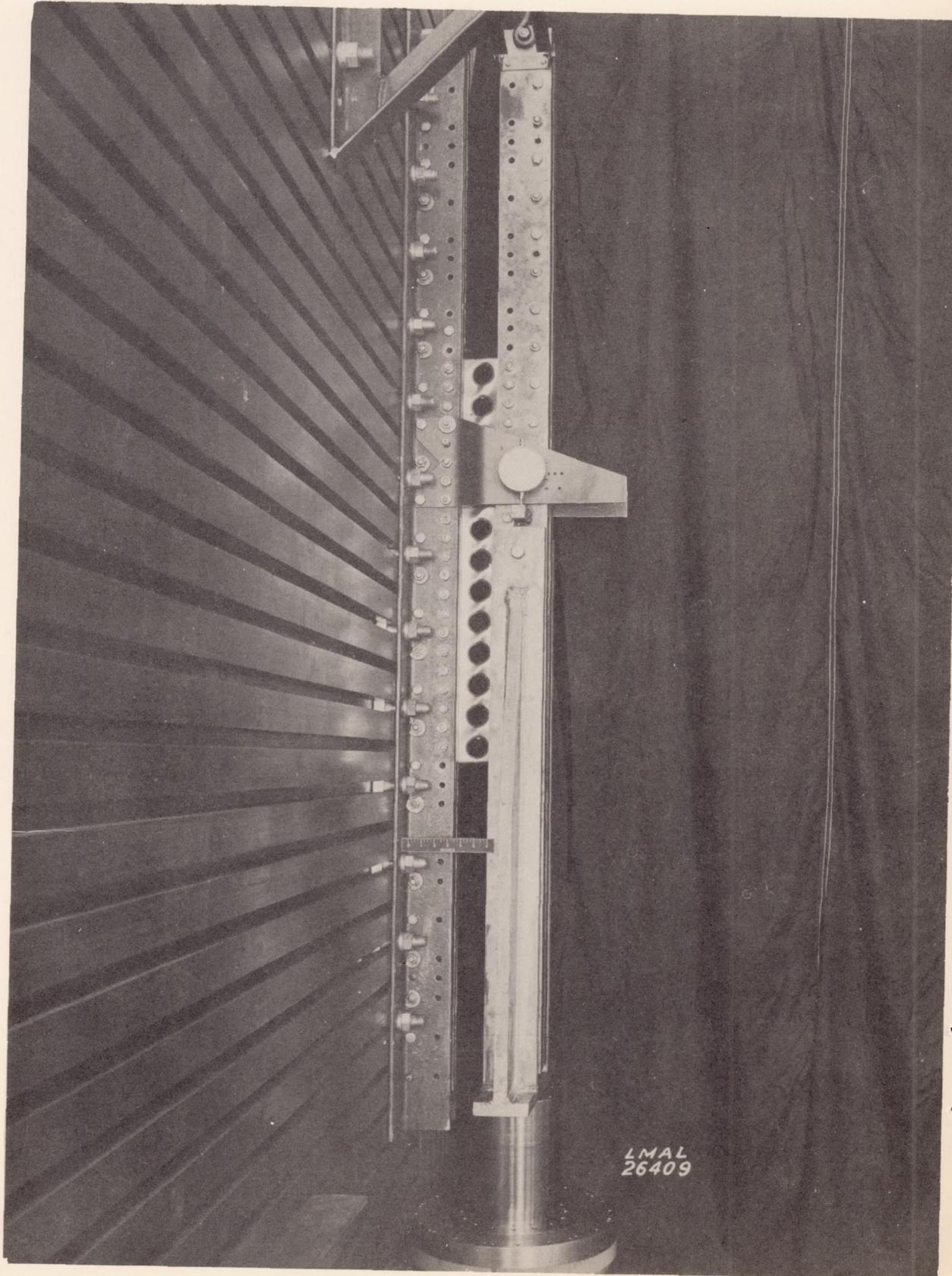
Figure 4.- Test jig in operation.

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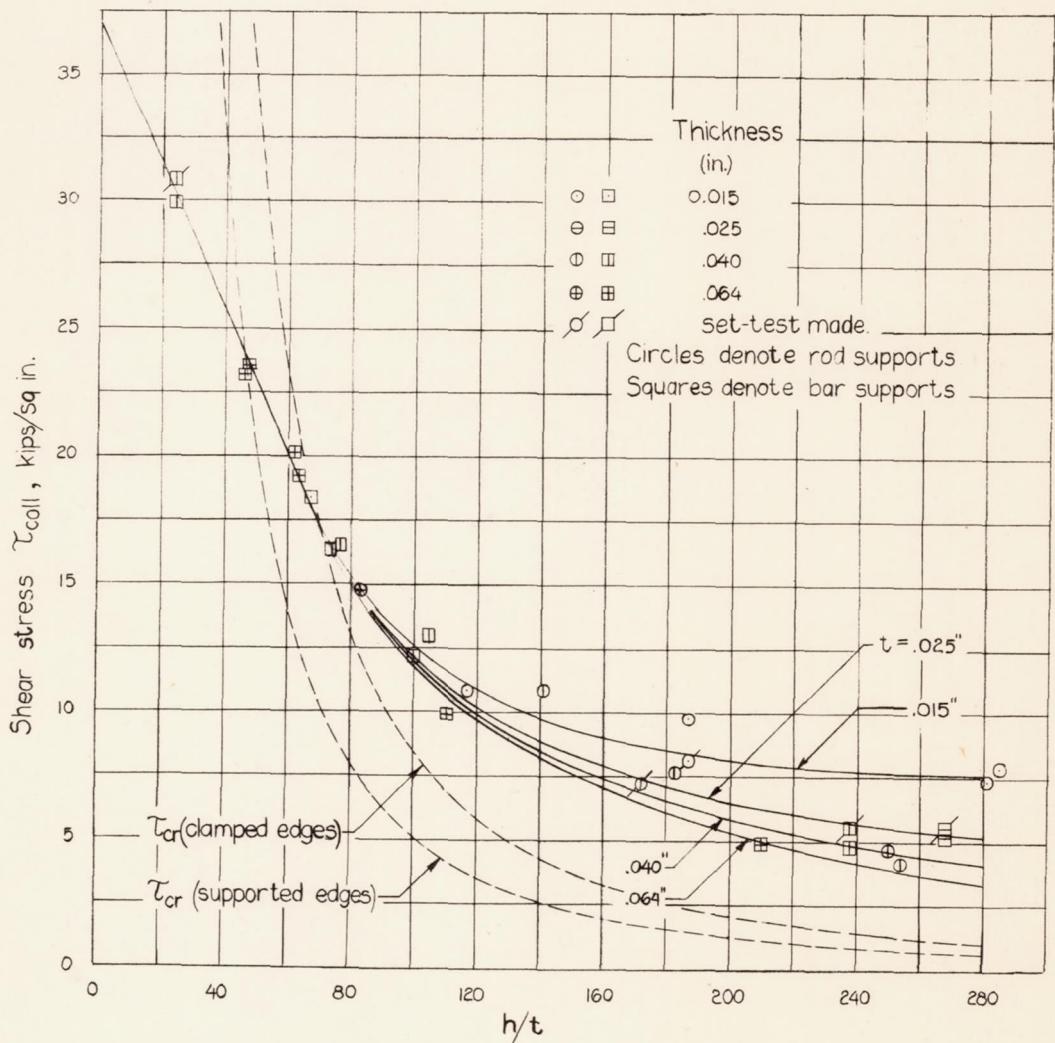
Figure 6 - Experimental shear stresses causing collapse of solid flat sheets.
