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A METHOD FOR DETERMINING THE RATE OF HEAT TRANSFER FROM A WING OR STREAMLINE BODY

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ADVANCE CONFIDENTIAL REPORT

A METHOD FOR DETERMINING THE RATE OF HEAT TRANSFER FROM A WING OR STREAMLINE BODY

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SUMMARY

A method for calculating the rate of heat transfer from the surface of an airfoil or streamline body is presented. A comparison with the results of an experimental investigation indicates that the accuracy of the method is good.

This method may be used to calculate the heat supply necessary for heat de-icing or in ascertaining the heat loss from the fuselage of an aircraft operating at great altitude, for example.

To illustrate the method, the total rate of heat transfer from an airfoil is calculated and compared with the experimental result.

INTRODUCTION

During the design of the test model for an investigation reported in reference 1, it was necessary to extend the theory of heat transfer into the laminar flow along a flat plate (reference 2) to cover the problem of determining the local rate of heat transfer from the surface of an airfoil into its laminar boundary layer. Using the same general method of attack, a solution was also obtained for the turbulent flow region, making possible the calculation of the total rate of transfer from the airfoil.

Since the boundary-layer characteristics of a streamline body may be calculated in the same manner as for two-dimensional flow if the spreading or crowding together of the boundary layer due to the change in body dimensions along the axis is considered, the heat-transfer equations developed for an airfoil were extended to make possible the computation of the total heat loss of a streamline body. The derivation of the method is given in the appendix.

A limited experimental investigation of the method was made. These tests were conducted with the same test model as for reference 1 to determine the accuracy of the method in calculating the local rate of heat transfer into both the laminar and the turbulent boundary layer of an airfoil, and also to obtain a check on the computed total rate of heat transfer for the wing.

It is hoped that this method will facilitate a more accurate determination of the heat losses from wings in designing heat de-icing systems as well as from fuselages in the design of cabin-heating systems for aircraft operating at great altitudes.

SYMBOLS

The symbols used throughout this report and in the appendix are defined as follows:

V free-stream velocity

U local velocity just outside the boundary layer

u local velocity inside the boundary layer

c wing chord

L length of streamline body

x distance along the chord from the leading edge for an airfoil, or along the axis for a streamline body

y distance normal to the surface

s distance along the surface from the stagnation point

r radius to surface of streamline body at any point along the axis

ρ air density

μ absolute viscosity

ν kinematic viscosity, μ/ρ

τ_o surface shear

ζ turbulent boundary-layer parameter, $\zeta = \sqrt{\frac{\rho U^2}{\tau_o}}$

R_c Reynolds number based on wing chord, Vc/ν

R_L Reynolds number based on body length, VL/ν

M_L local Mach number, ratio of velocity just outside the boundary layer to local velocity of sound

M_o free-stream Mach number, ratio of velocity of free stream to the velocity of sound in the free stream

θ momentum thickness of boundary layer,

$$\theta = \int_0^h \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$$

δ_L heat-transfer characteristic length for a laminar boundary layer

δ_T heat-transfer characteristic length for a turbulent boundary layer

t_o free-stream air temperature, ^oF

T_o free-stream air temperature, ^oF absolute

t_L local temperature outside the boundary layer, ^oF

T_L local temperature outside the boundary layer, ^oF absolute

t local temperature inside boundary layer, ^oF

T local temperature inside boundary layer, ^oF absolute

t_ϵ surface temperature

t_p surface temperature corrected for compressibility

$(t_p - t_o)$ heat-transfer temperature difference

c_p specific heat at constant pressure

k heat conductivity

γ ratio of specific heats

σ Prandtl number, $c_p\mu/k$

ϵ eddy viscosity

β eddy heat conductivity

h_x heat-transfer coefficient, Btu/sq ft, $^{\circ}$F, sec

q_x local rate of heat transfer, Btu/sq ft, sec

Method

The detailed analysis given in the appendix develops the following formulas, by which the local rate of heat transfer into both turbulent and laminar boundary layers may be computed. This method is applicable either to an airfoil or to a streamline body. The local rate for laminar flow is

$$q_x = 0.700 \frac{k}{\delta_L} (t_p - t_o) \tag{1)a}$$

or the local heat-transfer coefficient is

$$h_x = 0.700 \frac{k}{\delta_L} \tag{1)b}$$

and for turbulent flow,

$$q_x = 0.760 \frac{k}{\delta_T} (t_p - t_o) \tag{2)a}$$

or the local heat-transfer coefficient is

$$h_x = 0.760 \frac{k}{\delta_T} \tag{2)b}$$

Heat transfer from an airfoil.- For the laminar boundary layer of an airfoil, δ_L is computed as in reference 3 from the pressure distribution, as follows:

$$\delta_L = c \sqrt{\frac{5.3}{R_c} \left(\frac{V}{U_1}\right)^{9.17} \int_0^{s/c} \left(\frac{U}{V}\right)^{8.17} d\,\frac{s}{c}} \qquad (3)$$

For the turbulent boundary layer of an airfoil, δ_T is computed as

$$\delta_T = \frac{\zeta^2 c}{R_c \left(\frac{U_1}{V}\right)} \qquad (4)$$

where ζ is the value of the turbulent boundary-layer parameter as determined by a step-by-step solution of the relationship of reference 4, given as

$$\frac{d\zeta}{dx} + \frac{6.13}{U}\frac{dU}{dx} = \frac{U}{\nu} f(\zeta) \qquad (5)$$

The value of $f(\zeta)$ is given in table I as taken from reference 4, and may be plotted on semilogarithmic paper for ease in using.

Heat transfer from a streamline body.- For the laminar boundary layer of a streamline body, δ_L is computed from reference 3 as

$$\delta_L = L \sqrt{\frac{5.3}{R_L} \left(\frac{L}{r_1}\right)^2 \left(\frac{V}{U_1}\right)^{9.17} \int_0^{s/L} \left(\frac{r}{L}\right)^2 \left(\frac{U}{V}\right)^{8.17} d\,\frac{s}{L}} \qquad (6)$$

The turbulent boundary-layer heat-transfer length may be computed as

$$\delta_T = \frac{\zeta^2 L}{R_L \left(\frac{U_1}{V}\right)} \qquad (7)$$

where ζ is determined as for an airfoil from the equation

$$\frac{d\zeta}{dx} + \frac{6.13}{U}\frac{dU}{dx} + \frac{2.557}{r}\frac{dr}{dx} = \frac{U}{\nu} f(\zeta) \qquad (8)$$

by the step-by-step process mentioned for calculating δ_T for an airfoil.

Compressibility correction.- If the heat flow is to be obtained at free-stream Mach numbers such that the aerodynamic temperature rise is an appreciable portion of the total temperature difference, a correction for aerodynamic heating should be made. The "heat-transfer temperature difference" to be used for a laminar boundary layer is

$$(t_p - t_o) = (t_\epsilon - t_o) - 0.20\, M_o^2\, T_o \left[1 - 0.13 \left(\frac{U}{V}\right)^2\right] \quad (9)$$

and for turbulent flow

$$(t_p - t_o) = (t_\epsilon - t_o) - 0.20\, M_o^2\, T_o \quad (10)$$

where $(t_\epsilon - t_o)$ is the desired temperature rise for heat de-icing.

The total rate of heat transfer from an airfoil or streamline body may be found as follows:

1. Estimate the location of the transition point by the method of reference 5 for an increasing pressure gradient, or by reference 3 for a falling pressure gradient.

2. Calculations for the laminar region ahead of the transition point.

   a) Compute the values of δ_L along the surface to the transition point by equation (3) for an airfoil, or by equation (6) for a streamline body.

   b) With these values and the desired temperature distribution corrected for compressibility, compute the local rates of heat transfer along the surface by equation (1)a.

3. Calculations for the turbulent region behind the transition point.

   a) From the value of δ_L at the transition point compute θ, the momentum thickness as

$$\theta = 0.289\, \delta_L$$

Using this value of θ, find the initial value of ζ at the transition point as

$$\zeta = 2.557 \log_e \left(4.057 \frac{U\theta}{\nu}\right)$$

b) With this initial value of ζ, calculate the values of ζ^2 along the surface by equation (5) for an airfoil, or by equation (8) for a streamline body. With these values of ζ^2, compute δ_T along the surface by equation (4) for an airfoil, or by equation (7) for a streamline body.

c) Using these values of δ_T and the desired temperature difference across the boundary layer corrected for compressibility, compute the local rate of heat transfer along the surface by equation (2)a.

4. Integrate these local rates of heat transfer along the chord for both laminar and turbulent regions to obtain the <u>total</u> rate of heat transfer.

Heat-Transfer Measurements

As mentioned in the Introduction, a limited number of heat-transfer tests were made on the heated wing model of reference 1 in the 7- by 10-foot wind tunnel of the Ames Aeronautical Laboratory to check the accuracy of the theoretical method.

