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PERFORMANCE CHARTS FOR A TURBOJET SYSTEM

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SUMMARY

Convenient charts are presented for computing the thrust, fuel consumption, and other performance values of a turbojet system. These charts take into account the effects of ram pressure, compressor pressure ratio, ratio of combustion-chamber-outlet temperature to atmospheric temperature, compressor efficiency, turbine efficiency, combustion efficiency, discharge-nozzle coefficient, losses in total pressure in the inlet to the jet-propulsion unit and in the combustion chamber, and variation in specific heats with temperature. The principal performance charts show clearly the effects of the primary variables and correction charts provide the effects of the secondary variables.

The performance of illustrative cases of turbojet systems is given. It is shown that maximum thrust per unit mass rate of air flow occurs at a lower compressor pressure ratio than minimum specific fuel consumption. The thrust per unit mass rate of air flow increases as the combustion-chamber discharge temperature increases. For minimum specific fuel consumption, however, an optimum combustion-chamber discharge temperature exists, which in some cases may be less than the limiting temperature imposed by the strength temperature characteristics of present materials.

INTRODUCTION

The jet-propulsion system consisting of a compressor, a combustion chamber, a turbine, and a discharge nozzle, which is generally known as the turbojet, is now under extensive development for the propulsion of high-speed airplanes.

An analysis was made of the performance of such a system at the NACA Cleveland laboratory during 1944 for the purpose of providing convenient charts from which the performance of this system can be quickly and accurately obtained for any given set of operating conditions and system parameters. An attempt was made to predict accurate values of actual performance by the introduction of factors that account for the change in physical properties of the gas as it passes through the cycle and the effect of the change in mass by the addition of fuel. The charts take into account turbine efficiency, compressor efficiency, combustion efficiency, discharge-nozzle coefficient, losses in total pressure in the inlet duct and combustion chamber, ambient atmospheric conditions, flight velocity, compressor pressure ratio, and combustion-chamber-outlet total temperature. These variables are grouped in a few simple charts from which their effects on performance can be readily obtained. The charts and the analysis are presented herein.

The performance of the subject jet-propulsion system is given for several interesting cases to illustrate some of the characteristics of the system.

ANALYSIS

A diagram of the turbojet is shown in figure 1. Air is inducted into the intake of the unit and delivered to the compressor inlet. Part of the dynamic pressure of the free air stream is converted into static pressure at the compressor inlet by the diffusing action of the inlet duct. The air is further compressed in passing through the compressor and is delivered to the combustion chamber where fuel is injected and burned. The products of combustion then pass through the turbine nozzles and buckets where an appreciable drop in pressure occurs and finally are discharged rearwardly through the discharge nozzle to provide thrust.

The variables affecting the performance are divided into a primary group and a secondary group. The variables of the primary group are shown on the principal charts for determining the performance of the jet-propulsion unit. The variables of the secondary group are shown on an auxiliary chart for determining a factor ϵ usually close to unity, which also appears as a variable on the principal performance charts.

The primary group of variables includes:

(a) Compressor efficiency η_c

(b) Compressor total-pressure ratio p_2/p_1

(c) Burner efficiency η_f

(d) Ratio of combustion-chamber-outlet total temperature to free atmospheric temperature T_4/T_0

(e) Turbine efficiency η_t

(f) Airplane velocity V_0

(g) Atmospheric temperature T_0

(h) Discharge-nozzle velocity coefficient C_v, which includes losses in the tail pipe following the turbine

The secondary group includes:

(a) Drop in total pressure across the inlet ducting caused by friction and turbulence Δp_d

(b) Drop in total pressure across the combustion chamber caused by both the mechanical obstruction of the burners and the momentum increase of the gases during combustion $\Delta p_{(2-4)}$

(c) Effect of the difference between the physical properties of hot exhaust gases during the expansion processes and cold air (The effect of change in specific heat of the gas during the other processes is included in the principal charts.)

A chart is given from which a factor ϵ can be obtained corresponding to the values of the secondary group of variables. This factor ϵ appears in the parameters on the principal performance charts.

The compressor efficiency η_c in this report is defined as the isentropic work done in the compressor, including the difference between the kinetic energy of the air at the compressor outlet and at the compressor inlet, divided by the compressor shaft work. The turbine efficiency η_t as defined in this report is the shaft work divided by the difference between the isentropic work available in expanding the gas from turbine inlet conditions to the static pressure at turbine discharge and the kinetic energy of the gas at the turbine discharge. It is emphasized that, in these definitions of compressor and turbine efficiencies, the kinetic energy of the gas leaving the compressor or turbine is not charged against the respective unit as an energy loss.

The symbols used solely in the derivations of performance equations are listed in appendix A. The significance of the symbols appearing in the charts and in the subsequent discussion are as follows:

A	ratio of compressor pressure ratio p_2/p_1 to reference pressure ratio $(p_2/p_1)_{ref}$
a, b, c	factors that measure effects produced by secondary variables
C_v	velocity coefficient of discharge nozzle
c_{pa}	specific heat of air at constant pressure at $T_0 = 519^\circ$ R, 7.73 (Btu)/(slug)(°F)
F	net jet thrust, (lb)
f	fuel-air ratio
h	lower heating value of fuel, (Btu/lb)
J	mechanical equivalent of heat, 778 (ft-lb/Btu)
M	mass rate of air flow, (slug/sec)
P_c	compressor-shaft horsepower input
p_0	atmospheric free-air static pressure, (lb/sq ft absolute)
p_1	total pressure at compressor inlet, (lb/sq ft absolute)
p_2	total pressure at compressor outlet, (lb/sq ft absolute)
Δp_d	drop in total pressure across inlet duct, (lb/sq ft)
$\Delta p_{(2-4)}$	over-all drop in total pressure across combustion chamber due to mechanical obstruction of the burners and momentum increase of gases during combustion, (lb/sq ft)
T_0	atmospheric temperature, (°R)
T_1	compressor-inlet total temperature, (°R)
T_2	compressor-outlet total temperature, (°R)
T_4	combustion-chamber-outlet total temperature, (°R)
V_0	airplane velocity, (ft/sec)

V_5 gas velocity at turbine discharge, (ft/sec)

V_j jet velocity, (ft/sec)

ΔV_j increase in jet velocity due to effect of turbine-loss reheat, (ft/sec)

W_f weight flow of fuel, (lb/hr)

Y ratio of ram temperature rise to free-air atmospheric temperature, $V_o^2/2\ J\ c_{pa}\ T_o$

Z ratio of compressor power per unit mass rate of air flow to enthalpy of air at temperature T_o, $550\ P_c/J\ c_{pa}\ M\ T_o$

γ_a ratio of specific heats of air

ϵ correction factor that accounts for over-all effects produced by secondary variables

η_c compressor efficiency

η_f efficiency of combustion of fuel in combustion chamber

η_t turbine efficiency

$$(p_2/p_1)_{ref} = \left[\left(\frac{1}{1+Y}\right)^2 \eta_c \eta_t \epsilon \frac{T_4}{T_o}\right]^{\frac{\gamma_a}{2(\gamma_a - 1)}}$$

$(p_2/p_1)_{ref}$ also equal to the compressor pressure ratio for maximum thrust per unit mass rate of air flow when the rate of change of ϵ with compressor pressure ratio is negligible.

All velocities are axial and all except V_o are relative to the unit.

The equations from which the charts are prepared are listed in appendix B and are derived in appendix C.

In some cases, when a large pressure drop occurs across the final jet-discharge nozzle, reheat associated with the energy losses in the turbine has an appreciable effect on the jet velocity. A chart is given whereby the effect of reheat on the jet velocity can be readily determined.

DISCUSSION OF CHARTS

Useful equations. - The net thrust of the turbojet, when the effect of the fuel weight is neglected, is given by the equation

$$F = M\ (V_j - V_o) \tag{1a}$$

When the effect of fuel weight is included, the thrust is given by

$$F = M\ (V_j - V_o) + f\ M\ V_j \tag{1b}$$

The net thrust horsepower thp is given by

$$thp = F\ V_o/550 \tag{2}$$

The compressor-shaft horsepower per slug per second of air is expressed as

$$\left.\begin{aligned} P_c/M &= J\ c_{pa}\ T_o\ Z/550 \\ &= 5675\ Z\ (T_o/519) \end{aligned}\right\} \tag{3}$$

The compressor-inlet total temperature is obtained from

$$T_1/T_o = 1 + Y \tag{4}$$

The fuel consumption per unit mass rate of air flow is given in terms of the fuel-air ratio by the following relation

$$W_f/M = 115,920\ f \tag{5}$$

By means of equations (1) to (5) and the curves of figures 2 to 7 the performance of the turbojet engine and some associated quantities of interest can be readily determined. The curves are given in a form which shows the effects of the important variables and enables either very accurate computations or rapid but less accurate computations to be made.

Curves for obtaining the flight Mach number, the values of Y, and the compressor-inlet total pressure for various values of the factor $V_o\sqrt{519/T_o}$ are shown in figure 2. The compressor-inlet total temperature is obtained from the value of Y and equation (4).

The quantity $\eta_c Z$ is plotted against the compressor total-pressure ratio and Y in figure 3. The compressor power (and hence the turbine power) is computed from equation (3) and the value of Z.

The effect of the variation in the specific heat of air during compression is neglected in this plot, the error introduced being less than 1 percent for the range of compressor pressure ratios shown in figure 3 and for compressor inlet temperatures up to 550° R.

The value of $(p_2/p_1)_{ref}$ plotted against the factor $\eta_c\eta_t\epsilon\frac{T_4}{T_0}\left(\frac{1}{1+Y}\right)^2$ is also given in figure 3. The actual compressor pressure ratio p_2/p_1 divided by the quantity $(p_2/p_1)_{ref}$ defines the value of the factor A used in figure 4(a). This quantity $(p_2/p_1)_{ref}$ is useful in that it is equal to the compressor pressure ratio for maximum thrust per unit mass rate of air flow for any given value of $\eta_c\eta_t\epsilon\frac{T_4}{T_0}\left(\frac{1}{1+Y}\right)^2$, if the rate of change of the factor ϵ with respect to a change in pressure ratio is negligible. The factor ϵ is one which accounts for the effects of pressure losses in the inlet duct to the system, pressure drop in the combustion chamber, and the deviation from the value of the specific heat of air at 519° R of the specific heats of the gases during the expansion through the turbine and the nozzle. In a well designed system the value of ϵ is close to or slightly greater than unity and does not vary appreciably with p_2/p_1.

When the change in ϵ with p_2/p_1 is appreciable, then $(p_2/p_1)_{ref}$ is less than the compressor pressure ratio giving maximum thrust per unit mass rate of air flow; however, even in this case the thrust per unit mass rate of air flow corresponding to $(p_2/p_1)_{ref}$ is generally within 1 percent of the true maximum. Hence figure 3 permits a rapid approximation of the pressure ratio for maximum thrust per unit mass rate of air flow.

The main performance chart for determining the jet velocity is shown in figure 4(a). From the left-hand set of curves of figure 4(a), the jet-velocity factor $V_j\sqrt{\frac{\eta_c\eta_t}{c_v^2}}\sqrt{\frac{519}{T_0}}$ can be determined as a function of $\eta_c\eta_t\epsilon\frac{T_4}{T_0}$ and the parameter A or 1/A for zero flight speed. (When A is less than unity, the value of 1/A is used in reading values from fig. 4(a).) The jet-velocity factor can be obtained from airplane velocities other than zero by moving horizontally across the graph to the desired velocity curve on the right-hand set of curves and then reading the value on the lower abscissa. The thrust can then be computed from the value of V_j and equation (1a). As previously mentioned, the value of A is found by dividing the compressor pressure ratio p_2/p_1 by the value of $(p_2/p_1)_{ref}$ obtained from figure 3 corresponding to the values of the parameters η_c, η_t, ϵ, T_4, T_o, and Y being investigated.