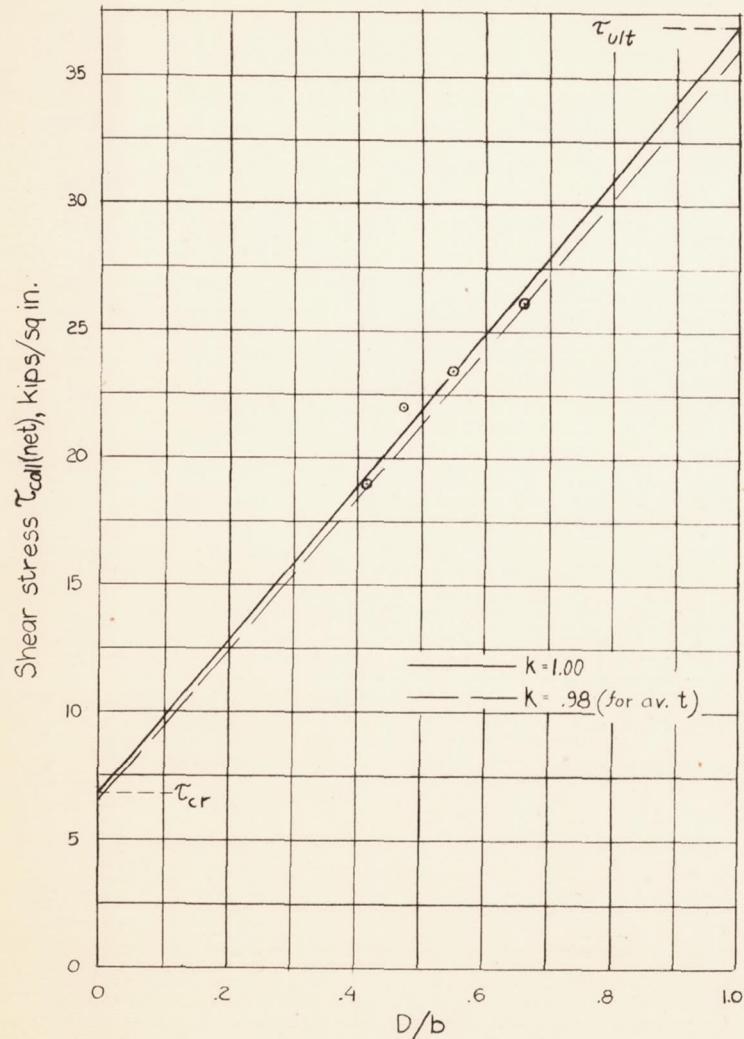
Figure 7.- Experimental shear stresses for flanged-hole webs with rod supports and reamed bolt holes (specimens 73B, 74B, 75B, and 76B).
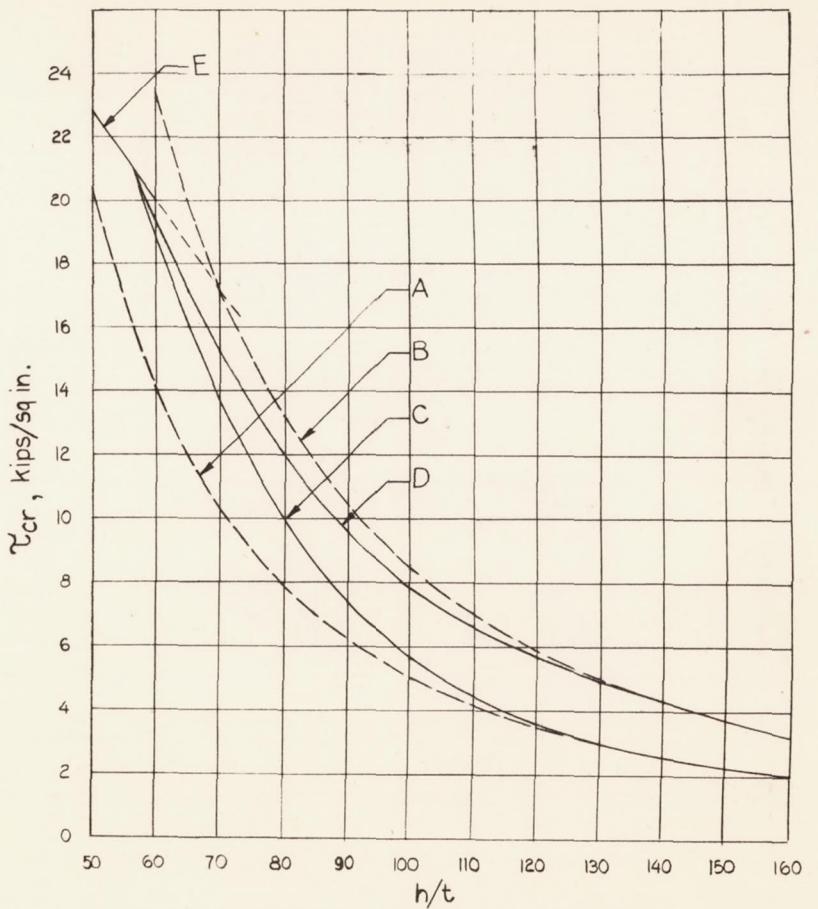
Curve A Ideally supported edges
Curve B Ideally clamped edges
Curve C Rod supports
Curve D Bar supports
Curve E Cut-off (τ_{coll} for solid sheet)

Figure 8.- Critical shear stresses used for computing τ_{coll}.

	D	b	
⊙	1.6	3.5	
□	1.6	4.0	Rod supports
▽	1.05	3.0	
▽	1.05	3.5	
△	1.05	4.0	
▲	1.05	3.0	Bar supports
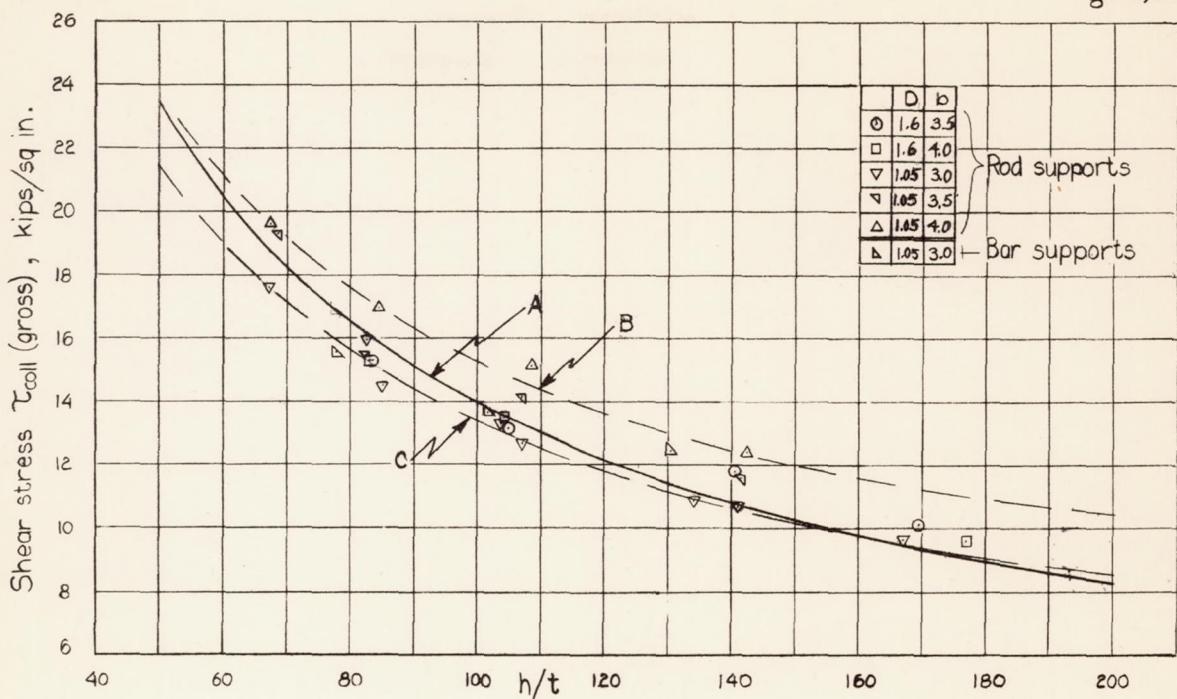
Figure 9.- Experimental shear stresses causing collapse of webs with beaded holes.