Since the wing was not designed specifically for heat-transfer tests, the experimental results are subject to several sources of error. All computed rates of heat transfer are based on the temperature distributions obtained at the center of the span, assuming that the spanwise variation is negligible. This is essentially true except for a small portion at each end of the wing. Precautions were taken to minimize the heat losses at the ends of the wing. These are not believed large since the design of the heating system allowed only slight transfer by convection, and the conduction of heat from the wing to its supports is negligible. Losses due to radiation from the wing have been computed as a maximum of 5 percent for the whole surface heated to 100° F above the surroundings. This loss is not considered in the heat-transfer data.

For the purpose of computing heat transfer, the chordwise temperature distribution was computed and plotted as "heat-transfer temperature difference," $(t_p - t_o)$. This is the observed temperature difference corrected for compressibility effect. That is, t_p is the temperature measured by a thermocouple in the skin minus the computed aerodynamic heating temperature rise. The value t_o is the free-stream air temperature (distinguished from the local temperature just outside the boundary layer of the wing, which will be higher or lower than t_o due to adiabatic variations caused by the velocity field of the wing).

RESULTS AND DISCUSSION

Heat-transfer tests of the wing were made in two parts, the first concerned with comparing computed and measured values of the local rate of transfer from the airfoil surface to laminar and turbulent boundary layers, and the second with checking the total rate of heat transfer from the airfoil.

The tests to measure the local rate of heat transfer were made at $c_l = 0$ for two test Reynolds numbers, with free transition to obtain the heat-flow rate into a laminar boundary layer, and with transition fixed at 5-percent chord to determine the flow rate into a turbulent boundary layer. Figure 1 presents the pressure distribution over the wing at $c_l = 0$. The experimental procedure consisted in adjusting the heat input so that the skin temperatures were nearly constant along the chord. With this temperature distribution achieved, it was assumed that the second compartment of the wing, extending from 14.6-percent to 26.3-percent chord, was thermally isolated so that no flow of heat occurred in the skin or through the bulkheads. The power input to this compartment was then measured by means of a voltmeter and an ammeter for comparison with the calculated rate of heat flow.

Figure 2 shows that the desired constant chordwise temperature distribution was attained for the laminar boundary layer, but the data of figure 3 for turbulent flow indicate that while the distribution was nearly constant from 10- to 30-percent chord, covering the region under consideration, the temperatures over the nose were excessively high. This came about through the heating difficulties resulting from the sudden change in heat-transfer coefficient at the point where transition was fixed. This type of distribution may have resulted in some change in the local values of the temperature gradient at the wing surface, though this effect should be small since no appreciable temperature gradient existed over the portion of the surface concerned.

To obtain the computed values, the variation of the heat-transfer coefficient along the chord was calculated for each case as outlined under the section Method (p. 4) (results plotted in figs. 4 and 5). The heat input into the second compartment was then computed from the heat-transfer coefficient and experimentally measured temperature difference (corrected for compressibility heating effects). Both the measured and the computed values of heat input into the second compartment are listed in the following table:

α (deg)	$R_c \times 10^{-6}$	Boundary layer	Calculated heat transfer, (kw)	Measured input, (kw)	Error input, (percent)
0	6.71	Laminar	0.861	0.875	1.6
0	10.72	--do---	1.048	1.028	1.8
0	6.90	Turbulent	3.76	3.75	.3
0	11.17	--do-----	4.01	4.01	.0

The check is considered quite satisfactory, and is taken to indicate that the method for the computations of the heat-transfer coefficient involves no serious errors despite the assumptions involved.

Further tests for the purpose of establishing the validity of the method as regards the total rate of heat flow from a wing were made at a lift coefficient of 0.55 and 8.60 million Reynolds number. In addition to a test with free transition, a second condition simulating the formation of ice near the stagnation point by fixing transition at 5-percent chord on the lower surface was investigated. Heat input in both conditions was maintained at the maximum available from the apparatus, and no attempt was made to achieve a predetermined temperature rise or chordwise distribution. The chordwise temperature distribution obtained, corrected for the effects of compressibility heating, is plotted in figures 6 and 7.

The chordwise variation of heat-transfer coefficient was computed for both the transition-fixed and transition-free conditions, as outlined in Method. Tables II and III present the computations for the transition-free condition and serve as an illustrative example. The pressure distribution used for these calculations is given in figure 8. The value of heat input was then computed by use of the experimental temperature distributions (corrected for compressibility) and the calculated heat-transfer coefficients of figures 9 and 10. The computed and measured heat input are compared in the table below. Results indicate satisfactory agreement.

α (deg)	$R_c \times 10^{-6}$	Boundary layer	Calculated heat transfer	Measured input, (kw)	Error input, (percent)
5	8.69	Upper surface turbulent Lower surface laminar	19.60	23.52	16.8
5	8.60	Upper surface turbulent Lower surface turbulent	22.30	23.55	5.2

The experimental temperature distributions for the tests of the total rate of heat transfer show that the heat-transfer temperature difference varies to a marked degree along the chord (figs. 6 and 7). This variation violates one of the assumptions underlying the development of the method; that is, that the temperature difference is constant along the chord, which must be true if the transfer of heat at all points along the surface is analogous to the transfer of momentum. To what degree this assumption may be ignored has not been determined analytically since the problem of considering the variation of temperature along the chord presents difficulties which have so far prevented a solution. The experimental results, however, indicate that the accuracy with which the total rate of heat transfer can be computed is not greatly impaired by the temperature variations experienced. Generalization of this result must await further experimental checks.

The accuracy with which the local rate of heat transfer may be computed in a falling pressure gradient is dependent upon the accuracy with which the surface shear may be determined. Squire and Young's method (reference 4) assumes that the turbulent boundary layer in a falling pressure gradient exhibits the same characteristics as the fully developed turbulent layer of a flat plate. The extent to which the relationship between the surface shear, the momentum thickness, and the local velocity so derived remains valid is shown by the accuracy of the Squire and Young method in determining friction drag. It must be realized, however, that the method will fail if turbulent separation is imminent.

The thickening of the turbulent boundary layer due to the fall in pressure acting on the displaced mass of fluid also is ignored by the assumption that the heat-transfer rate is proportional to the surface shear computed by Squire and Young's method. Actually, the heat capacity of the boundary layer is increased by this thickening which tends to increase the rate of heat flow at the surface. This counteracts the effect of the profile distortion, resulting from the same cause, which reduces the surface shear since it tends to cause separation. But this effect, too, is negligible for all cases where Squire and Young's method may be applied.

In concluding, it must be stated that while the method presented herein is subject to a number of broad assumptions in its development, the experimental evidence presented shows the total rate of heat flow may be calculated with reasonable accuracy.

CONCLUSIONS

The accuracy of the method for determining the rate of heat transfer from an airfoil is shown to be good by the results of a limited experimental investigation. Since the correctness with which the heat transfer can be computed is dependent mainly on the accuracy with which the boundary-layer characteristics may be determined, it is expected that the method possesses the same accuracy for computing heat-transfer rates from a streamline body.

Although the development of the heat-transfer formulas is based on the assumption that the skin temperature remains constant along the surface, the experimental results show that for moderate temperature variations the precision is still good.

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APPENDIX

1. Heat transfer into a laminar boundary layer.- The theory of heat transfer into a laminar boundary layer was first investigated by E. Pohlhausen for the case of incompressible flow along a flat plate maintained at a constant temperature (reference 2). Pohlhausen's solution is developed by solving the differential equation for the temperature boundary layer by using Blasius' solution for the velocity boundary layer.

In order to arrive at a solution for an airfoil in an incompressible fluid, it is necessary

(1) to assume that the temperature boundary-layer and the velocity boundary-layer profiles for the airfoil are related in the same manner as for Pohlhausen's solution. (This is true if the temperature of the skin remains constant along the surface and if the thinning of the friction layer in a favorable pressure gradient due to the change in pressure acting on the displaced mass of fluid is negligible.)

(2) to calculate the value of $\left(\frac{du}{dy}\right)_{y=o}$ for the velocity boundary layer and then determine $\left(\frac{dt}{dy}\right)_{y=o}$ with

of temperature along the chord presents difficulties which have so far prevented a solution. The experimental results, however, indicate that the accuracy with which the total rate of heat transfer can be computed is not greatly impaired by the temperature variations experienced. Generalization of this result must await further experimental checks.

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(2) to calculate the value of $\left(\frac{du}{dy}\right)_{y=o}$ for the velocity boundary layer and then determine $\left(\frac{dt}{dy}\right)_{y=o}$ with

the relationship resulting from (1). (The solution of the problem for the temperature of the skin varying along the chord has been prevented because the difficulties so far have been found insurmountable.)