It is noted in figure 4(a) that for given values of η_c, η_t, T_4, and T_o, if ϵ remains constant as p_2/p_1 or A varies, then the variation of jet velocity with pressure ratio occurs along the constant $\eta_c\eta_t\,\epsilon\,\frac{T_4}{T_o}$ line. In this case, V_j has a maximum value when A is equal to unity, which occurs at a pressure ratio equal to $(p_2/p_1)_{ref}$. Actually, however, for a given unit as p_2/p_1 varies, the value of ϵ changes slightly and hence $\eta_c\eta_t\,\epsilon\,\frac{T_4}{T_o}$ changes, with the result that V_j has a maximum value for a value of p_2/p_1 somewhat greater than $(p_2/p_1)_{ref}$. It should also be noted that $(p_2/p_1)_{ref}$ is changed by the change in ϵ and this new value must be used in computing the new value of A when p_2/p_1 is varied. In any event, the value of V_j corresponding to $A = 1$ is a close approximation to the jet velocity for maximum thrust per unit mass rate of air flow M for a given set of values of T_4, T_o, and component efficiencies.

The losses in kinetic energy in the turbine passages appear as heat energy in the gas leaving the turbine. This energy will be termed "turbine-loss reheat." If there is further expansion of the gas in passing through the jet nozzle (caused by a reduction in static pressure in passing from the turbine exit to the jet-nozzle exit), a conversion of part of the turbine-loss reheat to kinetic energy occurs in the jet. If, however, the velocity at the turbine exit is substantially equal to the final jet velocity, no further expansion occurs and no kinetic energy is recovered from the turbine-loss reheat. The curves of figure 4(a) correspond to this case. The ratio of the increase in jet velocity to the final jet velocity $\Delta V_j/V_j$ obtained when the velocity at the turbine discharge V_5 is less than the final jet velocity is shown in figure 4(b).

Figure 4(b) shows that $\Delta V_j/V_j = 0$ when $C_vV_5/V_j = 1$ for all values of turbine efficiency. It is also noted that $\Delta V_j/V_j$ approaches 0 as turbine efficiency approaches 1 for all values of C_vV_5/V_j because the turbine-loss reheat approaches 0 with increase in turbine efficiency.

It is evident from figure 4(b) that, for a given turbine efficiency, the smaller the ratio of $C_v V_5/V_j$, the greater is the recovery of turbine-loss reheat. Decrease in turbine-discharge velocity V_5 is obtained by increase in annular area swept by the turbine buckets. Bucket stress is one of the principal limitations on bucket height and thus on bucket-annulus area.

The compressor-outlet total temperature T_2 plotted against the factor $T_0 (1 + Y + Z)$ is shown in figure 5. This curve includes the variation in the specific heat of the air during compression and was computed using reference 1.

The fuel-air ratio factor $\eta_f f$ is plotted in figure 6 against $T_4 - T_2$ (the rise in total temperature in the combustion chamber) for various values of T_4. These curves were constructed using data on specific heats of air and exhaust-gas mixtures given in reference 2 and are for a fuel having a lower heating value of 18,900 Btu per pound and a hydrogen-carbon ratio of 0.185. For fuels having other values of h, the value of f given in figure 6 is corrected accurately by multiplying it by the factor $18{,}900/h$. The effect of the hydrogen-carbon ratio of the fuel on f is generally small and for a range of hydrogen-carbon ratios from 0.16 to 0.21 the error due to the deviation from the value of 0.185 is less than one-half of 1 percent. The fuel consumption per unit mass rate of air flow is obtained from the value of f and equation (5).

The value of ϵ, which takes care of the effect of the secondary group of variables, is obtained from figure 7. The quantity ϵ is given by the relation $\epsilon = 1 - a - b + c$, where a, b, and c are given in figure 7. The effect of the drop in total pressure across the inlet duct Δp_d is shown in figure 7(a). The effect of the over-all drop in total pressure across the combustion chamber $\Delta p_{(2-4)}$ is introduced in figure 7(b). Reference 3, which discusses combustion in a chamber of constant flow area, is useful in evaluating the momentum-pressure drop in the combustion chamber. A correction for the difference between the physical properties of the hot gases and the cold air, involved in the computation of the expansion processes through the turbine and the jet nozzle is given in figure 7(c). Although ϵ does not differ appreciably from unity, a change in ϵ of 1 percent in some cases may introduce a change of several percent in the thrust.

In the discussion of the charts, the effect of the weight of injected fuel was not mentioned. It is shown in appendix C that the effect of the weight of fuel on the jet velocity can be taken into account by using for the value of η_t in the charts the product of the actual turbine efficiency and (1 + f). This term appears in the factor $\eta_c \eta_t \epsilon \frac{T_4}{T_0}\left(\frac{1}{1+Y}\right)^2$ in figure 3 used in finding $(p_2/p_1)_{ref}$ and in the factors $\eta_c \eta_t \epsilon \frac{T_4}{T_0}$ and $V_j \sqrt{\eta_c \eta_t / C_v^2} \sqrt{519/T_0}$ of figure 4(a). The value of V_j determined is then used in equation (1b) which takes into account the additional weight of fuel introduced.

As an example of the use of these figures, consider a system having the following performance and operating parameters:

1. Compressor efficiency η_c 0.80
2. Turbine efficiency η_t 0.90
3. Combustion efficiency η_f 0.97
4. Discharge-nozzle velocity coefficient C_v 0.96
5. Airplane velocity V_0, (ft/sec) 733
6. Compressor total-pressure ratio p_2/p_1 6
7. Atmospheric free-air static pressure p_0, (in. Hg) 29.9
8. Atmospheric temperature T_0, ($^{\circ}$R) 519
9. Combustion-chamber-outlet total temperature T_4, ($^{\circ}$R) . . . 1960
10. Drop in total pressure across inlet duct Δp_d, (in. Hg). . 0.5
11. Drop in total pressure across combustion chamber $\Delta p_{(2-4)}$, (in. Hg) 3
12. h, (Btu/lb) 18,500

(a) Determination of Y and flight Mach number

From items 5 and 8

13. $V_0 \sqrt{\frac{519}{T_0}}$, (ft/sec) 733

From item 13 and figure 2

14. Y 0.0861
15. Flight Mach number 0.656

(b) Determination of Z and compressor power

Using items 6 and 14, read on figure 3

16. $\eta_c Z$ 0.726

From items 16 and 1

17. Z 0.908

Using items 17 and 8 in equation (3) the compressor power per unit mass rate of air flow is

18. P_c/M, (hp)/(slug/sec) 5153

(c) Determination of fuel-air ratio and fuel consumption

From items 8, 14, and 17

19. $T_o (1 + Y + Z)$, (°R) 1035

Using item 19 and figure 5

20. T_2, (°R) 1025

From items 20 and 9

21. $T_4 - T_2$, (°F) 935

From items 21 and 9 and figure 6

22. $\eta_f f$ 0.01372

Using items 22 and 3

23. f 0.01414

Since the lower heating value of the fuel is equal to 18,500 Btu per pound (item 12), item 23 has to be multiplied by the factor $\frac{18,900}{18,500}$ and the adjusted value is

24. f 0.01445

From item 24 and equation (5)

25. W_f/M, (lb/hr)/(slug/sec) 1675

(d) Determination of the factor ϵ

From figure 2 and item 13

26. $\dfrac{p_1 + \Delta p_d}{p_o}$ 1.335

From items 26, 10, and 7

27. $\frac{\Delta p_d}{p_1}$ 0.013

while from items 7 and 11

28. $\frac{\Delta p_2}{p_0}$ 0.10

and from items 14 and 16

29. $Y + \eta_c Z$ 0.812

Using items 27 and 29 in figure 7(a)

30. a 0.005

Using items 28 and 29 in figure 7(b)

31. b 0.004

From items 26, 10, 7, and 6

32. $\frac{p_2}{p_0}$ 7.91

which when used with items 9 and 24 in figure 7(c) gives

33. c 0.034

From items 30, 31, and 33

34. $\epsilon = 1 - 0.005 - 0.004 + 0.034$ 1.025

(e) Determination of $(p_2/p_1)_{ref}$ and A

Using items 1, 2, 34, 9, 8, and 14

35. $\eta_c \eta_t \epsilon \frac{T_4}{T_0} \left(\frac{1}{1 + Y}\right)^2$ 2.363

From item 35 and figure 3

36. $(p_2/p_1)_{ref}$ 4.50

Dividing item 36 by item 6

37. A 1.333

(f) Determination of jet velocity, net thrust per unit mass rate of air flow, and other performance quantities

Using items 1, 2, 34, 9, and 8

38. $\eta_c \eta_t \epsilon \frac{T_4}{T_o}$ 2.787

From items 38, 37, 13, and figure 4(a) the jet-velocity factor is

39. $V_j \sqrt{\frac{\eta_c \eta_t}{C_v^{\,2}}}\ \sqrt{\frac{519}{T_o}}$, (ft/sec) 1806

and from items 39, 1, 2, 4, and 8

40. V_j, (ft/sec) 2044

The net thrust per unit mass rate of air flow is obtained from items 40, 5, and equation (1a)

41. F/M, (lb)/(slug/sec) 1311

The thrust horsepower per unit mass rate of air flow is calculated from items 41, 5, and equation (2)

42. thp/M, (thp)/(slug/sec) 1747

From items 25 and 41

43. W_f/F, (lb/hr)/(lb thrust) 1.278

and from items 25 and 42

44. W_f/thp, (lb)/(thp-hr) 0.959

(g) Effect of the weight of injected fuel and turbine-loss reheat on jet velocity and thrust

Where more accurate results are desired, the calculations are made taking into account the effect of the weight of fuel introduced and the effect of turbine-loss reheat. The effect of the fuel on jet velocity is handled by using for the value of η_t the product of the turbine efficiency and $(1 + f)$. This will now be done for the case just considered.

From items 24 and 35

45. $\eta_c \eta_t \epsilon \frac{T_4}{T_o}\left(\frac{1}{1+Y}\right)^2$ 2.396

From figure 3 the corresponding

46. $(p_2/p_1)_{ref}$ 4.61

From items 6 and 46

47. A 1.30

Similarly accounting for fuel flow, item 38 becomes

48. $\eta_c \eta_t \epsilon \frac{T_4}{T_o}$ 2.827

so that from items 47, 48, and 13, and figure 4(a)

49. $V_j \sqrt{\frac{\eta_c \eta_t}{{c_v}^2}} \sqrt{\frac{519}{T_c}}$, (ft/sec) 1838

Again taking into account the effect of fuel by adjusting the η_t term

50. V_j, (ft/sec) 2065

which differs from item 40 by 1 percent

The effect of reheat may be important when η_t is considerably less than unity and the velocity at turbine discharge is appreciably less than the final jet velocity. Let it be assumed in the example being discussed that the turbine is designed to have a discharge velocity of

51. V_5, (ft/sec) 700

Then from items 4, 50, and 51

52. $c_v V_5/V_j$ 0.33

From items 8, 9, and 17

53. $\frac{T_4}{T_o\ Z}$ 4.16

From figure 4(b) corresponding to items 2, 52, and 53

54. $\Delta V_j/V_j$ 0.012

and from items 50 and 54

55. ΔV_j, (ft/sec) 25

Using items 55 and 50

56. Corrected V_j, (ft/sec) 2090

Thus in this case, reheat provides an additional 1 percent increase in the value of V_j.