Curve A from formula (9); curve B for b=4 inches; curve C for b=3 inches.
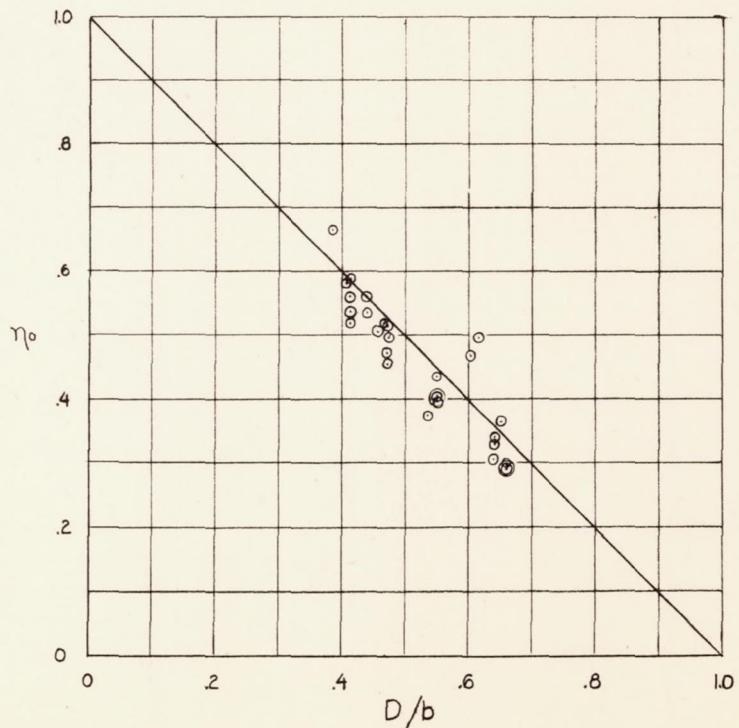
Figure 10.- Shear-stiffness factor η_0 for webs with
flanged holes 0.040, 0.051, and 0.064 inch thick.
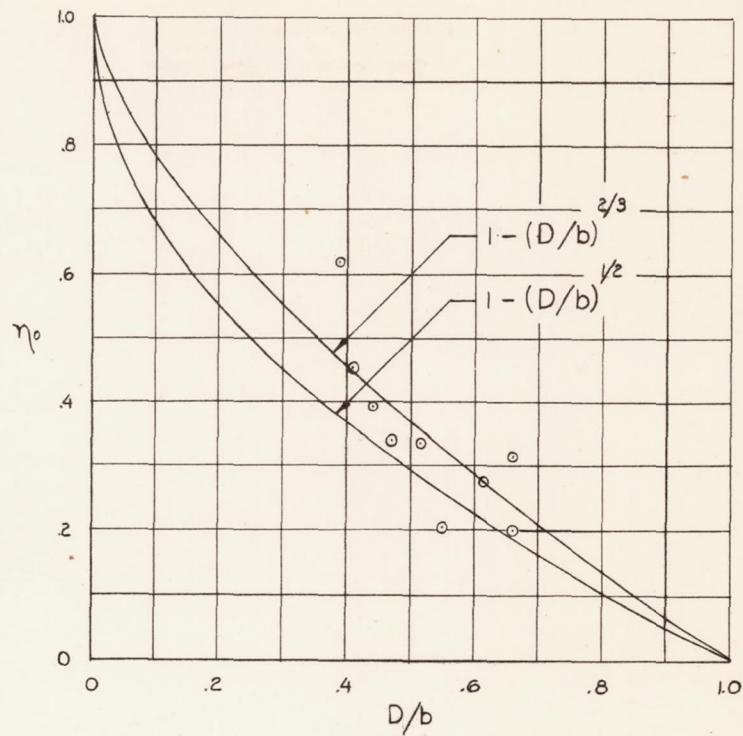
Figure 11.- Shear-stiffness factor η_0 for webs with flanged holes 0.032 inch thick.
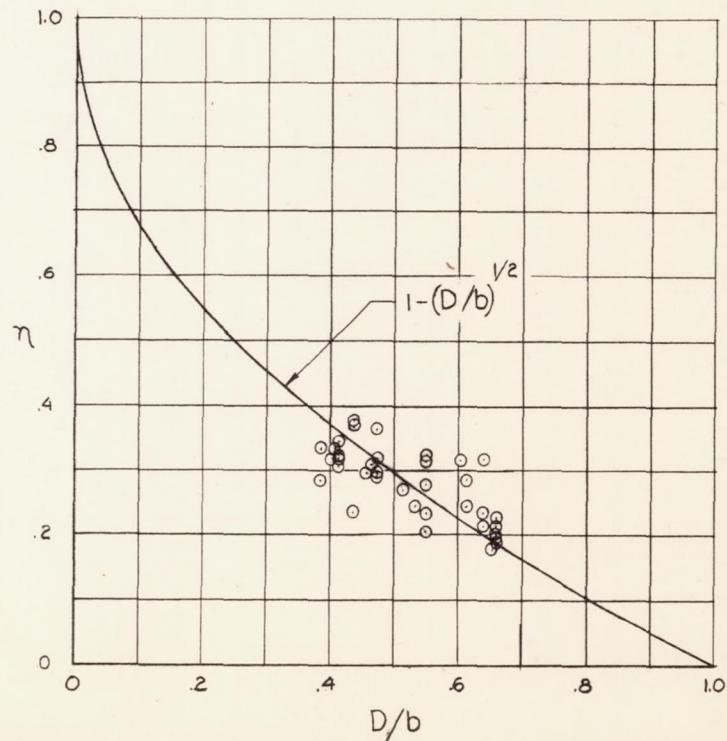
Figure 12.- Shear-stiffness factor η for webs with flanged holes 0.032 to 0.064 inch thick.