Pohlhausen's expression for the temperature gradient in the boundary layer at the surface of the plate is given as

$$\left(\frac{dt}{dy}\right)_{y=o} = -\frac{1}{2}\,\alpha(\sigma)\sqrt{\frac{U\rho}{\mu x}}\;(t_p - t_o)$$

The function $\alpha(\sigma)$ is the first derivative of Pohlhausen's function defining the temperature boundary layer which, for $\sigma = 1$, is equivalent to the second derivative of Blasius' function for the velocity boundary layer. Pohlhausen found that $\alpha(\sigma)$ is accurately given by the relationship

$$\alpha(\sigma) = 0.664\sqrt[3]{\sigma}$$

then

$$\left(\frac{dt}{dy}\right)_{y=o} = -0.332\sqrt[3]{\sigma}\sqrt{\frac{U\rho}{\mu x}}\;(t_p - t_o)$$

Now, for the Blasius boundary-layer distribution

$$\left(\frac{du}{dy}\right)_{y=o} = 0.332\sqrt{\frac{U\rho}{\mu x}}\;U$$

so that

$$\left(\frac{dt}{dy}\right)_{y=o} = -\left(\frac{du}{dy}\right)_{y=o}\sqrt[3]{\sigma}\;\frac{(t_p - t_o)}{U}\;.$$

It is now necessary to determine $\left(\frac{du}{dy}\right)_{y=o}$ for the airfoil at any chordwise position. For the Blasius profile

$$\left(\frac{du}{dy}\right)_{y=o} = \frac{0.765}{\delta_L}\,U$$

where δ_L is the thickness of the boundary layer where $u = 0.707\ U$. Substituting in equation (2)

$$\left(\frac{dt}{dy}\right)_{y=o} = 0.765\sqrt[3]{\sigma}\ \frac{(t_p - t_o)}{\delta_L}$$

or, taking $\sigma = 0.760$ for air, the local rate of heat transfer is

$$q_x = k\left(\frac{dt}{dy}\right)_{y=o} = 0.700\ \frac{k}{\delta_L}\ (t_p - t_o) \qquad (1)a$$

or the heat-transfer coefficient is

$$h_x = 0.700\ \frac{k}{\delta_L} \qquad (1)b$$

The development in reference 6 of an expression for the heat-transfer rate based on Reynolds analogy gives results which are in complete agreement with the above if $\sigma = 1$. However, the experimental results of reference 7 indicate that the expression $\sqrt[3]{0.760}$, that is, $\sqrt[3]{\sigma_{air}}$, properly relates the velocity and temperature gradients in the laminar layer. The two methods give results within 10 percent of each other, which is sufficient for practical cases.

The values of δ_L for laminar flow may be determined both for an airfoil and a streamline body by the method of reference 3.

2. Turbulent boundary layer.- The theory of heat transfer in eddying flow as given by Dryden (reference 8) requires the introduction of several new concepts. If the equations of motion for turbulent flow are written by placing $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, where the bars indicate mean values and the primes indicate fluctuations, and these values substituted into the equations of motion for steady flow, similarity between equations so developed and the steady-flow equations can be shown by introducing a value of eddy viscosity, ϵ. Similarly, the concept of eddy heat conductivity, β, is introduced by placing $t = \bar{t} + t'$ in the equations of the temperature field.

Those values of eddy viscosity, ϵ, and eddy conductivity, β, however, do not have the same properties as μ and k since they vary from point to point in the flow.

Nevertheless, ϵ and β can be shown to vary in the same manner from point to point in the fluid. This is done by introducing Prandtl's concept of a mixing length; that is, a length of path followed by a fluid particle before it becomes lost in the mass of eddying fluid.

It is therefore shown if the shear, $\tau' = -\rho\overline{u'v'}$, then

$$\tau = \epsilon \frac{d\overline{u}}{dy} = \rho l^2 \left|\frac{d\overline{u}}{dy}\right| \frac{d\overline{u}}{dy}$$

or that

$$\epsilon = \rho l^2 \left|\frac{d\overline{u}}{dy}\right|$$

in which the mixing length, l, varies from point to point in the fluid. Now, if the eddy heat transfer is considered to be $-c_p\overline{v't'}$, then the eddy heat conductivity is equal to $c_p\rho l^2 \left|\frac{d\overline{u}}{dy}\right|$ or equal to $c_p\,\epsilon$, provided the mixing length for heat transfer is considered to be the same as the mixing length for the transfer of shearing stress. Dryden states that available experimental data show that the mixing lengths near a wall are closely equivalent for transfer of heat and momentum, but that the relationship falls down, for instance, in the wake of a heated body. Since the present case concerns heat transfer from a wall to eddying flow in a boundary layer, it is believed that this relationship is acceptable.

For turbulent flow, it has been shown that the Prandtl number is equal to unity; that is,

$$\frac{c_p\mu_{turb}}{k_{turb}} = 1$$

If the Prandtl number is unity, then the thermal and dynamic boundary layers have the same profile (reference 7). If we make the same assumptions as in step (1) for the laminar boundary layer, we may write

$$\left(\frac{dt}{dy}\right)_{y=o} = -\left(\frac{du}{dy}\right)_{y=o} \frac{(t_p - t_o)}{U}$$

where U is the velocity outside the friction layer. (This relationship is dependent on the assumption that the temperature along the surface remains constant as for step (1) for the laminar boundary layer, and that the thickening of the boundary layer due to the increasing pressures acting on the displaced mass of fluid and the distortion of the profile thus resulting is negligible.)

Now $$q_x = -\beta \left(\frac{dt}{dy}\right)_{y=o}$$

but $$\beta = c_p \epsilon$$

so that $$q_x = -c_p \epsilon \left(\frac{dt}{dy}\right)_{y=o}$$

or $$q_x = +c_p \epsilon \left(\frac{du}{dy}\right)_{y=o} \frac{(t_p - t_o)}{U}$$

since the surface shear

$$\tau_o = \epsilon \left(\frac{du}{dy}\right)_{y=o}$$

$$q_x = c_p \tau_o \frac{(t_p - t_o)}{U}$$

This is the same formula developed by Reynolds for flow in pipes (reference 8).

So it is seen that the problem of calculating the rate of heat transfer in turbulent flow is primarily a problem of calculating the surface shear along the airfoil. This may be done by the method of reference 4, in which Squire and Young write the relationship

$$\tau_o = \frac{\rho U^2}{\zeta^2}$$

where $$\zeta^2 = 2.557 \log_e \left(4.075 \frac{U\theta}{\nu}\right)$$

θ being the momentum thickness of the boundary layer. (This relationship is developed from von Kármán's formula for the skin friction experienced by a flat plate with a fully developed turbulent boundary layer. This assumption becomes less and less true as the turbulent boundary-layer profile of an airfoil becomes changed in shape and approaches separation in a steep pressure recovery.) Substituting for τ_o

$$q_x = c_p \frac{\rho U^2}{\zeta^2} \frac{(t_p - t_o)}{U}$$

$$q_x = c_p \frac{\rho U}{\zeta^2} (t_p - t_o)$$

or

$$q_x = c_p \rho \frac{k}{k} \frac{U}{\zeta^2} \frac{\mu}{\mu} \frac{V}{V} \frac{c}{c} (t_p - t_o)$$

$$q_x = \frac{c_p \mu}{k} \frac{k}{c \zeta^2} \times \frac{Vc}{\nu} \frac{U}{V} (t_p - t_o)$$

where μ and k are values for uneddying flow. Substituting for $\frac{c_p \mu}{k}$ the value for air, 0.760,

$$q_x = 0.760 \frac{k}{c \zeta^2} R_c \left(\frac{U}{V}\right) (t_p - t_o)$$

or considering $\frac{\zeta^2 c}{R_c \left(\frac{U}{V}\right)} = \delta_T$, a characteristic length for the turbulent boundary layer,

then $$q_x = 0.760 \frac{k}{\delta_T} (t_p - t_o) \qquad (2)a$$

and $$h_x = 0.760 \frac{k}{\delta_T} \qquad (2)b$$

In calculating δ_T, ζ may be computed by the step-by-step solution of the equations of reference 4 for an airfoil or streamline body.

3. <u>Compressibility effects on heat transfer</u>.- Since the foregoing analysis has been made for incompressible flow, the effect of aerodynamic heating must be dealt with

if the heat transfer is to be accurately obtained. The effect of compressibility may be considered simply as influencing the heat-transfer temperature difference to be used in the above-developed equations; that is, a part of any desired increase in the skin temperature will result from aerodynamic heating, and this part of the temperature increase involves no expenditure of heat.

The temperature field near the heated surface of an airfoil or body of revolution operating at high Mach numbers may be determined by superposing the heat-transfer temperature field on that due to the friction heating as in reference 9. Eckert (reference 10) has shown that the temperature field due to aerodynamic heating for $\sigma = 1$ may be expressed as

$$t = t_L + \frac{\gamma - 1}{2} M_L^2 T_L \left[1 - \left(\frac{u}{U}\right)^2\right]$$

or

$$t = t_o + \frac{\gamma - 1}{2} M_o^2 T_o - \frac{\gamma - 1}{2} M_L^2 T_L \left(\frac{u}{U}\right)^2$$

Superposing this on the temperature field for heat transfer, which may be given as

$$t = t_p - (t_p - t_o) \frac{u}{U}$$

the combined temperature field is

$$t = t_p - (t_p - t_o) \left(\frac{u}{U}\right) + \frac{\gamma - 1}{2} M_o^2 T_o - \frac{\gamma - 1}{2} M_L^2 T_L \left(\frac{u}{U}\right)^2$$

It is evident that at $y = 0$

$$\left(\frac{dt}{dy}\right)_{y=o} = - \frac{(t_p - t_o)}{U} \left(\frac{du}{dy}\right)_{y=o}$$

which indicates that the heat transfer corresponds to the heat-transfer temperature field, so that for compressible flow the only correction necessary is that of correcting the skin temperature for the rise in temperature due to aerodynamic heating.