The thrust per unit mass rate of air flow is obtained from items 56, 5, and equation (1b)

57. F/M, (lb)/(slug/sec) 1357

compared with 1311 where the effects of fuel and reheat were neglected.

From equation (2) and items 57 and 5

58. thp/M, (thp)/(slug/sec) 1808

and using items 25 and 57

59. W_f/F, (lb/hr)/(lb) 1.234

and items 25 and 58 give

60. W_f/thp, (lb/thp-hr) 0.926

(h) Optimum thrust per unit mass flow of air

The value of V_j corresponding to $(p_2/p_1)_{ref}$ is very close to the value of V_j giving maximum thrust per unit mass rate of air flow. The compressor pressure ratio p_2/p_1 for maximum F/M is slightly greater than $(p_2/p_1)_{ref}$ because of the increase in ϵ with pressure ratio. The value of the maximum F/M and the corresponding value of p_2/p_1 can be obtained by computing V_j for a range of values of p_2/p_1 in the vicinity of and greater than $(p_2/p_1)_{ref}$ by the method previously illustrated for a compressor pressure ratio of 6. From a plot of V_j against p_2/p_1 the maximum value of V_j (and hence F/M) and the corresponding value of p_2/p_1 can be read. This computation for the previously illustrated case was made and the results are presented in the following table.

The effect of the weight of fuel and the turbine-loss reheat were neglected in calculating the values given in the table. Since item 36 gave a value for $(p_2/p_1)_{ref}$ of 4.5, the range of compressor pressure ratios chosen started at this value. In the calculation of ϵ the values of Δp_d and $\Delta p_{(2-4)}$ were assumed to remain constant at the values given in items 10 and 11, as p_2/p_1 varied.

$\frac{p_2}{p_1}$	ϵ	$\eta_c \eta_t \epsilon \frac{T_4}{T_0}$	$\left(\frac{p_2}{p_1}\right)_{ref}$	A	V_j $\left(\frac{ft}{sec}\right)$	F/M $\left(\frac{lb}{slug/sec}\right)$	W_f/M $\left(\frac{lb/hr}{slug/sec}\right)$	W_f/F $\left(\frac{lb/hr}{lb}\right)$
4.5	1.017	2.765	4.44	1.014	2049	1316	1820	1.383
4.8	1.019	2.771	4.46	1.076	2051	1318	1790	1.358
5.0	1.020	2.773	4.46	1.120	2052	1319	1770	1.342
5.2	1.021	2.776	4.47	1.163	2051	1318	1751	1.329
5.6	1.023	2.782	4.49	1.247	2049	1316	1713	1.302
6.0	1.025	2.787	4.50	1.333	2044	1311	1675	1.278

The table shows the increase in ϵ with increase in p_2/p_1. This causes an increase in the value of $\eta_c \eta_t \epsilon \frac{T_4}{T_0}$ and the corresponding value of $(p_2/p_1)_{ref}$. The percentage increase in A is slightly less than the percentage increase in p_2/p_1 because of the increase in $(p_2/p_1)_{ref}$. The maximum value of F/M is 1319 as compared with a value of F/M of 1316 obtained at a compressor pressure ratio of 4.5 which was the $(p_2/p_1)_{ref}$ for the previous example (see item 36). The values of V_j and F/M varied so slightly over the range of compressor pressure ratios from 4.5 to 6.0 that they were calculated using the formulas given in the appendixes rather than using the charts in order to detect the variation. It is noted that the true optimum occurs at a p_2/p_1 of about 5.0 which is about 11 percent greater than the p_2/p_1 of 4.5. If a maximum value of F/M is the main design consideration, it is doubtful that the additional complication to obtain the higher compressor pressure ratio is warranted by the small increase in F/M obtained. However, for the case where a higher compressor-discharge pressure results in an increased mass flow of gases through the engine (for example, when sonic flow in the turbine nozzles instead of in the compressor limits the gas flow through an engine), the increase in F is greater than the increase in F/M, so that higher values of p_2/p_1 may be justified. When fuel consumption is also an important consideration, the increase in compressor pressure ratio may be desirable as indicated by the values of W_f/F in the table.

JET-PROPULSION-UNIT PERFORMANCE

For illustration of the performance and some of the characteristics of the turbojet system, several cases of interest will be discussed.

The following parameters are assumed:

Compressor efficiency η_c	0.85
Turbine efficiency η_t	0.90
Discharge-nozzle velocity coefficient C_v	0.97
Combustion efficiency η_f	0.96
Heating value of fuel h, (Btu/lb)	18,900
ϵ	1.00

These compressor and turbine efficiencies are not unreasonably high when it is considered that in the definition of efficiency in this report the compressor and the turbine are credited with the kinetic energy of the gases at the compressor and turbine exits, respectively.

The computed turbojet performance in this illustrative case includes the contribution of the fuel weight.

The values of component efficiencies and ϵ for any given turbojet engine vary with altitude and flight speed. In the present computations, the component efficiencies and ϵ were assumed constant at the values listed; hence, the illustrative curves represent the performance of a series of turbojet engines having the listed characteristics. One curve is also given for a case in which the variation of ϵ with compressor pressure ratio is considered.

When $V_o = 0$, $T_o = 519°$ R, figure 8 shows the rate of fuel consumption per unit thrust and the static thrust per unit mass rate of air flow plotted against the compressor pressure ratio for various values of the gas total temperature at the combustion-chamber exit. It is noted that minimum specific fuel consumption occurs at a higher compressor pressure ratio than maximum thrust per unit mass rate of air flow. A curve for $T_4 = 1960°$ R where the variation in ϵ with p_2/p_1 is considered is also shown in figure 8. For this curve, values of $\Delta p_d/p_o = 0.04$ and $\Delta p_{(2-4)}/p_o = 0.10$ were chosen and assumed to remain constant. (For a given unit, however, $\Delta p_{(2-4)}$ will also vary with p_2/p_1 so that the determination of the actual variation in ϵ with compressor pressure ratio becomes quite complex.) It is seen from figure 8 that the value of compressor pressure ratio for a maximum value of F/M is greater for the case where ϵ varies with pressure ratio than for the case where ϵ is assumed constant; and that the peak value of F/M for the first case is slightly higher than that for the second case.

Figure 9(a) is a replot of figure 8 and shows compressor pressure ratio and fuel consumption per unit thrust plotted against thrust per unit mass rate of air flow. Similar curves are presented in figures 9(b) and 9(c) for other combinations of atomospheric temperature and airplane velocity. A scale of specific fuel consumption in pounds per thrust horsepower-hour is added on figures 9(b) and 9(c).

The amount of air handled by a unit is limited by the diameter of the unit. When high thrust per unit mass rate of air flow rather than low specific fuel consumption is the primary consideration, it is apparent from figure 9 that high combustion-chamber discharge temperatures should be used. High thrust is the more important consideration in take-off, climb, and maximum-speed operation.

The curves of figure 9 show that, with no limitation on compressor pressure ratio, higher thrust per unit mass rate of air flow and lower specific fuel consumption can be obtained by increasing the combustion-chamber-outlet temperature until the value giving minimum specific fuel consumption is reached. For figures 9(a), 9(b), and 9(c), this temperature is less than 1460° R, about 2210° R, and 1710° R, respectively. Further increase in temperature permits an increase in thrust at the cost of increase in specific fuel consumption. As the gas temperature at the combustion-chamber outlet is increased, a large increase in compressor pressure ratio is required to maintain nearly minimum specific fuel consumption.

If the available compressor pressure ratio is limited, the combustion-chamber-outlet temperature for minimum specific fuel consumption is very sensitive to the other operating conditions. For example, at a limiting compressor pressure ratio of 4, minimum specific fuel consumption occurs at a temperature below the lowest values shown in figure 9. If the limiting compressor pressure ratio is 8, the combustion-chamber discharge temperature for minimum specific fuel consumption is still less than the lowest temperature shown in figure 9(c) for an atmospheric temperature of 412° R but approaches an intermediate value of approximately 1710° R for an atmospheric temperature of 519° R (fig. 9(b)). The optimum combustion-gas temperature is also very sensitive to the efficiencies of the components of the jet-propulsion units.

In figure 10(a) the specific fuel consumption and the thrust per unit mass rate of air flow are plotted against airplane velocity for the conditions listed in the figure for the following cases:

(a) Compressor pressure ratio chosen to give values of $A = 1$

(b) Compressor pressure ratio chosen to give minimum specific fuel consumption

It is noted that the specific fuel consumption for case (a) is between 15 and 23 percent higher than for case (b) for airplane velocities between 300 and 800 feet per second; the percentage difference in specific fuel consumption is greater at the lower airplane velocities and at the lower atmospheric temperatures.

The thrust per unit mass rate of air flow is between 21 and 31 percent higher for case (a) than for case (b) for airplane velocities between 300 and 800 feet per second; the greater percentage difference in thrust per unit mass rate of air flow occurs at the lower airplane velocities and the lower atmospheric temperature.

Figure 10(b) shows the compressor pressure ratios and the values of A that are associated with the performance values given in figure 10(a). The large increase in required pressure ratio from the condition of $A = 1$ to the condition of minimum specific fuel consumption is noted.

CONCLUSIONS

The following conclusions are based on an analysis of a turbojet system:

1. Maximum thrust per unit mass rate of air flow occurs at a lower compressor pressure ratio than minimum specific fuel consumption.

2. Increase in combustion-chamber discharge temperature causes an increase in thrust. An optimum temperature, however, exists at which minimum specific fuel consumption is obtained. This temperature for minimum specific fuel consumption is at some conditions less than the temperature limit imposed by the strength-temperature characteristics of the materials of present turbojet units.

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APPENDIX A

ADDITIONAL SYMBOLS USED IN THE DERIVATIONS OF PERFORMANCE EQUATIONS

Symbols used in the derivations of performance equations, in addition to those given in the report, are:

A' factor defined as equal to $\left[\frac{p_2/p_1}{(p_2/p_1)_{ref}}\right]^{\frac{\gamma_a-1}{\gamma_a}}$ or $A^{\frac{\gamma_a-1}{\gamma_a}}$

c_p average specific heat at constant pressure of the exhaust gases during the expansion process. This term, when used with the temperature change accompanying the expansion, gives the change in enthalpy per unit mass. (Btu)/(slug)($^{\circ}$F)

$\bar{c}_p$ average specific heat at constant pressure of the gases during the combustion process. This term, when used with the temperature change during combustion, is used to determine the fuel consumption. (Btu)/(slug)($^{\circ}$F)

c_{pa2} specific heat of air at constant pressure at compressor-outlet total temperature. It is equal to the enthalpy per unit mass (zero enthalpy arbitrarily fixed at absolute zero temperature) divided by the total temperature. (Btu)/(slug)($^{\circ}$F)

K, K' ratios of functions expressed in terms of physical properties of exhaust gas to same functions expressed in terms of physical properties of cold air. These functions are described in appendix C.

p_4 total pressure at turbine inlet, (lb/sq ft absolute)

p_{5s} static pressure at turbine discharge, (lb/sq ft absolute)

P_t turbine-shaft horsepower output

R gas constant of exhaust gas, (ft-lb)/(slug)($^{\circ}$F)

R_a gas constant of air, (ft-lb)/(slug)($^{\circ}$F)

T_{5s} gas temperature at turbine discharge, ($^{\circ}$R)

ΔT_{5s} reheat due to net turbine loss, (^{o}F)

W_{th} work obtainable from isentropic expansion of exhaust gas, (ft-lb)/(slug)

γ ratio of specific heats of exhaust gas

ρ_o density of atmospheric air, (slug/cu ft)

The subscript i refers to the hypothetical case of no burning, no turbine in system, compressor-shaft power input $\eta_c P_c$, compressor efficiency 100 percent, and no losses in system beyond compressor.