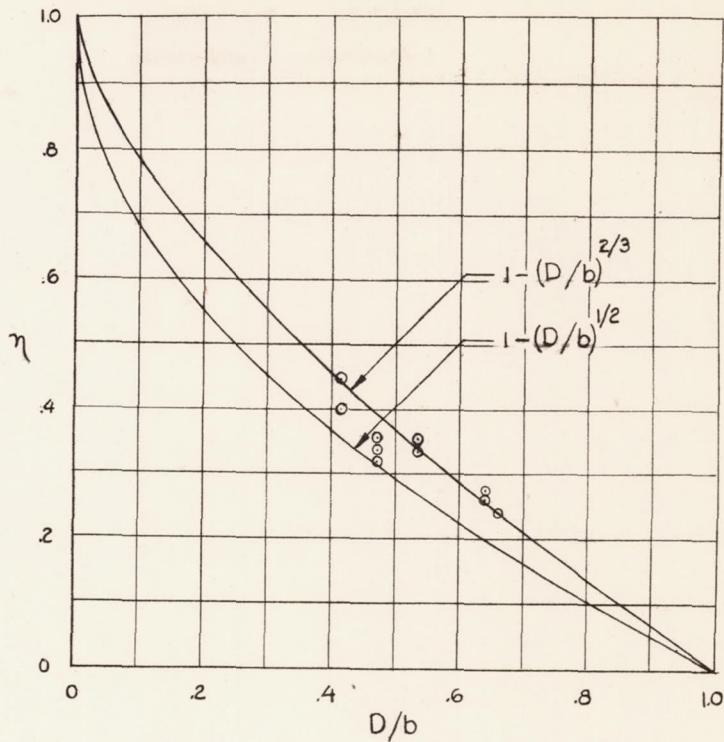
Figure 13.- Shear-stiffness factor η for webs with flanged holes; webs preloaded twice to 2/3 P_{coll}.
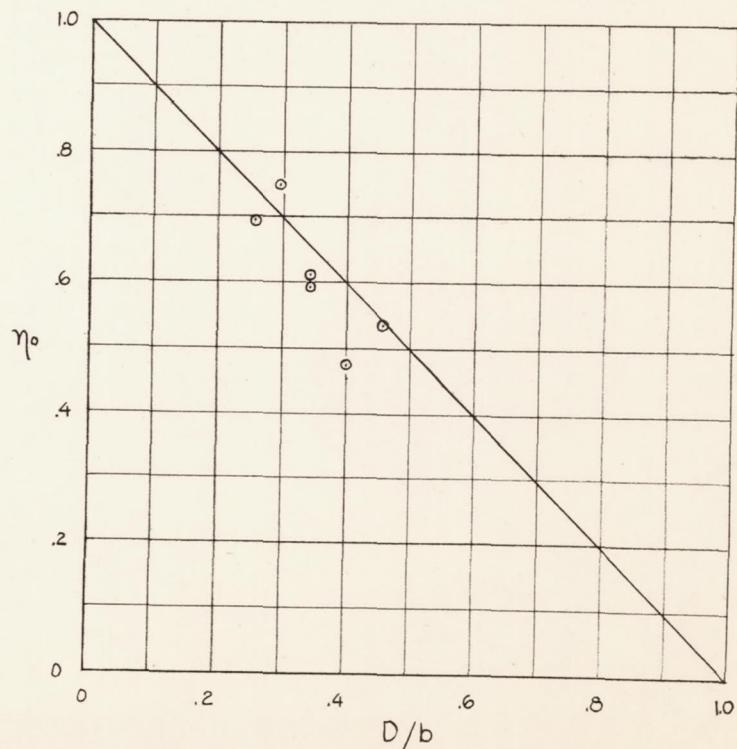
Figure 14.- Shear-stiffness factor η_o for webs with beaded holes 0.064 inch thick.
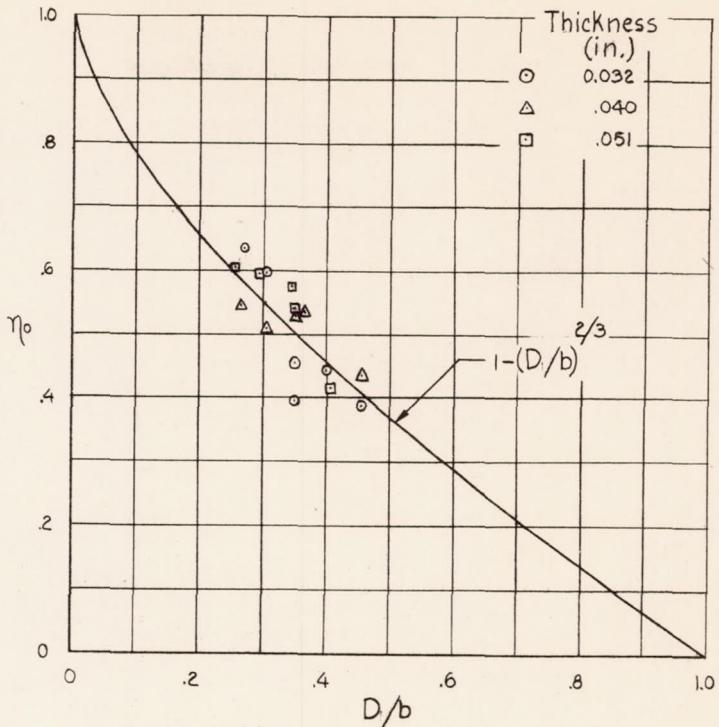
Figure 15.- Shear-stiffness factor η_0 for webs with beaded holes 0.032 to 0.051 inch thick.
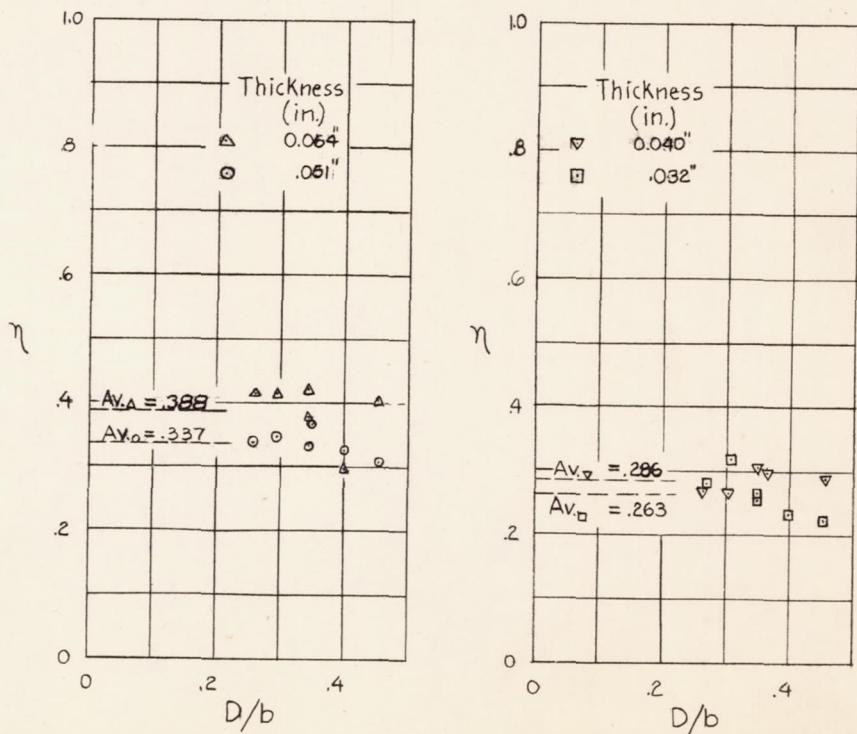
Figure 16.- Shear-stiffness factor η for webs with beaded holes.