Eckert has shown that

$$t_\epsilon = t_L + \sqrt{\sigma}\ \frac{\gamma-1}{2} M_L^2 T_L$$

$$t_\epsilon = t_o + \frac{\gamma-1}{2} M_o^2 T_o - \frac{\gamma-1}{2} M_L^2 T_L + \sqrt{\sigma}\ \frac{\gamma-1}{2} M_L^2 T_L$$

or

$$t_\epsilon = t_o + \frac{\gamma-1}{2} M_o^2 T_o \left[1 - \left(1 - \sqrt{\sigma}\right)\left(\frac{u}{V}\right)^2\right]$$

We may write for the laminar region, $\sigma = 0.760$

$$(t_p - t_o) = (t_\epsilon - t_o) - 0.2\ M_o^2 T_o \left[1 - 0.13\left(\frac{u}{V}\right)^2\right]$$

and for the turbulent region, $\sigma = 1$

$$(t_p - t_o) = (t_\epsilon - t_o) - 0.2\ M_o^2 T_o$$

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TABLE I

$$f(\zeta) = 10.411\ \zeta^{-2}\ e^{-0.3914\ \zeta}$$

Numerical values

ζ	$10^6\ f(\zeta)$	ζ	$10^6\ f(\zeta)$
14	221.6	24	1.502
15	130.39	25	.937
16	77.47	26	.585
17	46.11	27	.367
18	27.97	28	.2308
19	16.96	29	.1454
20	10.35	30	.0917
21	6.35	31	.0582
22	3.906	32	.0369
23	2.418	33	.0235

TABLE II

COMPUTATION OF HEAT-TRANSFER COEFFICIENT

Airfoil 65,2-016 $c_l=0.55$ $R_c=8.69\times10^6$ $M_0=0.183$

Upper surface Transition at 1% chord $\theta_{1\%c}=154.5\times10^{-6}$ $\zeta_{1\%c}=18.25$

Position $\frac{x}{c}$	Position (ft)	Δx	$\left(\frac{U}{V}\right)$	$\frac{dU/V}{dx}$	$f(\zeta)$ $\times10^6$	$\frac{d\zeta}{dx}$	$\Delta\zeta$	ζ	$10^6\delta_T$	h_x
0.01	0.07		1.61					18.25	166.5	0.0191
		0.07		-0.667	24.6	51.8	3.63			
.02	.14		1.57					21.88	245.4	.0130
		.07		-.572	4.17	10.36	.73			
.03	.21		1.54					22.61	267.1	.0120
		.07		-.443	2.92	7.34	.51			
.04	.28		1.52					23.12	283.2	.0113
		.07		-.300	2.29	5.53	.39			
.05	.35		1.50					23.51	296.9	.0108
		.14		-.270	1.89	4.62	.65			
.07	.49		1.48					24.16	317.4	.0100
		.14		-.185	1.39	3.32	.46			
.09	.63		1.44					24.62	335.3	.00952
		.14		-.133	1.12	2.57	.36			
.11	.77		1.422					24.98	353.4	.00903
		.35		-.069	.940	1.96	.69			
.16	1.12		1.399					25.67	379.5	.00841
		.35		-.046	.685	1.39	.49			
.21	1.47		1.382					26.16	399.0	.00800
		.35		-.0372	.545	1.10	.39			
.26	1.82		1.386					26.55	410.0	.00778
		.35		-.0286	.453	.906	.32			
.31	2.17		1.356					26.87	430.9	.00741
		.35		-.0171	.393	.739	.26			
.36	2.52		1.350					27.13	439.8	.00725
		.35		-.0143	.345	.643	.23			
.41	2.87		1.345					27.36	448.2	.00712
		.35		-.0343	.312	.677	.24			
.46	3.22		1.330					27.60	461.0	.00682
		.35		-.0915	.278	.918	.32			
.51	3.57		---					27.92	472.1	.00676
		--		--	--	--	--			

TABLE III

COMPUTATION OF HEAT-TRANSFER COEFFICIENT

Airfoil 65,2-016 c_l=0.55 R_c=8.69x10 M_0=0.183

Lower surface Transition at minimum pressure

$\frac{x}{c}$	$\left(\frac{U}{V}\right)^2$	$\left(\frac{U}{V}\right)^{8.17}$	$\int_0^{s/c} \left(\frac{U}{V}\right)^{8.17} d\frac{s}{c}$	$\left(\frac{U_1}{V}\right)^{9.17}$	$10^4 \frac{\delta}{c} L$	$10^4\delta_L$ (ft)	$10^4 h_x$
0.010	0.0	---	---	---	---	---	---
.020	.185	0.00094	.4x10^{-6}	0.00041	0.775	5.42	54.2
.030	.290	.0060	34x10^{-6}	.0032	.809	5.66	52.0
.040	.370	.0165	144x10^{-6}	.0100	.942	6.59	49.6
.050	.455	.0388	416x10^{-6}	.0261	.992	6.94	42.4
.075	.60	.122	2340x10^{-6}	.094	1.234	8.64	34.1
.100	.71	.243	0.0056	.205	1.293	9.06	29.7
.150	.85	.512	.0248	.472	1.767	12.37	23.8
.200	.92	.708	.0572	.680	2.267	15.86	18.5
.250	1.00	1.00	.1012	1.00	2.495	17.46	16.9
.300	1.07	1.30	.1610	1.34	2.710	18.97	15.1
.350	1.12	1.60	.2340	1.69	2.940	20.47	14.3
.400	1.16	1.81	.3272	1.95	3.20	22.40	13.3
.450	1.17	1.93	.4306	2.10	3.54	24.80	11.8
.500	1.18	1.98	.5392	2.15	3.91	27.40	11.0

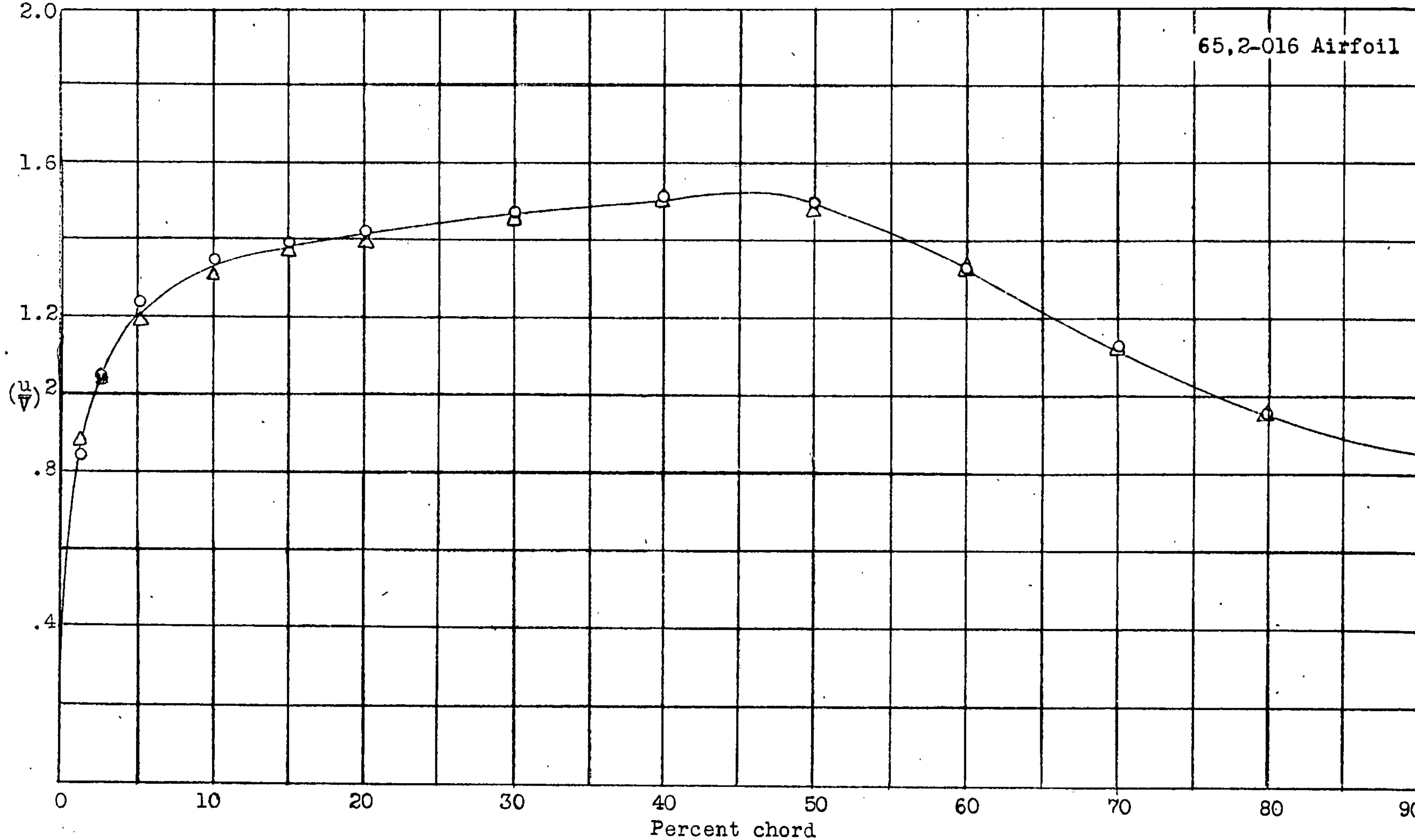

Figure 1.- Chordwise pressure distribution $\alpha=0^{\circ}$ $c_l=0$ $R_c=11\times10^6$