APPENDIX B

EQUATIONS FOR THE PERFORMANCE FIGURES

The equation numbers correspond to those in the derivation given in appendix C.

Figure 2:

$$Y = \frac{V_o^2}{2\ J\ c_{pa}\ T_o} = \frac{1}{2\ J\ c_{pa}\ 519}\left(V_o\sqrt{\frac{519}{T_o}}\right)^2 \tag{C6}$$

$$\frac{p_1 + \Delta p_d}{p_o} = \left[1 + \frac{1}{2\ J\ c_{pa}\ 519}\left(V_o\sqrt{\frac{519}{T_o}}\right)^2\right]^{\frac{\gamma_a}{\gamma_a - 1}} \tag{C70}$$

$$\text{Flight Mach number} = \sqrt{\frac{1}{(\gamma_a - 1)\ J\ c_{pa}\ 519}}\left(V_o\sqrt{\frac{519}{T_o}}\right) \tag{C72}$$

Figure 3:

$$\frac{p_2}{p_1} = \left(1 + \frac{\eta_c\ Z}{1 + Y}\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{C67}$$

$$\left(\frac{p_2}{p_1}\right)_{ref} = \left[\left(\frac{1}{1 + Y}\right)^2 \eta_c\eta_t\ \epsilon\ \frac{T_4}{T_o}\right]^{\frac{\gamma_a}{2(\gamma_a - 1)}} \tag{C68}$$

Figure 4(a):

$$V_j\sqrt{\frac{\eta_c\eta_t}{C_v^2}\ \frac{519}{T_o}} = \sqrt{\frac{519}{T_o}\ V_o^2 + 2Jc_{pa}519\left[\eta_c\eta_t\ \epsilon\ \frac{T_4}{T_o} - \left(A' + \frac{1}{A'}\right)\sqrt{\eta_c\eta_t\ \epsilon\ \frac{T_4}{T_o} + 1}\right]} \tag{C37}$$

where $A' = A^{\frac{\gamma_a - 1}{\gamma_a}}$

Figure 4(b):

$$\frac{\Delta V_j}{V_j} = \frac{\frac{1}{2}\left[1 - \left(\frac{C_v V_5}{V_j}\right)^2\right]\left(\frac{1}{\eta_t} - 1\right)}{\frac{T_4}{T_o Z}\frac{c_p}{c_{pa}} - 1} \tag{C62}$$

Figure 5:

$$T_2 = \frac{c_{pa}}{c_{pa2}} T_o (1 + Y + Z) \tag{C29}$$

Figure 6:

$$\eta_f f = \frac{\bar{c}_p (T_4 - T_2)}{32.2 h} \tag{C27}$$

where $\bar{c}_p$ is determined from unpublished data

Figure 7(a):

$$a = \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y + \eta_c Z} \tag{C43}$$

Figure 7(b):

$$b = \frac{\Delta p_{(2-4)}}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y + \eta_c Z}\left(\frac{1}{1 + Y + \eta_c Z}\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{C45}$$

Figure 7(c):

$$c = \frac{W_{th}/R T_4}{\left(\frac{\gamma_a}{\gamma_a - 1}\right)\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]}\frac{R}{R_a} - 1 \tag{C50}$$

APPENDIX C

DERIVATION OF EQUATIONS FOR JET VELOCITY, THRUST, THRUST HORSEPOWER, FUEL CONSUMPTION, SPECIFIC FUEL CONSUMPTION, AND MISCELLANEOUS EXPRESSIONS

From the momentum equation the net jet thrust, when the effect of the mass of fuel is neglected, is

$$F = M\,(V_j - V_o) \tag{C1a}$$

and when the mass of fuel is included

$$F = M\,(V_j - V_o) + f\,M\,V_j \tag{C1b}$$

The thrust horsepower developed by the jet is

$$thp = F\,V_o/550 \tag{C2}$$

Jet Velocity and Thrust

Consider the hypothetical case of a unit running with a compressor efficiency of 100 percent but with a compressor-shaft power input equal to $\eta_c P_c$ (that is, the product of the actual compressor efficiency by the actual shaft power input). Also assume no turbine in the system and no burning (that is, the compressor is considered to be driven by an engine). The available jet kinetic energy, assuming no losses after the compressor but accounting for the losses in the intake system leading to the compressor, is

$$\frac{1}{2} M\,V_{ji}^2 = \frac{1}{2} M\,V_o^2 + 550\,\eta_c P_c - \frac{\Delta p_d}{\rho_o} M \tag{C3}$$

The following approximation is accurate for a wide range of T_{2i} and p_2/p_o.

$$\frac{1}{2} M\,V_{ji}^2 = M\,J\,c_{pa2}\,T_{2i}\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right] \tag{C4}$$

From the conservation of energy,

$$\frac{c_{pa2}}{c_{pa}} T_{2i} - T_o = \frac{550\ \eta_c P_c}{M\ J\ c_{pa}} + \frac{V_o^2}{2\ J\ c_{pa}} \tag{C5}$$

By definition

$$Y = V_o^2/2\ J\ c_{pa}\ T_o \tag{C6}$$

$$Z = 550\ P_c/J\ c_{pa}\ M\ T_o \tag{C7}$$

then

$$\frac{c_{pa2}}{c_{pa}} T_{2i} = T_o\ (1 + Y + \eta_c\ Z) \tag{C8}$$

and

$$V_{ji}^2 = V_o^2 \left(1 + \eta_c \frac{Z}{Y} - \frac{\Delta p_d}{\frac{1}{2}\ \rho_o\ V_o^2}\right) \tag{C9}$$

Now

$$\frac{\Delta p_d}{\frac{1}{2}\ \rho_o\ V_o^2} = \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{2\ J\ c_{pa}\ T_o}{V_o^2} = \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y} \tag{C10}$$

and equation (C9) becomes

$$V_{ji}^2 = V_o^2 \left[1 + \eta_c \frac{Z}{Y} - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y}\right] \tag{C11}$$

The compressor energy transferred to the gas in this hypothetical case is equal to the useful energy transferred to the gas in the actual case where the shaft power input is P_c and the compressor efficiency is η_c. Thus, the compressor-discharge pressure p_2 is the same in both cases. The compressor-discharge temperature for the hypothetical case T_{2i} differs from the true compressor-discharge temperature. When V_{ji} and T_{2i} are eliminated from equations (C4), (C8), and (C11), the following relation is obtained:

$$2\,J\,c_{pa}\,T_o\,(1+Y+\eta_c\,Z)\left[1-\left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a-1}{\gamma_a}}\right]=V_o^{\,2}\left[1+\eta_c\,\frac{Z}{Y}-\frac{\Delta p_d}{p_o}\left(\frac{\gamma_a-1}{\gamma_a}\right)\frac{1}{Y}\right] \tag{C12}$$

which is used later to evaluate the compressor outlet pressure p_2. By definition

$$\eta_t=\frac{550\,P_t}{(1+f)\,M\,J\,c_p\,T_4\left[1-\left(\frac{p_{5s}}{p_4}\right)^{\frac{\gamma-1}{\gamma}}\right]-\frac{(1+f)\,M\,V_5^{\,2}}{2}} \tag{C13}$$

Now consider the actual system with burning taking place and turbine power being removed to drive the compressor. The jet velocity (when the effect of reheat due to the turbine loss, which occurs in the further expansion of the gases from turbine-discharge static pressure to atmospheric pressure, is neglected) is given by

$$V_j=c_v\sqrt{2\,J\,c_p\,T_4\left[1-\left(\frac{p_o}{p_4}\right)^{\frac{\gamma-1}{\gamma}}\right]-\frac{550\,P_t}{\frac{1}{2}\,M\,\eta_t\,(1+f)}} \tag{C14}$$

For simplification, the effect of the weight of the fuel injected will be neglected by dropping the term f in equation (C14). The effect of the presence of the fuel on the jet velocity V_j can be taken into account in the subsequent equations and charts for V_j by using, for the value of η_t, the product of the turbine efficiency and $1+f$, as the quantities η_t and f appear only as the product $\eta_t\,(1+f)$ in equation (C14). Now

$$\left[1-\left(\frac{p_o}{p_4}\right)^{\frac{\gamma-1}{\gamma}}\right]=1-\left(\frac{p_o}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\left(1-\frac{\Delta p_{(2-4)}}{p_2}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} \tag{C15}$$

When the last term of equation (C15) is expanded into a series,

$$\left(1 - \frac{\Delta p_{(2-4)}}{p_2}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} = 1 + \frac{\gamma - 1}{\gamma}\,\frac{\Delta p_{(2-4)}}{p_2} \tag{C16}$$

for small $\Delta p_{(2-4)}/p_2$. Since only enough turbine power is removed to drive the compressor

$$P_t = P_c \tag{C17}$$

When equations (C15), (C16), and (C17) are substituted into equation (C14),

$$V_j = C_v \sqrt{2Jc_pT_4\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right] - 2Jc_pT_4\left(\frac{p_o}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{\gamma - 1}{\gamma}\right)\frac{\Delta p_{(2-4)}}{p_2} - \frac{550\ P_c}{\frac{1}{2}M\ \eta_t}} \tag{C18}$$

Let

$$K = \frac{\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]}{\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a-1}{\gamma_a}}\right]}\frac{c_p}{c_{pa}} \tag{C19}$$

and

$$K' = \frac{\left(\frac{p_o}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{\gamma - 1}{\gamma}\right)c_p}{\left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a-1}{\gamma_a}}\left(\frac{\gamma_a - 1}{\gamma_a}\right)c_{pa}} \tag{C20}$$

When equations (C7), (C12), (C19), and (C20) are used in equation (C18),

$$V_j = C_v \sqrt{\frac{T_4}{T_o} \frac{V_o^2\left[1 + \eta_c \frac{Z}{Y} - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y}\right]}{\left(1 + Y + \eta_c Z\right)}\left[K - K' \frac{\frac{\Delta p_{(2-4)}}{p_2}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{2Jc_{pa}T_o}{V_o^2}\left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\left(1 + Y + \eta_c Z\right)}{1 + \eta_c \frac{Z}{Y} - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y}}\right] - \frac{V_o^2 Z}{\eta_t Y}} \qquad (C21)$$

or

$$V_j = C_v \sqrt{\frac{T_4}{T_o} \frac{V_o^2\left(1 + \eta_c \frac{Z}{Y}\right)}{1 + Y + \eta_c Z}\,\epsilon - \frac{V_o^2 Z}{\eta_t Y}} \qquad (C22)$$

where ϵ is defined by the relation

$$\epsilon = \left[1 - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{\left(Y + \eta_c Z\right)}\right]\left[K - K' \frac{\frac{\Delta p_{(2-4)}}{p_2}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\left(\frac{2Jc_{pa}T_o}{V_o^2}\right)\left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\left(1 + Y + \eta_c Z\right)}{1 + \eta_c \frac{Z}{Y} - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{Y}}\right] \qquad (C23)$$

Equation (C22) can be written

$$V_j = V_o \sqrt{C_v^2 \epsilon \frac{T_4}{T_o} \frac{[1 + \eta_c (Z/Y)]}{(1 + Y + \eta_c Z)} - \frac{C_v^2 Z}{\eta_t Y}} \qquad \text{(C24)}$$

When equation (C24) is substituted in equation (C1a)

$$F = M (V_j - V_o) = M V_o \left[\sqrt{C_v^2 \epsilon \frac{T_4}{T_o} \frac{[1 + \eta_c (Z/Y)]}{(1 + Y + \eta_c Z)} - \frac{C_v^2 Z}{\eta_t Y}} - 1 \right] \qquad \text{(C25)}$$

and equation (C6) is used in equation (C25)

$$\frac{F}{M}\sqrt{\frac{519}{T_o}} = \sqrt{2 J c_{pa} 519} \left[\sqrt{C_v^2 \epsilon \frac{T_4}{T_o} \frac{(Y + \eta_c Z)}{(1 + Y + \eta_c Z)} - \frac{\eta_c Z}{\left(\frac{\eta_c \eta_t}{C_v^2}\right)}} - \sqrt{Y} \right] \qquad \text{(C26)}$$