(a) h = 4 inches
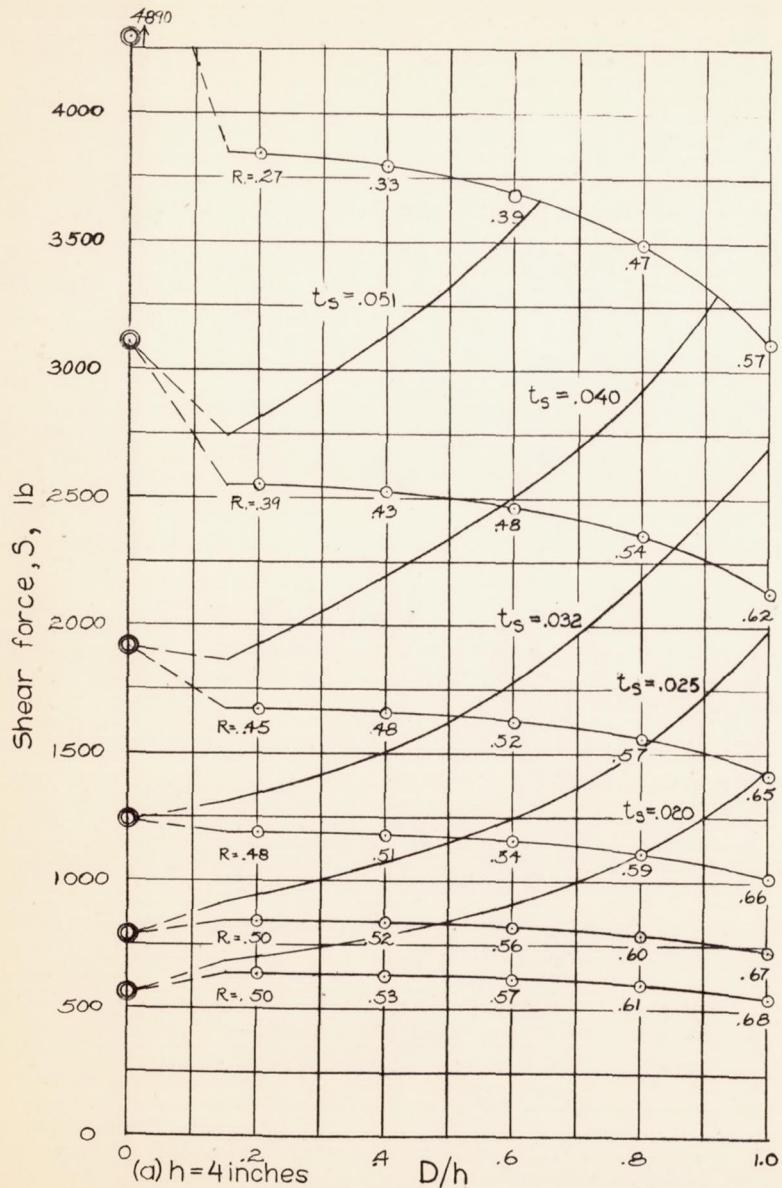
Figure 17.— Design chart for shear webs of 24S-T alloy with round flanged holes.

(b) h = 8 inches. Experimental verification limited to h/t < 180.
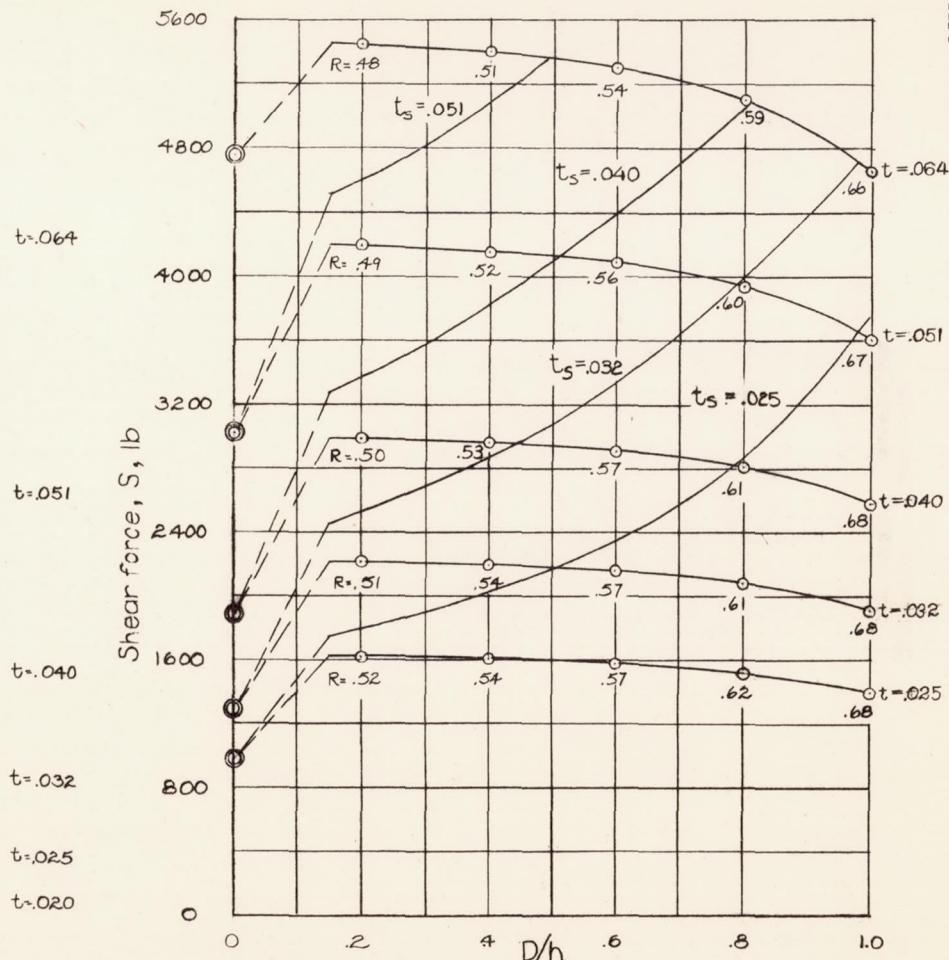
Figure 17.— Concluded.