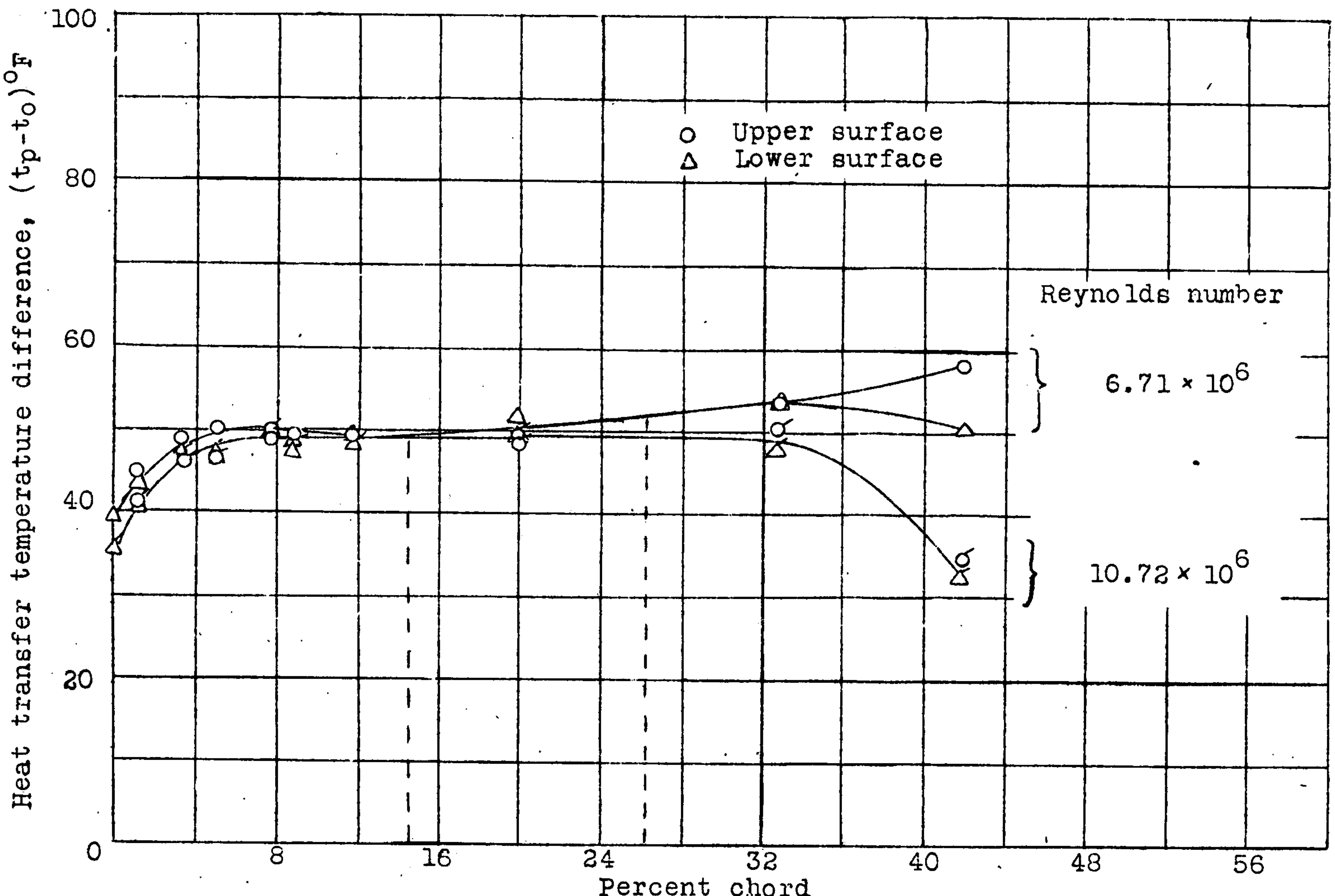

Figure 2. Chordwise distribution of temperature difference, $(t_p - t_o)$ $\alpha=0^\circ$ transition free.

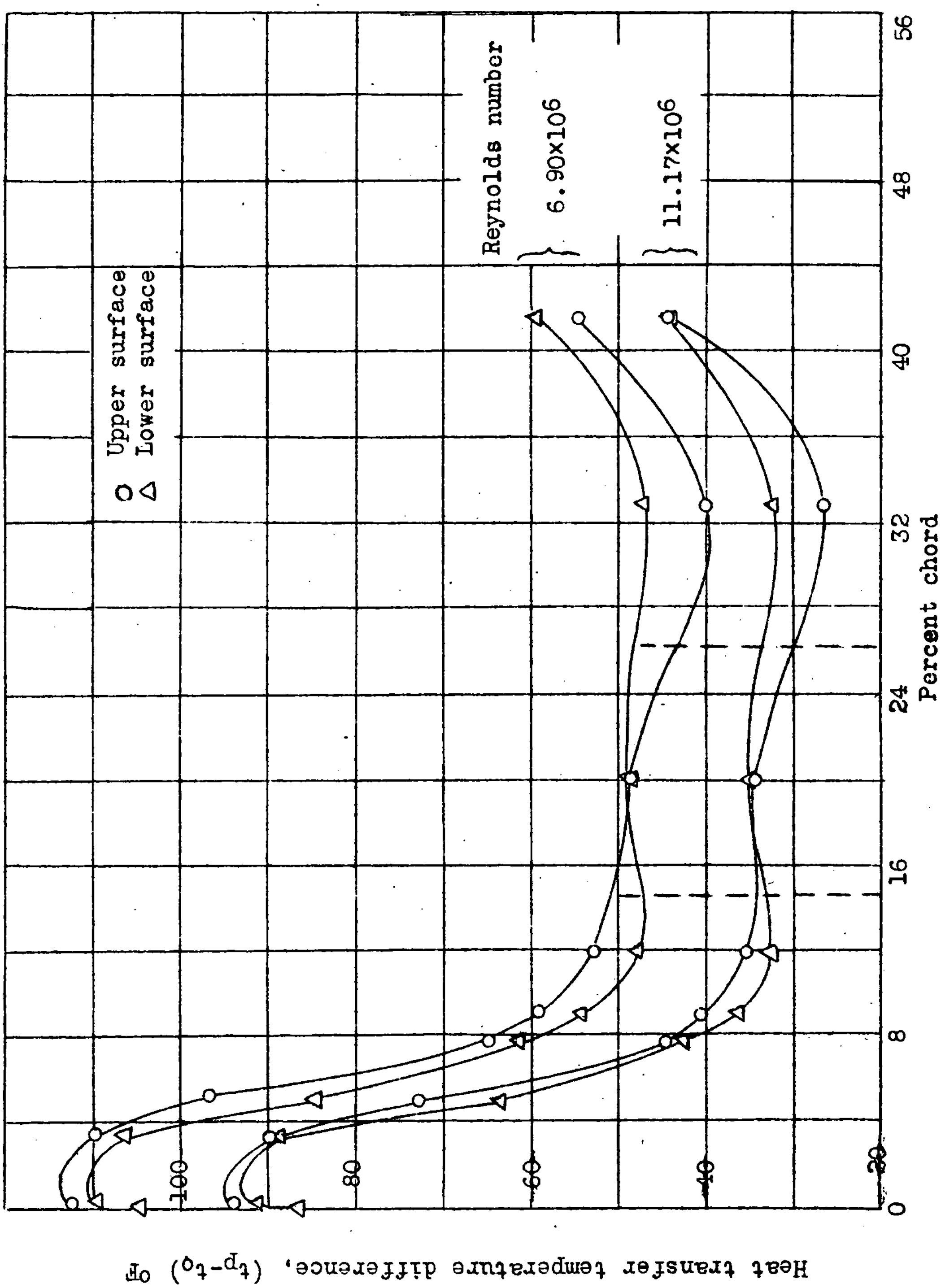

Figure 3.- Chordwise distribution of temperature difference, (t_p-t_o) $\alpha=0^o$ transition fixed at 5% chord.

