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AN ANALYSIS OF JET-PROPULSION SYSTEMS MAKING DIRECT USE OF THE WORKING SUBSTANCE OF A THERMODYNAMIC CYCLE

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ADVANCE CONFIDENTIAL REPORT

AN ANALYSIS OF JET-PROPULSION SYSTEMS MAKING DIRECT USE OF THE WORKING SUBSTANCE OF A THERMODYNAMIC CYCLE

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SUMMARY

An analysis has been made of jet-propulsion systems deriving their entire thrust from jet reaction of the products of combustion, with a view to aiding visualization of the possibilities and limitations of this class of jet propulsion. Equations are developed for ideal cycle, propulsive, and combined efficiencies and are extended to include provision for inefficiency in the components of the system.

The results of the analysis showed that the combined efficiency, which is unacceptably low at speeds less than 300 miles per hour, becomes nearly equal to that of conventional power plants of current design at 500 miles per hour. It was also shown that the efficiency will be increased considerably as the physical limitations on cycle temperature and blower speed are raised and as the efficiencies of the system components are improved. A brief supplementary analysis indicated that, for at least some conditions, the constant-volume-combustion intermittent-operation ram jet possesses an advantage over the constant-pressure-combustion continuous-flow ram jet of about $2\frac{1}{2}$:1 with respect to theoretical combined efficiency and about 2:1 with regard to theoretical power per unit mass air flow.

INTRODUCTION

Thrust for aircraft propulsion is always obtained as the force reaction to rearward acceleration of a gaseous mass, and propulsive systems may be classified according to the nature of the gases employed and the method by which they are accelerated. Rockets derive both the gases and the energy necessary for their acceleration from chemical reaction between constituents of the rocket charge. Thrust in the conventional engine-propeller propulsive system is obtained mostly from the rearward acceleration of air engaged by the propeller, and the energy to drive the propeller is obtained from combustion of a hydrocarbon fuel in the engine. The thrust derived from the propeller may be supplemented by that obtained from rearward acceleration of the engine exhaust gases, which is referred to as "jet thrust." "Jet propulsion" is a broad term covering a class of propulsive systems that, like conventional systems, derive their energy from combustion supported by the air in which the system operates. These systems differ from conventional systems in that all air accelerated to produce thrust is handled by internal-flow systems. Furthermore, much or all of the thrust developed by jet propulsion is due to rearward acceleration of gases that have acted as working substances for the thermodynamic cycle from which the energy for propulsion is obtained.

Jet propulsion is not a new idea, nor can the failure to employ it heretofore be attributed to a lack of understanding or appreciation of its merits. Airplane performance and the characteristics of the components of jet-propulsion engines formerly were such as to make jet propulsion hopelessly inefficient if not entirely impossible. Both the airplane and the jet engine components, however, have undergone continuous development and have recently attained such a degree of refinement that jet propulsion is rapidly becoming not only possible but also, in some respects, definitely advantageous.

The functions of converting heat into mechanical work and applying this mechanical work to producing thrust by rearwardly accelerating a mass of air are clearly separated in the conventional engine-propeller propulsive system. This clear-cut division greatly facilitates separate study of the efficiency of conversion of heat to work by the engine and of the efficiency of utilization of this work for propulsion by the propeller, and extensive research has been conducted in these fields. This independence of thermodynamic and propulsive efficiencies is lost when the gas that performs the heat engine cycle serves also as the mass accelerated to produce thrust and, consequently, the propulsive efficiency becomes intimately related to the thermodynamic efficiency without, however, impairing the usefulness of separate study of the thermodynamic and propulsive efficiencies.

Analyses of propulsion in which the working substance of the thermodynamic cycle is the only mass accelerated to produce thrust are presented in this paper, with a view to aiding visualization of the possibilities and limitations of this class of jet propulsion. The treatment is divided into three sections: a basic analysis of an ideal system in which the cycle, propulsive, and combined efficiencies and their interrelation are developed; an extension of the basic analysis to incorporate the effects of deviations from the ideal in the execution of the several phases of the thermodynamic cycle; and a study and discussion of the implications of the results of these analyses with regard to the possibilities and limitations of jet propulsion.

In the analyses when parameters such as blower or turbine efficiency, blower tip speed, or limiting temperature have been assumed for the purpose of illustrating the effects of the several variables, values have been selected that are considered reasonable or representative of current practice. All necessary equations, however, have been included, so that similar analyses may be performed with whatever values are most in accord with the requirements of any specific case. Whenever possible, blower temperature rises have been interpreted in terms of the impeller tip speed of a single-stage centrifugal blower for purposes of illustration, as an aid to visualization of the actual physical conditions. No simple relation between temperature rise and tip speed can be applied to axial compressors as a class, however, and the blower speeds used throughout this report must be considered as restricted to those of centrifugal blowers of usual construction. All the efficiency relations, however, have been developed in terms of flight speed and temperature and are equally applicable to centrifugal blowers and axial compressors.

SYMBOLS

a_0 velocity of sound at atmospheric temperature, feet per second $\left(49.0\sqrt{T_0}\right)$

a_r velocity of sound at any reference temperature T_r, feet per second $\left(49.0\sqrt{T_r}\right)$

c_p specific heat at constant pressure of air (0.24 Btu/lb/°F)

c_v specific heat at constant volume of air (0.17 Btu/lb/°F)

F thrust, pounds

g ratio of weight to mass (32.2 lb/slug)

H altitude, feet

J Joule's mechanical equivalent of heat (778 ft-lb/Btu)

M_0 Mach number of flight $\left(V_0/a_0\right)$

m mass air-flow rate, slugs per second

p absolute static pressure, pounds per square foot

P_F thrust power, foot-pounds per second

P_w power to wake, foot-pounds per second

P_{in} input power, foot-pounds per second

Q quantity of heat added per unit weight of gas, Btu per pound

R ratio of static pressure at combustion-chamber outlet to pressure at combustion-chamber inlet

r thrust-power ratio (ratio of $\eta\,\Delta T_c$ to the value of $\eta\,\Delta T_c$ for the datum conditions of $V_0 = 500$ mph, $H = 30,000$ ft, $T_0 = 411^\circ$ F abs., $T_{max} = 1500^\circ$ F abs., $\eta_B = 0.80$, $\eta_T = 0.75$, $R = 0.916$, and $V_t = 1200$ fps)

S entropy, Btu per pound per °F

T temperature, °F absolute

T_{max} maximum allowable temperature, °F absolute $\left(T_0 + \Delta T_s + \Delta T_B + \Delta T_c\right)$

T_0 atmospheric temperature, °F absolute

T_r reference temperature, °F absolute

V_0 flight speed, feet per second unless otherwise specified

V_t blower tip speed, feet per second

E_w mechanical output of a thermodynamic cycle in heat units, Btu per pound

ΔT_B blower temperature rise, °F

ΔT_c combustion temperature rise, °F

ΔT_N nozzle temperature drop, °F

ΔT_s stagnation temperature rise, °F

$$\left(\frac{1}{2gJc_p}V_0^2 = 0.832\left(\frac{V_0}{100}\right)^2\right)$$

ΔT_T turbine temperature drop, °F

ΔV net velocity change imposed on propulsive air in passing through the system, measured between stations of equal static pressure ahead of and behind the airplane

Δ increment

$\gamma = \frac{c_p}{c_v}$ (1.4)

η_B blower efficiency

η_c thermodynamic-cycle efficiency

η_p	propulsive efficiency
η_T	turbine efficiency
η	combined efficiency $(\eta_c \eta_p)$
0,...5	stations in jet-propulsion system (see fig. 1)

Condition designations:

0,...5	conditions at corresponding stations in jet-propulsion system when compression and expansion are isentropic and combustion is at constant pressure (see fig. 2)
2',...5'	same as conditions 2,...5 except compression and expansion are not isentropic (see fig. 9)
2",...5"	same as conditions 2',...5' except pressure loss occurs during combustion (see fig. 11)
2a',2b',3a' 3b',3a",3b"	special conditions in figures 9 and 11

Subscripts:

in	input
out	output
out_c	basic heat rejection associated with performance of cycle
out_B	heat rejection associated with increase of entropy in blower
out_T	heat rejection associated with increase of entropy in turbine
out_C	heat rejection primarily associated with excess entropy increase during combustion as a result of pressure loss in combustion process

Symbols given as condition designations are used as identifying subscripts to symbols denoting properties of the working substance at the conditions designated.

ANALYSIS

Basic Assumptions

For simplicity, the entire analysis has been idealized in two important respects. Only the heating effect of the fuel is considered; that is, the contribution of the fuel to the mass of the exhaust gas is disregarded. The working substance of the cycle, in addition, is assumed to be a perfect gas having the constant specific heats of air under standard conditions. The efficiency relations are considerably simplified through the adoption of these assumptions, and it is believed that the consequent greater ease of visualization of the effects of dominant variables outweighs the loss of accuracy involved. In the excellent treatment of combustion-turbine efficiencies given in reference 1, the effect of these simplifying assumptions is shown to be small and to lead to slightly conservative results.

Further simplification of the analysis has been obtained by assuming that the velocities at the combustion-chamber inlet and outlet are sufficiently low to permit interchangeable use of total pressure and static pressure. Installation losses, such as those in ducting external to the essential parts of the jet unit, have been disregarded because of their relative unimportance in efficient installations.

Ideal-System Efficiencies

Thermodynamic-cycle efficiencies.- Of the many different thermodynamic cycles employed for converting heat to mechanical energy, only two are at present of importance in jet propulsion. One of the cycles, in which combustion occurs at constant volume, is employed only for highly specialized applications, and the discussion of this cycle is limited to a treatment in the appendix of its more important characteristics. The other cycle, which is the basic ideal cycle of almost all jet-propulsion systems, is composed of four phases: isentropic compression, constant-pressure addition of heat, isentropic expansion, and constant-pressure rejection of heat. This thermodynamic cycle was first applied in the Brayton complete-expansion engine and is called the Brayton cycle.

The method by which the Brayton cycle is utilized for jet propulsion is illustrated schematically in figure 1. Air from the free stream is inducted and compressed, heat is added in the combustion chamber, and the heated air is expanded in the exit nozzle to produce a high-velocity propulsive jet. The compression may be obtained entirely from the dynamic pressure of flight, in which case the device is called a ram jet. The dynamic compression is more commonly supplemented by the use of an axial-flow compressor, or a centrifugal blower, usually driven directly by a turbine that extracts the necessary energy from the heated air before it is delivered to the propulsive jet. For a discussion of the ideal cycle, however, it is unnecessary to distinguish between the various means of accomplishing the desired compression and expansion.

The Brayton cycle is illustrated thermodynamically by a pressure-volume diagram in figure 2(a) and by a temperature-entropy diagram in figure 2(b). The cycle efficiency is readily obtained from a study of the quantities of heat supplied and rejected. The heat input to a pound of gas is given by

$$\begin{aligned} Q_{in} &= c_p \, \Delta T_c = c_p\left(T_3 - T_2\right) \\ &= \int_2^3 T \, dS \end{aligned} \tag{1}$$

where ΔT_c is the temperature rise in the combustion phase of the cycle. This heat input is represented graphically in figure 2(b) by the area A23BA. The heat rejected is

$$\begin{aligned} Q_{out} &= c_p\left(T_5 - T_0\right) \\ &= \int_0^5 T \, dS \end{aligned} \tag{2}$$

indicated in figure 2(b) by the area A05BA. The difference between heat supplied and heat rejected is the amount of heat converted to mechanical work; that is,

$$E_W = c_p\left[\left(T_3 - T_2\right) - \left(T_5 - T_0\right)\right] \tag{3}$$

represented in figure 2(b) by the area 02350. The cycle efficiency is the ratio of the mechanical-work output to the heat input:

$$\eta_c = \frac{\text{Work output}}{\text{Heat input}} = \frac{c_p\left[\left(T_3 - T_2\right) - \left(T_5 - T_0\right)\right]}{c_p\left(T_3 - T_2\right)}$$

$$= 1 - \frac{T_5 - T_0}{T_3 - T_2} \tag{4}$$

Now, from the thermodynamic relations for isentropic processes,

$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_5} = \left(\frac{p_3}{p_5}\right)^{\frac{\gamma-1}{\gamma}}$$

and, since $p_2 = p_3$ and $p_5 = p_0$,

$$\frac{T_2}{T_0} = \frac{T_3}{T_5}$$

Thus

$$\eta_c = \frac{T_2 - T_0}{T_2}$$

$$= \frac{\Delta T_{0 \text{ to } 2}}{T_0 + \Delta T_{0 \text{ to } 2}} \tag{5}$$

that is, the thermodynamic efficiency of the ideal cycle is the ratio of the compression temperature rise to the absolute temperature at the end of compression. It should be noted that the cycle efficiency is independent of the amount of heat added.

The compression temperature rise $\Delta T_{0 \text{ to } 2}$ is the sum of the dynamic compression or stagnation temperature rise ΔT_s and the blower temperature rise ΔT_B; that is,

$$\Delta T_{0 \text{ to } 2} = \Delta T_s + \Delta T_B$$

The stagnation temperature rise is given by

$$\Delta T_s = \frac{1}{2gc_pJ} V_0^2$$

$$= 0.832\left(\frac{V_0}{100}\right)^2 \tag{6}$$

Another convenient expression for ΔT_s is

$$\frac{\Delta T_s}{T_0} = \frac{\gamma - 1}{2} M_0^2 \tag{7}$$

in which the Mach number M_0 is the ratio of flight speed to sonic velocity at atmospheric temperature. A convenient and reasonably accurate, although not always precise, expression for the temperature rise in a conventional centrifugal blower is (reference 2)

$$\Delta T_B = \frac{1}{gc_pJ} V_t^{\,2}$$

$$= 1.664\left(\frac{V_t}{100}\right)^2 \tag{8}$$

Also

$$\frac{\Delta T_B}{T_r} = (\gamma - 1)\left(\frac{V_t}{a_r}\right)^2 \tag{9}$$

where V_t/a_r is the ratio of impeller tip speed to sonic velocity at the reference temperature T_r. If an axial-flow compressor is used instead of a centrifugal blower, equations (8) and (9) require a factor determined by the characteristics of the compressor. Inasmuch as the principles involved are not altered, this factor is omitted for simplicity.

The special case of the ram jet is of interest. Because of the absence of a blower, the compression temperature rise depends only upon the flight speed, according to equation (6), and consequently from equation (5) the ideal cycle efficiency depends only upon flight speed and atmospheric temperature. Equation (5) may be rearranged to give, for the cycle efficiency of a ram jet,

$$\eta_c = \frac{1}{1 + \frac{T_0}{\Delta T_s}}$$

By substituting in this equation the expression for $T_0/\Delta T_s$ given by equation (7),

$$\eta_c = \frac{1}{1 + \frac{1}{\frac{\gamma - 1}{2} M_0^2}}$$

$$= \frac{1}{1 + \frac{2}{(\gamma - 1)M_0^2}} \tag{10}$$

which for $\gamma = 1.4$ reduces to

$$\eta_c = \frac{1}{1 + \frac{5}{M_0^2}} \tag{11}$$

The ideal cycle efficiency for a ram jet operating on the Brayton cycle, obtained from equation (11), is plotted as a function of flight Mach number in figure 3. The cycle efficiencies of Brayton cycle ram jets at subsonic flight speeds do not compare favorably with the ideal efficiencies of internal-combustion engines of current design, which may be as high as 50 percent.

The limitation imposed by flight speed on compression temperature rise is removed when the dynamic compression from flight speed is supplemented with additional compression from a blower. Now

$$\Delta T_{0\ \text{to}\ 2} = \Delta T_s + \Delta T_B$$

so that the cycle efficiency from equation (5) becomes

$$\eta_c = \frac{\Delta T_s + \Delta T_B}{T_0 + \Delta T_s + \Delta T_B}$$

$$= \frac{1}{1 + \frac{1}{\frac{\Delta T_s}{T_0} + \frac{\Delta T_B}{T_0}}} \tag{12}$$

By substituting for $\Delta T_s/T_0$ the expression given by equation (7), equation (12) becomes

$$\eta_c = \frac{1}{1 + \frac{1}{\frac{\gamma - 1}{2}M_0^2 + \frac{\Delta T_B}{T_0}}} \tag{13}$$

Inspection of equation (9) shows that the temperature by which ΔT_B is divided in equation (9) can be any temperature whatsoever, provided the sonic velocity corresponds to the temperature chosen. Therefore

$$\frac{\Delta T_B}{T_0} = 2\frac{\gamma - 1}{2}\left(\frac{V_t}{a_0}\right)^2$$

The equation for efficiency then becomes

$$\eta_c = \frac{1}{1 + \frac{1}{\frac{\gamma - 1}{2}M_0^2 + 2\frac{\gamma - 1}{2}\left(\frac{V_t}{a_0}\right)^2}} \tag{14}$$

which for $\gamma = 1.4$ reduces to

$$\eta_c = \frac{1}{1 + \frac{5}{M_0^2 + 2\left(\frac{V_t}{a_0}\right)^2}} \tag{15}$$

By similar reasoning, equation (15) can be extended to treat the case of multiple-stage compression, with the term $\left(V_t/a_0\right)^2$ replaced by the sum of the individual terms for the several stages of compression.

The variation of η_c with flight Mach number is shown for several values of V_t/a_0 in figure 4. The curve for $\frac{V_t}{a_0} = 0$ is the special case for straight dynamic compression or ram jet and is identical with the curve of figure 3. The ratio V_t/a_0 should not be confused with the true Mach number of flow at the impeller tips, since the velocity of sound at the impeller tip is considerably higher than that in the free stream because of the higher temperature. Although the limiting tip speeds of present-day compressors may lie in the neighborhood of sonic velocity, multistaging may be resorted to in order to obtain compression temperature rises as high as desired. The use of a single stage of compression at high but attainable tip speeds, however, offers at zero flight speed an efficiency over 50 percent greater and at sonic flight speed an efficiency about twice as great as the best efficiency possible with a Brayton cycle ram jet at any subsonic flight speed.

Propulsive efficiency.- When airplane thrust is developed by rearward acceleration of a mass of air, the energy required is expended in two parts, one useful and one wasted. The part that is spent in forcing the airplane forward may be considered useful, but the kinetic energy acquired by the propulsive mass of air is an unavoidable waste. In the subsequent discussion, power usefully expended on thrust is referred to as "thrust power" and that wasted as kinetic energy of the wake is referred to as "wake power." The ratio of thrust energy to the sum of thrust energy and waste energy is called the propulsive efficiency η_p and represents that fraction of the mechanical power available for propulsion which can be used in overcoming airplane drag.

An expression for propulsive efficiency in terms of flight speed and the velocity change imposed on the propulsive jet, measured between stations at equal static pressure ahead of and behind the airplane, is easily derived by considering the power expenditures. From Newton's law, the thrust is equal to the rate of change of momentum of the propulsive mass of air and may be written as

$$F = m\,\Delta V \tag{16}$$

where

F thrust, pounds

m mass flow of propulsive air, slugs per second

ΔV velocity change, feet per second

The thrust power is therefore

$$P_F = mV_0 \, \Delta V \tag{17}$$

where V_0 is the speed of the airplane in feet per second. The kinetic energy acquired by the air is one-half the product of the mass and the square of the velocity imposed on it and, accordingly, the power represented by kinetic energy of the wake is

$$P_w = \frac{1}{2}m(\Delta V)^2$$

The total power input required is therefore

$$P_{in} = mV_0 \, \Delta V + \frac{1}{2}m(\Delta V)^2 \tag{18}$$

and propulsive efficiency, which is the ratio of thrust power to input power, becomes

$$\eta_p = \frac{mV_0 \, \Delta V}{mV_0 \, \Delta V + \frac{1}{2}m(\Delta V)^2} \tag{19}$$

$$= \frac{1}{1 + \frac{1}{2}\frac{\Delta V}{V_0}} \tag{20}$$

When considered in conjunction with equation (16), this expression for propulsive efficiency takes on special significance. The thrust, which is the product of the mass flow and the velocity change, can be obtained either by small masses of air strongly accelerated or by large masses of air undergoing small acceleration. Propulsive efficiency, however, is independent of the mass flow of air and approaches unity as the velocity change approaches zero. Attainment of high propulsive efficiencies consequently demands a maximum mass flow and a minimum acceleration.

<u>Relation of propulsive efficiency to cycle efficiency in jet-propulsion systems.</u>- When the working substance of the thermodynamic cycle from which energy for propulsion is obtained is distinct from the propulsive mass of air, it is possible to select a propulsive device best suited to the requirements of efficiency and available space. In the jet-propulsion systems discussed herein, however, in which the cycle working substance is also the propulsive air mass, the velocity change imposed on the propulsive mass of air, and therefore the propulsive efficiency, is no longer independent of the thermodynamic cycle but becomes a function of the energy supplied and the thermodynamic efficiency of the cycle. An expression for propulsive efficiency in terms of cycle efficiency and factors dependent upon airplane flight speed and the amount of heat supplied can be obtained by equating the mechanical power available from the cycle to the sum of the thrust power and the wake power.