Fig. 17
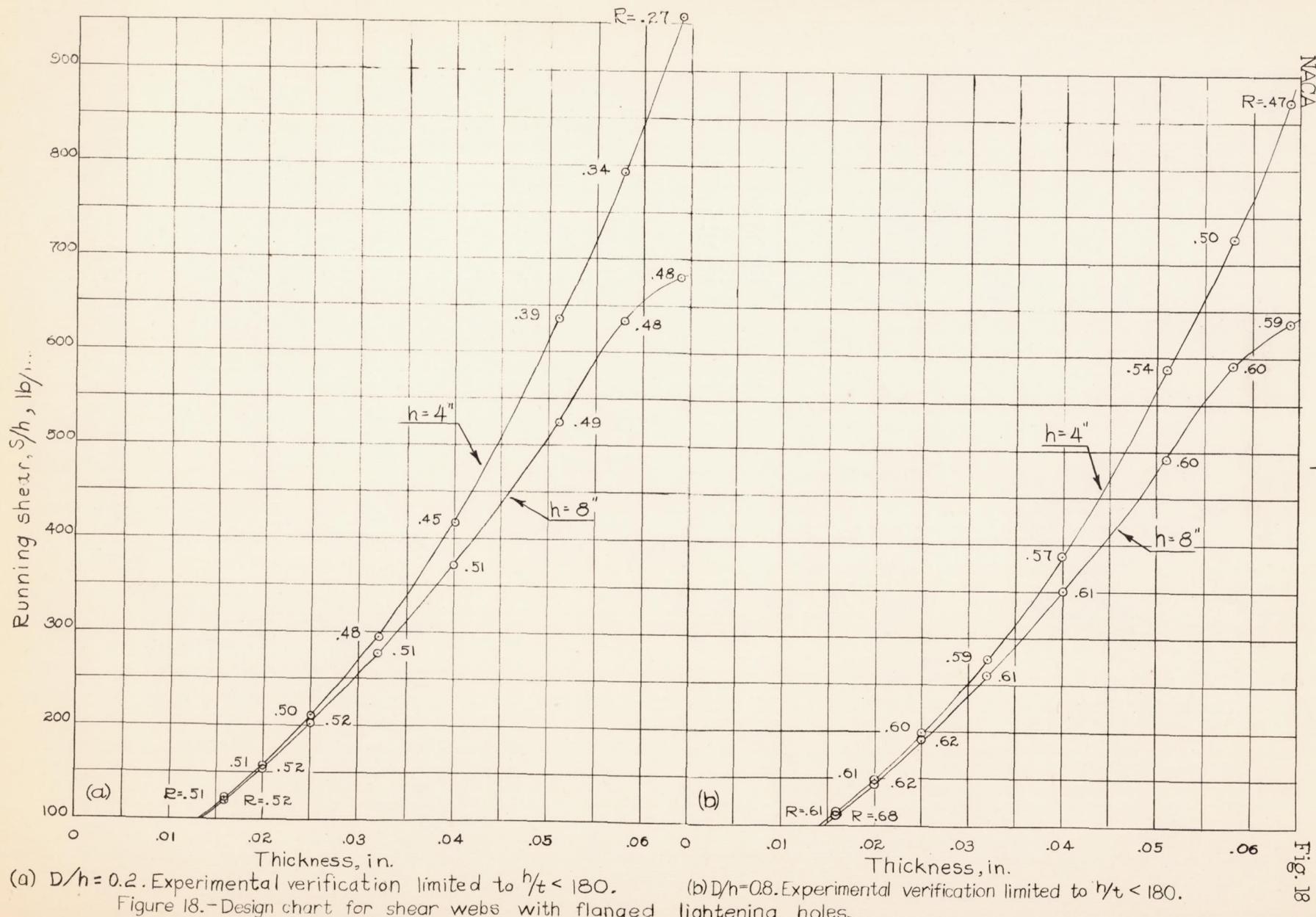
(a) D/h=0.2. Experimental verification limited to $h/t < 180$.
(b) D/h=0.8. Experimental verification limited to $h/t < 180$.
Figure 18.-Design chart for shear webs with flanged lightening holes.

(a) Specimens 73-76.

Figure 19.- Shear strength and strength-
volume ratio.
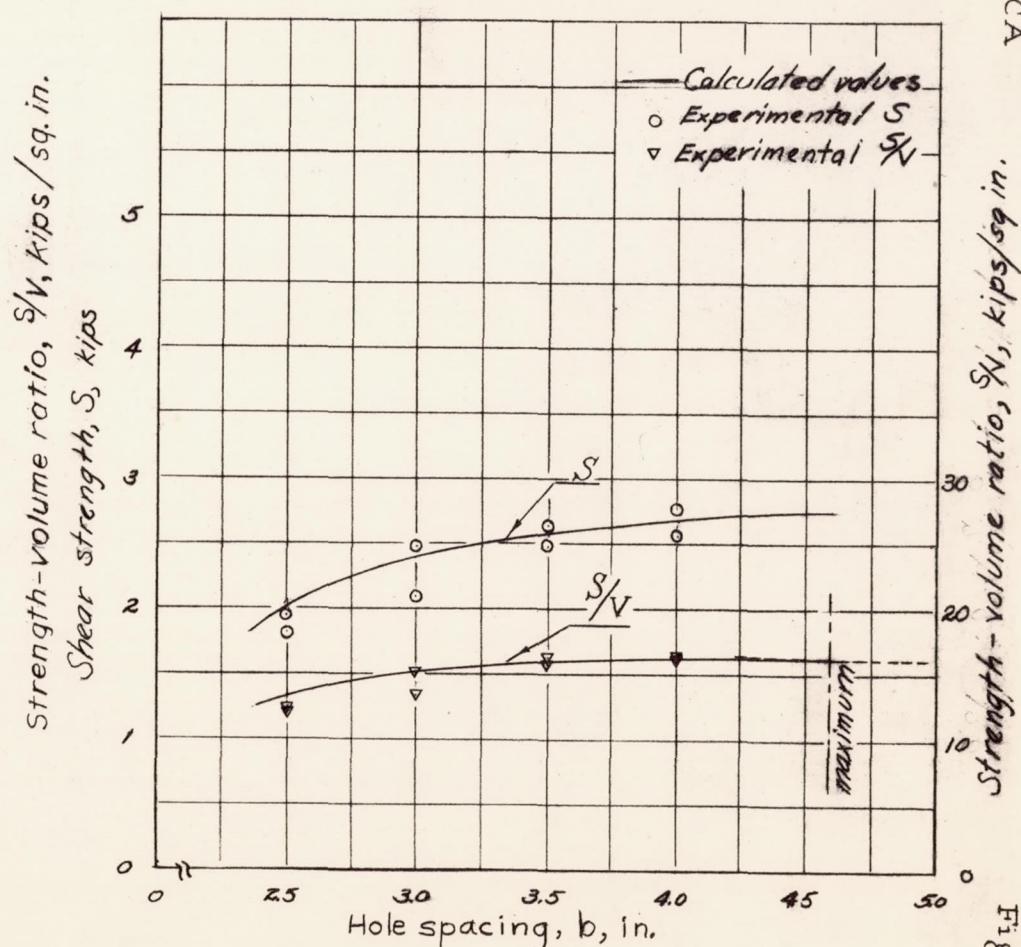
(b) Specimens 77-80 A and B.

Figure 19.- Continued.