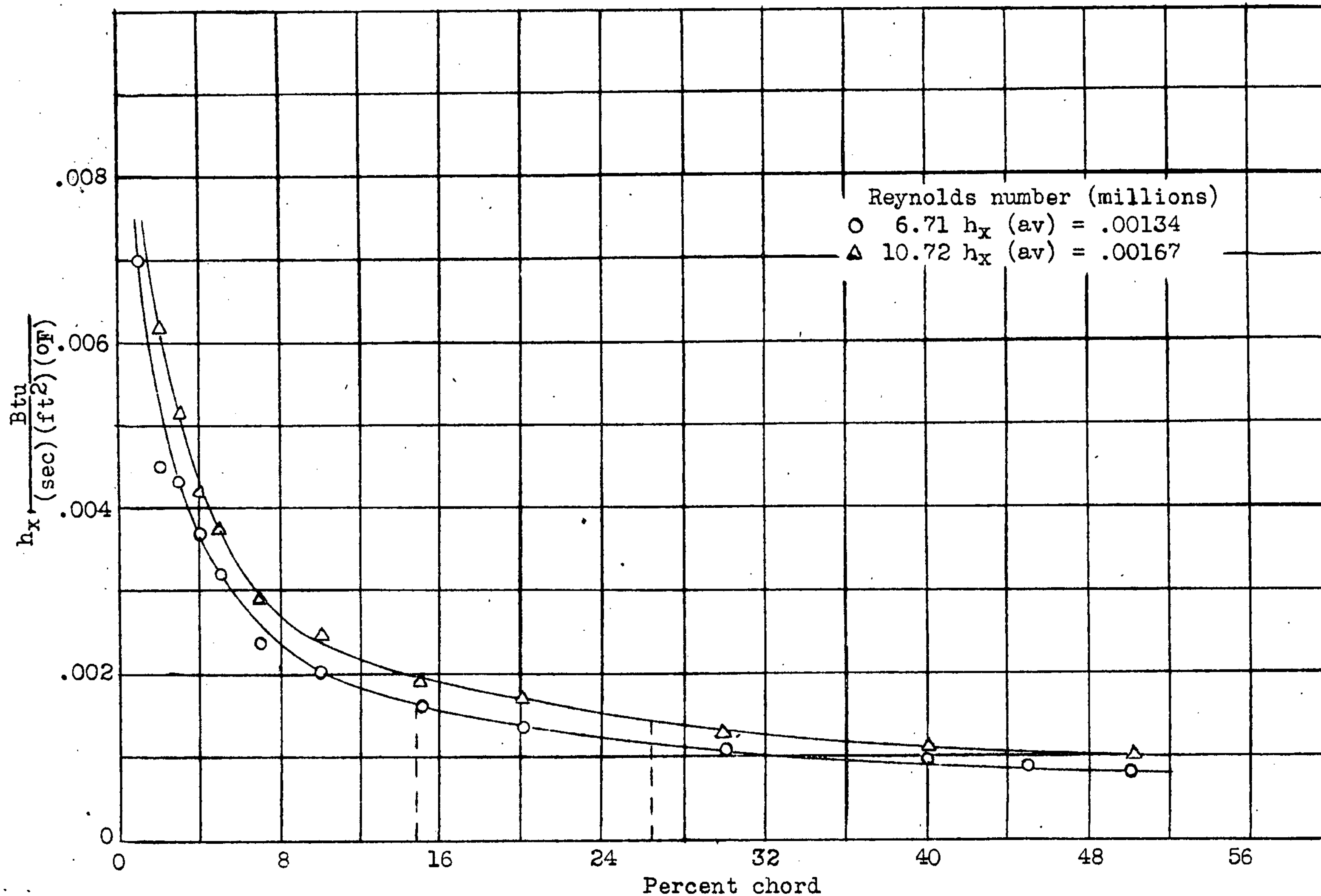

Figure 4.- Chordwise distribution of heat transfer coefficient, $h_x, \alpha=0°$ transition free.

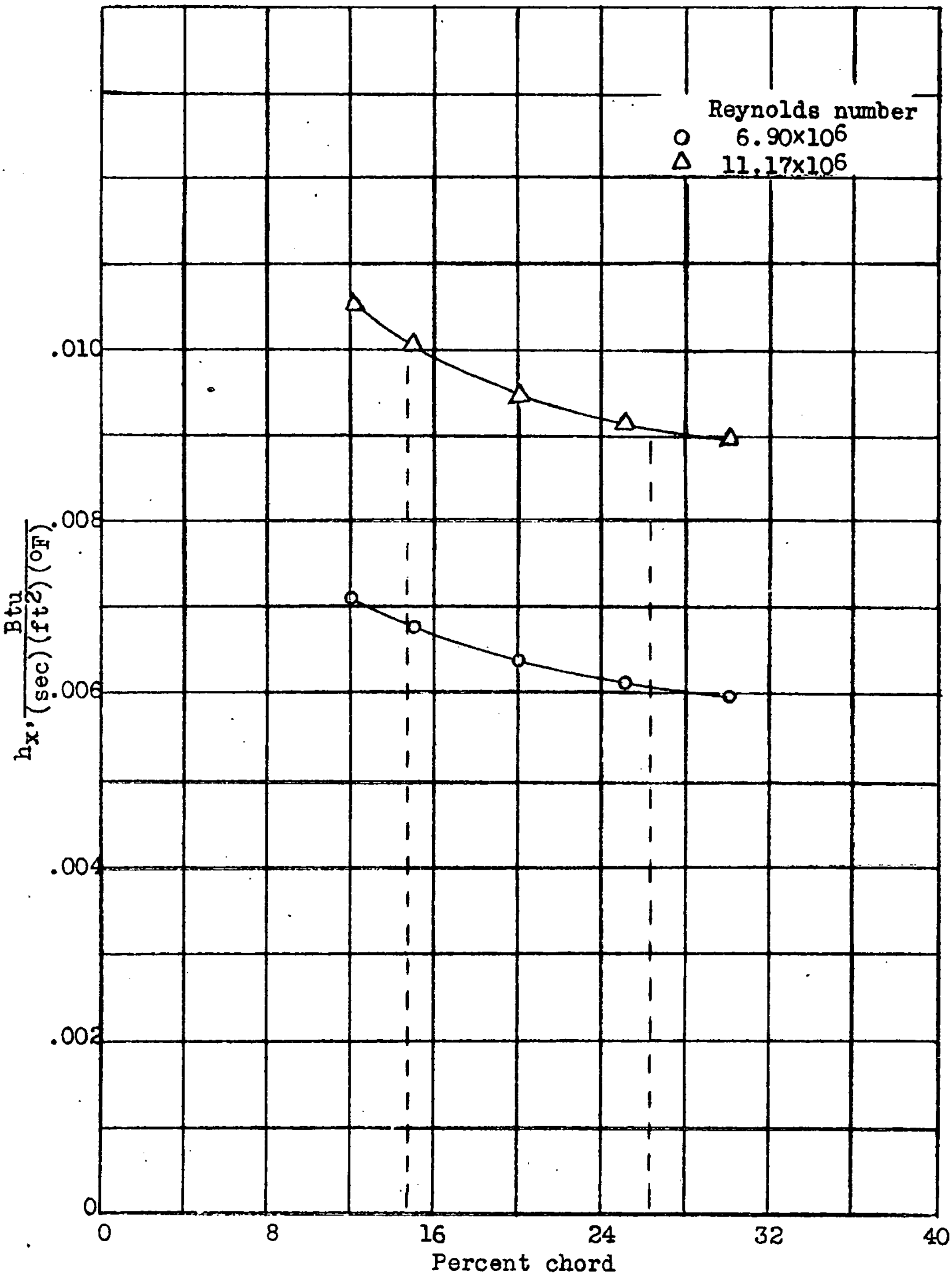

Figure 5.- Chordwise distribution of heat transfer coefficient, h_x, $\alpha = 0^{\circ}$, transition fixed at 5% chord.