Equating the available power, which is the product of the heat supplied and the cycle efficiency, to the total power required for propulsion (equation (18)) yields

$$mgc_pJ\,\Delta T_c\,\eta_c = mV_0\,\Delta V + \frac{m}{2}(\Delta V)^2 \tag{21}$$

where ΔT_c is the temperature rise occurring in the constant-pressure addition of heat. Dividing through by $V_0^2/2$, substituting ΔT_s for $\dfrac{V_0^2}{2gc_pJ}$ in accord with equation (6), completing the square, and extracting the root yield

$$1 + \frac{\Delta V}{V_0} = \sqrt{1 + \frac{\Delta T_c}{\Delta T_s}\eta_c} \tag{22}$$

Equation (20) can be rewritten

$$\eta_p = \frac{2}{1 + \left(1 + \dfrac{\Delta V}{V_0}\right)} \tag{23}$$

which by substitution from equation (22) results in

$$\eta_p = \frac{2}{1 + \sqrt{1 + \frac{\Delta T_c}{\Delta T_s}\eta_c}} \tag{24}$$

Substituting the value of η_c from equation (12) in equation (24) results in an expression for the propulsive efficiency of the ideal Brayton cycle:

$$\eta_p = \frac{2}{1 + \sqrt{1 + \frac{\Delta T_c}{\Delta T_s} \frac{\Delta T_s + \Delta T_B}{T_0 + \Delta T_s + \Delta T_B}}} \tag{25}$$

Combined efficiency.- The efficiency of greatest interest is that which measures the percentage of heat supplied that can be converted to useful thrust. This efficiency, which is referred to as "combined efficiency," is the product of cycle efficiency and propulsive efficiency:

$$\eta = \eta_c \eta_p \tag{26}$$

or, from equations (26), (12), and (25),

$$\eta = \frac{\Delta T_s + \Delta T_B}{T_0 + \Delta T_s + \Delta T_B} \frac{2}{1 + \sqrt{1 + \frac{\Delta T_c}{\Delta T_s} \frac{\Delta T_s + \Delta T_B}{T_0 + \Delta T_s + \Delta T_B}}} \tag{27}$$

For ram jets, in which $\Delta T_B = 0$, equation (27) reduces to

$$\eta = \frac{\Delta T_s}{T_0 + \Delta T_s} \frac{2}{1 + \sqrt{1 + \frac{\Delta T_c}{T_0 + \Delta T_s}}} \tag{28}$$

which shows that the ideal combined efficiency is a function of only three variables: the stagnation temperature rise ΔT_s, which is solely a function of flight speed; the atmospheric temperature T_0, which under standard conditions is a function of altitude; and the combustion temperature rise ΔT_c, which is a function of the ratio of air to fuel.

The relation of the combined efficiency of a ram jet to the three fundamental variables of flight speed, altitude, and air-fuel ratio is shown in figure 5, in which the combined efficiency at several altitudes is plotted against flight speed for a number of representative air-fuel ratios. These curves show how rapidly the ideal combined efficiency falls off with diminishing flight speed because of the reduction in propulsive efficiency. The increase of efficiency with altitude, entirely due to the decrease in atmospheric temperature, is more clearly delineated in figure 6, which is a cross plot of combined efficiency against altitude for an air-fuel ratio of 40, corresponding to a combustion temperature rise of 1700° F. The efficiency ceases to vary with altitude at the tropopause, above which under standard conditions the atmospheric temperature is constant. The effect of varying the air-fuel ratio, and consequently the combustion temperature rise, is brought out in figure 7, which is a cross plot from figure 5 of combined efficiency against air-fuel ratio, and corresponding combustion temperature rise, at three flight speeds. Figure 7 shows clearly that the combined efficiency of a ram jet diminishes as the heat input per pound of air increases. This decrease in the combined efficiency is entirely due to the diminishing propulsive efficiency, since the ideal cycle efficiency is independent of heat addition.

Often it is preferable to express the combined efficiency in terms of the limiting maximum temperature T_{max} instead of the combustion temperature rise ΔT_c. Inasmuch as the maximum temperature occurs at the end of the combustion phase of the cycle,

$$\Delta T_c = T_{max} - \left(T_0 + \Delta T_s + \Delta T_B\right) \tag{29}$$

and the desired form is obtained directly from equation (27) by substituting the expression for ΔT_c given in equation (29). Thus

$$\eta = \frac{\Delta T_s + \Delta T_B}{T_0 + \Delta T_s + \Delta T_B} \frac{2}{1 + \sqrt{1 + \frac{T_{max} - (T_0 + \Delta T_s + \Delta T_B)}{\Delta T_s} \frac{\Delta T_s + \Delta T_B}{T_0 + \Delta T_s + \Delta T_B}}} \tag{30a}$$

The comparable equation for combined efficiency of a ram jet in terms of maximum temperature is

$$\eta = \frac{\Delta T_s}{T_0 + \Delta T_s} \frac{2}{1 + \sqrt{\frac{T_{max}}{T_0 + \Delta T_s}}} \tag{30b}$$

Illustrative curves for cycle efficiency, propulsive efficiency, and combined efficiency are plotted in figure 8(a) against speed for flight at 30,000 feet in standard air, at a selected T_{max} of 1500° F absolute, and with an impeller tip speed of 1200 feet per second. The cycle efficiency increases with flight speed at an increasing rate, because of the rise in ΔT_s, which increases with the square of the flight speed. The propulsive efficiency increases rapidly at low speeds, as a direct consequence of speed increase as indicated by equation (20). At higher speeds, the rate of increase of propulsive efficiency diminishes - the propulsive efficiency being asymptotic to unity, as evident from equation (20). The characteristics of the cycle and propulsive efficiencies unite in the combined efficiency to produce a variation with flight speed that is almost linear.

Pressure ratios corresponding to the conditions of figure 8(a) are given in figure 8(b). The pressure ratio of the blower decreases with increasing flight speed because of the increase in inlet temperature. The rate of increase of the ratio of inlet pressure to free-stream pressure, however, is sufficient to outweigh the decreasing blower pressure ratio, so that the over-all pressure ratio increases with flight speed.

Real-System Efficiencies

In practice it is not possible to realize the isentropic compression and expansion of the ideal Brayton cycle, and it usually is not feasible to perform the heat addition without loss of pressure. Failure to meet the requirements of the ideal cycle can be expected to result in loss of efficiency. Inefficiency in execution of the cycle phases, however, not only reduces over-all efficiency but also alters the trends of cycle efficiency and combined efficiency with variation in the fundamental parameters, particularly maximum allowable temperature and impeller tip speed.

Real cycle efficiency in terms of blower and turbine efficiency with constant-pressure combustion.- An expression for cycle efficiency in which the effect of blower and turbine inefficiency is included may be derived by analyzing the heat supply and rejection in the execution of the cycle. For clarity of exposition, the effect of pressure loss in the combustion phase is taken up following the treatment of the effects of blower and turbine inefficiency. Figure 9 gives a temperature-entropy diagram of the constant-pressure-combustion cycle, in which the dynamic compression at the inlet and the nozzle expansion at the exit are assumed to be isentropic processes, and combustion is assumed to occur without loss of pressure, but the blower compression and turbine expansion are permitted to have adiabatic efficiencies less than unity. With good design and under favorable circumstances, the conditions of isentropic inlet compression and isentropic nozzle expansion may be approximated in practice.

The adiabatic efficiency of a blower is defined as the ratio of the work of an isentropic compression to the work of actual compression between initial and final total pressures equal to those of the isentropic compression. For flow processes, the work input in each case is measured by the temperature rise of the air, so that

$$\eta_B = \frac{\Delta T_{\text{Isentropic compression}}}{\Delta T_{\text{Actual compression}}} \tag{31}$$

From figure 9,

$$\eta_B = \frac{\Delta T_{1 \text{ to } 2a'}}{\Delta T_{1 \text{ to } 2'}} \tag{32}$$

The adiabatic efficiency of a turbine is defined as the ratio of the mechanical work that is taken out of the expanding air to the work that could have been taken out by isentropic expansion between initial and final pressures equal to those of the actual process. In this case, the work taken out is measured by the actual temperature drop, and the work that could have been taken out is measured by the temperature drop for an isentropic expansion; thus

$$\eta_T = \frac{\Delta T_{\text{Actual expansion}}}{\Delta T_{\text{Isentropic expansion}}} \tag{33}$$

In figure 9,

$$\eta_T = \frac{\Delta T_{3' \text{ to } 4'}}{\Delta T_{3' \text{ to } 3a'}} \tag{34}$$

In the same way as for the analysis of the ideal cycle, the heat supplied is given by

$$\begin{aligned} Q_{in} &= c_p \Delta T_c = c_p\left(T_{3'} - T_{2'}\right) \\ &= \int_{2'}^{3'} T\,dS \end{aligned} \tag{35}$$

denoted by the area $A'2'3'C'A'$. As before, there is an associated heat rejection

$$Q_{out_C} = c_p\left(T_{3b'} - T_{2b'}\right)$$

$$= \int_{2b'}^{3b'} T\ dS \tag{36}$$

denoted by the area A'2b'3b'C'A'. This time, however, an additional heat rejection results from the entropy increase in the blower:

$$Q_{out_B} = c_p\left(T_{2b'} - T_0\right)$$

$$= \int_{0}^{2b'} T\ dS \tag{37}$$

denoted by the area A02b'A'A. Furthermore, there is a third heat rejection associated with the increase in entropy in the turbine:

$$Q_{out_T} = c_p\left(T_{5'} - T_{3b'}\right)$$

$$= \int_{3b'}^{5'} T\ dS \tag{38}$$

denoted by the area C'3b'5'B'C'. The total heat rejection thus is given by

$$Q_{out} = c_p\left[\left(T_{3b'} - T_{2b'}\right) + \left(T_{2b'} - T_0\right) + \left(T_{5'} - T_{3b'}\right)\right]$$

$$= \int_{0}^{5'} T\ dS \tag{39}$$

denoted by the area A05'B'A.

The efficiency of the cycle is equal to the ratio of the difference of heat supplied and rejected to the heat supplied:

$$\eta_c = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} \tag{40}$$

Substituting equations (35) to (39) in equation (40) and grouping give

$$\eta_c = \underbrace{\frac{\left(T_{3'} - T_{2'}\right) - \left(T_{3b'} - T_{2b'}\right)}{T_{3'} - T_{2'}}}_{\text{I}}$$

$$- \underbrace{\frac{T_{2b'} - T_0}{T_{3'} - T_{2'}}}_{\text{II}} - \underbrace{\frac{T_{5'} - T_{3b'}}{T_{3'} - T_{2'}}}_{\text{III}} \tag{41}$$

Group I of equation (41) is the efficiency of an ideal constant-pressure-combustion cycle operating between the same pressures as the real cycle but having isentropic compression and expansion; that is,

$$\frac{\left(T_{3'} - T_{2'}\right) - \left(T_{3b'} - T_{2b'}\right)}{T_{3'} - T_{2'}} = \frac{\Delta T_s + \eta_B\,\Delta T_B}{T_0 + \Delta T_s + \eta_B\,\Delta T_B} \tag{42}$$

The numerator of group II of equation (41) can be treated as the heat rejection from an ideal Brayton cycle operating between the same pressures as the real cycle - therefore having an efficiency equal to that of equation (42) - and having a heat input equal to the part of the blower input in excess of that required for isentropic compression. The denominator is ΔT_c, so that group II of equation (41) becomes

$$\frac{T_{2b'} - T_0}{T_{3'} - T_{2'}} = \frac{\Delta T_B \left(1 - \eta_B\right)}{\Delta T_c} \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B} \tag{43}$$

The numerator of group III of equation (41) can be considered as the heat rejection from an ideal Brayton cycle operating between the same pressures as the expansion nozzle and having as heat input the difference between the actual shaft output of the turbine and the output that could have been obtained from the same turbine pressure drop if the expansion had been isentropic. The actual temperature drop through the turbine is a measure of the turbine shaft output and must be equal to the blower shaft input, which is measured by the blower temperature rise ΔT_B. Since the turbine output for actual expansion is therefore measured by ΔT_B, the output if the expansion had been isentropic would have been measured by $\Delta T_B/\eta_T$ and the difference is $\frac{\Delta T_B}{\eta_T}\left(1 - \eta_T\right)$. The numerator of group III of equation (41) is therefore

$$\Delta T_B \frac{1 - \eta_T}{\eta_T} \frac{T_{3b'}}{T_{3a'}}$$

The denominator again is ΔT_c, so that

$$\frac{T_{5'} - T_{3b'}}{T_{3'} - T_{2'}} = \frac{\Delta T_B}{\Delta T_c} \frac{1 - \eta_T}{\eta_T} \frac{T_{3b'}}{T_{3a'}} \tag{44}$$

Now

$$\begin{aligned} T_{3a'} &= T_{max} - \frac{\Delta T_B}{\eta_T} \\ &= \frac{T_{max}}{\eta_T}\left(\eta_T - \frac{\Delta T_B}{T_{max}}\right) \end{aligned} \tag{45}$$

and

$$T_{3b'} = T_{max} \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B} \tag{46}$$

so that group III of equation (41) is given as

$$\frac{T_{5'} - T_{3b'}}{T_{3'} - T_{2'}} = \frac{\Delta T_B}{\Delta T_c} \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B} \frac{1 - \eta_T}{\eta_T - \frac{\Delta T_B}{T_{max}}} \tag{47}$$

Substituting expressions (42), (43), and (47) for groups I, II, and III, respectively, in equation (41) gives

$$\eta_c = \underbrace{\frac{\Delta T_s + \eta_B \Delta T_B}{T_0 + \Delta T_s + \eta_B \Delta T_B}}_{\mathrm{I}}$$

$$- \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B} \frac{\Delta T_B}{\Delta T_c} \left(\underbrace{1 - \eta_B}_{\mathrm{II}} + \underbrace{\frac{1 - \eta_T}{\eta_T - \frac{\Delta T_B}{T_{max}}}}_{\mathrm{III}} \right) \tag{48}$$

Further condensation of equation (48) is possible, and derivation by other methods leads more directly to the condensed form. The form used was selected because it brings out more clearly the effects of the several component efficiencies on the over-all cycle efficiency.

Group I of equation (48) shows the primary effect of blower efficiency; is the efficiency of an ideal cycle operating at the reduced blower-pressure-rise ratio, which results from the blower losses; and is independent of the heat addition to the cycle. Since groups II and III are both multiplied by $\Delta T_B/\Delta T_c$, which approaches zero as a limit as ΔT_c becomes infinite, group I is therefore the cycle efficiency approached as a limit as the heat input becomes infinite. Group II of equation (48) is a subtractive term that represents the reduction of useful output chargeable to compressor losses. The compressor losses are determined by the process within the compressor and are independent of the rest of the cycle. Their importance therefore diminishes as the output of the cycle is increased by increasing ΔT_c. Group III of equation (48) is a subtractive term that represents the reduction of useful output chargeable to turbine inefficiency. The amount of this loss is determined primarily by the turbine efficiency, but secondary effects stem from the blower loss and the amount of heat addition. At any given turbine efficiency less than unity, an increased demand on the turbine as a result of low blower efficiency means an increase in the absolute value of the turbine loss. Not all the difference, however, between the heat withdrawn from the gases in the turbine and the heat that would have been required if the expansion had been isentropic is rejected, and the percentage of energy recovered increases with maximum temperature of the cycle. Whether the greater improvement in cycle efficiency results from a given increase in blower efficiency or from the same increase in turbine efficiency depends on the operating condition. Partial differentiation of equation (48) with respect to η_B and η_T leads to an expression for the relative effectiveness in changing the cycle efficiency of changes in blower efficiency or turbine efficiency:

$$\frac{\dfrac{\text{Change of } \eta_c}{\text{Change of } \eta_B}}{\dfrac{\text{Change of } \eta_c}{\text{Change of } \eta_T}} = \frac{\eta_T\left(\eta_T T_{max} - \Delta T_B\right)}{T_0 + \Delta T_s + \eta_B \Delta T_B} \tag{49}$$

Equation (49) shows that the effectiveness in changing the cycle efficiency of changes in blower efficiency increases rapidly with increasing turbine efficiency but usually is not greatly affected by changes in the blower efficiency itself. This effect is illustrated in figure 10 for flight at 500 miles per hour at an altitude of 30,000 feet with a maximum temperature of 1500° F absolute. The applicability of equation (49) to design studies is illustrated by figure 10(b), in which the relative importance of blower and turbine efficiencies is given for a constant pressure ratio instead of at constant blower speed.

<u>Effect on cycle efficiency of pressure loss in combustion</u>.- If the ratio of pressure after combustion to pressure before combustion R is less than unity, equation (48) is modified in two respects. One modification is an additional heat rejection due to the excess entropy increase during combustion, which results from the pressure loss and is given by

$$Q_{out_C} = c_p\left(T_{3b''} - T_{3b'}\right)$$

$$= \int_{3b'}^{3b''} T\ dS \qquad (50)$$

and represented in figure 11 by the area C'3b'3b"C"C'. The heat rejection associated with turbine loss becomes

$$Q_{out_T} = c_p\left(T_{5''} - T_{3b''}\right)$$

$$= \int_{3b''}^{5''} T\ dS \qquad (51)$$

denoted in figure 11 by the area C"3b"5"B"C". The total heat rejection including the effect of pressure loss in combustion then is

$$Q_{out} = c_p\left[\left(T_{3b'} - T_{2b'}\right) + \left(T_{2b'} - T_0\right) + \left(T_{5''} - T_{3b''}\right) + \left(T_{3b''} - T_{3b'}\right)\right]$$

$$= \int_0^{5''} T\ dS \qquad (52)$$

denoted by the area A05"B"A. Dividing through by $c_p\left(T_{3'} - T_{2'}\right)$ gives as the cycle efficiency

$$\eta_c = \underbrace{\frac{\left(T_{3'} - T_{2'}\right) - \left(T_{3b'} - T_{2b'}\right)}{T_{3'} - T_{2'}}}_{\mathrm{I}} - \underbrace{\frac{T_{2b'} - T_0}{T_{3'} - T_{2'}}}_{\mathrm{II}}$$

$$- \underbrace{\frac{T_{5''} - T_{3b''}}{T_{3'} - T_{2'}}}_{\mathrm{III}} - \underbrace{\frac{T_{3b''} - T_{3b'}}{T_{3'} - T_{2'}}}_{\mathrm{IV}} \tag{53}$$

Group I of equation (53) is identical with group I of equation (41) and therefore with the form used in equation (42). Group II of equation (53) is the same as group II of equation (41) and can be rewritten as in equation (43). The heat rejection associated with turbine loss, given by group III of equation (53), is

$$\frac{T_{5''} - T_{3b''}}{T_{3'} - T_{2'}} = \frac{\Delta T_B}{\Delta T_c} \frac{1 - \eta_T}{\eta_T} \frac{T_{3b''}}{T_{3a''}} \tag{54}$$

Comparison of equation (54) with equation (44) shows that the second modification of equation (48) due to combustion pressure loss is an increase in the turbine-loss heat rejection. Now

$$T_{3a''} = T_{3a'}$$

so that, by use of equation (45),

$$T_{3a''} = \frac{T_{max}}{\eta_T} \left(\eta_T - \frac{\Delta T_B}{T_{max}}\right) \tag{55}$$

Also

$$\frac{T_{3b''}}{T_{max}} = \frac{T_{3b'}}{T_{max}} \frac{1}{R^{\frac{\gamma-1}{\gamma}}}$$

$$= \frac{1}{R^{\frac{\gamma-1}{\gamma}}} \frac{T_0}{T_0 + \Delta T_s + \eta_B \, \Delta T_B} \tag{56}$$

where R is the ratio $p_{3''}/p_{2'}$. Thus

$$\frac{T_{3b''}}{T_{3a''}} = \frac{1}{R^{\frac{\gamma-1}{\gamma}}} \frac{\eta_T}{\eta_T - \frac{\Delta T_B}{T_{max}}} \frac{T_0}{T_0 + \Delta T_s + \eta_B \, \Delta T_B} \tag{57}$$

and group III of equation (53) reduces to

$$\frac{T_{5''} - T_{3b''}}{T_{3''} - T_{2'}} = \frac{\Delta T_B}{\Delta T_c} \frac{T_0}{T_0 + \Delta T_s + \eta_B \, \Delta T_B} \frac{1}{R^{\frac{\gamma-1}{\gamma}}} \frac{1 - \eta_T}{\eta_T - \frac{\Delta T_B}{T_{max}}} \tag{58}$$

The expression for group III of equation (53), given in equation (58), differs from group III of equation (41), as expressed in equation (47), by the presence of a factor $\frac{1}{R^{\frac{\gamma-1}{\gamma}}}$.

By substituting the expression for $T_{3b''}$ from equation (56) and the expression for $T_{3b'}$ from equation (46) in group IV of equation (53), the efficiency correction for excess entropy increase during combustion that results from pressure loss reduces to

$$\frac{T_{3b''} - T_{3b'}}{T_{3'} - T_{2'}} = \left(\frac{1}{R^{\frac{\gamma-1}{\gamma}}} - 1\right) \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B} \frac{T_{max}}{\Delta T_c} \tag{59}$$

Substitution in equation (53) of the expressions from equations (42), (43), (58), and (59) for groups I to IV yields the general equation for the thermodynamic efficiency of a real, or modified, Brayton cycle in which provision is made for inefficiency of compressor and turbine and for pressure loss in combustion:

$$\eta_c = \underbrace{\frac{\Delta T_s + \eta_B \Delta T_B}{T_0 + \Delta T_s + \eta_B \Delta T_B}}_{\mathrm{I}}$$

$$- \frac{\Delta T_B}{\Delta T_c} \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B} \left(\underbrace{1 - \eta_B}_{\mathrm{II}} + \frac{1}{R^{\frac{\gamma-1}{\gamma}}} \underbrace{\frac{1 - \eta_T}{\eta_T - \frac{\Delta T_B}{T_{max}}}}_{\mathrm{III}}\right)$$

$$- \underbrace{\left(\frac{1}{R^{\frac{\gamma-1}{\gamma}}} - 1\right) \frac{T_{max}}{\Delta T_c} \frac{T_0}{T_0 + \Delta T_s + \eta_B \Delta T_B}}_{\mathrm{IV}} \tag{60}$$

Propulsive efficiency and combined efficiency of the real cycle.- In deriving equation (24) for the propulsive efficiency of an ideal thermodynamic cycle, the heat input was measured by $c_p \Delta T_c$. This condition must be met for equation (24) to be valid. For the case under consideration, in which the heat is added in a process subject to some loss of pressure, the condition that $c_p \, \Delta T_c$ be a measure of the heat addition is satisfied and equation (24) is valid, provided the expression for η_c used in equation (24) is that from equation (60). The effect of blower losses, combustion-chamber pressure drop, and turbine inefficiency on the propulsive efficiency is entirely the result of changes in the amount of heat converted to mechanical work. Since the combined efficiency is the product of the cycle efficiency and the propulsive efficiency, the final expression for combined efficiency can be stated as

$$\eta = \eta_c \eta_p \tag{61}$$

where η is for the real cycle, η_c is obtained from equation (60), and η_p is obtained from equation (24) through use of equation (60).