Fuel Consumption

$$\eta_f f = \frac{\bar{c}_p (T_4 - T_2)}{32.2 h} \qquad \text{(C27)}$$

From the conservation of energy

$$c_{pa2} T_2 = c_{pa} T_o + \frac{V_o^2}{2J} + \frac{550 P_c}{M J} \qquad \text{(C28)}$$

so that

$$T_2 = \frac{c_{pa}}{c_{pa2}} T_o (1 + Y + Z) \qquad \text{(C29)}$$

Pressure Ratio for Optimum Thrust

For a given V_o, T_o, T_4, η_c, η_t, and C_v, neglecting the change in ϵ due to a change in $\eta_c Z$, the maximum thrust per unit mass rate of air flow with respect to compressor power input (or pressure ratio) is obtained when

$$\frac{\partial\left(\frac{F}{M}\sqrt{\frac{519}{T_o}}\right)}{\partial\,(\eta_c\, Z)} = 0 = C_V{}^2\,\epsilon\,\frac{T_4}{T_o}\left(\frac{1}{1 + Y + \eta_c\, Z}\right)^2 - \frac{C_V{}^2}{\eta_c\,\eta_t} \tag{C30}$$

from which

$$1 + Y + (\eta_c\, Z)_{ref} = \sqrt{\eta_c\,\eta_t\,\epsilon\,\frac{T_4}{T_o}} \tag{C31}$$

Define A' by the relation

$$A' = \frac{1 + Y + \eta_c\, Z}{1 + Y + (\eta_c\, Z)_{ref}} \tag{C32}$$

$$1 + Y + \eta_c\, Z = A'\sqrt{\eta_c\,\eta_t\,\epsilon\,\frac{T_4}{T_o}} \tag{C33}$$

Jet Velocity, Thrust, and Specific Fuel Consumption

in Terms of the Factor A'

Equation (C24) can be written

$$V_j = \frac{V_o}{\sqrt{Y}}\sqrt{C_V{}^2\,\epsilon\,\frac{T_4}{T_o}\,\frac{(Y + \eta_c\, Z)}{(1 + Y + \eta_c\, Z)} - \frac{C_V{}^2\,\eta_c\, Z}{\eta_c\,\eta_t}} \tag{C34}$$

When equation (C33) is used in equation (C34),

$$V_j = \frac{V_o}{\sqrt{Y}}\sqrt{\left(\frac{C_V{}^2}{\eta_c\eta_t}\right)\left(\eta_c\eta_t\,\epsilon\,\frac{T_4}{T_o}\right)\frac{\left(A'\sqrt{\eta_c\eta_t\,\epsilon\,\frac{T_4}{T_o}} - 1\right)}{A'\sqrt{\eta_c\,\eta_t\,\epsilon\,\frac{T_4}{T_o}}} - \frac{C_V{}^2}{\eta_c\eta_t}\left(A'\sqrt{\eta_c\eta_t\epsilon\frac{T_4}{T_o}} - 1 - Y\right)} \tag{C35}$$

and

$$V_j\sqrt{\frac{\eta_c\eta_t}{C_v^{\ 2}}}=\frac{V_o}{\sqrt{Y}}\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}-\left(A'+\frac{1}{A'}\right)\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}}+1+Y} \tag{C36}$$

When equation (C6) is substituted in equation (C36),

$$V_j\sqrt{\frac{\eta_c\eta_t}{C_v^{\ 2}}}\sqrt{\frac{519}{T_o}}=\sqrt{\frac{519}{T_o}\,V_o^{\ 2}+2Jc_{pa}\,519\left[\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}-\left(A'+\frac{1}{A'}\right)\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}}+1\right]} \tag{C37}$$

Equation (C26) becomes in terms of A'

$$\frac{F}{M}\sqrt{\frac{519}{T_o}}=\sqrt{2Jc_{pa}\,519}\left[\sqrt{\frac{C_v^{\ 2}}{\eta_c\eta_t}}\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}-\left(A'+\frac{1}{A'}\right)\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}}+1+Y}-\sqrt{Y}\right] \tag{C38}$$

The fuel consumption per unit thrust is obtained from equations (C27) and (C38) and is

$$\eta_f\,\frac{W_f}{F}\sqrt{\frac{T_o}{519}}=\frac{3600\,\bar{c}_p}{h\sqrt{2Jc_{pa}\,519}}\left[\frac{T_4-T_2}{\sqrt{\frac{C_v^{\ 2}}{\eta_c\eta_t}}\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}-\left(A'+\frac{1}{A'}\right)\sqrt{\eta_c\eta_t\,\epsilon\frac{T_4}{T_o}}+1+Y}-\sqrt{Y}}\right] \tag{C39}$$

Evaluation of the Correction Factor ϵ

From equations (C12) and (C6)

$$\left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a-1}{\gamma_a}}=1-\frac{Y+\eta_c\,Z-\frac{\Delta p_d}{p_o}\left(\frac{\gamma_a-1}{\gamma_a}\right)}{1+Y+\eta_c\,Z} \tag{C40}$$

When equation (C40) is used in equation (C23),

$$\epsilon = \left[1 - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{1}{(Y + \eta_c Z)}\right]\left\{K - K'\frac{\dfrac{\Delta p_{(2-4)}}{p_2}\left(\dfrac{\gamma_a - 1}{\gamma_a}\right)\left[1 + \dfrac{\Delta p_d}{p_o}\left(\dfrac{\gamma_a - 1}{\gamma_a}\right)\right]}{Y + \eta_c Z - \dfrac{\Delta p_d}{p_o}\left(\dfrac{\gamma_a - 1}{\gamma_a}\right)}\right\} \tag{C41}$$

or

$$\epsilon = K\left[1 - \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\left(\frac{1}{Y + \eta_c Z}\right)\right] - K'\frac{\Delta p_{(2-4)}}{p_2}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{\left[1 + \left(\dfrac{\Delta p_d}{p_o}\right)\left(\dfrac{\gamma_a - 1}{\gamma_a}\right)\right]}{(Y + \eta_c Z)} \tag{C42}$$

let

$$a = \frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\left(\frac{1}{Y + \eta_c Z}\right) \tag{C43}$$

and

$$b = \frac{\Delta p_{(2-4)}}{p_2}\left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{\left[1 + \dfrac{\Delta p_d}{p_o}\left(\dfrac{\gamma_a - 1}{\gamma_a}\right)\right]}{Y + \eta_c Z} \tag{C44}$$

When equation (C40) is used in equation (C44) and the $\frac{\Delta p_d}{p_o}\left(\frac{\gamma_a - 1}{\gamma_a}\right)$ term in the numerator is neglected because it is small in comparison with unity,

$$b = \frac{\dfrac{\Delta p_{(2-4)}}{p_o}\left(\dfrac{\gamma_a - 1}{\gamma_a}\right)}{Y + \eta_c Z}\left(\frac{1}{1 + Y + \eta_c Z}\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{C45}$$

When equations (C43) and (C45) are substituted into equation (C42),

$$\epsilon = K\ (1 - a) - K'\ b$$

The terms K and K' are close to unity in value whereas the values of a and b are small in comparison with unity; therefore, only a very small error is introduced by letting

$$\epsilon = K - a - b \tag{C46}$$

Defining the quantity c as

$$c = K - 1 \tag{C47}$$

then

$$\epsilon = 1 - a - b + c \tag{C48}$$

Now

$$K = \frac{W_{th}/R\,T_4}{\left(\frac{\gamma_a}{\gamma_a - 1}\right)\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]} \frac{R}{R_a} \tag{C49}$$

where the values of $W_{th}/R\,T_4$ are obtained from reference 5. These values correspond to the required temperature T_4 and pressure ratio p_2/p_o. Therefore,

$$c = \frac{W_{th}/R\,T_4}{\left(\frac{\gamma_a}{\gamma_a - 1}\right)\left[1 - \left(\frac{p_o}{p_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]} \frac{R}{R_a} - 1 \tag{C50}$$

Correction for Reheat Accompanying Irreversibility in the Turbine

The actual jet velocity including the reheat in the turbine is given by the equation

$$\frac{V_j^2}{C_v^2} - V_5^2 = 2\,J\,c_p\,T_{5s}\left[1 - \left(\frac{p_o}{p_{5s}}\right)^{\frac{\gamma - 1}{\gamma}}\right] \tag{C51}$$

from which the following equation in terms of differentials is obtained:

$$2\,\frac{V_j}{C_v^{\,2}}\,dV_j = 2\,J\,c_p\left[1-\left(\frac{p_o}{p_{5s}}\right)^{\frac{\gamma-1}{\gamma}}\right]dT_{5s} \tag{C52}$$

When equation (C51) is used in equation (C52),

$$2\,\frac{V_j}{C_v^{\,2}}\,dV_j = \left(\frac{V_j^{\,2}}{C_v^{\,2}} - V_5^{\,2}\right)\frac{dT_{5s}}{T_{5s}} \tag{C53}$$

T_{5s} is the independent variable, therefore

$$\Delta T_{5s} \equiv dT_{5s}$$

For small values of ΔT_{5s} the following equation is very nearly true:

$$\Delta V_j = dV_j \tag{C54}$$

If these expressions for dT_{5s} and dV_j are used in equation (C53)

$$\frac{\Delta V_j}{V_j} = \frac{1}{2}\left[1-\left(\frac{C_v V_5}{V_j}\right)^2\right]\frac{\Delta T_{5s}}{T_{5s}} \tag{C55}$$

ΔT_{5s} is the amount of reheat and is equal to

$$\Delta T_{5s} = \frac{550\ P_t}{M\ J\ c_p}\left(\frac{1}{\eta_t} - 1\right) \tag{C56}$$

whereas the gas temperature at the turbine discharge

$$T_{5s} = T_4 - \frac{550\ P_t}{M\ J\ c_p} - \frac{V_5^{\,2}}{2\ J\ c_p} \tag{C57}$$

When equations (C6), (C7), and (C17) are used in equations (C56) and (C57),

$$\Delta T_{5s} = Z\ T_o\left(\frac{c_{pa}}{c_p}\right)\left(\frac{1}{\eta_t} - 1\right) \tag{C58}$$

$$T_{5s} = T_4 - Z\,T_o\left(\frac{c_{pa}}{c_p}\right) - \frac{V_5^{\,2}}{V_o^{\,2}}\,Y\,T_o\,\frac{c_{pa}}{c_p} \tag{59}$$

and, when equations (C58) and (C59) are substituted into equation (C55),

$$\frac{\Delta V_j}{V_j} = \frac{\frac{1}{2}\left[1 - \left(\frac{C_v V_5}{V_j}\right)^2\right] Z\,T_o\,\frac{c_{pa}}{c_p}\left(\frac{1}{\eta_t} - 1\right)}{T_4 - Z\,T_o\,\frac{c_{pa}}{c_p} - \frac{V_5^{\,2}}{V_o^{\,2}}\,Y\,T_o\,\frac{c_{pa}}{c_p}} \tag{C60}$$

or

$$\frac{\Delta V_j}{V_j} = \frac{\frac{1}{2}\left[1 - \left(\frac{C_v V_5}{V_j}\right)^2\right]\left(\frac{1}{\eta_t} - 1\right)}{\frac{T_4}{T_o\,Z}\,\frac{c_p}{c_{pa}} - 1 - \frac{V_5^{\,2}\,Y}{V_o^{\,2}\,Z}} \tag{C61}$$