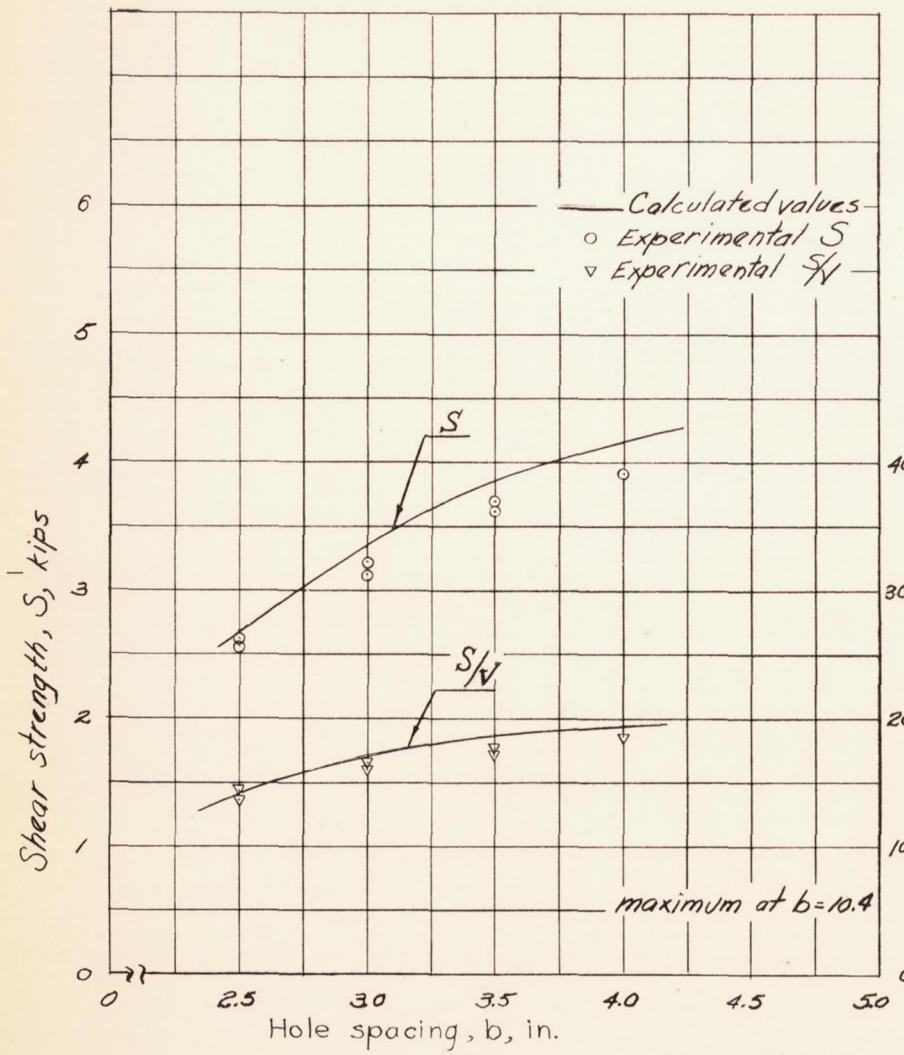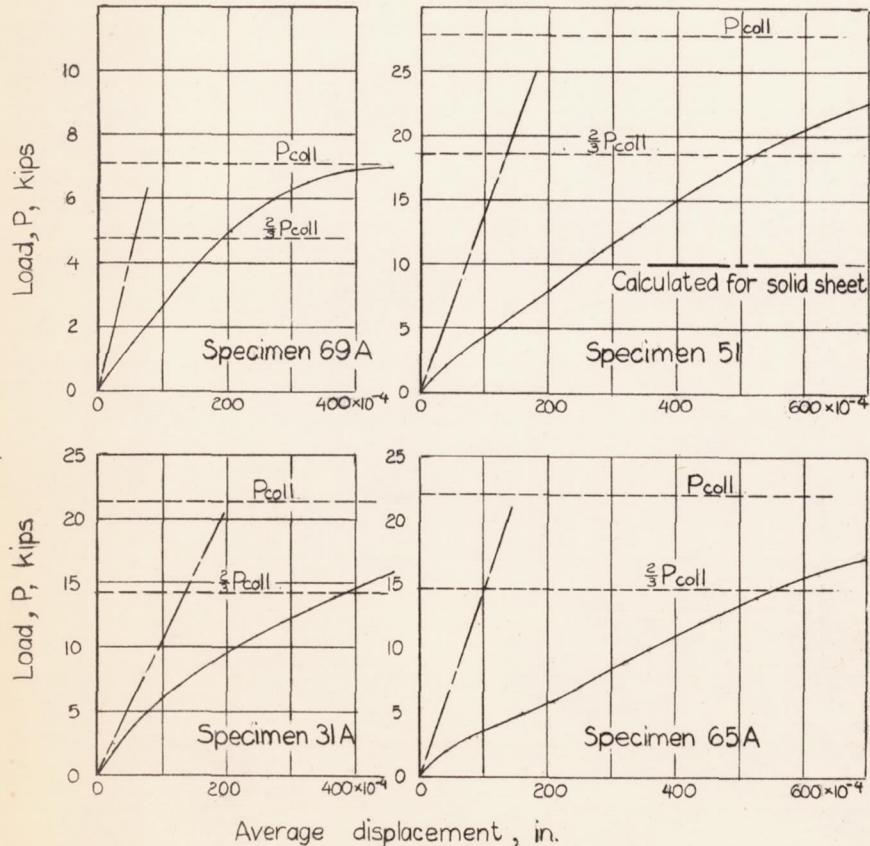
(c) Specimens 81-84 A and B.

Figure 19. - Concluded.

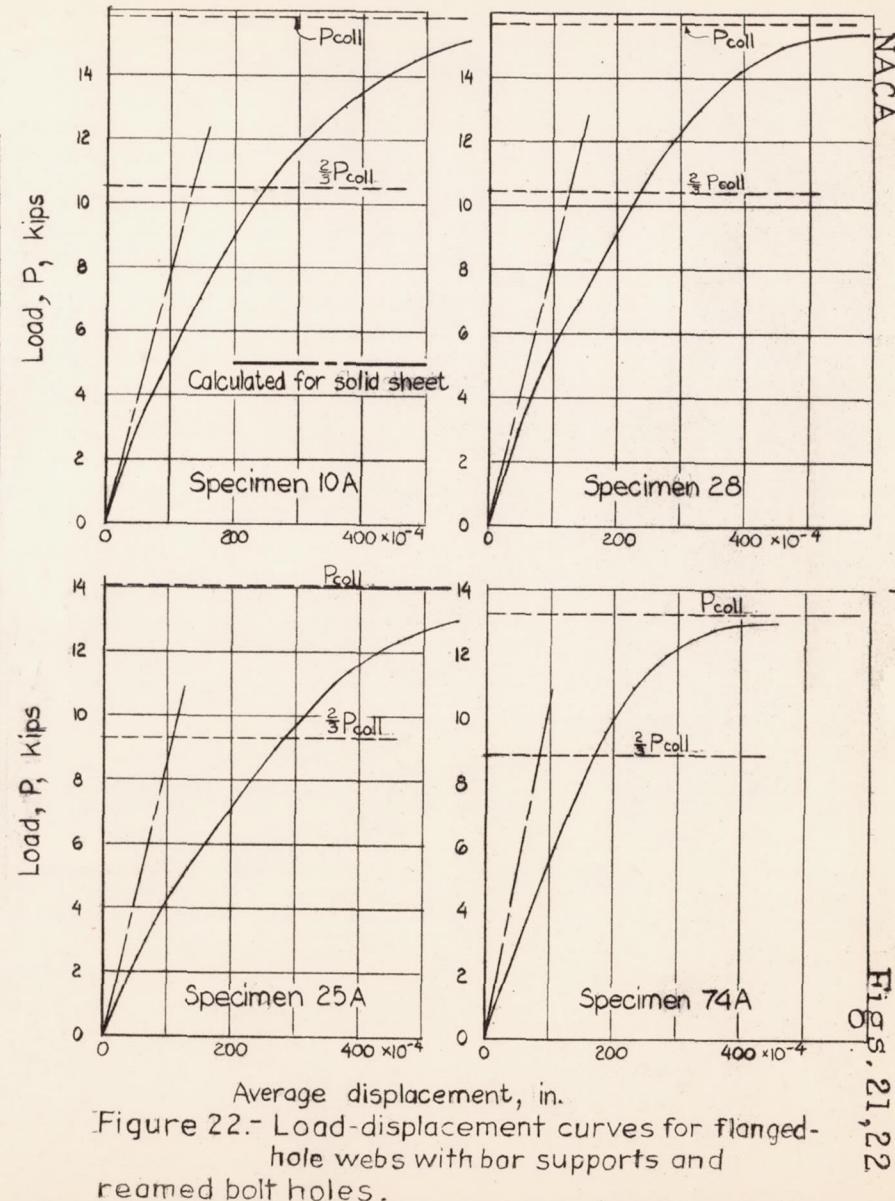Figure 20.- Load-displacement curves for solid webs with bar supports.