RESULTS AND DISCUSSION

Classification of variables.- The cycle efficiency of equation (60), the propulsive efficiency of equation (24), and their product, the combined efficiency, are functions of seven variables or parameters: T_0, ΔT_s, η_B, η_T, R, T_{max}, and ΔT_B. These seven variables may be classified in three groups according to the nature of the control over them that is available to the jet-power-plant designer. In the subsequent discussion, following establishment of this classification, the effects on combined efficiency and thrust power of changes in each of the seven variables are taken up in turn. Because of the large number of interrelated variables, in most cases these effects are illustrated by curves showing the result of deviation in one variable at a time from the following set of conditions selected as a reference datum and considered either reasonable or representative of current practice:

Condition	Corresponding to:
Altitude, 30,000 ft	$T_0 = 411^\circ$ F abs.
Flight speed, 500 mph	$\Delta T_s = 45^\circ$ F
Blower efficiency, 80 percent	$\eta_B = 0.80$
Turbine efficiency, 75 percent	$\eta_T = 0.75$
Combustion-chamber pressure-drop ratio	$R = 0.916$
Maximum temperature	$T_{max} = 1500^\circ$ F abs.
Blower tip speed, 1200 fps	$\Delta T_B = 240^\circ$ F

Thrust-power variations are presented nondimensionally as the ratio of the thrust power at the condition in question to the thrust power of the datum condition.

The atmospheric temperature T_0 for standard air is a function of altitude alone and the stagnation temperature rise ΔT_s depends only on the speed of flight. The variables T_0 and ΔT_s therefore can be classified as flight conditions that are to be met by the designer instead of adjusted to his requirements. The component efficiencies η_B and η_T and the combustion-chamber pressure-drop ratio R can be classified as parameters expressing the characteristics of the components of the propulsive system. The other two variables T_{max} and ΔT_B are in the nature of variables on which operating limits may be set by physical limitations. The maximum allowable temperature T_{max} is determined by the properties of the materials employed and by the stresses to be carried. For a predetermined number of blower stages, the maximum permissible blower temperature rise ΔT_B is limited by the maximum allowable blower tip speed, which is in turn determined by the design and materials employed in the construction of the blower. Since attainment of maximum output, and usually maximum efficiency, requires utilization of maximum permissible heat inputs and blower speeds, T_{max} and ΔT_B at maximum output can be considered as design parameters expressing operating limits imposed by the design and choice of materials. If the maximum temperature is appreciably above that shown to be required for best efficiency when the blower is operating at its maximum tip speed, power output can be controlled quite efficiently by variation in combustion temperature rise. With this exception, however, reduction in maximum operating limits is associated with marked reductions in efficiency, regardless of the power output.

Flight conditions.- The combined efficiency of a jet-propulsion system is strongly influenced by the speed of flight, and the suitability of jet propulsion to any particular application is determined by the airplane performance requirements. Figure 12 shows the variation of combined efficiency with airplane speed for flight at several altitudes, at representative values for system-component efficiencies, and at operating limits. The combined efficiency increases almost linearly with flight speed and shows an appreciable advantage in operation at the lower atmospheric temperatures at high altitude. The greater physical dimensions required for handling the low-density air at high altitude must not be overlooked, however, in considering the advantages of high-altitude operation, since a decrease in component efficiency caused by insufficient size could easily outweigh the advantage of the lower temperatures.

A breakdown of the combined efficiency into cycle and propulsive efficiencies for flight at 30,000 feet with representative component and operating parameters is given in figure 13. Dashed lines of the corresponding efficiencies of an ideal system operating at the same maximum temperatures and blower tip speed have been added for comparison. The almost linear variation with flight speed of the combined efficiency is seen to be the net effect of an increasing rate of increase in cycle efficiency and a decreasing rate of increase in propulsive efficiency. The very low combined efficiency at low flight speeds is due largely to low propulsive efficiency.

The power developed per unit mass of air handled is proportional to the combined efficiency and the heat input. Variation in thrust power is indicated in figure 13 by a thrust-power ratio r, which is the ratio of $\eta\,\Delta T_c$ to the value of $\eta\,\Delta T_c$ for the datum condition at 500 miles per hour. The thrust-power factor increases with flight speed as does the combined efficiency but at a lesser rate because of a slight decrease in ΔT_c as the stagnation temperature rise increases with flight speed.

Jet-propulsion combined efficiencies for flight in standard air at 30,000 feet from figure 13 are compared in figure 14 with estimated efficiencies for a conventional engine-propeller system designed for efficient operation at high speed. Because this analysis extends to flight speeds beyond the range of current knowledge, the efficiency curves for the engine-propeller system are only rough estimates and must not be considered as other than such. These estimates are based on the best available propeller data and brake specific fuel consumptions of 0.76 pound per horsepower-hour at military power and 0.46 pound per horsepower-hour at cruising power. The lower curve of engine-propeller efficiency is for military power at all speeds, and the upper curve indicates roughly the efficiency that could be attained if it were possible to maintain the brake specific fuel consumption of 0.46 pound per horsepower-hour up to the highest speeds. The separation of the curves shows the difference between the engine specific fuel consumptions for cruising power and military power, and the curvature of the lines shows the variation in propeller efficiency.

On the basis of the curves of figure 14, jet propulsion at flight speeds less than 300 miles per hour is hard to justify because of the low efficiency. For special applications at speeds higher than 300 miles per hour, the simpler and more compact construction of the jet motor may warrant the sacrifice in efficiency. At a flight speed of 540 miles per hour, the jet motor can compete with conventional systems in high-power operations and, as the flight speed is further increased, the efficiency of the jet motor rapidly exceeds the military-power performance and approaches the efficiency of cruising operation of conventional systems. It is quite clear, therefore, that on the basis of efficiency the primary field of application of jet propulsion of the type discussed herein is to flight at speeds of 500 miles per hour or higher. At low speeds, the mass of air necessary for efficient propulsion greatly exceeds that required as a working substance in the thermodynamic cycle, and acceptable efficiencies cannot be obtained without engagement of auxiliary propulsive air which is actuated by some means other than performance of the main thermodynamic cycle.

Component efficiencies.- The components of a jet-propulsion engine - compressor, turbine, and combustion chamber - are still in a stage of development from which appreciable improvement in efficiencies can be expected. Gains in cycle efficiency through improvement of component efficiencies are offset to some extent by the reduction in propulsive efficiency that accompanies an increase in cycle efficiency. The combined efficiency of the real cycle illustrated in figure 13, however, is only about 65 percent of the ideal efficiency throughout the flight-speed range, and appreciable gains are to be expected from improved component efficiencies.

The extent to which the combined efficiency can be increased through improvement of the compressor alone is illustrated in figure 15, which shows the variation with blower efficiency of combined efficiency, propulsive efficiency, cycle efficiency, and thrust-power ratio at a flight speed of 500 miles per hour and with all other conditions constant at the datum values. The effect of variations in blower efficiency is very marked, because inefficient compression not only causes a direct loss of power in the compressor itself but also, by decreasing the pressure-rise ratio, decreases the cycle efficiency and, by requiring a greater turbine output, increases the power loss in the turbine. Attainment of an efficiency of 0.90 in the blower is shown in figure 15 to increase the combined efficiency from the datum value of 0.135 to 0.156 and, by reference to figure 14, it can be seen that the flight speed at which the efficiencies of conventional and jet-propulsion systems are equal is reduced from 540 miles per hour to about 500 miles per hour. The gains to be had from increasing the blower efficiency are interesting, but the most important aspect of figure 15 is the indication of the penalty incurred through failure to obtain creditable efficiency in the blower. It should be noted that the cycle and combined efficiencies go to zero at a blower efficiency of 40 percent. Inasmuch as improper installation can reduce the efficiency of a blower almost to this value, the vital importance of careful installation is obvious. Variation in blower efficiency does not affect the allowable combustion temperature rise, and the curve of thrust-power factor therefore shows directly the influence of variations of combined efficiency.

The influence of the turbine efficiency is shown in figure 16, in which the cycle, propulsive, and combined efficiencies and the thrust-power ratio are plotted against turbine efficiency with all other conditions at the reference datum. The turbine losses compound less rapidly than the blower losses in the higher efficiencies and, for the conditions chosen, the gain to be had from improvement of the turbine is somewhat less than the gain for improving the blower. That this fact is not necessarily true for other conditions was shown in figure 10. At low turbine efficiencies, the recoverable fraction of the turbine loss diminishes rapidly with decrease in turbine efficiency, and the cycle efficiency becomes zero at a turbine efficiency of 0.50, which is appreciably higher than the blower efficiency at which the cycle efficiency became zero. It therefore can be seen that attainment of reasonably high efficiencies is even more important for the turbine than for the compressor but that there is not so great a premium on getting the highest possible efficiency. The combustion temperature rise is independent of the turbine efficiency and, as a consequence, the curve of thrust-power ratio shows the effect of the variation in combined efficiency.

The relative importance of combustion-chamber pressure drop is shown in figure 17 for flight at 500 miles per hour in standard air at 30,000 feet and with other conditions as assumed in figure 13. Although there is a definite loss in efficiency chargeable to combustion-chamber pressure drop, this loss for the conditions illustrated is small compared with the losses associated with blower and turbine inefficiency, and only a moderate increase in efficiency is to be had from total elimination of combustion-chamber pressure losses. Since the combustion temperature rise is unaffected by the pressure loss in the combustion chamber, the thrust-power factor shows the effect of the variation in combined efficiency.

The combustion-chamber pressure-drop ratio at which the efficiency becomes zero corresponds to losses of absolute pressure far greater than those encountered in practice for compressor-turbine propulsive systems and, in this case, this condition is of little practical interest. As blower tip speed is decreased, however, the basic cycle efficiency decreases with little change in absolute magnitude of the loss associated with combustion-chamber pressure drop, which correspondingly increases in importance. For the limiting case of the ram jet, the efficiency is extremely sensitive to combustion-chamber pressure loss, as indicated in figure 18.

Operating limits.- The benefit to be derived from raising the limit on the maximum temperature of the cycle is intimately associated with the blower tip speed and the efficiencies of both blower and turbine. In the ideal cycle, the efficiency is independent of the maximum temperature. In the real cycle, however, the amount of heat that may be added is limited by the difference between the blower discharge temperature and the limiting maximum temperature. At a given blower speed, an increase in maximum temperature therefore permits an increase in heat supplied without in any way affecting the blower losses. Furthermore, the recoverable percentage of the turbine waste is slightly increased. Since the losses do not increase in absolute magnitude as the heat input increases, the losses diminish as a fraction of the total input, and the cycle efficiency is increased. This variation of cycle efficiency with limiting maximum temperature is illustrated in figure 19, in which the reference datum conditions have again been taken for a flight speed of 500 miles per hour. The continuous rise of cycle efficiency with increasing maximum temperatures, however, does not carry over to the combined efficiency. The rate at which the cycle efficiency increases diminishes rapidly and, at sufficiently high temperatures, the decrease of propulsive efficiency with heat addition outweighs the improvement in cycle efficiency so that further increase in maximum temperature begins to reduce the combined efficiency. It therefore should be noted that, for any particular combination of system characteristics and flight conditions, there exists an optimum maximum temperature above which the efficiency is decreased. The maximum power output, however, is by no means reached at this optimum temperature. The heat input to the cycle increases rapidly as the limit on maximum temperature increases, and as a result the thrust-power ratio continues to increase rapidly throughout the range. For the system characteristics chosen, a lowering of T_{max} below 1500° F absolute would sharply reduce the efficiency. On the other hand, raising T_{max} to 2000° F absolute would increase the power output by 65 percent with little loss of efficiency.

Blower speed also has an optimum value, as shown by the curves of figure 20 in which the efficiencies and thrust-power ratio are plotted as functions of blower tip speed. Although some increase in efficiency can be derived from an increase in blower speed, increases in blower speed to higher than about 1400 feet per second at fixed maximum temperature decrease the heat input sharply at the same time that the compression losses are being increased, and the efficiency falls off very rapidly. In the region of decreasing blower speeds, the curves level off and become asymptotic to the efficiencies of a ram jet as zero blower speed is approached. The decrease in allowable combustion temperature rise that accompanies an increase in blower speed has an appreciable effect on the thrust-power ratio, which reaches its peak value at blower speeds about 200 feet per second less than the speed for maximum efficiency.

Study of figures 19 and 20 shows that the optimum limiting maximum temperatures and blower speeds are so interconnected that an increase of one without an increase in the other may bring about a loss of efficiency instead of the increase which might have been expected without a thorough understanding of the situation. If, as blowers are improved and the limits on tip speed are raised, corresponding increases in allowable maximum temperatures are achieved, an appreciable gain in combined efficiency can be anticipated. This trend is shown in figure 21, which compares the efficiencies for $T_{max} = 1500^{\circ}$ F absolute with values calculated for $T_{max} = 2000^{\circ}$ F absolute at variable blower speed and with other conditions at the datum values. At a blower speed of 1200 feet per second $(\Delta T_B = 240^{\circ}$ F), there is little difference in combined efficiency for $T_{max} = 1500^{\circ}$ F absolute and $T_{max} = 2000^{\circ}$ F absolute because the higher cycle efficiency at the higher temperature is offset by a lower propulsive efficiency. The higher value of T_{max}, however, corresponds to a higher heat input and, for the same efficiency, the power output is much greater with the higher value of T_{max}. The higher maximum temperature shifts the blower temperature rise for maximum combined efficiency to a value corresponding to a blower speed of 1800 feet per second and thus results in a 20-percent increase in the maximum value of combined efficiency. The peak power output for $T_{max} = 2000^{\circ}$ F absolute occurs at a blower speed of 1400 feet per second and is 70 percent greater than the peak power for $T_{max} = 1500^{\circ}$ F absolute.

Values of combined efficiency, which cover a wider range of flight speed and limiting maximum temperatures, are plotted against ΔT_B in figure 22. This figure shows clearly the impossibility of obtaining acceptable efficiencies at low flight speeds with the type of jet propulsion discussed herein and also indicates the good efficiencies that can be obtained when the flight speed and operating limits are sufficiently high.

The interrelation of maximum temperature and blower speed is illustrated in figure 23, which is a plot of the peak efficiencies from figure 22. Figure 23 shows, for the datum values of the component characteristics, the maximum temperature necessary for best efficiency at any blower speed and the value of this maximum efficiency. The maximum temperature required for best efficiency at a given blower speed is seen to be quite insensitive to flight speed. In interpreting figure 23, reference should be made to figures 21 and 22, which show that the loss of combined efficiency due to a maximum temperature too low for the blower speed is acute. Reference to figure 19 shows that the loss of efficiency due to operation at a maximum temperature above that necessary for best efficiency is slight and that the power gain is quite large. The optimum maximum temperature from figure 23 may therefore be regarded as not merely desirable but essential to efficient operation. If the optimum value is higher than that permissible, a lower blower speed should be used.

SPECIAL CONSIDERATIONS

Jet augmentation.- The scope of the present report has been confined to an analysis of propulsion systems making direct use of the working substance and working on modifications of the Brayton cycle. Some mention has been made, however, of the possibility and necessity at low speeds of improving the propulsive efficiency by the engagement of additional masses of propulsive air. This may be done by using some of the turbine power to drive a propeller or fan or by using a jet pump.

Other cycles and regeneration.- Thermodynamic efficiencies exceeding those of the Brayton cycle may be obtained by various means. One approach is through use of the constant-volume-combustion cycle, as in the Holzwarth turbine, for which, however, the greater mechanical difficulties largely balanced out the theoretically higher cycle efficiencies. A discussion of the constant-volume-combustion cycle as applied to a ram jet is given in the appendix. A more promising approach for large power plants is through regeneration, in which some of the heat from the high-temperature exhaust is transferred to the air after compression and before combustion. The complications and weight inherent in the necessary heat exchangers are great, however, and such refinements in the thermodynamic cycle probably will be of more interest after the possibilities of improving the simpler system have been exhausted.

Supersonic jets.- For flight at subsonic speeds, the addition of heat alone cannot produce a supersonic jet and, in the absence of a blower, the jet is always subsonic. When a blower is used, however, the discharge velocity satisfying equation (22), and therefore equation (24), is supersonic at the higher blower speeds. Attainment of this supersonic velocity requires a convergent-divergent nozzle, the proportions of which should vary with conditions. The curves presented consequently involve the added assumption that, for each condition shown, the discharge nozzle is given the proper shape. In practice this variation in shape would hardly be feasible and, at conditions other than those for which the nozzle was designed, some reduction in performance might be expected. There is reason to believe, however, that the loss in performance through use of nozzles of inexact shape need not be excessive. This statement does not mean, however, that the nozzle area can be kept constant, because varying the nozzle area is essential to fully satisfactory engine-speed control.

CONCLUSIONS

A number of conclusions regarding the field of usefulness, limitations, and possibilities for improvement of jet-propulsion systems deriving their entire thrust from jet reaction of the products of combustion can be drawn from the analysis presented. Unless specifically otherwise stated, the following conclusions regarding the effect of changes in some of the variables are for the condition that all other variables are held constant:

1. On the basis of efficiency, the primary field of application of propulsion systems deriving their entire thrust from jet reaction of the products of combustion is to flight at speeds of 500 miles per hour or higher.

2. If applied to flight at speeds higher than about 550 miles per hour, the combined efficiency of such a jet-propulsion system exceeds the efficiency of conventional power plants of current design when operating at military power (bsfc = 0.76 lb/hp-hr) and approaches the efficiency of conventional power plants at cruising power (bsfc = 0.46 lb/hp-hr).

3. For flight at low speeds, engagement of supplementary air is necessary in order to obtain acceptable propulsive efficiencies.

4. The loss in efficiency attendant upon failure to achieve component efficiencies of the order of best current practice is very severe. Because unfavorable installation can seriously impair the blower efficiency, careful attention to the installation design is essential to satisfactory performance.

5. For any given combination of flight conditions and component characteristics, there is a blower speed (or compression temperature rise) for maximum combined efficiency above which the combined efficiency falls off rapidly.

6. Raising the limit on maximum temperature beyond that necessary to obtain efficient operation at maximum permissible blower speed (or compression temperature rise) does not greatly increase the combined efficiency and may decrease it slightly. When such a decrease occurs, however, the rate of decrease with increase in temperature

is small and increasing the maximum temperature permits large power increases at relatively small decrease in efficiency.

7. The gain in efficiency that can be expected from coordinated increases in operating limits on maximum temperature and blower speed (or compression temperature rise) at current component efficiencies is considerable, and the increase in output is large. Improper coordination of limiting temperatures and blower speed, however, may actually reduce both efficiency and power.

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APPENDIX

COMBINED EFFICIENCY OF CONSTANT-VOLUME-COMBUSTION RAM JET

EFFICIENCIES

Cycle efficiency.- The idealized cycle for ram-jet-propulsion systems employing constant-volume combustion is illustrated by a pressure-volume diagram in figure 24 and a temperature-entropy diagram in figure 25. The cycle consists of four phases: isentropic compression due to ram from conditions 0 to 1, constant-volume addition of heat from conditions 1 to 3, isentropic expansion in the nozzle from conditions 3 to 5, and a constant-pressure rejection of heat from conditions 5 to 0. The points 3 and 5 were used to denote conditions at the end of combustion and at the nozzle as in the constant-pressure diagrams given previously for the constant-pressure-combustion cycle. The actual conditions, however, at these two points for the two cycles are vastly different.

The efficiency of the constant-volume-combustion cycle is easily obtained from an analysis of the heat addition and rejection. The heat supplied per pound of gas is

$$Q_{in} = c_v \, \Delta T_c = c_v\left(T_3 - T_1\right)$$

$$= \int_1^3 T \, dS \qquad (62)$$

denoted in figure 25 by the area A13BA, and the heat rejection is

$$Q_{out} = c_p\left(T_5 - T_0\right)$$

$$= \int_0^5 T\ dS \tag{63}$$

denoted by A05BA.