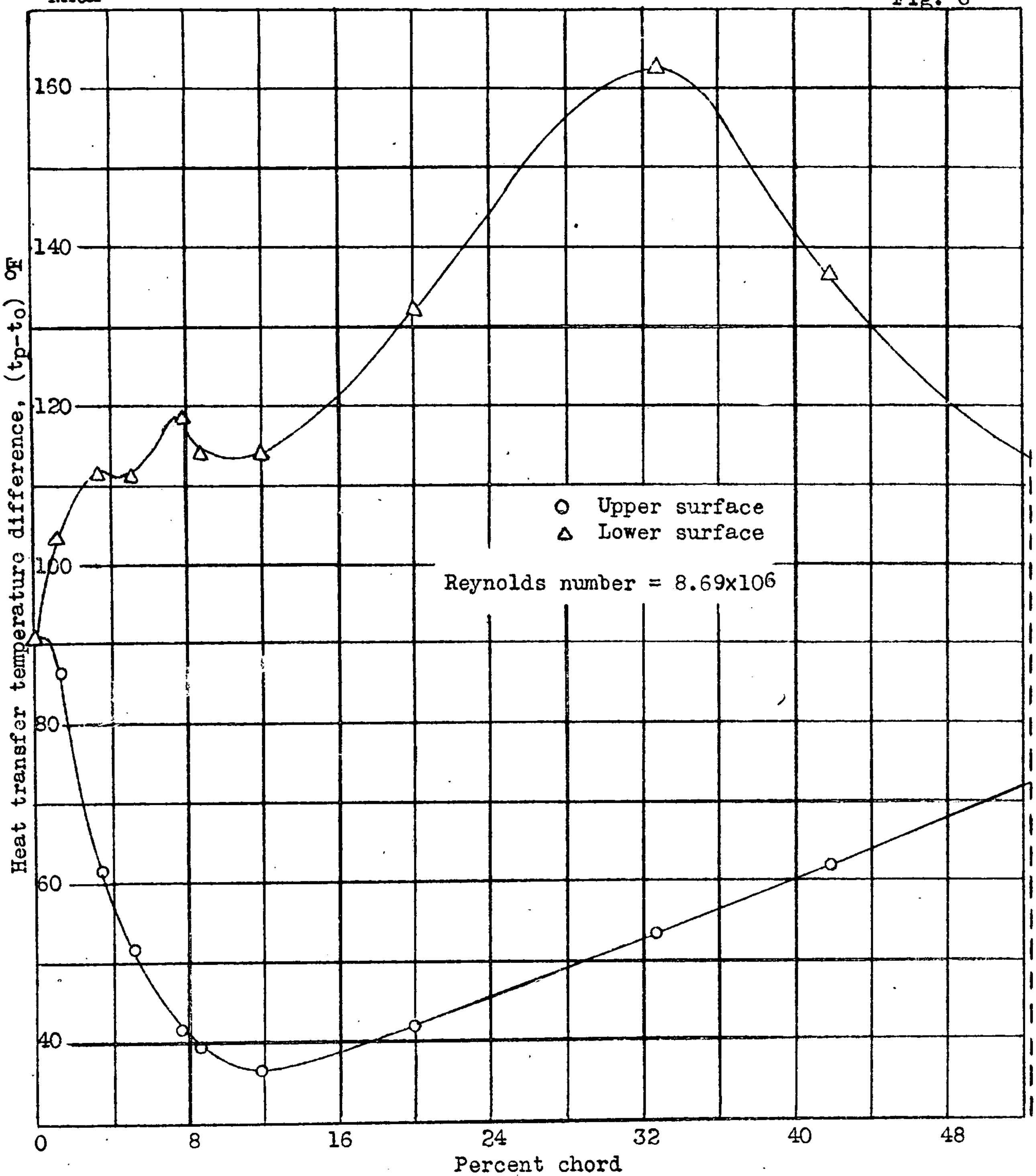

Figure 6.- Chordwise distribution of temperature difference, (t_p-t_o), $\alpha=5^o$, transition free.

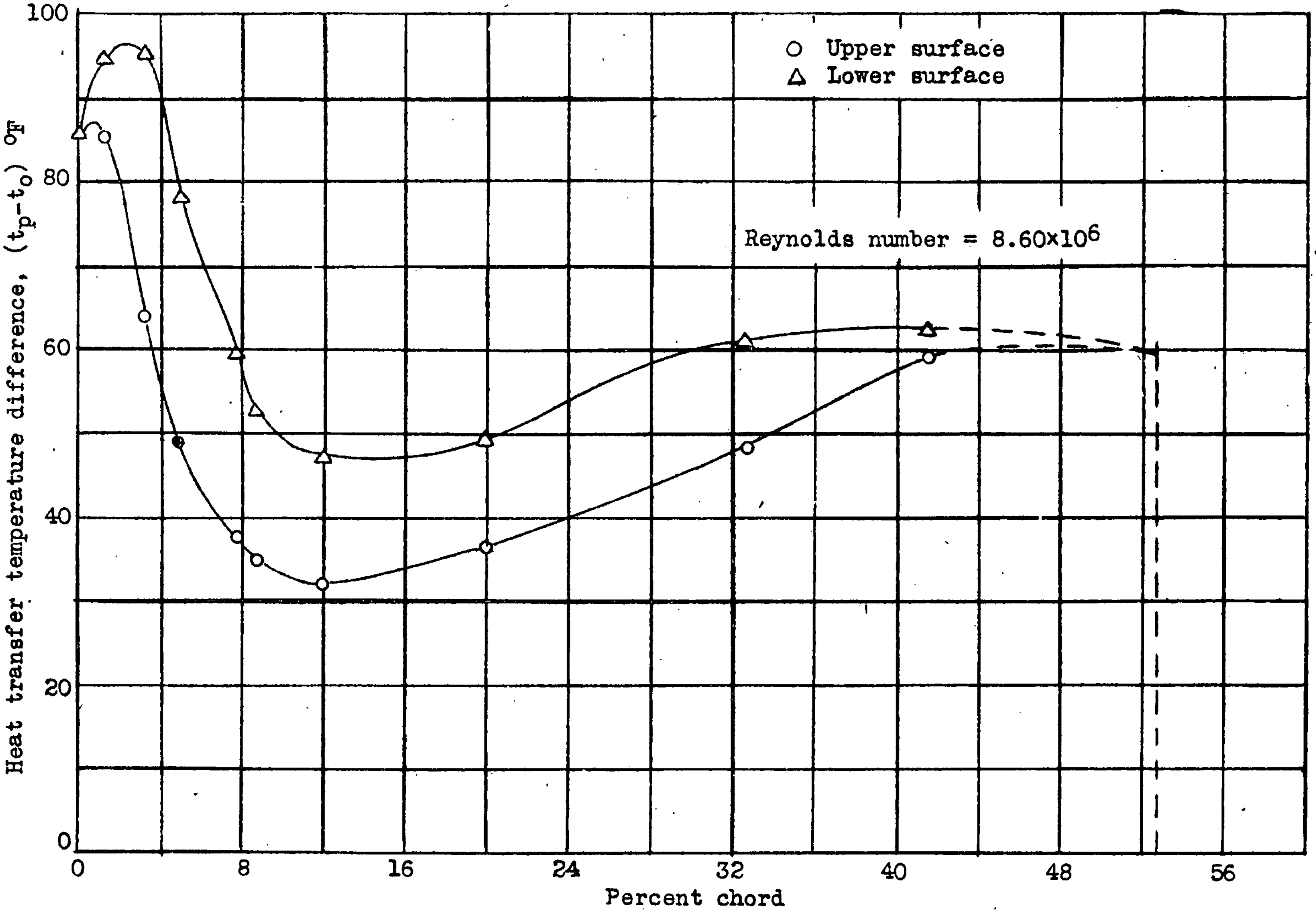

Figure 7.- Chordwise distribution of temperature difference, (t_p-t_o), $\alpha=5^o$, transition fixed at 5% chord.

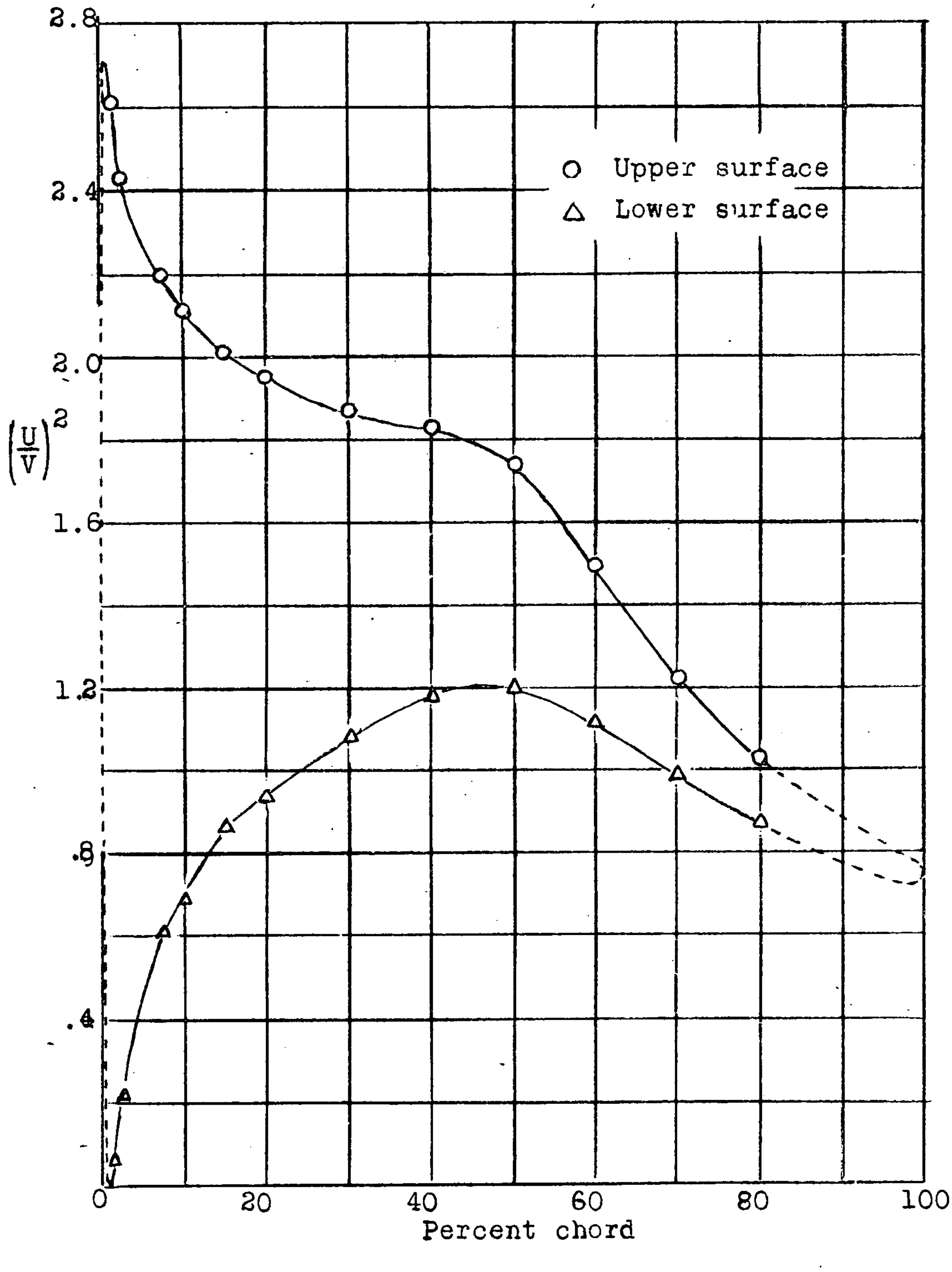

Figure 8.-Chordwise pressure distribution, $\alpha = 5^{\circ}$, R.N.$=8.60 \times 10^6$