The $V_5^{\,2}\,Y/V_o^{\,2}\,Z$ term in the denominator is small in comparison with $(T_4/T_o\,Z)\,(c_p/c_{pa}) - 1$ and can be neglected, resulting in

$$\frac{\Delta V_j}{V_j} = \frac{\frac{1}{2}\left[1 - \left(\frac{C_v V_5}{V_j}\right)^2\right]\left(\frac{1}{\eta_t} - 1\right)}{\frac{T_4}{T_o\,Z}\,\frac{c_p}{c_{pa}} - 1} \tag{C62}$$

Derivation of Miscellaneous Expressions

(a)
$$\eta_c\,Z = \eta_c\,\frac{550\,P_c}{M\,J\,c_{pa}\,T_o} \tag{C63}$$

where the compressor power is accurately given for a wide range of compressor pressure ratios and compressor inlet temperatures by the relation

$$P_c = \frac{M\ J\ c_{pa}\ T_1}{550\ \eta_c}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma_a-1}{\gamma_a}} - 1\right] \tag{C64}$$

and

$$T_1 = T_o + \frac{V_o^2}{2\ J\ c_{pa}} = T_o\ (1 + Y) \tag{C65}$$

When equations (C64) and (C65) are used in equation (C63),

$$\eta_c\ Z = (1 + Y)\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma_a-1}{\gamma_a}} - 1\right] \tag{C66}$$

or

$$\frac{p_2}{p_1} = \left(1 + \frac{\eta_c\ Z}{1 + Y}\right)^{\frac{\gamma_a}{\gamma_a-1}} \tag{C67}$$

(b) When equation (C31) is substituted into equation (C67),

$$\left(\frac{p_2}{p_1}\right)_{ref} = \left[\left(\frac{1}{1 + Y}\right)^2 \eta_c \eta_t\ \epsilon \frac{T_4}{T_o}\right]^{\frac{\gamma_a}{2(\gamma_a-1)}}, \tag{C68}$$

(c) The ideal ram pressure ratio is

$$\frac{p_1 + \Delta p_d}{p_o} = \left(\frac{T_1}{T_o}\right)^{\frac{\gamma_a}{\gamma_a-1}} \tag{C69}$$

and when equations (C65) and (C6) are used in equation (C69)

$$\frac{p_1 + \Delta p_d}{p_o} = (1 + Y)^{\frac{\gamma_a}{\gamma_a-1}} = \left[1 + \frac{1}{2Jc_{pa}\ 519}\left(V_o\sqrt{\frac{519}{T_o}}\right)^2\right]^{\frac{\gamma_a}{\gamma_a-1}} \tag{C70}$$

(d) The Mach number at the inlet to the unit (or the flight Mach number) is

$$\text{Flight Mach number} = V_o \Big/ \sqrt{\gamma_a R_a T_o} \tag{C71}$$

$$= \sqrt{\frac{1}{(\gamma_a - 1) J c_{pa} 519}} \left(V_o \sqrt{\frac{519}{T_o}} \right) \tag{C72}$$

or, when equation (C6) is used in equation (C71),

$$\text{Flight Mach number} = \frac{\sqrt{2 J c_{pa} T_o Y}}{\sqrt{\gamma_a R_a T_o}} = \sqrt{\left(\frac{2}{\gamma_a - 1}\right) Y} \tag{C73}$$

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1. Keenan, Joseph H., and Kaye, Joseph: Thermodynamic Properties of Air. John Wiley & Sons, Inc., 1945.

2. Turner, L. Richard, Lord, Albert M.: Thermodynamic Charts for the Combustion and Mixture Temperatures at Constant Pressure. NACA TN No. 1086, 1946.

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4. Pinkel, Benjamin, and Turner, L. Richard: Thermodynamic Data for the Computation of the Performance of Exhaust-Gas Turbines, NACA ARR No. 4B25, 1944.

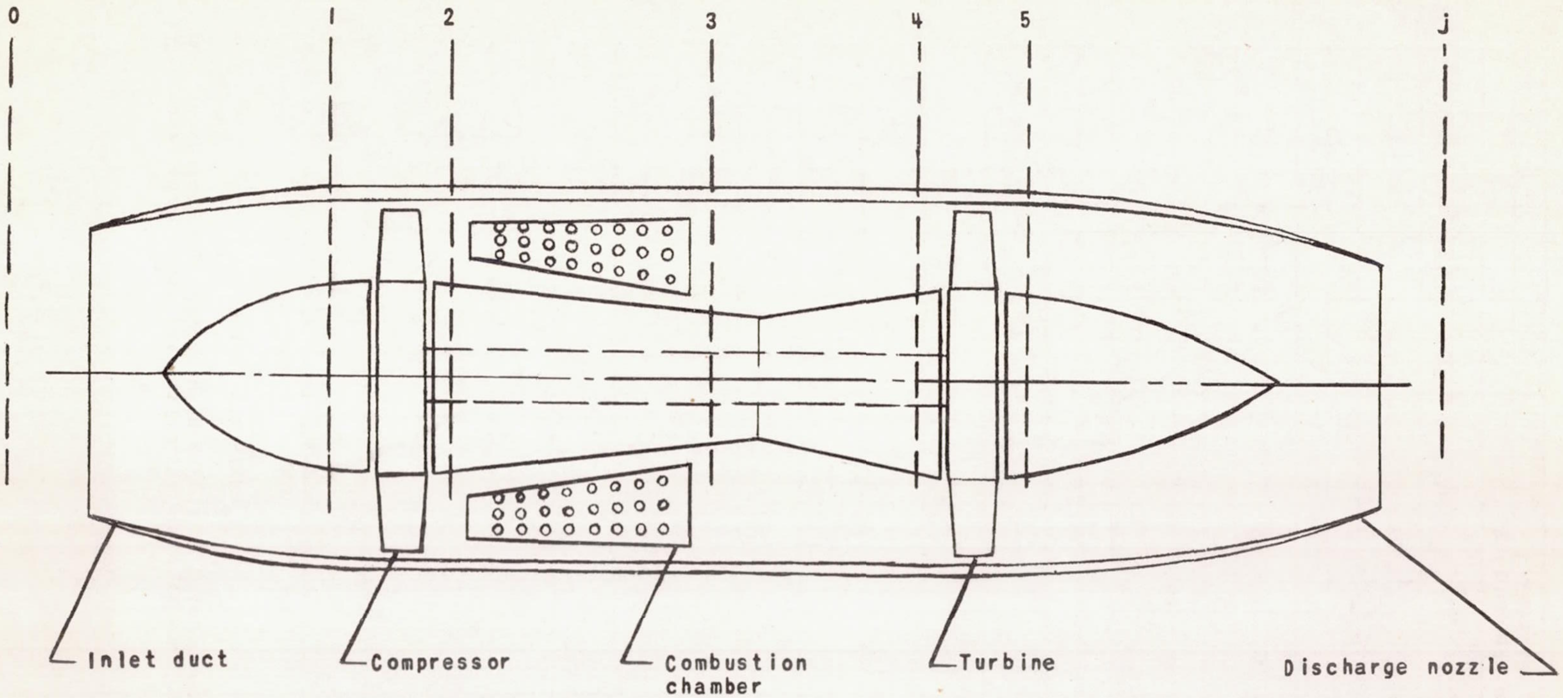

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Figure 1. - Schematic diagram of the turbojet system.

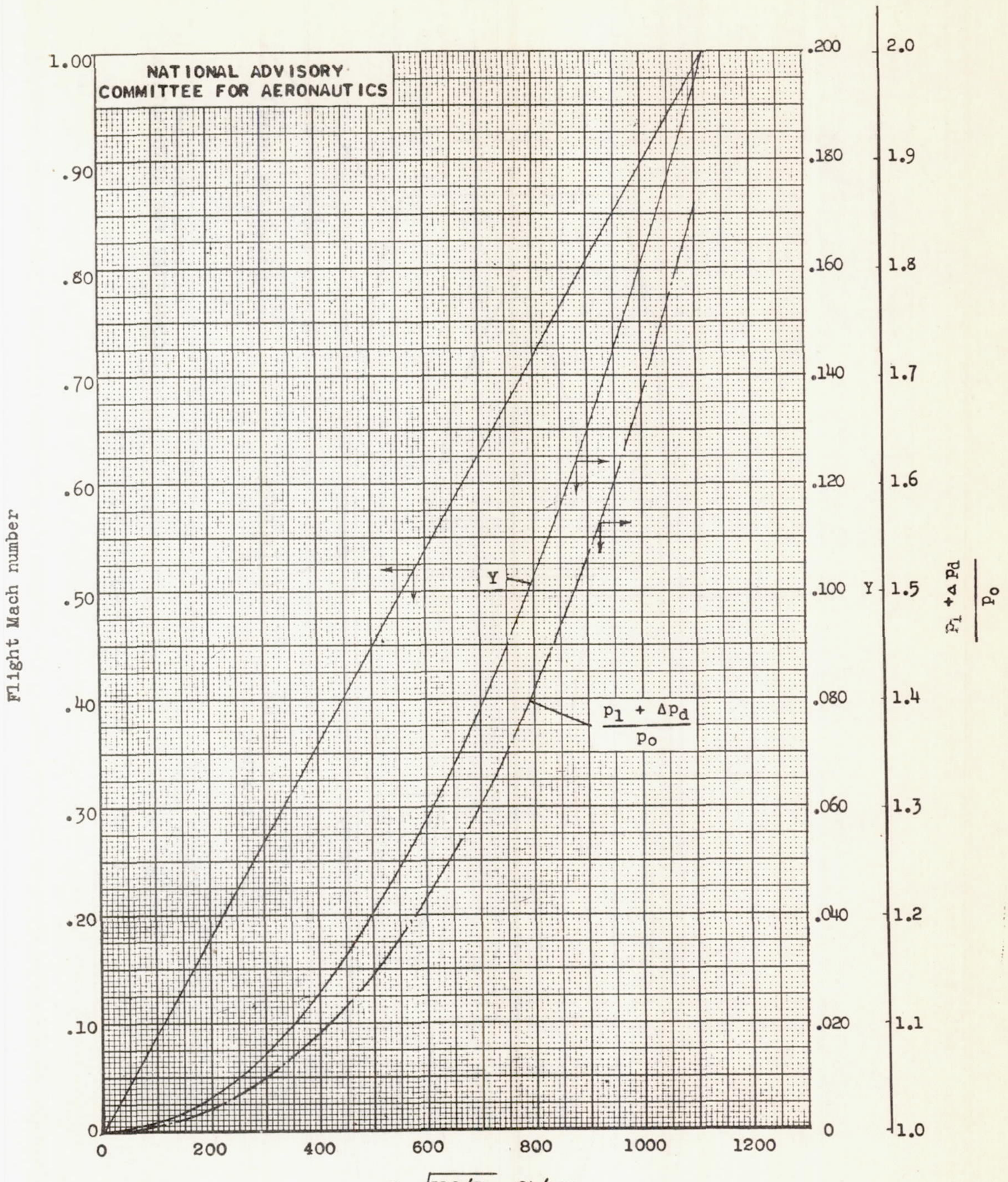

Figure 2.- Chart for determining Y, flight Mach number, and compressor inlet total pressure for various airplane velocities and atmospheric temperatures.