The difference between the heat input and the heat output is converted to work, so that the efficiency of the cycle is

$$\eta_c = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

$$= \frac{c_v\left(T_3 - T_1\right) - c_p\left(T_5 - T_0\right)}{c_v\left(T_3 - T_1\right)} \tag{64}$$

$$= 1 - \gamma\,\frac{T_5 - T_0}{T_3 - T_1} \tag{65}$$

Equation (65) can be reduced to

$$\eta_c = 1 - \gamma\,\frac{T_0}{T_3 - T_1}\left[\left(\frac{T_3}{T_1}\right)^{1/\gamma} - 1\right] \tag{66}$$

$$= 1 - \gamma\,\frac{T_0}{\Delta T_c}\left[\left(\frac{T_0 + \Delta T_s + \Delta T_c}{T_0 + \Delta T_s}\right)^{1/\gamma} - 1\right] \tag{67}$$

which is the expression for the thermodynamic efficiency of the ideal constant-volume-combustion cycle. In terms of maximum temperature T_{max} instead of combustion temperature rise ΔT_c,

$$\eta_c = 1 - \gamma \frac{T_0}{T_{max} - \left(T_0 + \Delta T_s\right)} \left[\left(\frac{T_{max}}{T_0 + \Delta T_s}\right)^{1/\gamma} - 1\right] \tag{68}$$

Combined efficiency.- As applied to jet propulsion, the constant-volume-combustion cycle is operated intermittently. The initial admission to the combustion chamber of a charge of air, compressed by the dynamic pressure of flight, is the same as for the Brayton cycle. In the constant-volume-combustion cycle, however, the combustion chamber, after being charged, is closed to prevent expansion of the gases during the combustion phase of the cycle and then opened to the propulsive nozzle only upon completion of combustion. The gas flows out through the nozzle until the pressure in the combustion chamber has dropped to its value previous to combustion (condition 4 in the cycle). The charging process is then repeated; the remaining hot gases are displaced by the incoming cold gases. Such a cycle can be approximated in practice through the use of automatic shutters at the inlet and a fixed exit nozzle. The closeness with which constant-volume combustion is approached depends upon the rate of flame propagation and the inertia of the gases.

The nozzle-discharge velocity of the intermittent system varies from a maximum immediately after combustion to a minimum just before recharging. The propulsive efficiency consequently changes continuously throughout the discharge. It becomes necessary, therefore, to integrate a varying discharge velocity to obtain the thrust power and, accordingly, only the equation for combined efficiency is developed. In deriving the expression for combined efficiency, the following supplementary symbols are used:

f cycle frequency, cps

m_t mass of charge in combustion chamber at any instant, slugs

m_3 mass of charge in combustion chamber at condition 3 of the cycle, slugs (see figs. 24 and 25)

m_4 mass of charge in combustion chamber at condition 4 of the cycle, slugs

$\overline{P_T}$ average power developed by thrust of discharge, ft-lb/sec

$\overline{P_D}$ average power deductible for intake drag, ft-lb/sec

$\overline{P}$ net average thrust power, ft-lb/sec $\left(\overline{P_T} - \overline{P_D}\right)$

V_{N_t} nozzle discharge velocity at any instant, fps

V_{N_4} nozzle discharge velocity during recharging phase of cycle, fps

$X_t = \dfrac{m_t}{m_3}$

$X_4 = \dfrac{m_4}{m_3}$

$X_5 = \dfrac{\rho_5}{\rho_3}$

ρ density of charge, slugs/cu ft

The average power developed by the thrust of the discharge, based on the assumption that the variation of flight speed V_0 during the cycle is negligible, is

$$\overline{P_T} = fV_0 \int_{m_4}^{m_3} V_{N_t}\, dm_t + fV_0 V_{N_4} \int_0^{m_4} dm_t \tag{69}$$

and the average power deductible for intake drag is

$$\overline{P_D} = fV_0^{\,2} \int_0^{m_3} dm_t \tag{70}$$

so that the net average thrust power is

$$\overline{P} = f\left(V_0\int_{m_4}^{m_3} V_{N_t}\ dm_t + V_0V_{N_4}\int_0^{m_4} dm_t - V_0^2\int_0^{m_3} dm_t\right) \tag{71}$$

By substituting the variable

$$X_t = \frac{m_t}{m_3} \qquad \text{or} \qquad m_t = X_t m_3$$

and

$$X_4 = \frac{m_4}{m_3}$$

and evaluating all but the first integral, equation (71) becomes

$$\overline{P} = fm_3\left[V_0\int_{X_4}^{1} V_{N_t}\ dX_t - V_0^2\left(1 - X_4\right) + V_0\left(V_{N_4} - V_0\right)X_4\right] \tag{72}$$

The nozzle velocity can be expressed as a function of the operating conditions and the instantaneous value of m_t and therefore is a function of the new variable X_t. From the thermodynamic relations for an adiabatic expansion, by assuming zero velocity at condition 3,

$$V_{N_t} = \sqrt{2gJc_p\ T_3}\ \sqrt{X_t^{\gamma-1} - X_5^{\gamma-1}} \tag{73}$$

It should be noted that V_{N_t} will exceed sonic velocity even at quite moderate heat additions and that the applicability of equation (73) involves assuming the existence at all times of the necessary form of discharge nozzle. By substituting V_{N_t} from equation (73) in equation (72) and rearranging, the net average thrust power becomes

$$\overline{P} = fm_3\left[2\sqrt{\frac{gJc_pV_0^2}{2}T_3}\int_{X_4}^{1}\sqrt{X_t^{\gamma-1} - X_5^{\gamma-1}}\,dX_t\right.$$

$$\left. + V_0^2\left(X_4\frac{V_{N_4}}{V_0} - 1\right)\right] \tag{74}$$

The average heat input in foot-pounds per second is

$$Q_{in} = fm_3\,\Delta T_c\,\frac{c_p}{\gamma}gJ \tag{75}$$

Dividing the thrust power from equation (74) by the heat input from equation (75) gives the combined efficiency:

$$\eta = \frac{\overline{P}}{Q_{in}}$$

$$= \left[\frac{2\gamma}{\Delta T_c}\sqrt{\frac{V_0^2}{2gJc_p}T_3}\int_{X_4}^{1}\sqrt{X_t^{\gamma-1} - X_5^{\gamma-1}}\,dX_t\right.$$

$$\left. + \frac{2\gamma}{\Delta T_c}\frac{V_0^2}{2gJc_p}\left(X_4\frac{V_{N_4}}{V_0} - 1\right)\right]$$

Substituting the stagnation temperature rise ΔT_s for $V_0^2/2gJc_p$ yields the following equation for combined efficiency:

$$\eta = 2\gamma\frac{\Delta T_s}{\Delta T_c}\left(\sqrt{\frac{T_3}{\Delta T_s}}\int_{X_4}^{1}\sqrt{X_t^{\gamma-1} - X_5^{\gamma-1}}\,dX_t + X_4\frac{V_{N_4}}{V_0} - 1\right) \tag{76}$$

Now

$$\frac{V_{N_4}}{V_0} = \sqrt{\frac{T_3}{\Delta T_s}}\sqrt{X_4{}^{\gamma-1} - X_5{}^{\gamma-1}} \tag{77}$$

so that

$$\eta = 2\gamma\,\frac{\Delta T_s}{\Delta T_c}\left[\sqrt{\frac{T_3}{\Delta T_s}}\left(\underbrace{\int_{X_4}^{1}\sqrt{X_t{}^{\gamma-1} - X_5{}^{\gamma-1}}\,dX_t}_{I} + X_4\sqrt{X_4{}^{\gamma-1} - X_5{}^{\gamma-1}}\right) - 1\right] \tag{78}$$

in which

$$X_4{}^{\gamma-1} = \left(\frac{T_1}{T_3}\right)^{\frac{\gamma-1}{\gamma}} \tag{79}$$

and

$$X_5{}^{\gamma-1} = \frac{T_0}{T_1}\left(\frac{T_1}{T_3}\right)^{\frac{\gamma-1}{\gamma}} \tag{80}$$

The value of the definite integral I of equation (78) is

$$I = \frac{5}{48}\left[\left(4 - 3X_5^{\gamma-1}\right)\left(2 + X_5^{\gamma-1}\right)\sqrt{1 - X_5^{\gamma-1}}\right.$$

$$- \left(4X_4^{\gamma-1} - 3X_5^{\gamma-1}\right)\left(2X_4^{\gamma-1} + X_5^{\gamma-1}\right)X_4^{\frac{\gamma-1}{2}}\sqrt{X_4^{\gamma-1} - X_5^{\gamma-1}}$$

$$- \frac{3}{2}X_5^{3(\gamma-1)} \log_e\left(\frac{1 + \sqrt{1 - X_5^{\gamma-1}}}{1 - \sqrt{1 - X_5^{\gamma-1}}}\right)$$

$$\left. + \frac{3}{2}X_5^{3(\gamma-1)} \log_e\left(\frac{X_4^{\frac{\gamma-1}{2}} + \sqrt{X_4^{\gamma-1} - X_5^{\gamma-1}}}{X_4^{\frac{\gamma-1}{2}} - \sqrt{X_4^{\gamma-1} - X_5^{\gamma-1}}}\right)\right] \tag{81}$$

DISCUSSION

The constant-volume-combustion ram jet has a considerably higher ideal combined efficiency and ideal power output per pound of air handled per second than a constant-pressure-combustion ram jet. For flight at 500 miles per hour at 30,000 feet with a combustion temperature rise of 1700° F, the combined efficiency of the constant-volume-combustion ram jet obtained from the preceding analysis is 0.15 and that of the constant-pressure-combustion ram jet is only 0.06 - a ratio of $2\frac{1}{2}$:1 for the conditions chosen. Because of the difference in specific heats at constant volume and constant pressure, the heat input to the constant-volume-combustion ram jet is less for the same combustion temperature rise than that to the constant-pressure-combustion ram jet in the ratio of 1:1.4, and the advantage in power per unit mass air flow of the constant-volume-combustion ram jet is slightly less than 2:1 for the conditions chosen.

The difficulty of attaining, in practice, efficiencies that approach the ideal is probably even greater for the constant-volume-combustion ram jet than for the constant-pressure-combustion ram jet. The very much higher pressures may introduce structural problems. Isolating stressed parts of the combustion chamber from the intense heat and providing necessary cooling, so readily accomplished in the constant-pressure-combustion ram jet with a simple internal shroud, become difficult. There is, in addition, the problem of obtaining sufficiently rapid flame propagation. The shutters and similar equipment necessary for intermittent operation add mechanical complication. It is reasonable to suppose that the added practical difficulties of the intermittently operated constant-volume-combustion ram jet will offset to some degree the pronounced theoretical advantages.

The lengthy expression (equation (81)) arising from the integral appearing in equation (78) makes evaluation slow and tedious. By use of a concept of an equivalent mean gain of velocity through the system $\overline{\Delta V}$ defined to be such that the average total power, or sum of average thrust power and average wake power, is given by the relation

$$\text{Average total power} = mV_0\ \overline{\Delta V} + \frac{m}{2}\ \overline{\Delta V}^2 \tag{82}$$

a simple although not entirely valid expression for propulsive efficiency may be derived in a manner similar to that used for the constant-pressure-combustion cycle:

$$\eta_p = \frac{2}{1 + \sqrt{1 + \frac{1}{\gamma}\frac{\Delta T_c}{\Delta T_s}\eta_c}} \tag{83}$$

The combined efficiency is then obtained as the product of the cycle efficiency from equation (67) and the propulsive efficiency from equation (83) through use of equation (67). Although use of the mean velocity gain $\overline{\Delta V}$ cannot be exact because of the several nonlinear relations involved, equation (83) has given in the range explored results surprisingly close to the results of the more correct procedure. For the case cited of flight at 500 miles per hour at 30,000 feet with a combustion temperature rise of 1700° F, equation (83) gave a propulsive efficiency of 0.489 in comparison with 0.488 obtained by dividing the combined efficiency from equation (78) by the cycle efficiency from equation (67). This agreement is regarded as much closer than the accuracy of the assumptions on which the analysis is based.

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2. Pye, D. R.: The Internal Combustion Engine. Vol. II. The Aero-Engine. Clarendon Press (Oxford), 1934, p. 271.

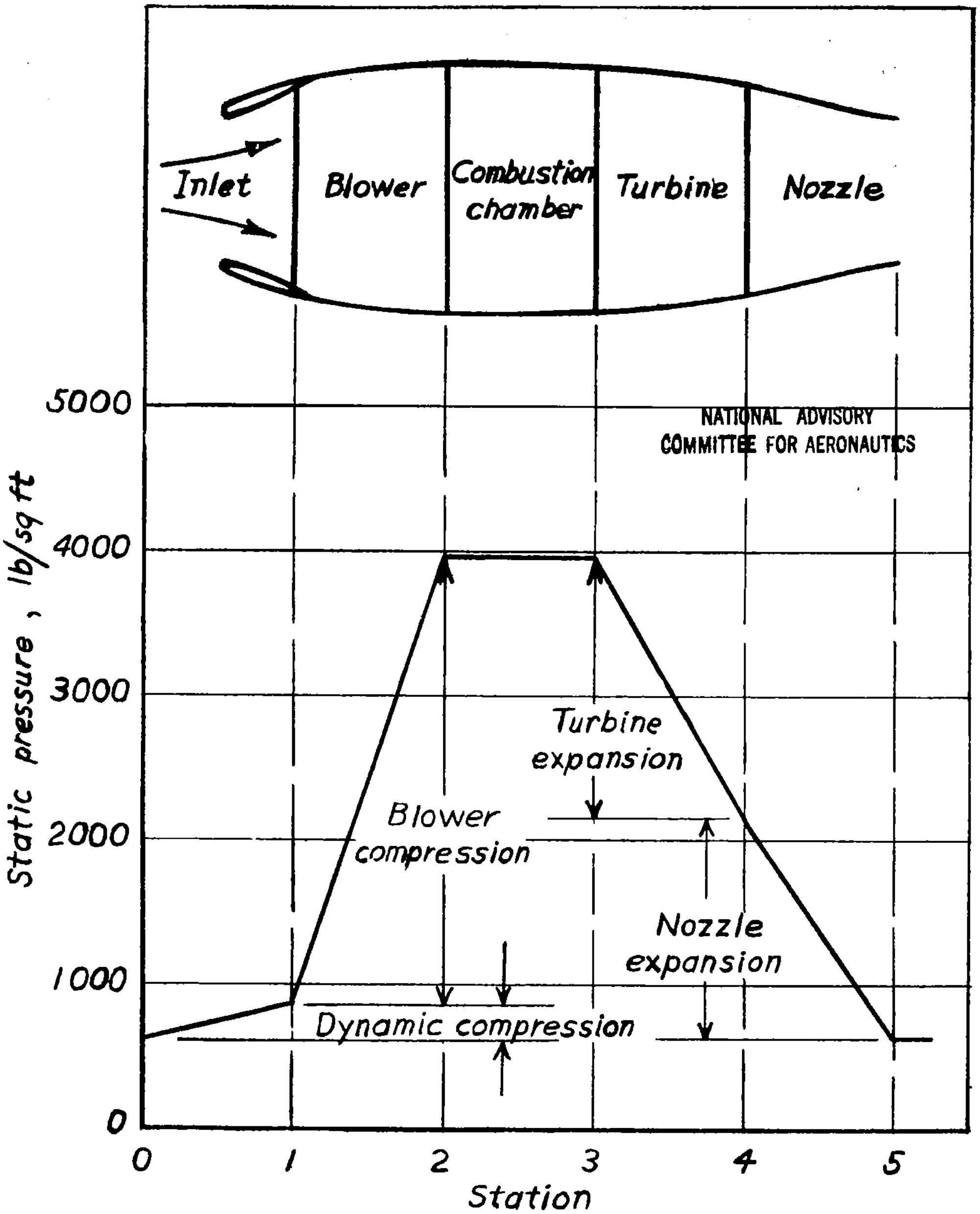
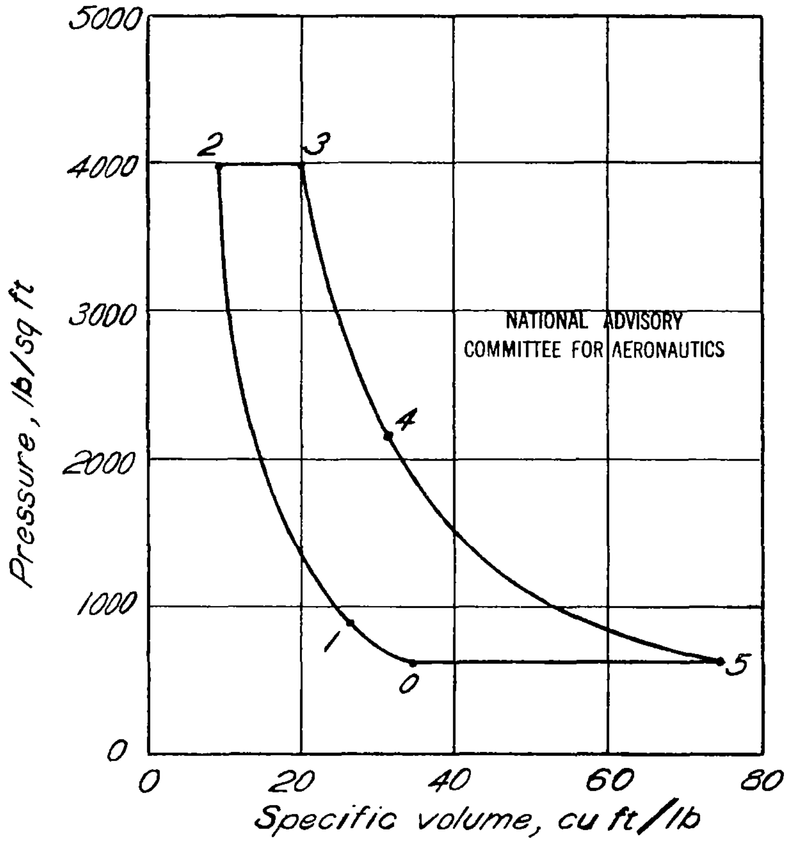
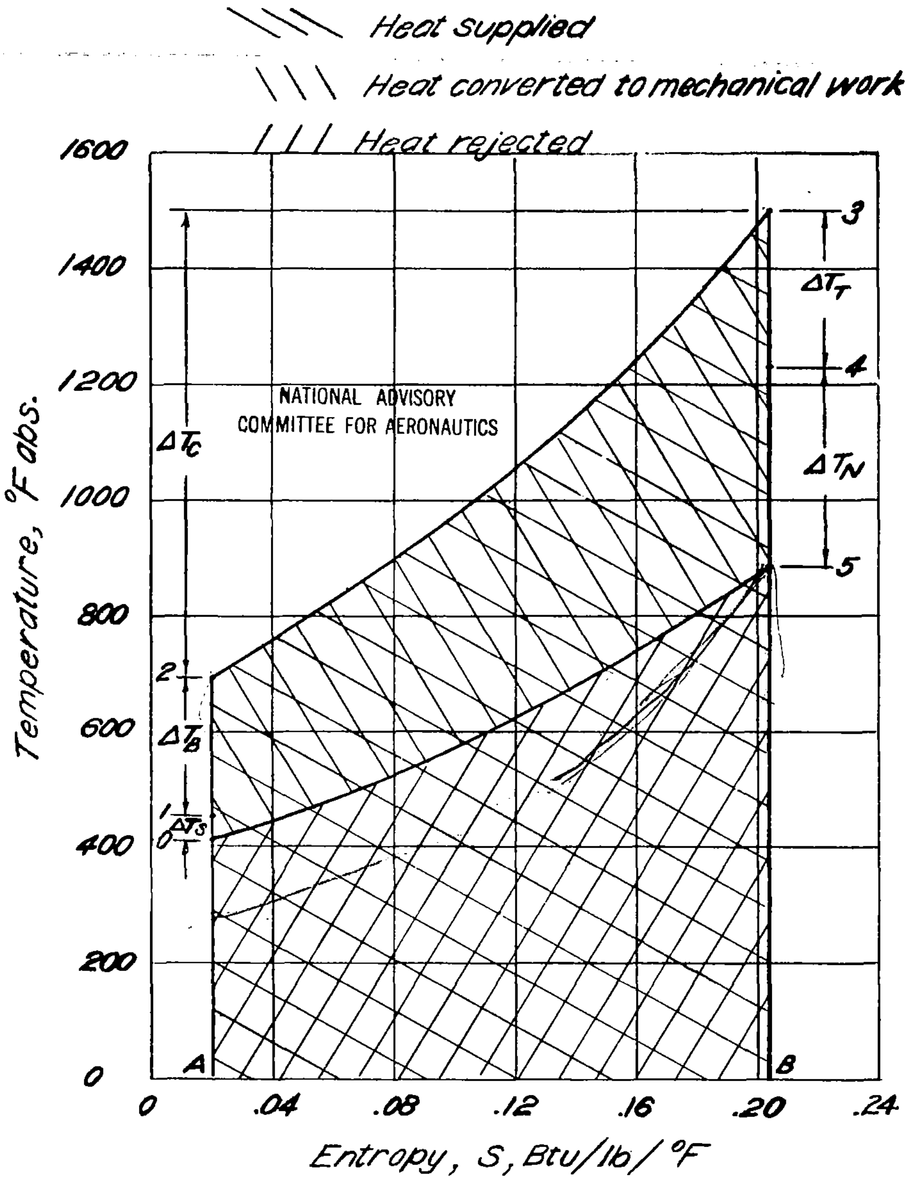


Figure 1.- Static-pressure variation through a jet-propulsion engine.



(a) Pressure-volume diagram.

Figure 2.- Ideal Brayton cycle.



(b) Temperature-entropy diagram.

Figure 2.- Concluded.