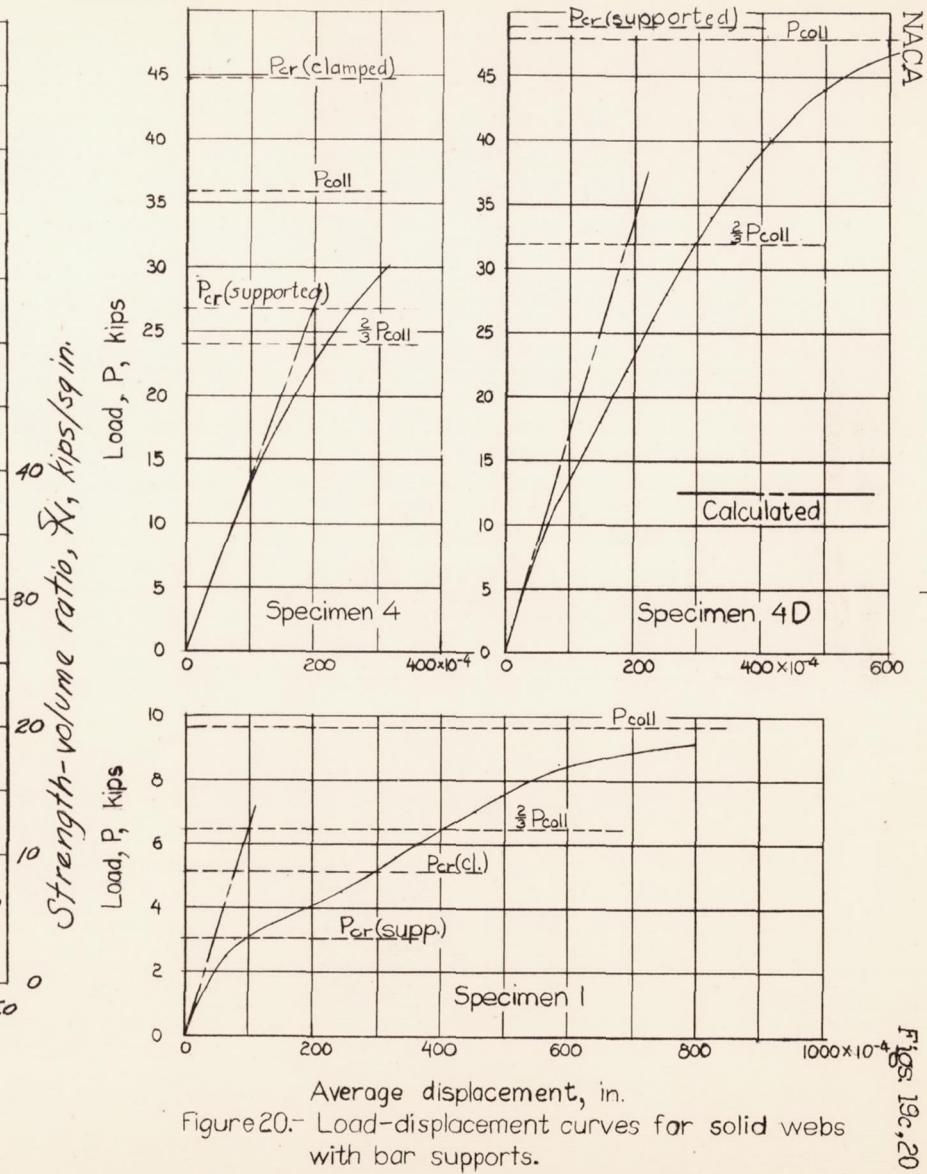
Figure 21.- Load-displacement curves for flanged-hole webs with bar supports and drilled bolt holes.

Figure 22.- Load-displacement curves for flanged-hole webs with bar supports and reamed bolt holes.
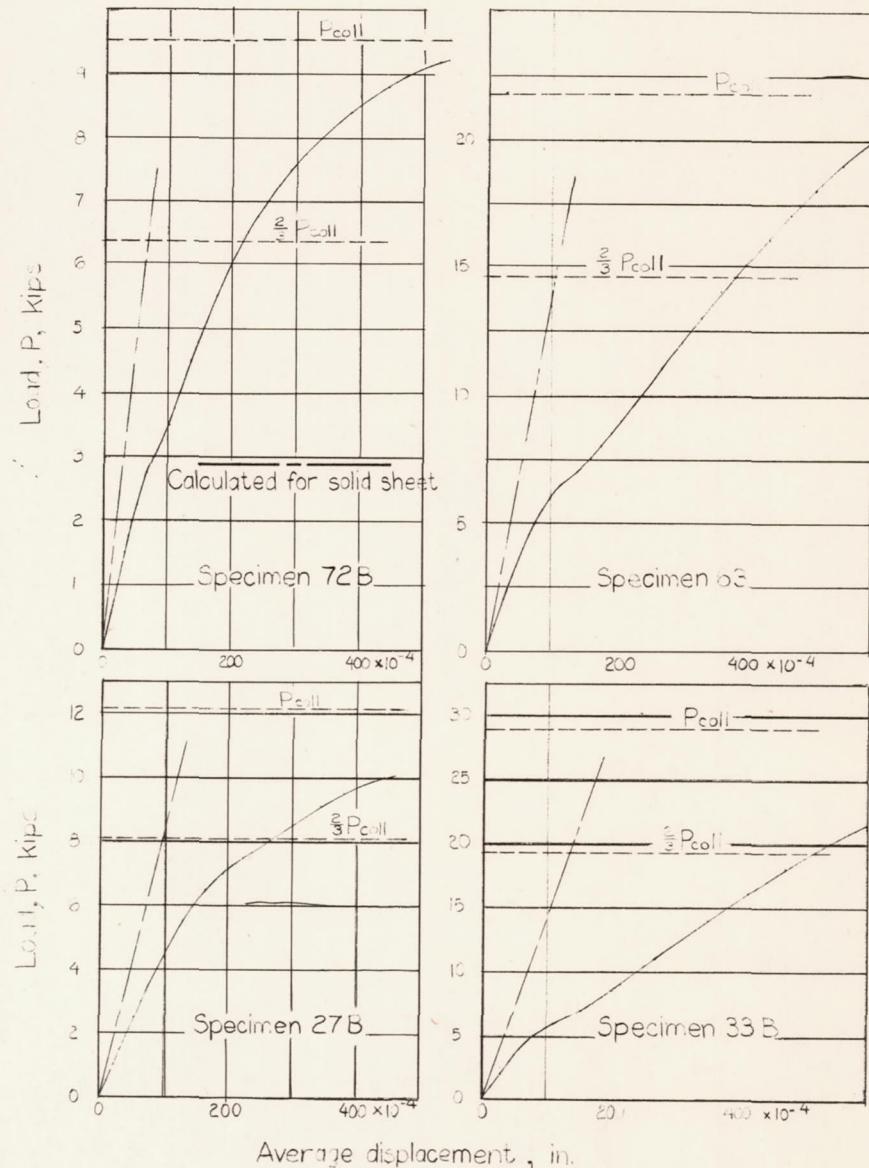
Figure 23.- Load-displacement curves for flanged-hole webs with rod supports and reamed bolt holes.