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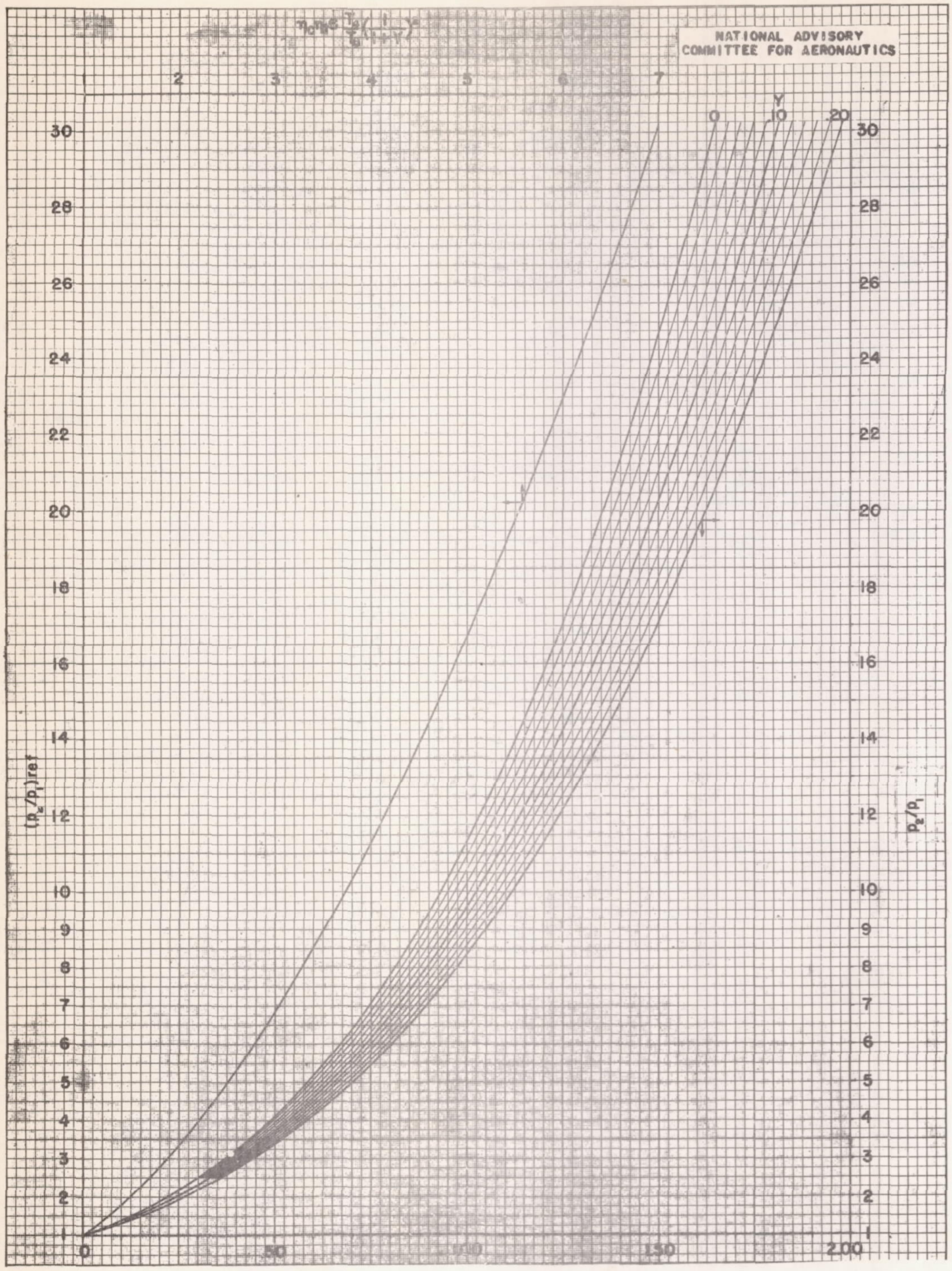
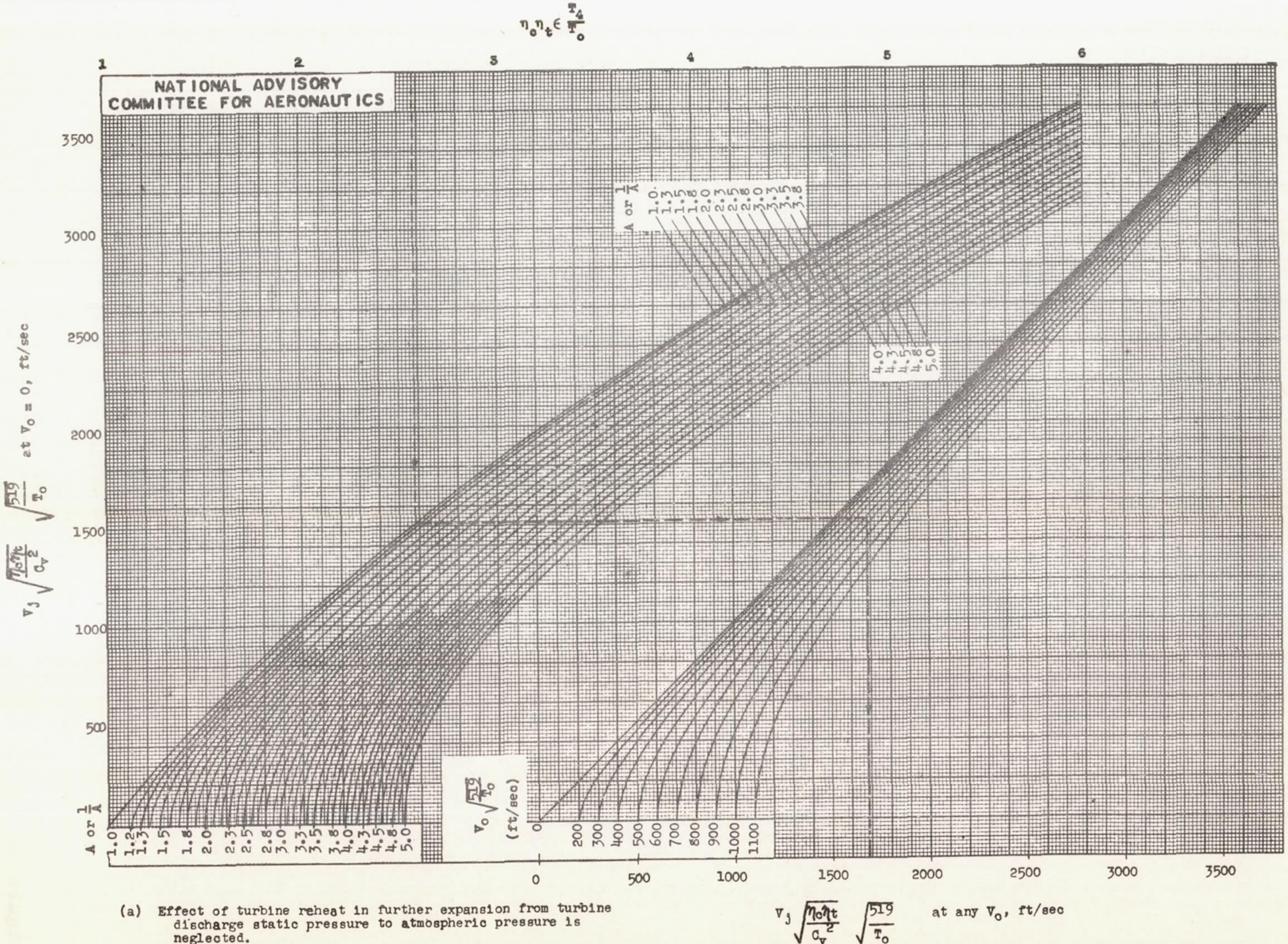
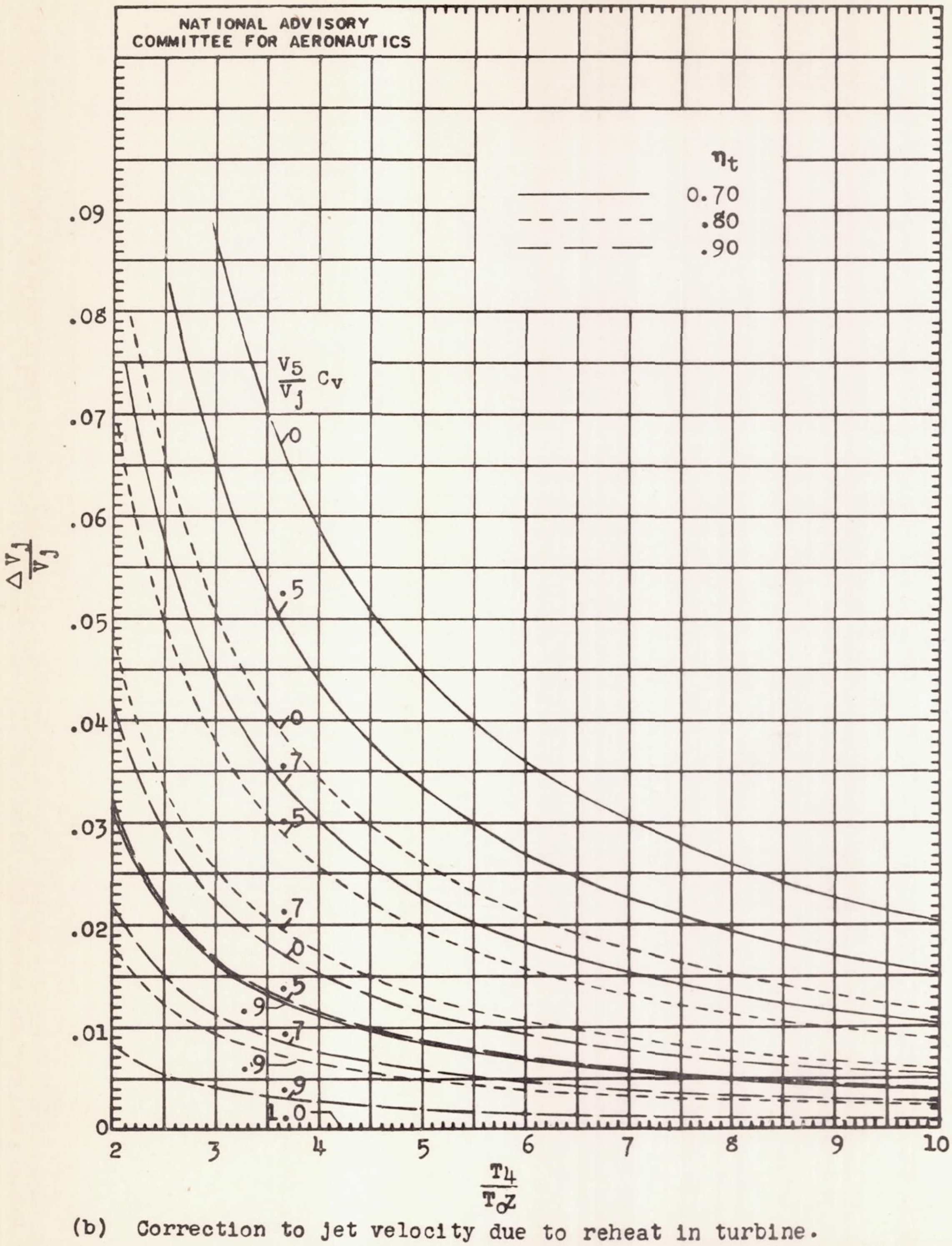


Figure 3. - Chart for determining the reference compressor pressure ratio for various values of $\eta_c\eta_t\epsilon\frac{T_4}{T_0}\left(\frac{1}{1+Y}\right)^2$ and the compressor pressure ratio for various values of Y and $\eta_c Z$. (A $19\frac{1}{2}$-in. by 28-in. print of this chart is enclosed with the report.)



(a) Effect of turbine reheat in further expansion from turbine discharge static pressure to atmospheric pressure is neglected.