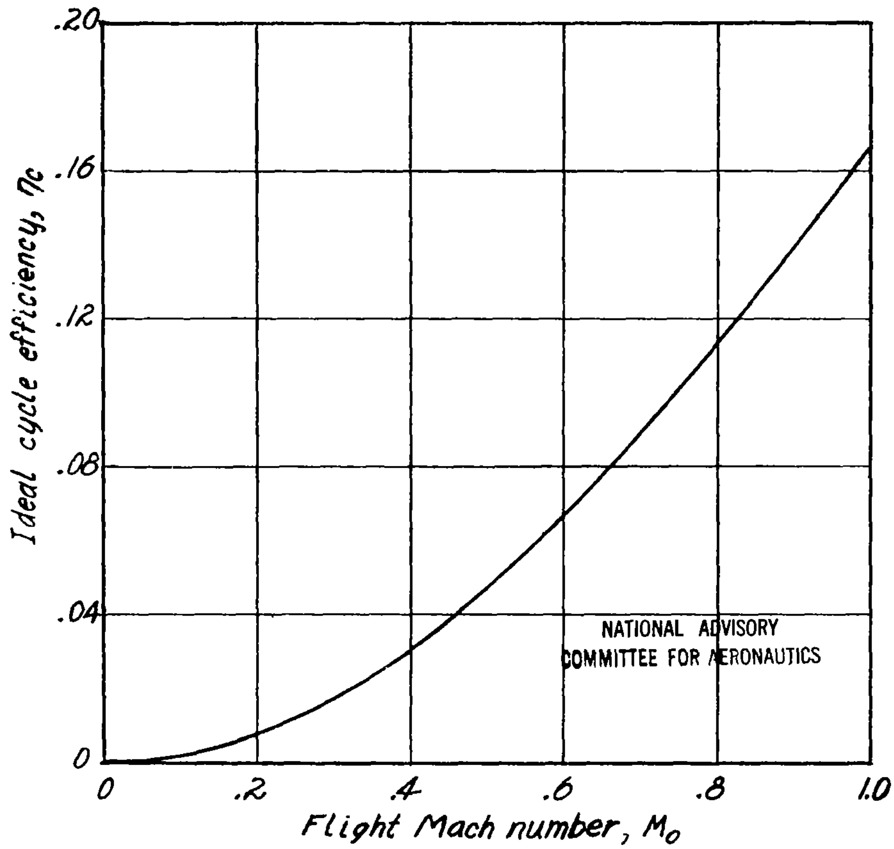


Figure 3.- Ideal Brayton cycle efficiency of a ram jet as a function of flight Mach number.

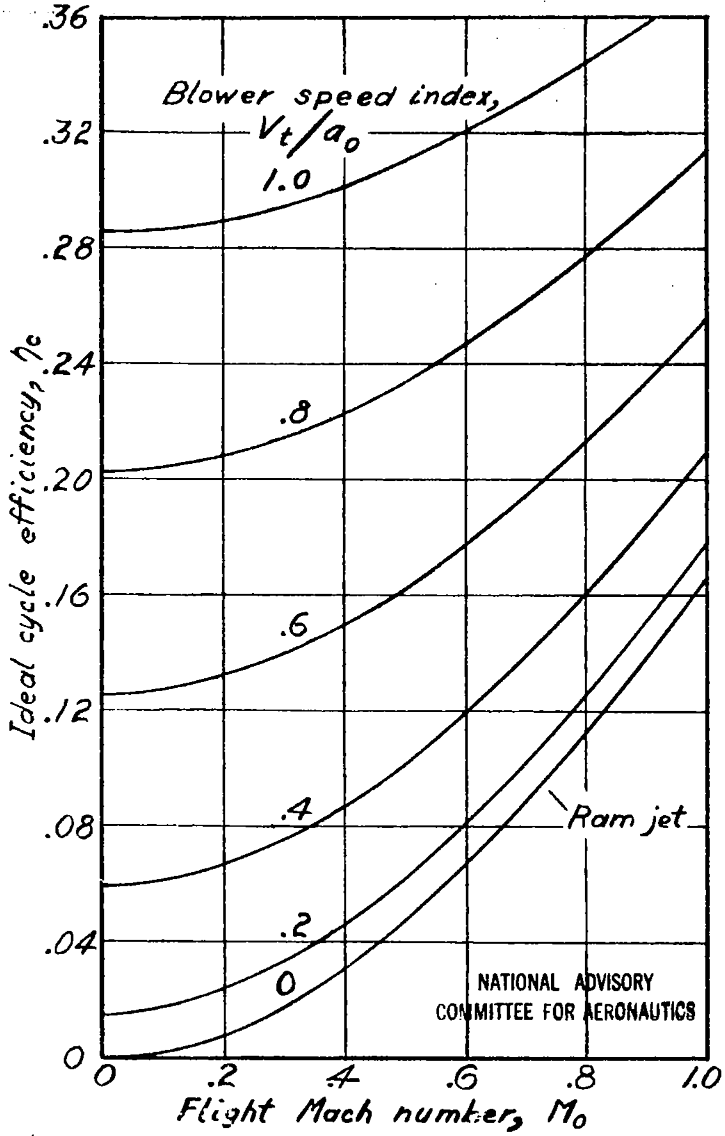


Figure 4.- Ideal Brayton cycle efficiency of a jet-propulsion engine as a function of flight and blower Mach numbers.

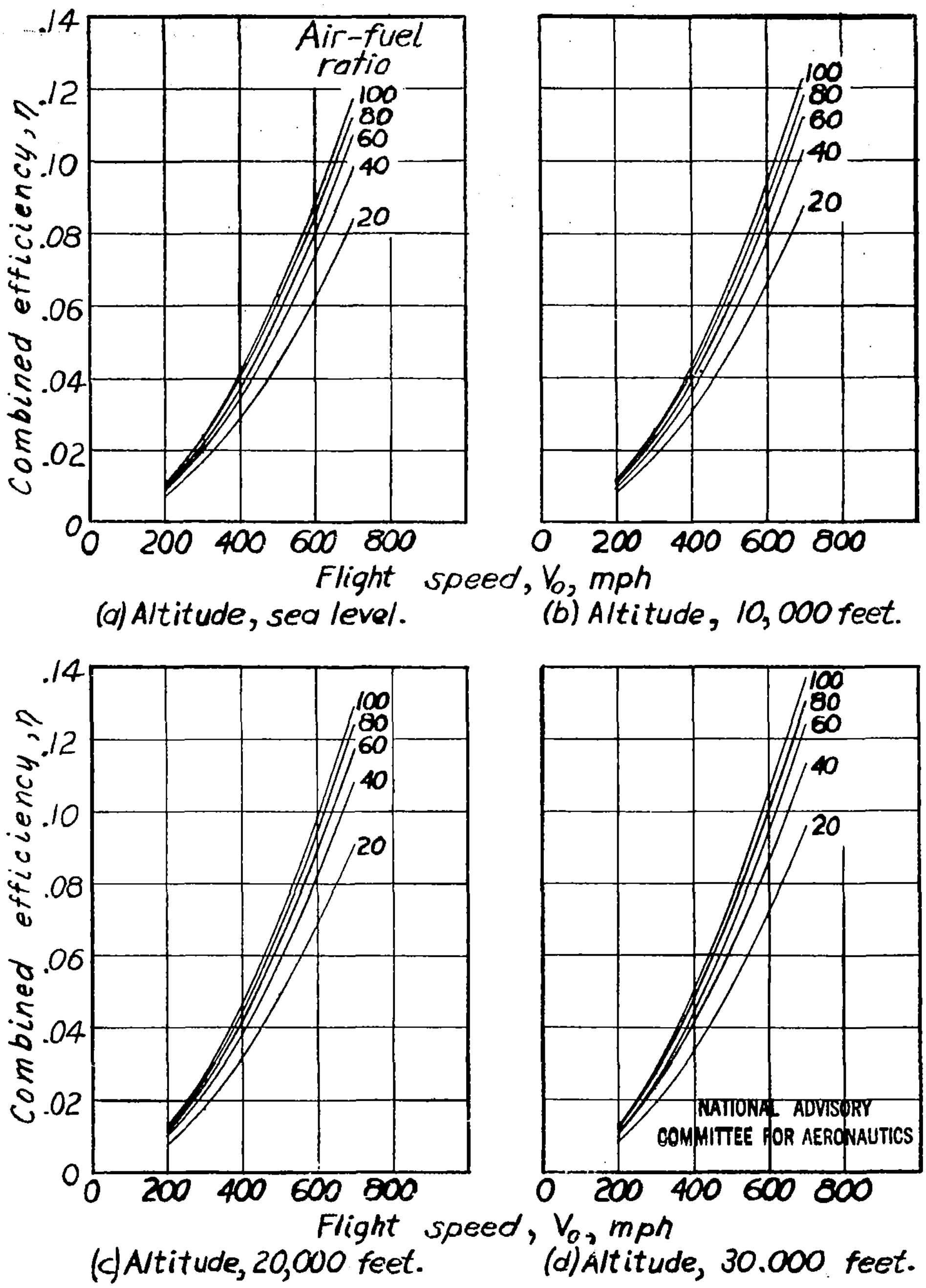


Figure 5.- Ideal combined efficiency of a Brayton cycle ram jet as a function of flight speed.

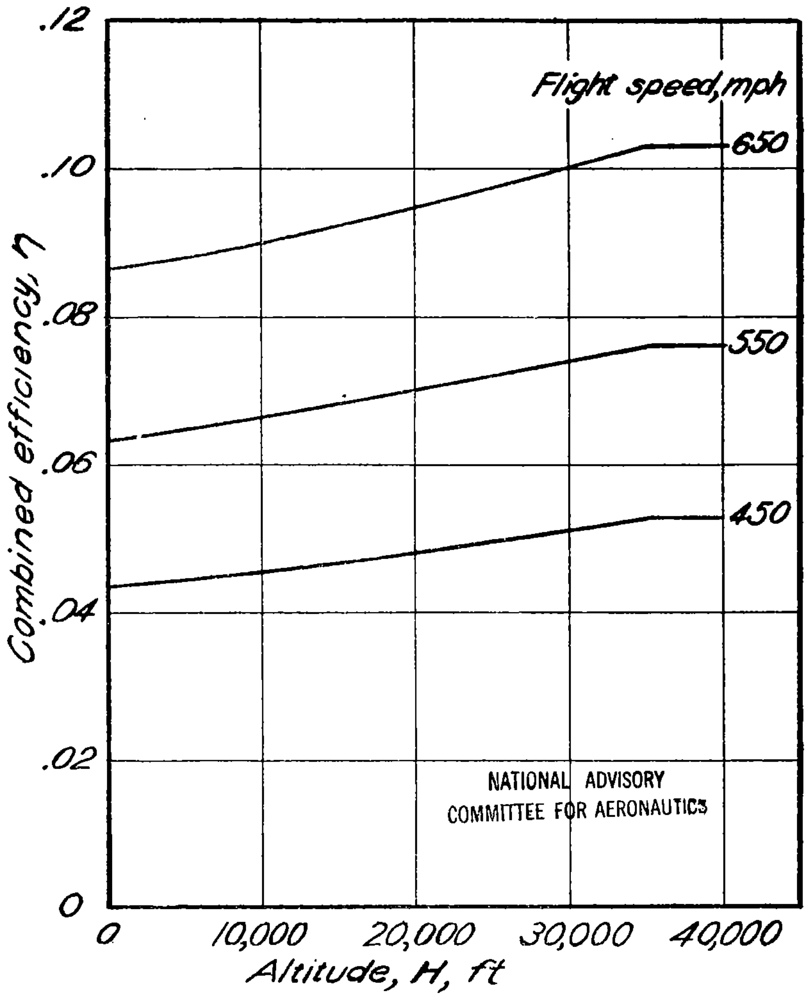


Figure 6.- Ideal combined efficiency of a Brayton cycle ram jet as a function of altitude for three flight speeds and an air-fuel ratio of 40.

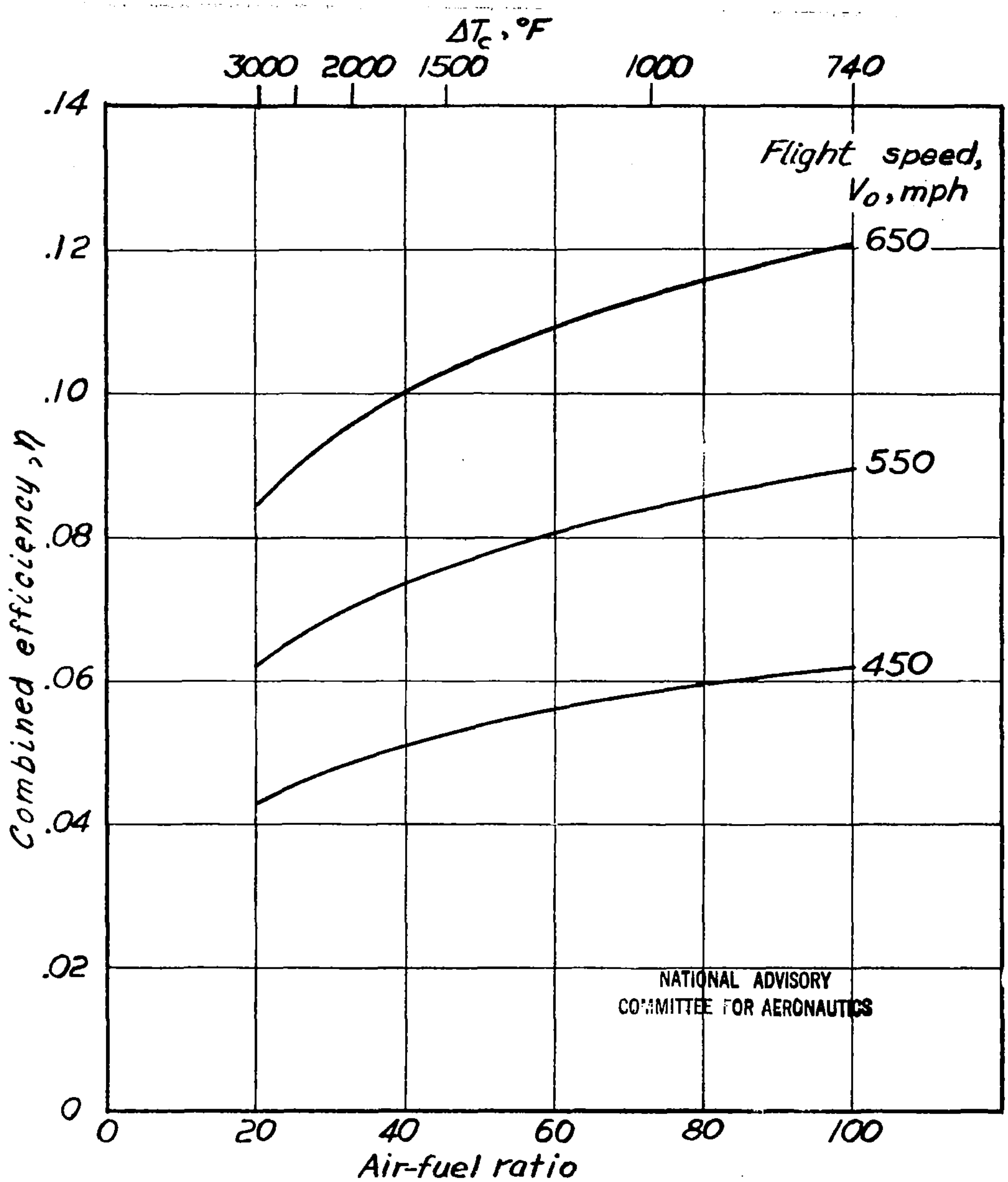
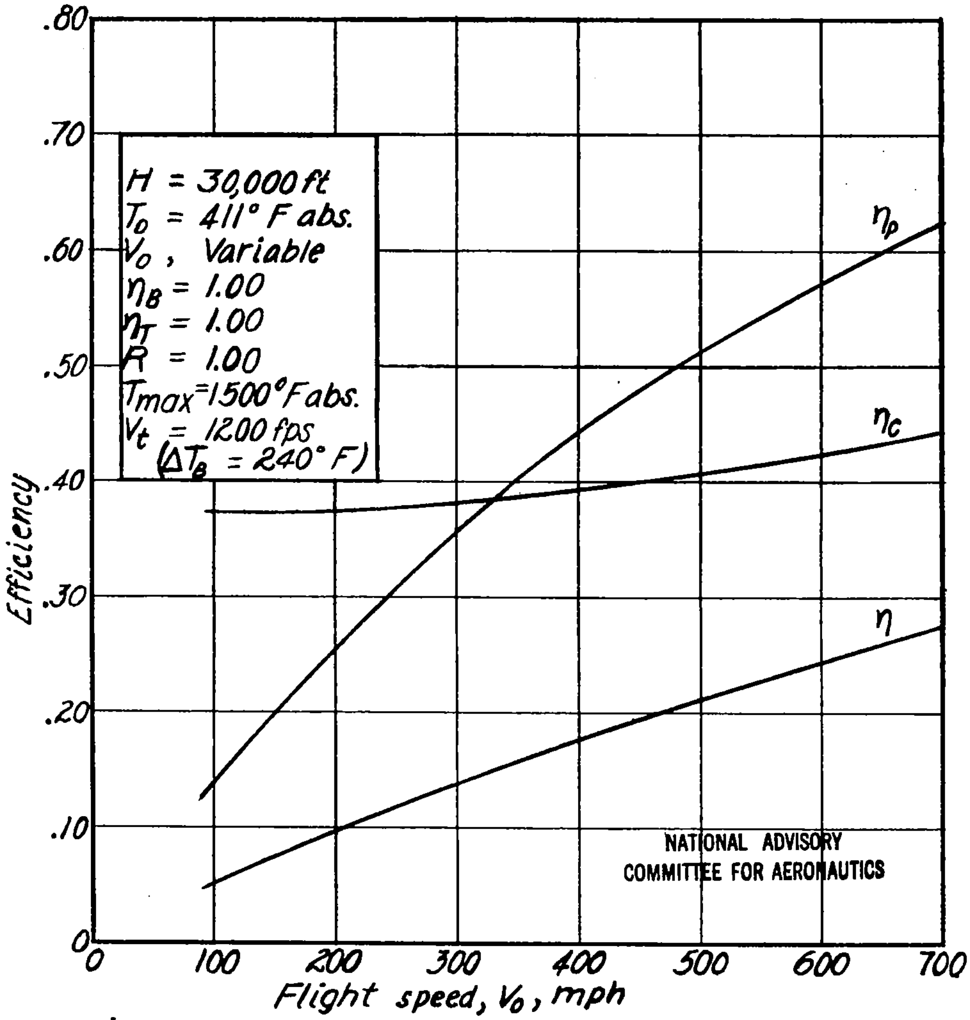
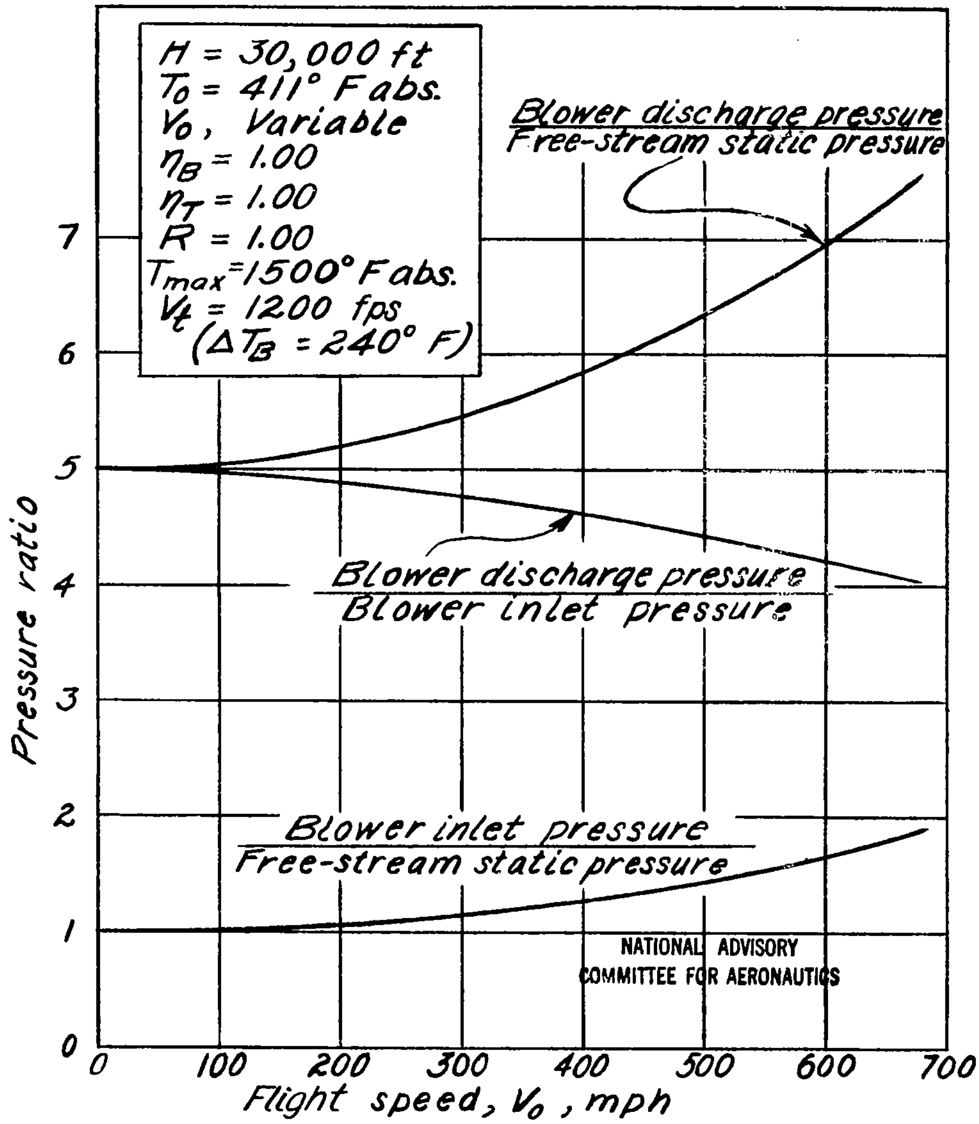


Figure 7.- Ideal combined efficiency of a Brayton cycle ram jet as a function of air-fuel ratio for three flight speeds and an altitude of 30,000 feet.