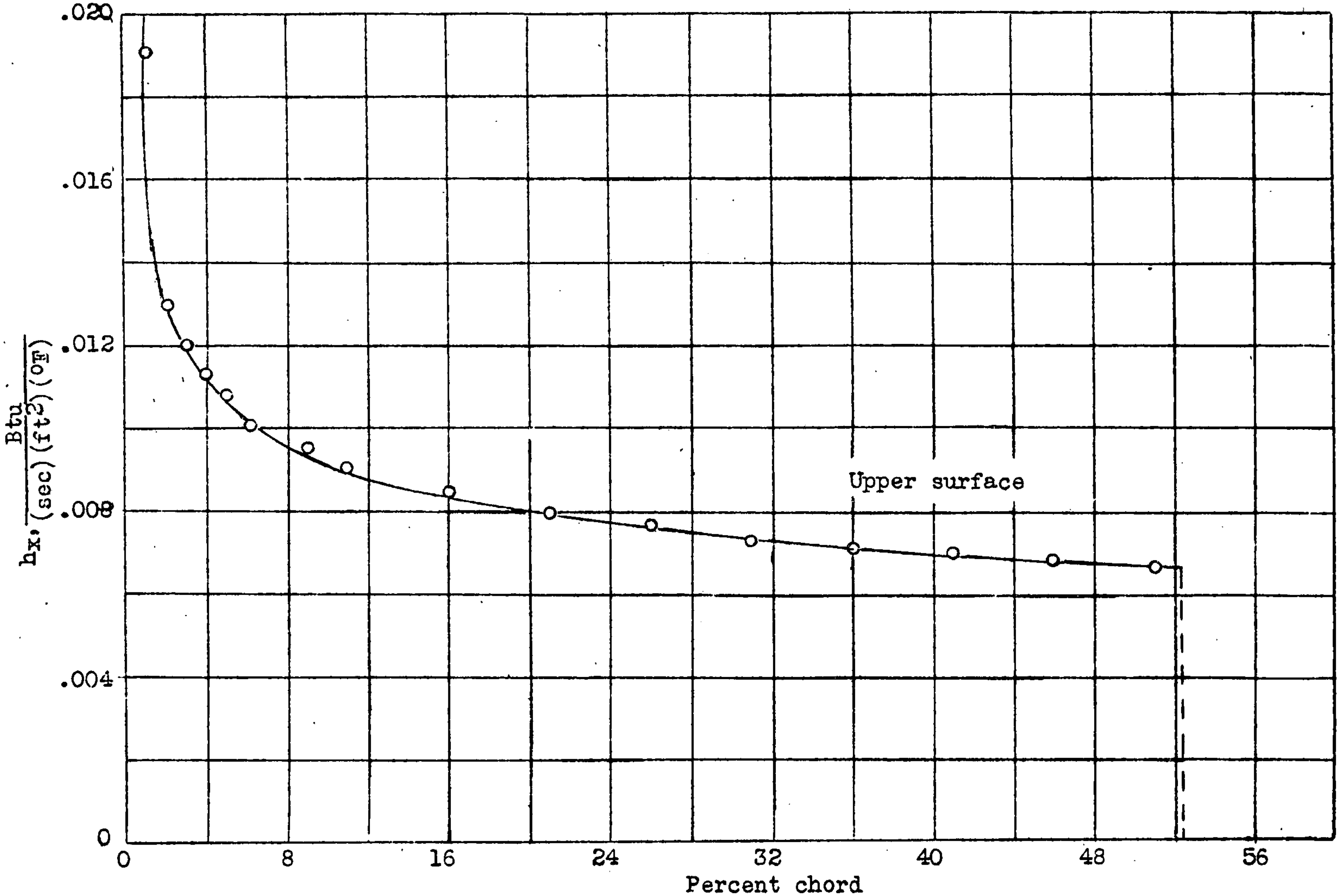

Figure 9.- Chordwise distribution of heat transfer coefficient, h_x, $\alpha=5^o$, transition free and transition fixed at 5% chord, Reynolds number = 8.69×10^6 and 8.60×10^6

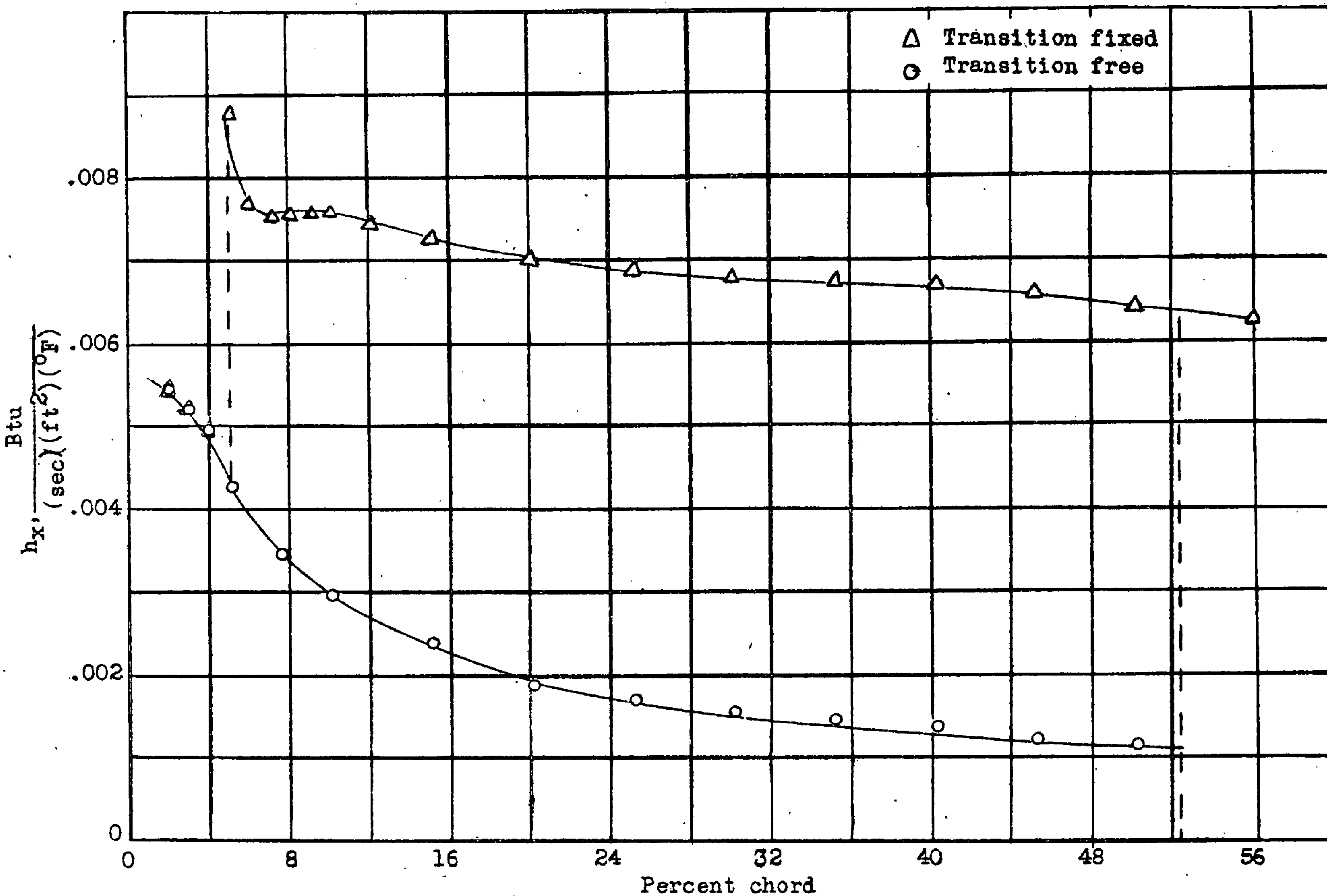

Figure 10.- Chordwise distribution of heat transfer coefficient, h_x, for lower surface, $\alpha=5^o$, transition free and transition fixed at 5% chord, Reynolds number = 8.69 and 8.60×10^6.