(a) Efficiencies.

Figure 8.- Typical characteristics of an ideal Brayton cycle jet-propulsion engine.



(b) Pressure ratios.

Figure 8.- Concluded.

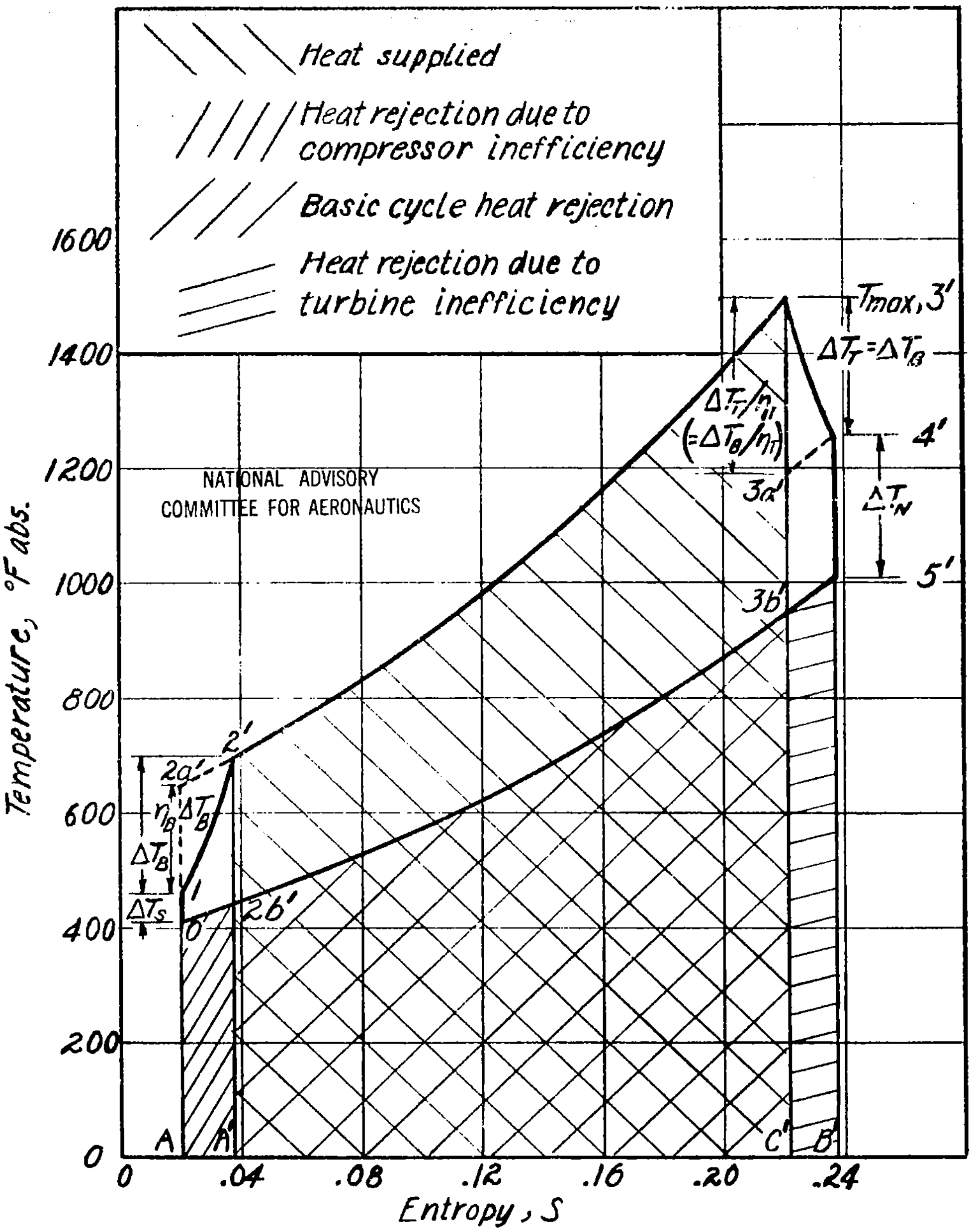
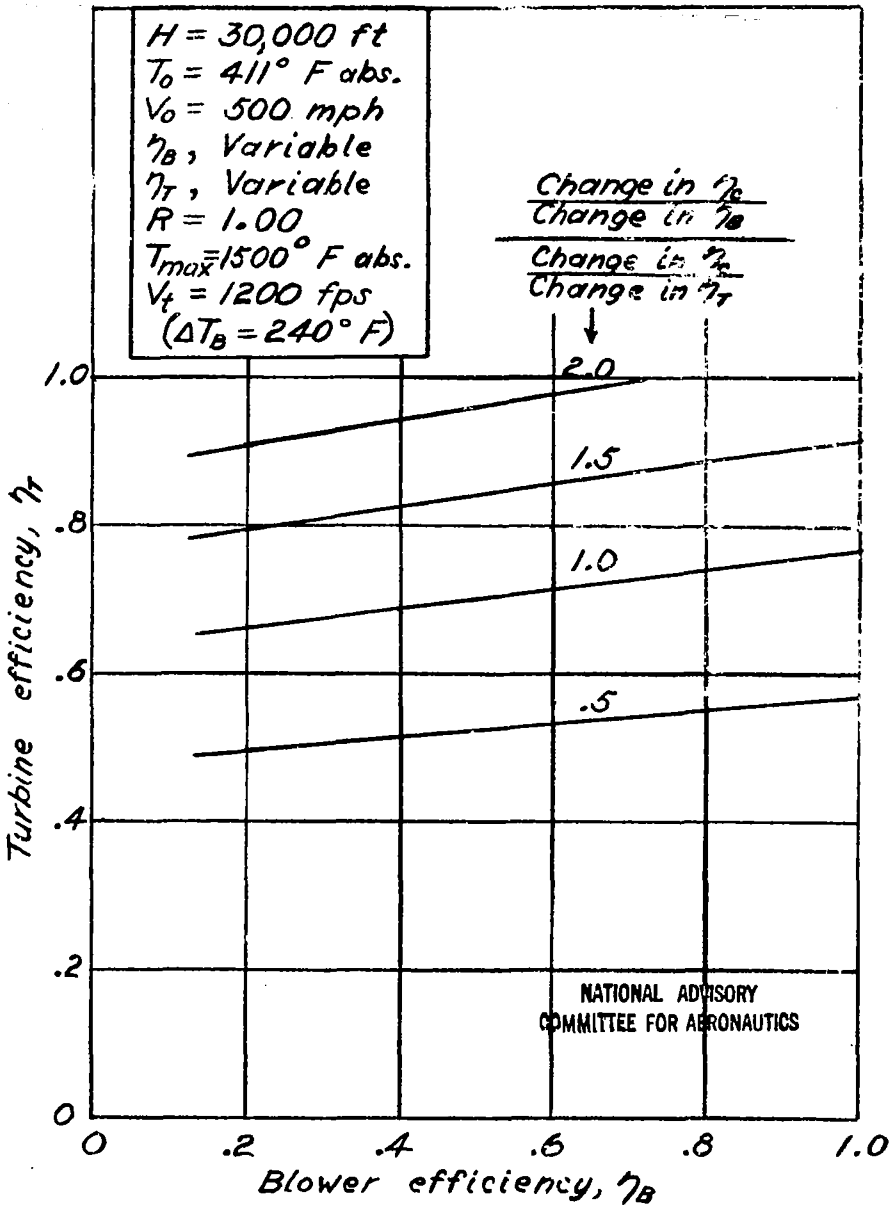
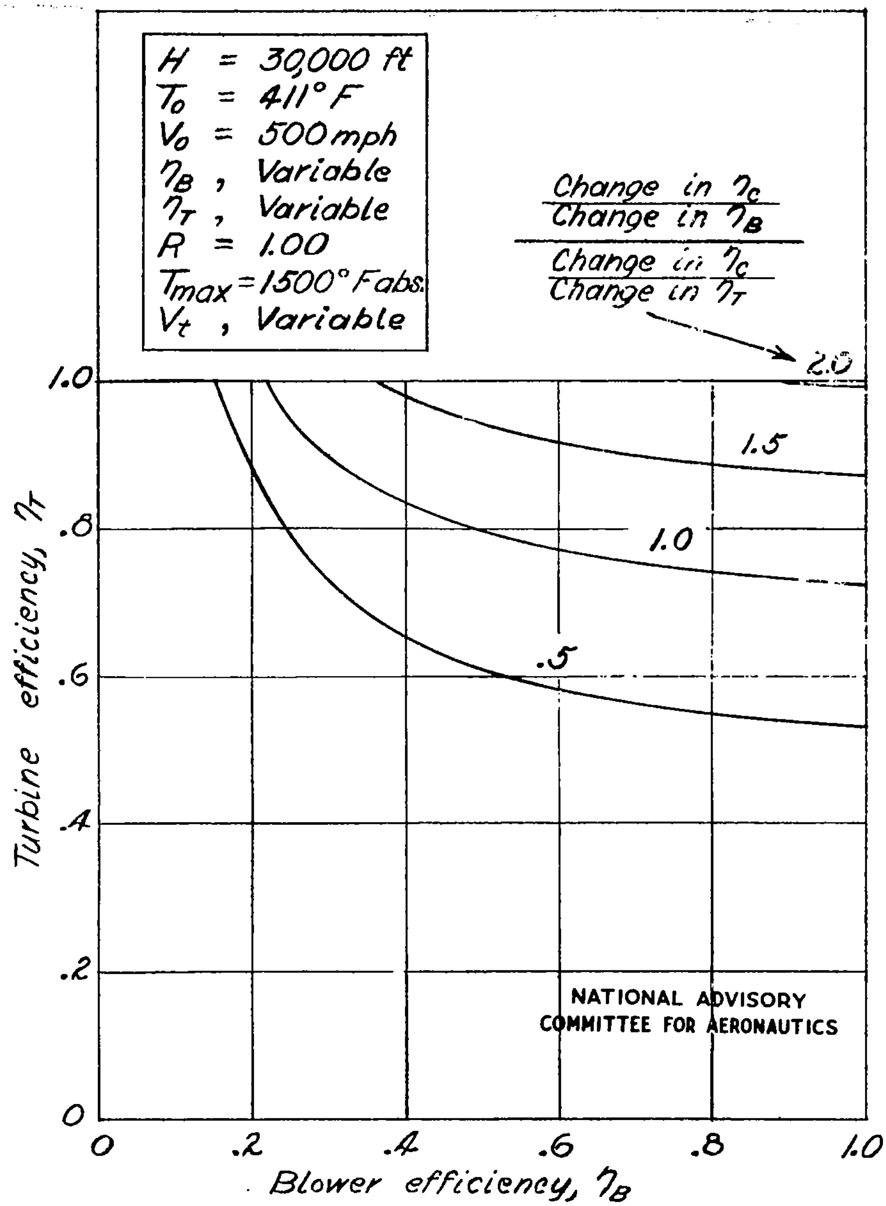


Figure 9.- Temperature-entropy diagram for a real Brayton cycle having constant-pressure combustion but nonisentropic compression and expansion.



(a) Blower speed constant.

Figure 10.- Relative effectiveness on real cycle efficiency of changes in blower and turbine efficiencies.



(b) Blower speed variable with $\eta_B \, \Delta T_B$ constant at 192° F.

Figure 10.- Concluded.

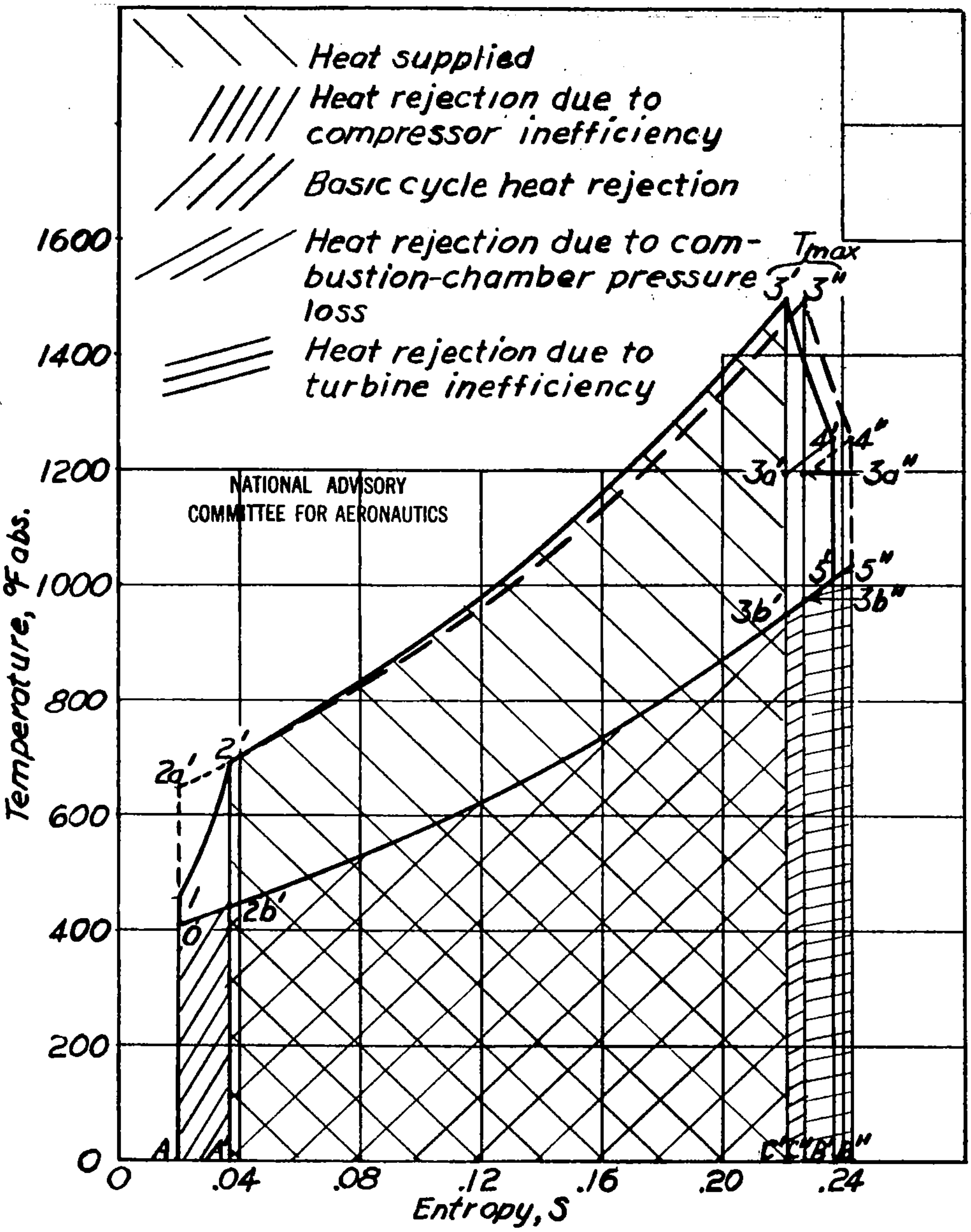


Figure 11.- Temperature-entropy diagram for a real Brayton cycle having pressure loss in the combustion process and nonisentropic compression and expansion.

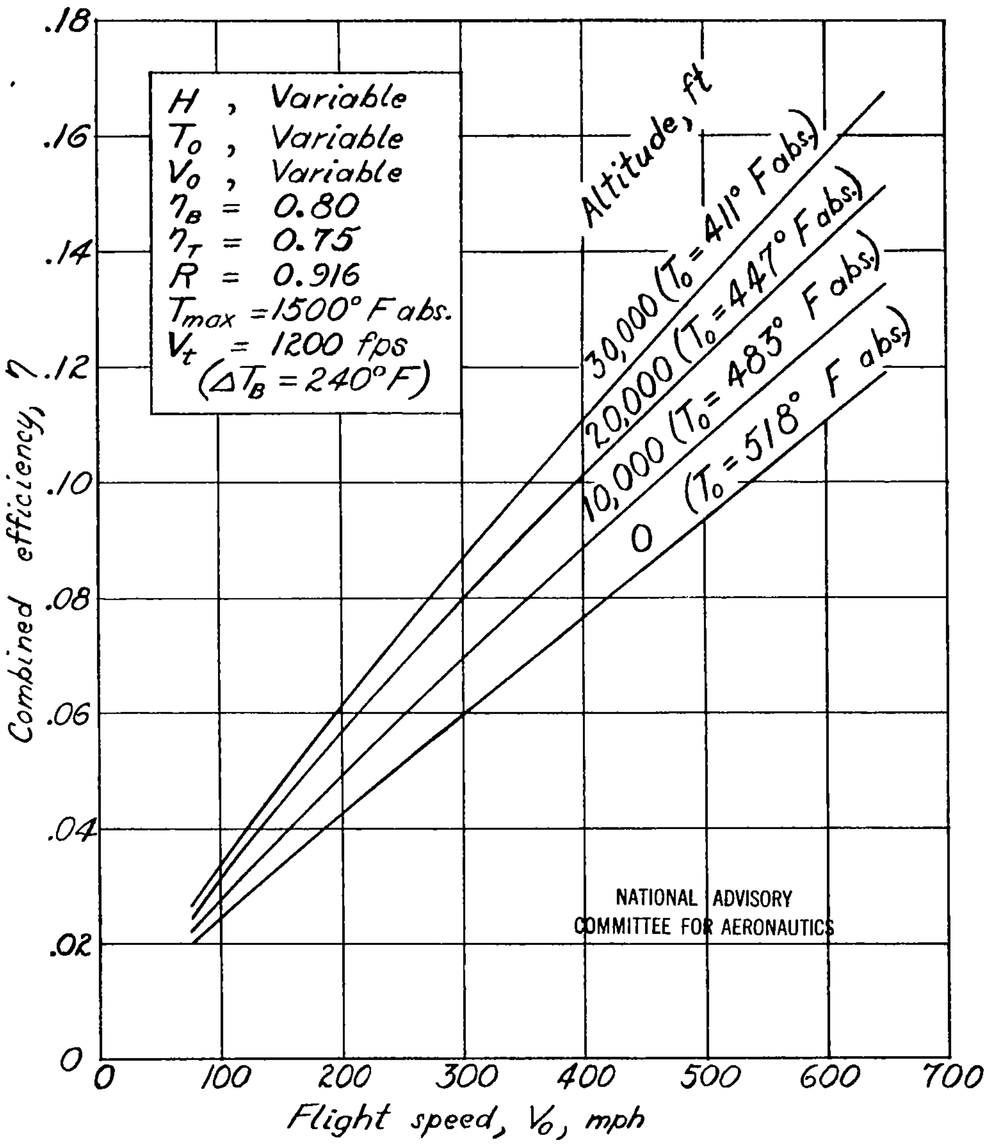


Figure 12.- Combined efficiency of a jet-propulsion engine as a function of flight speed for four altitudes.

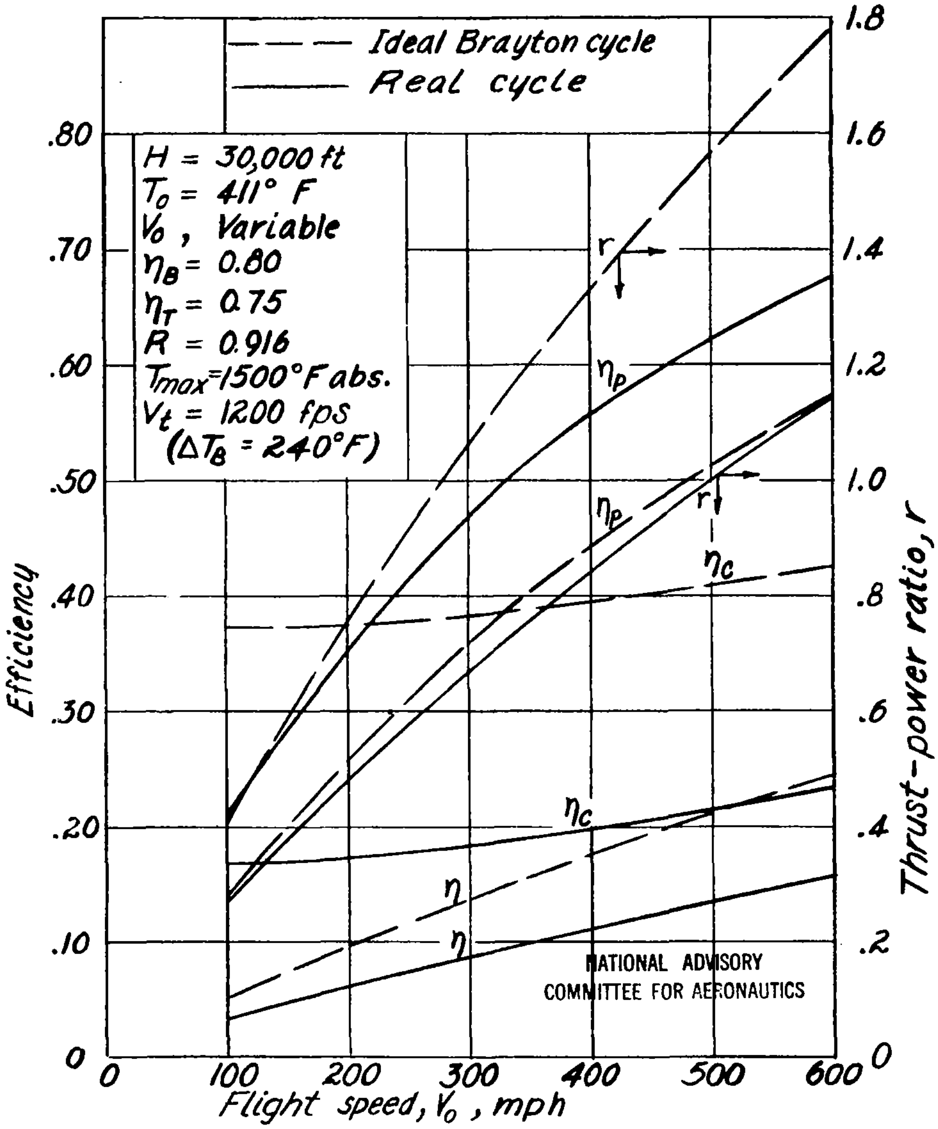


Figure 13.- Comparison of ideal and real powers and efficiencies of jet-propulsion engines as a function of flight speed.

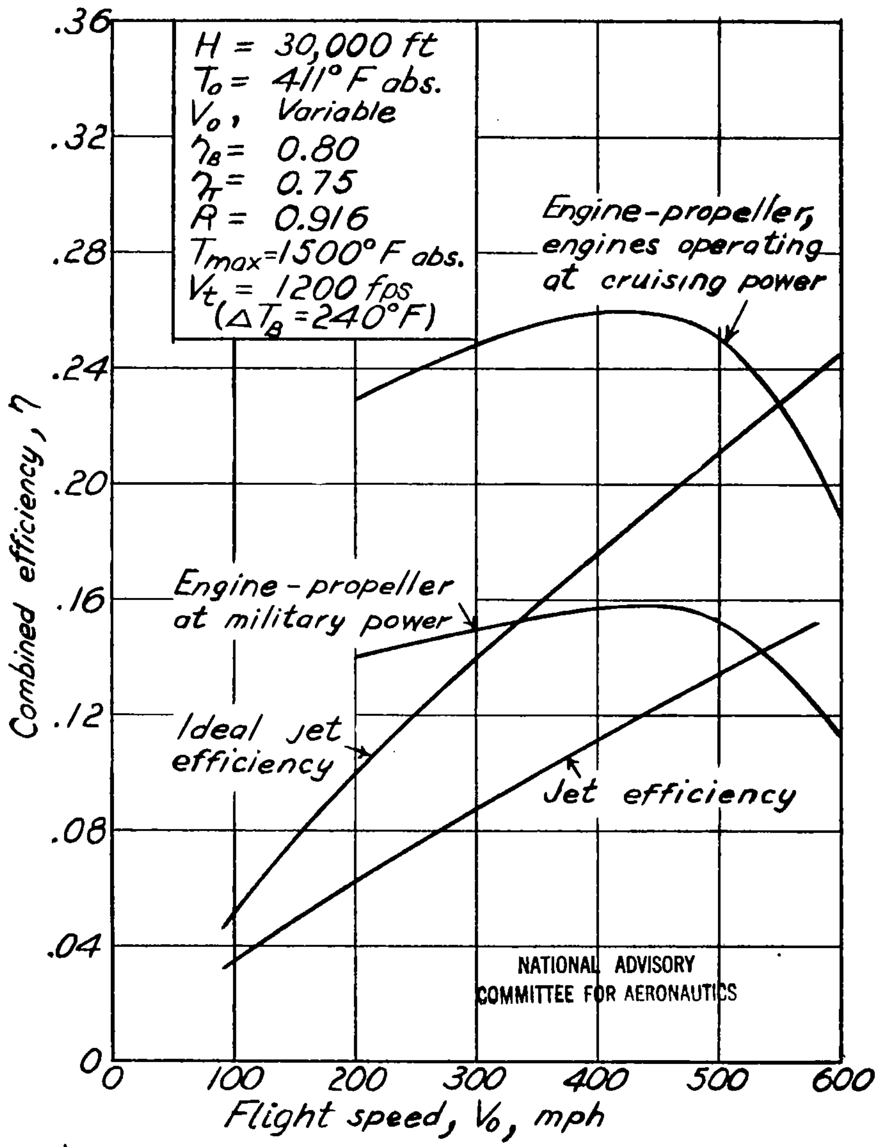


Figure 14.- Comparison of combined efficiency of a jet-propulsion engine with estimated over-all efficiency of an engine propeller combination.

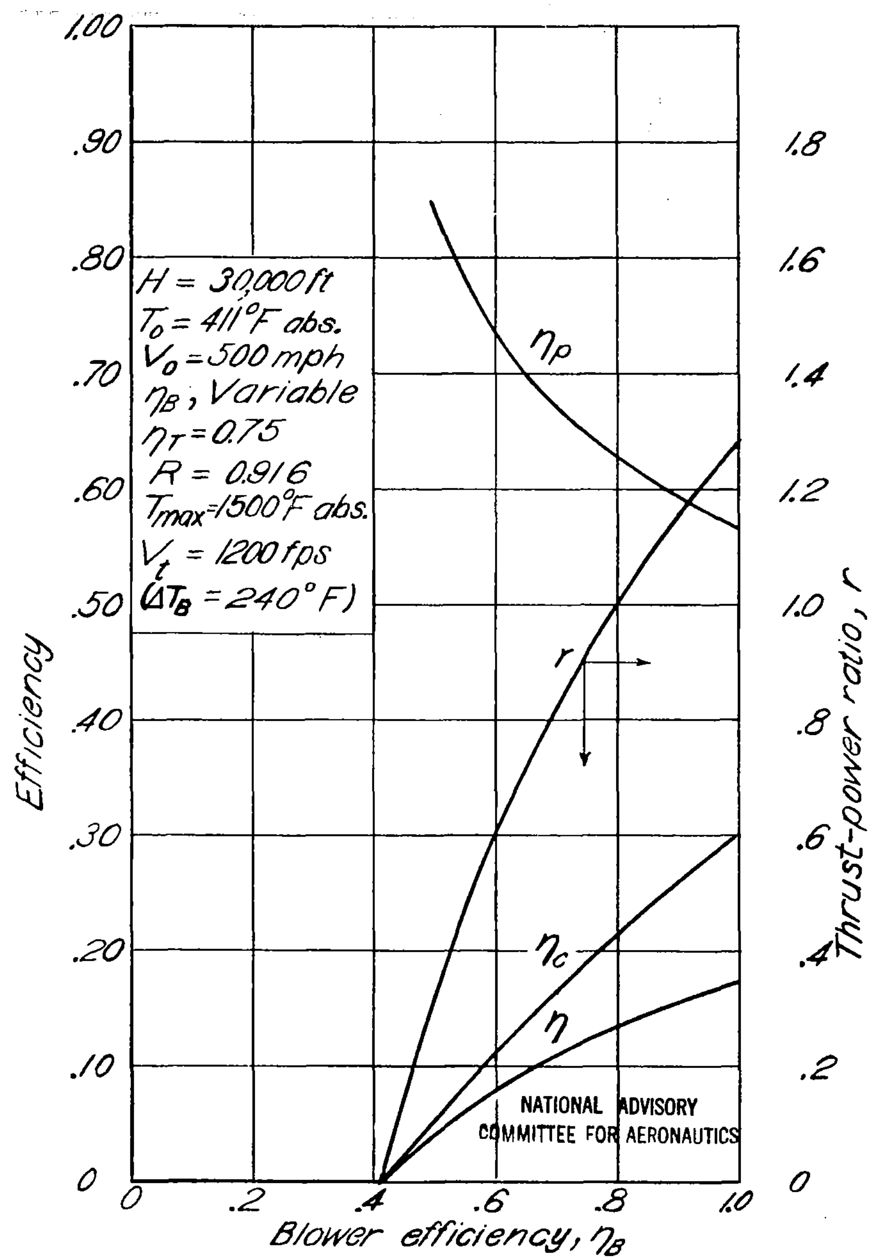


Figure 15.- Variation with blower efficiency of the power and the efficiencies of a jet-propulsion engine.

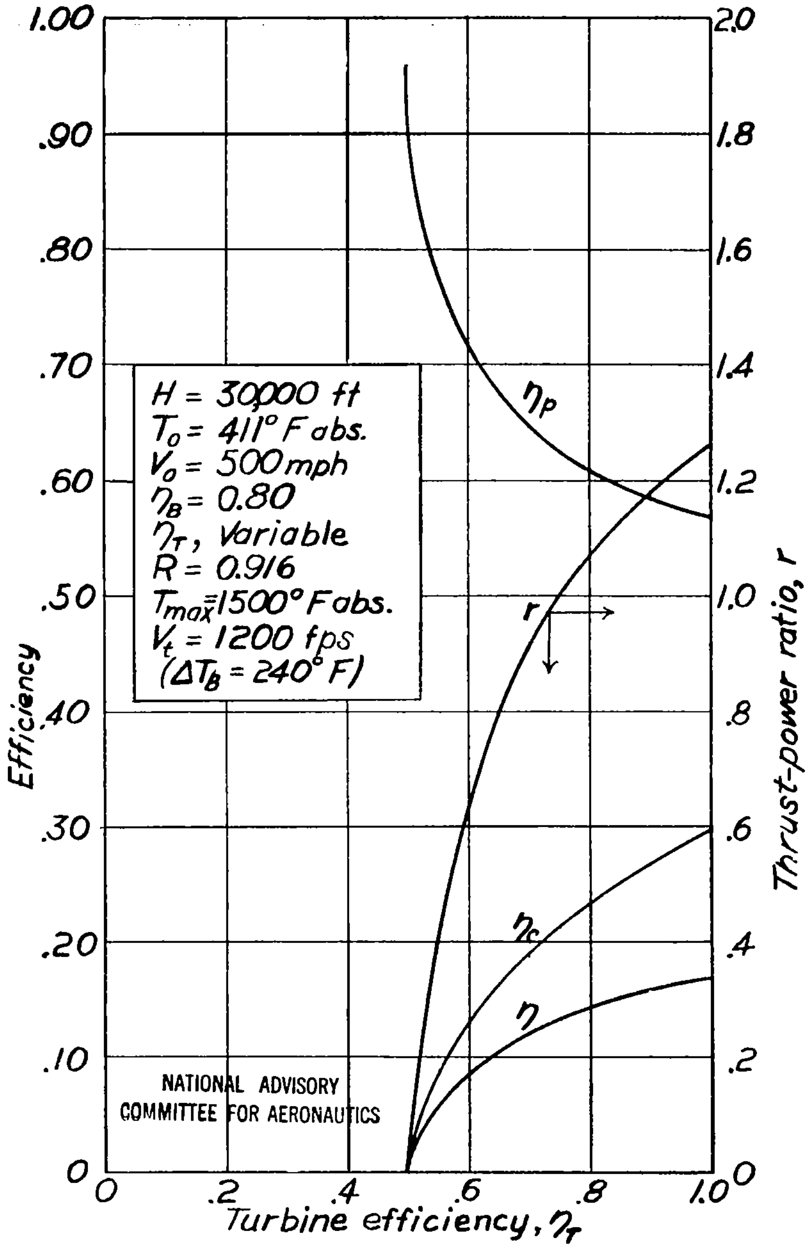


Figure 16.- Variation with turbine efficiency of the power and the efficiencies of a jet-propulsion engine.

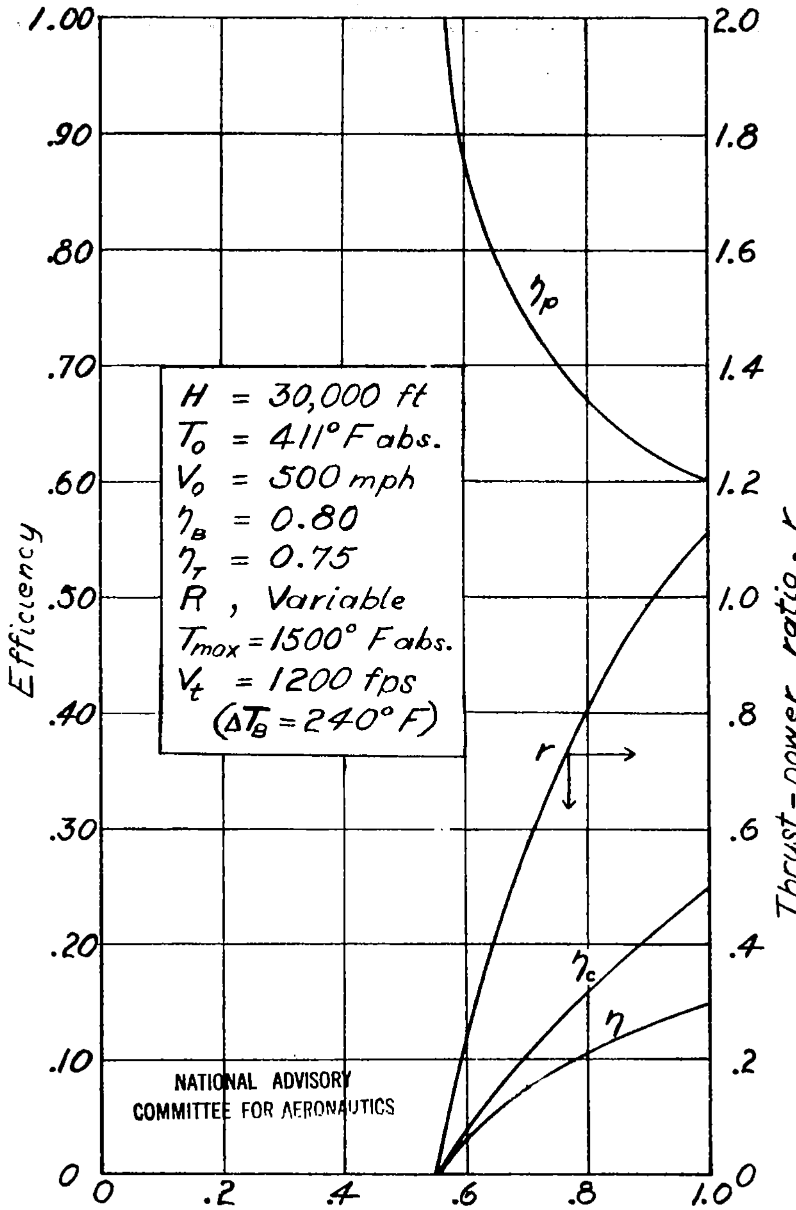


Figure 17.- Variation with combustion-chamber pressure-loss ratio R of the power and the efficiencies of a jet-propulsion engine.

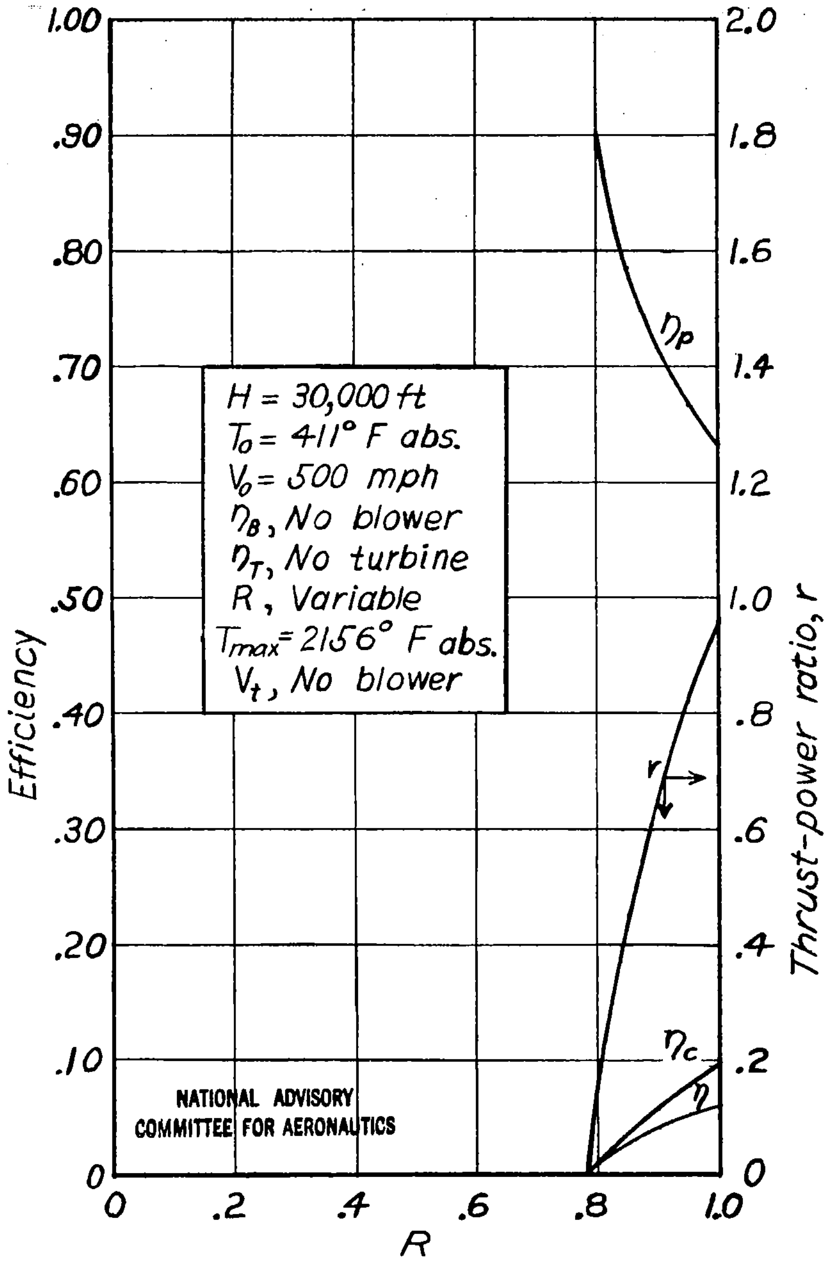


Figure 18.- Variation with combustion-chamber pressure-loss ratio R of the power and the efficiencies of a ram jet.

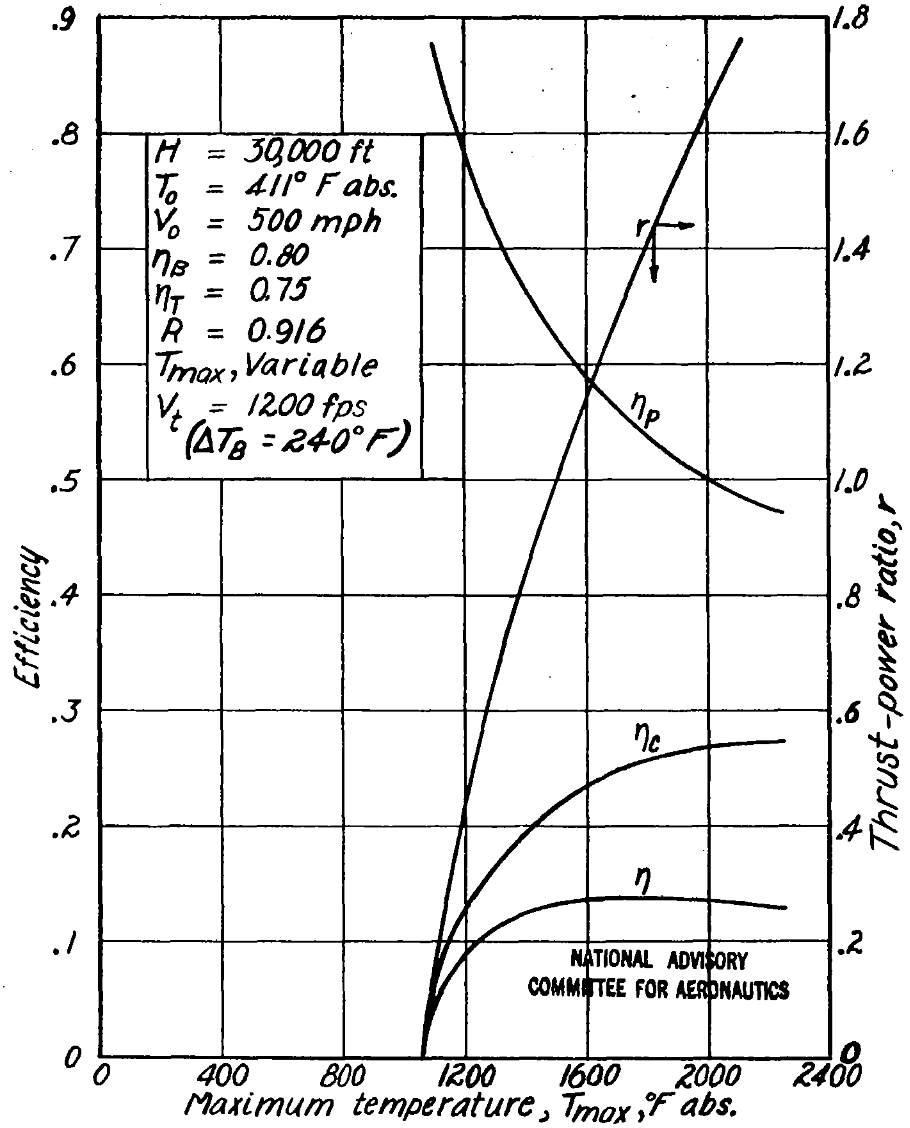


Figure 19.- Variation with maximum temperature of the power and the efficiencies of a jet-propulsion engine.

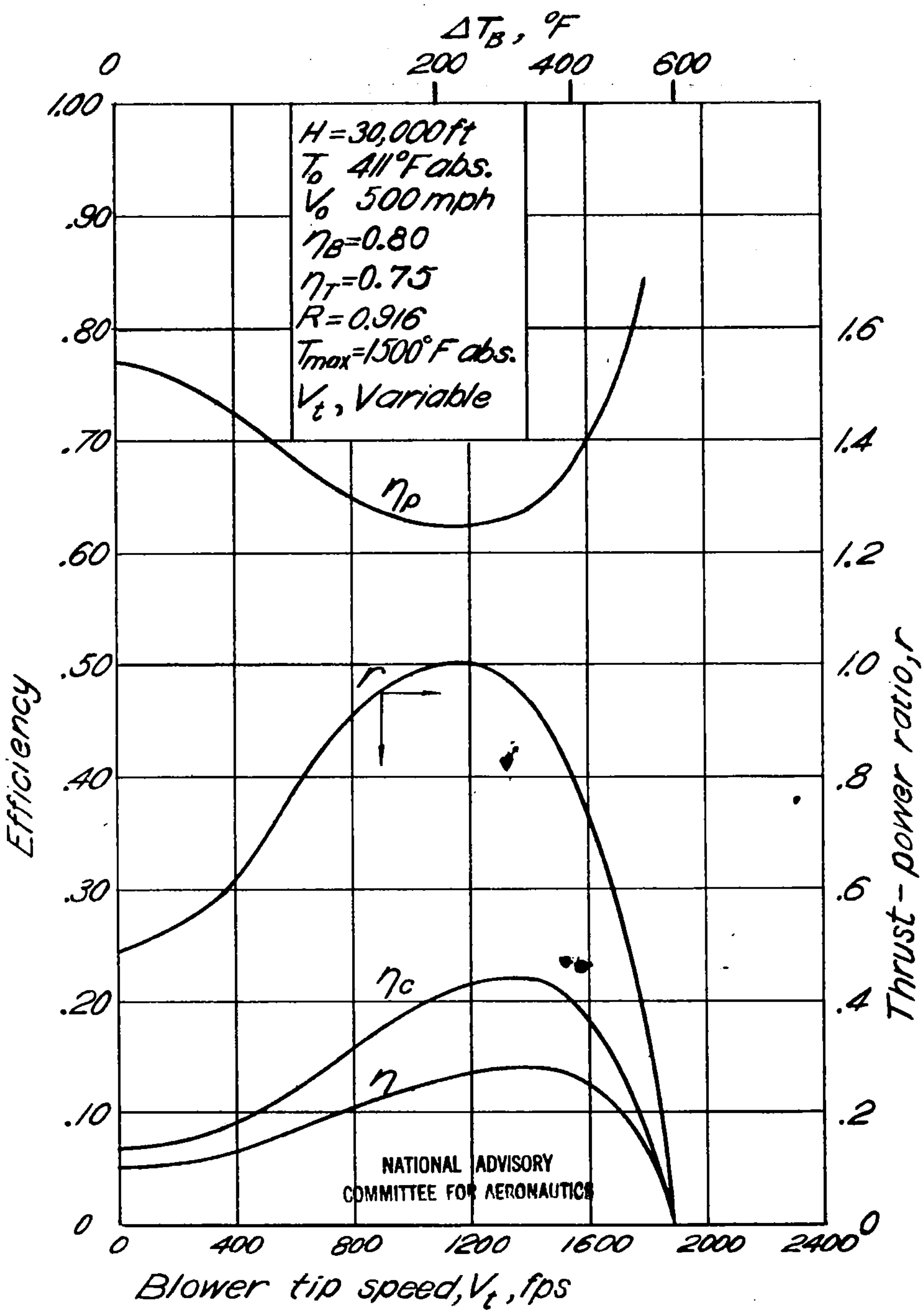


Figure 20.- Variation with blower speed of the power and the efficiencies of a jet-propulsion engine.

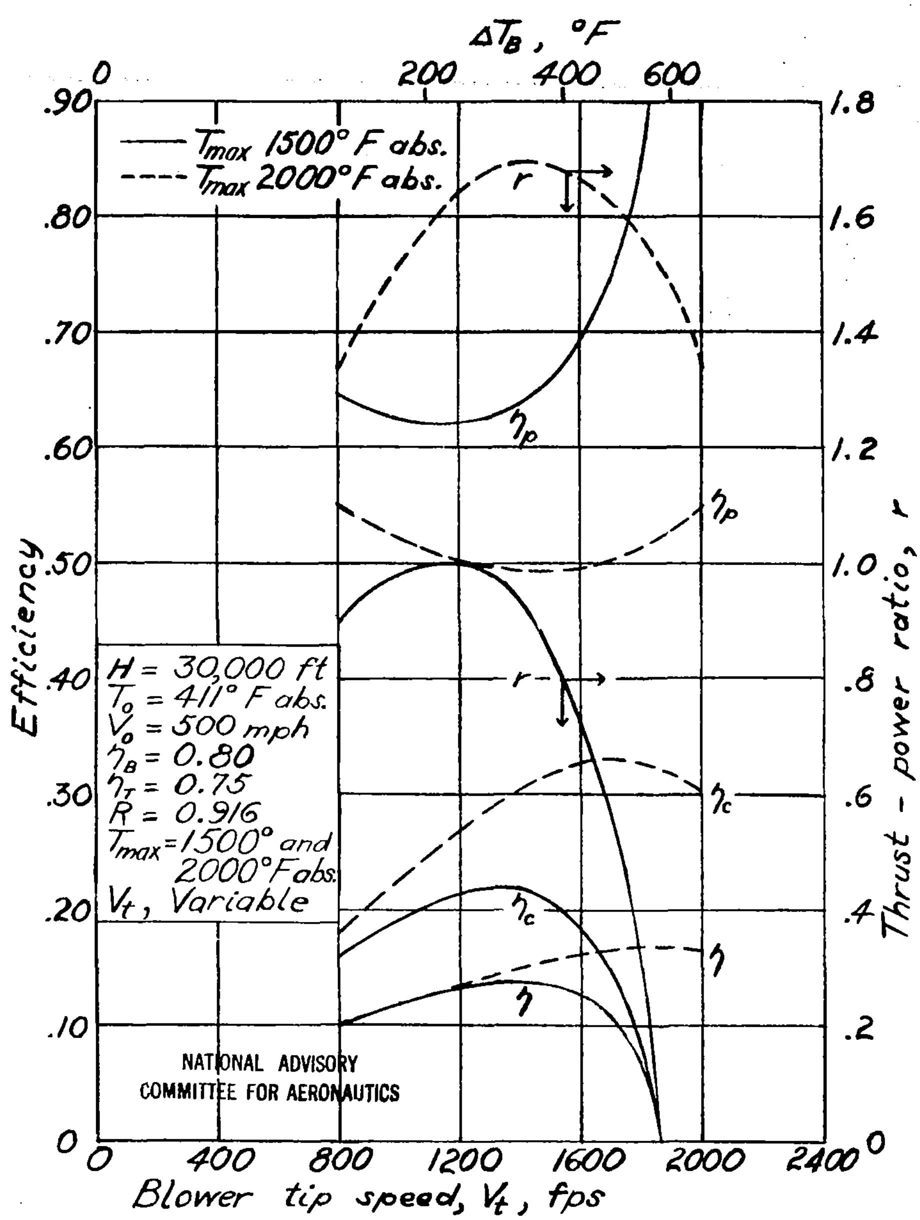


Figure 21.- Comparison of the variation with blower speed of the power and the efficiencies of a jet-propulsion engine at two maximum temperatures.

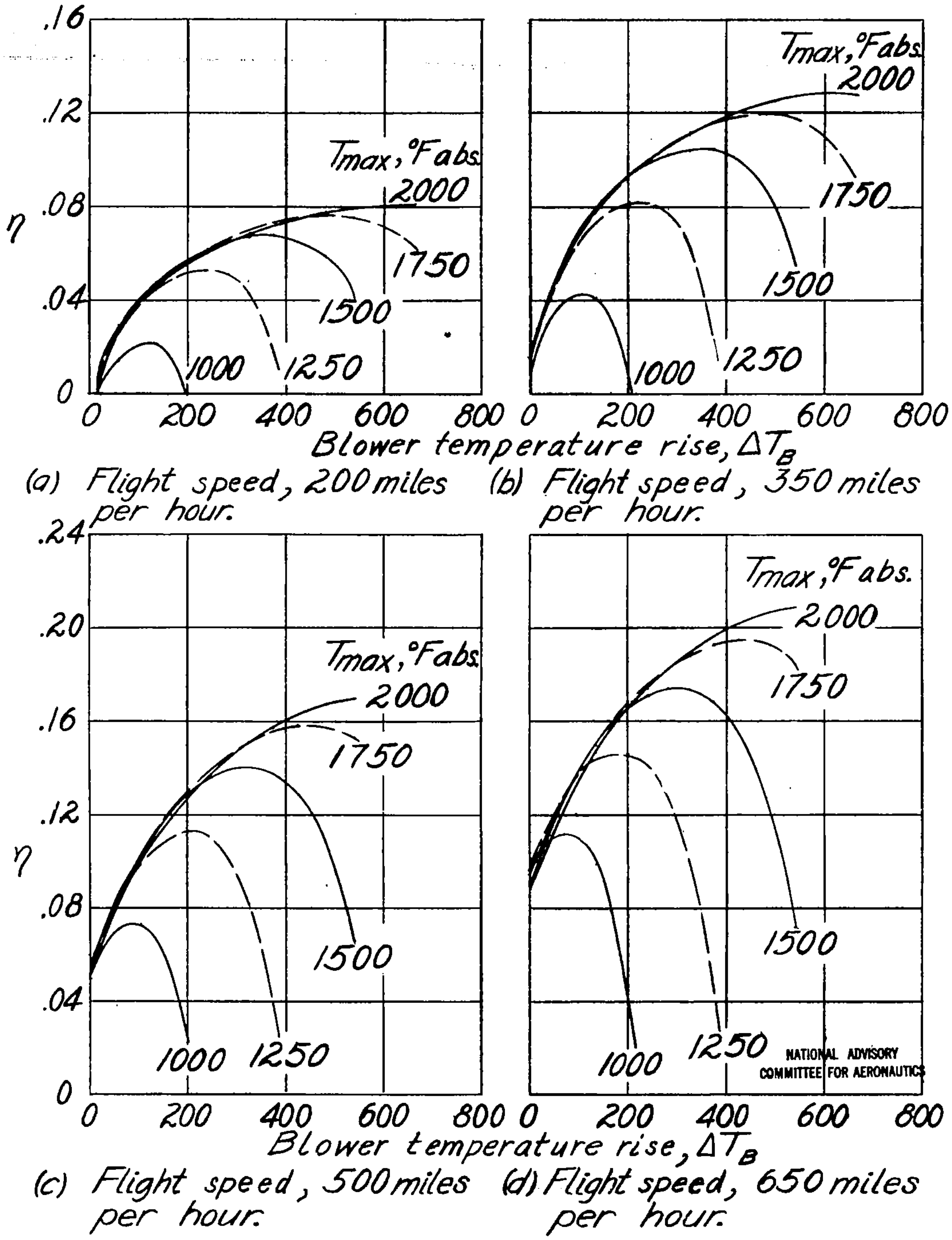


Figure 22.- Combined efficiency of a jet-propulsion engine over a range of blower temperature rise, flight speed, and maximum temperature. Altitude, 30,000 feet.

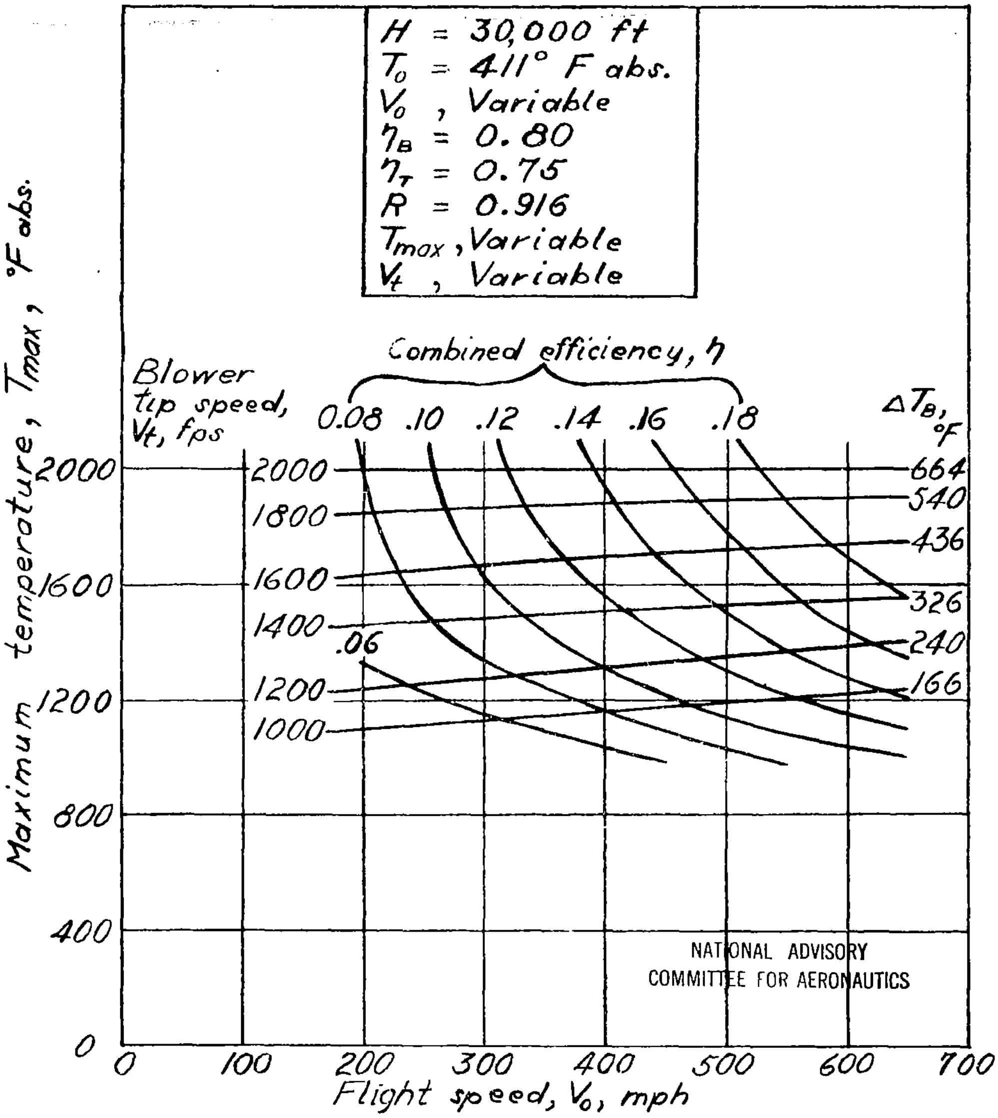


Figure 23.- Maximum temperature for maximum combined efficiency as a function of flight speed and blower speed. Altitude, 30,000 feet.

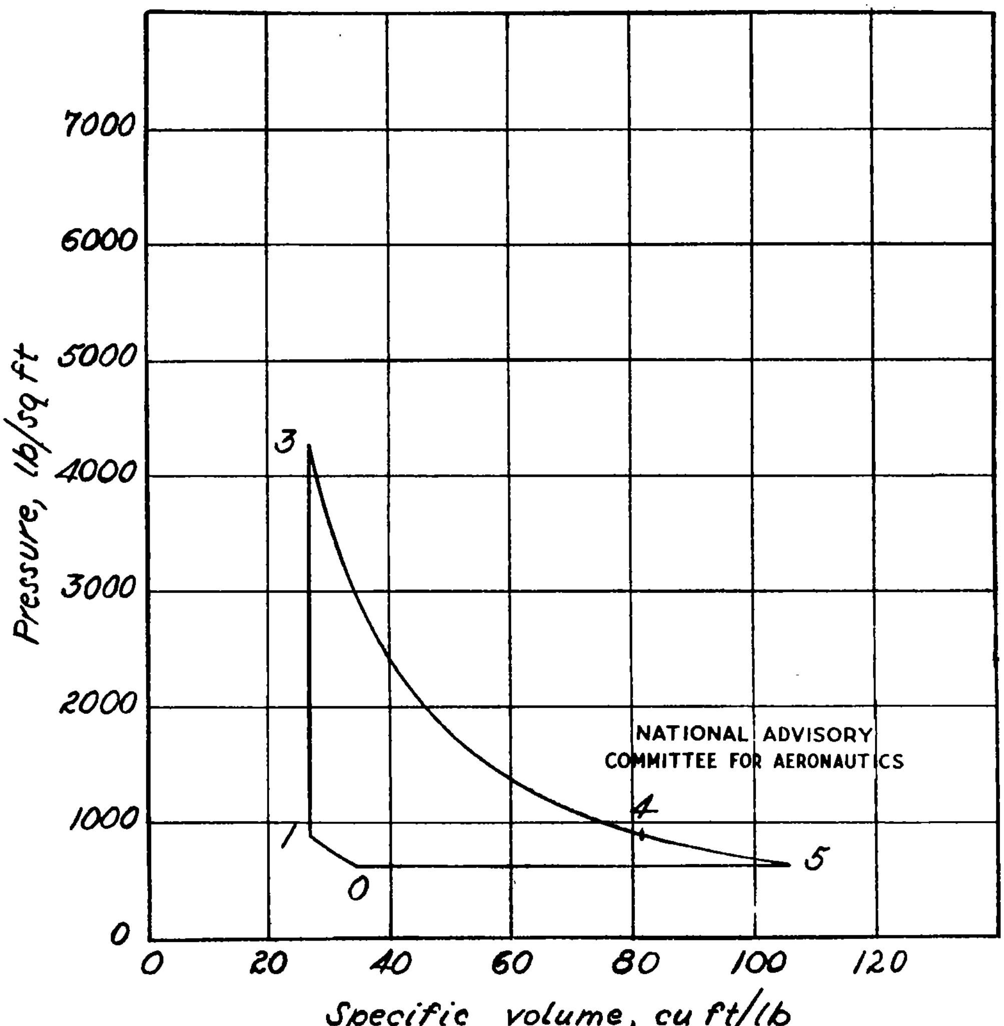


Figure 24.- Pressure-volume diagram for constant-volume-combustion cycle.

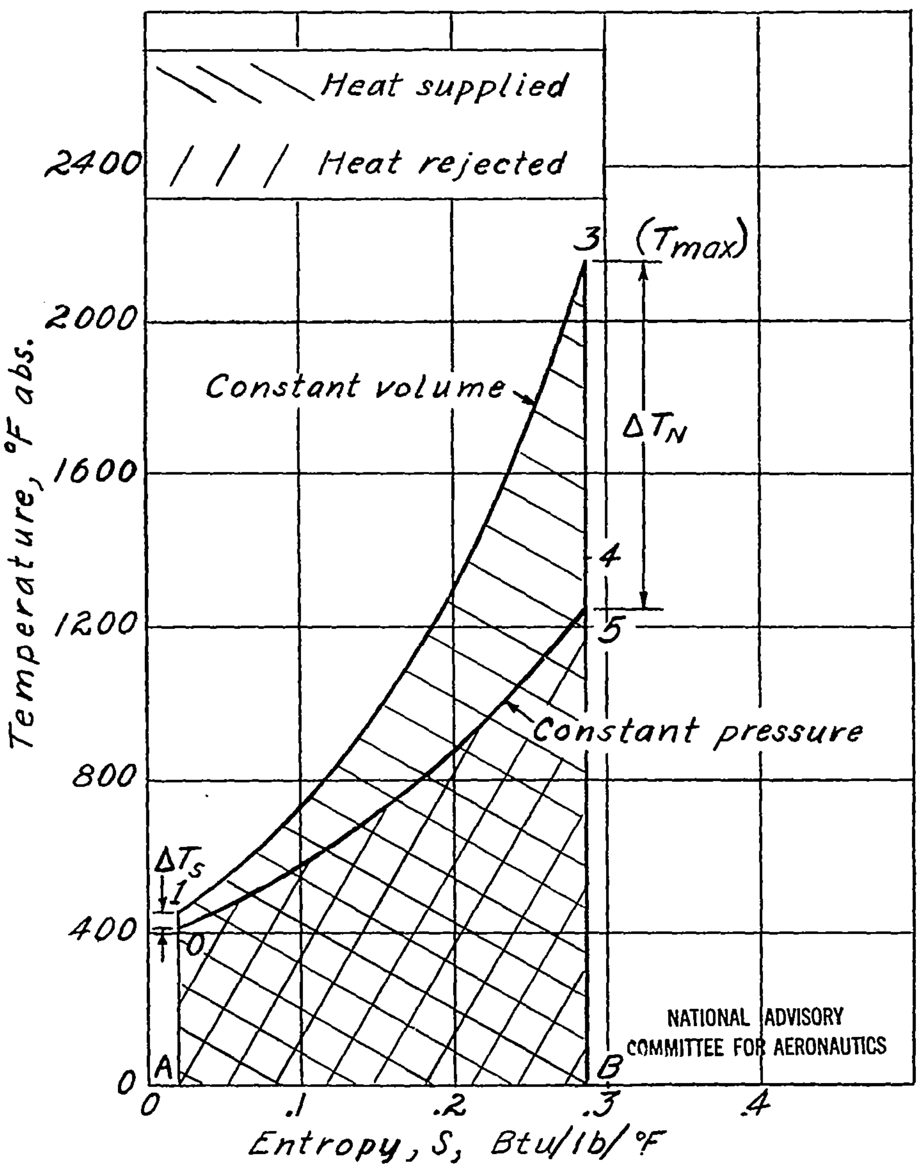


Figure 25.- Temperature-entropy diagram for constant-volume-combustion cycle.