Figure 4. - Chart for determining jet velocity. (This chart has been divided into two sections, an 18-in. by $35\frac{1}{2}$-in. section and a $32\frac{1}{2}$-in.-by 36-in. section, both of which are enclosed with the report.)



(b) Correction to jet velocity due to reheat in turbine.

Figure 4. - Concluded.

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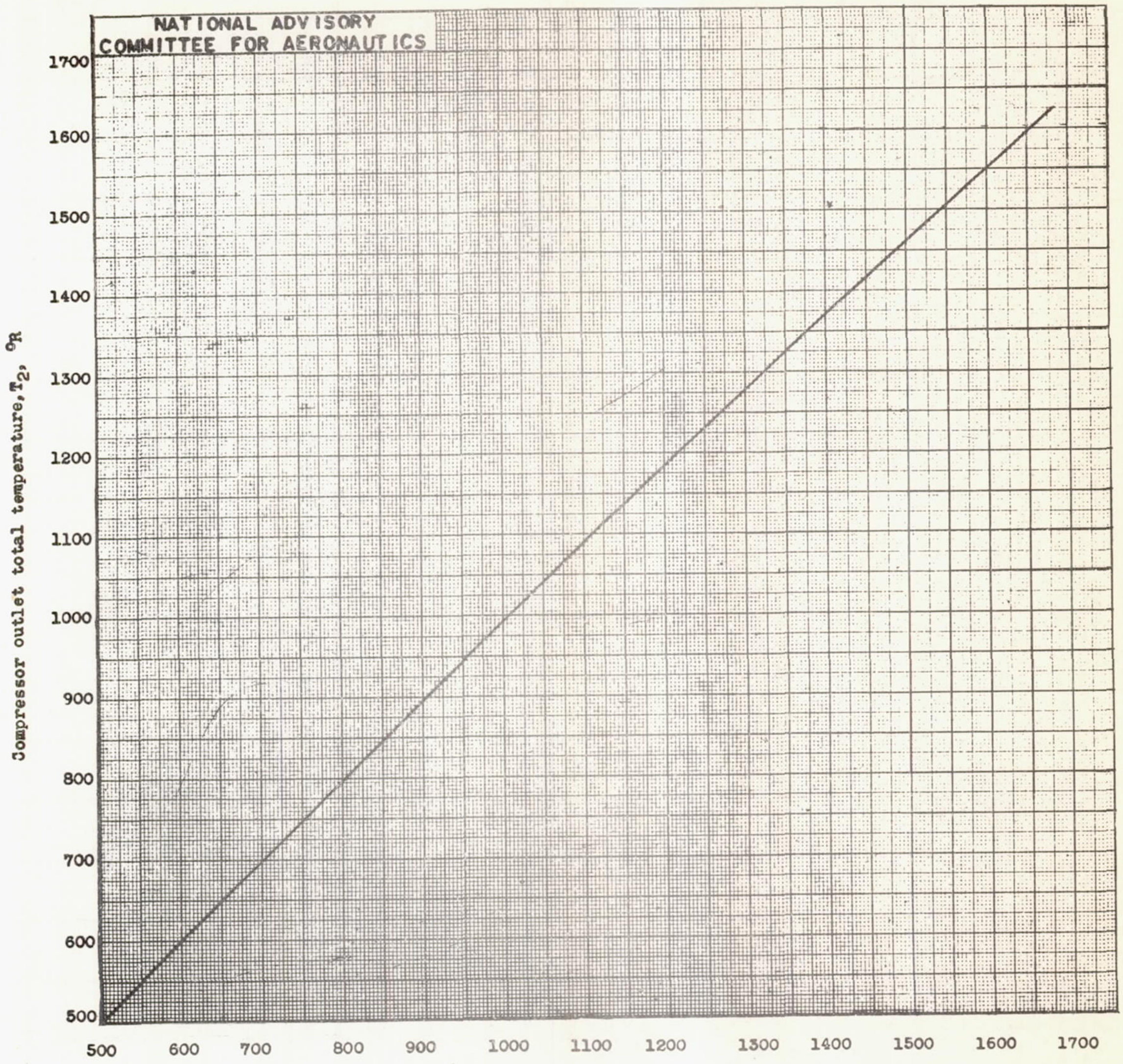

Figure 5. - Chart for determining the compressor outlet total temperature for various values of the factor T_o (1 + Y + Z).

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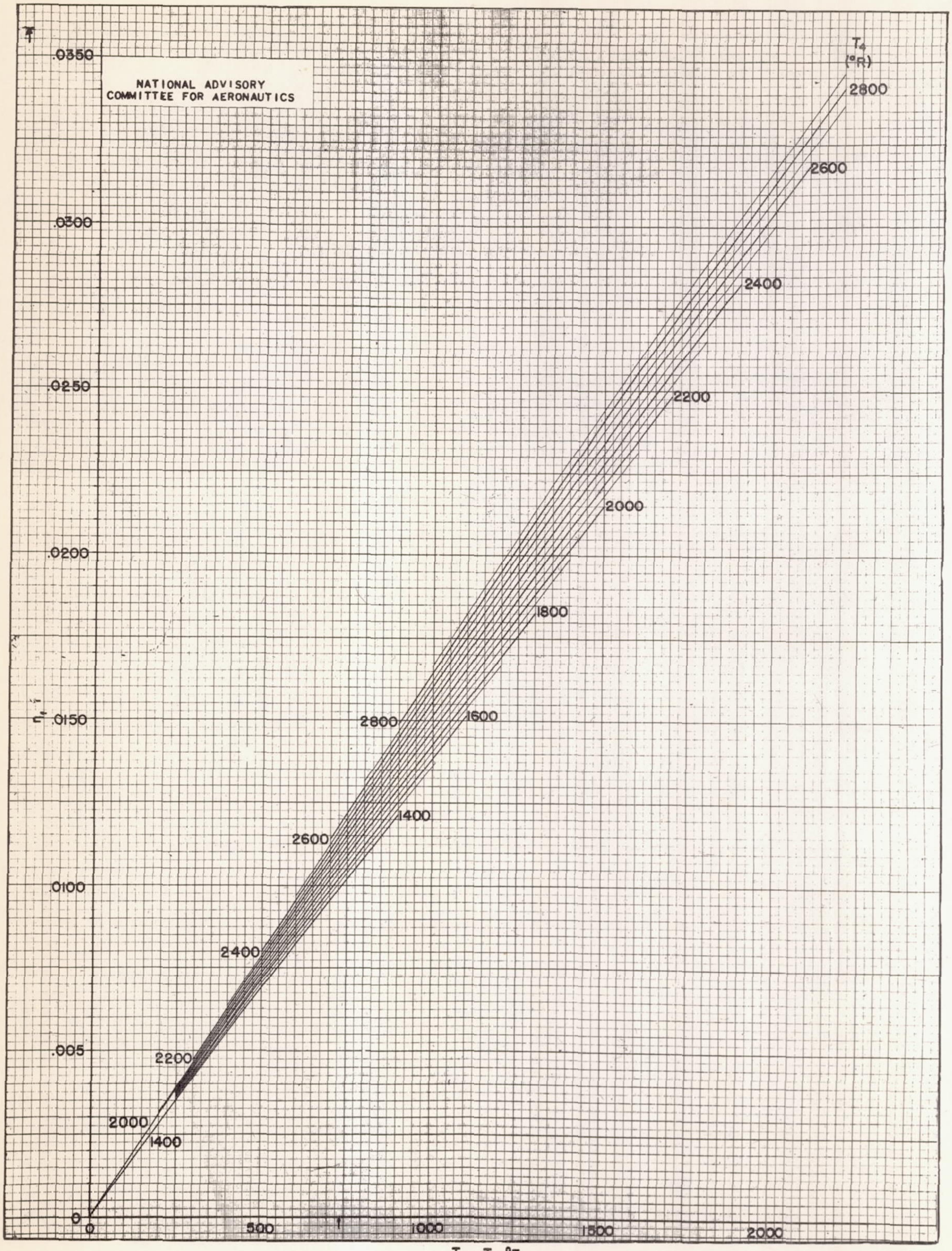
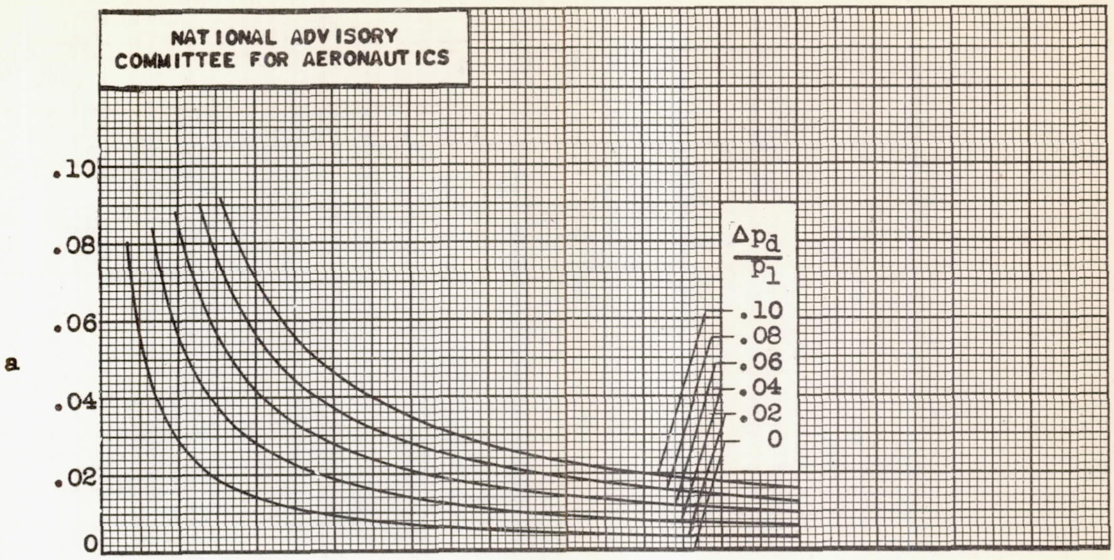
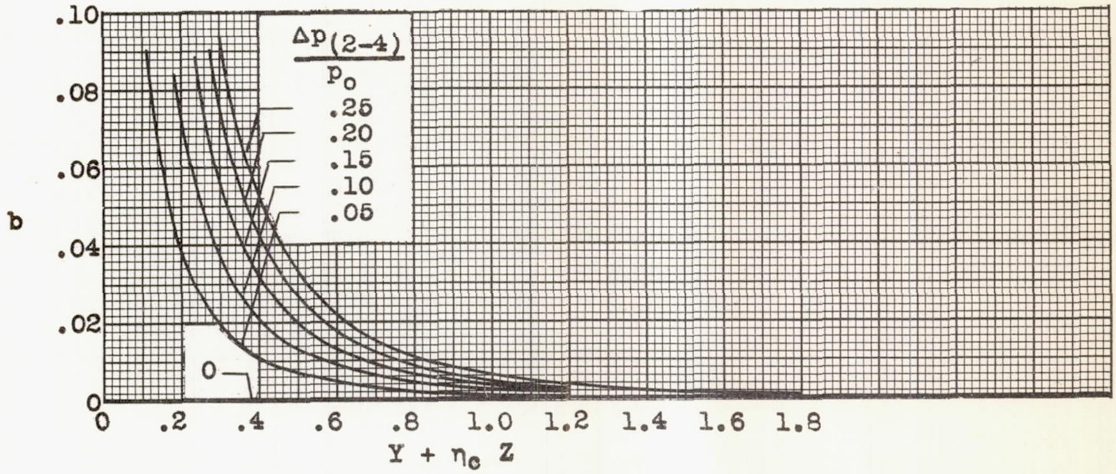


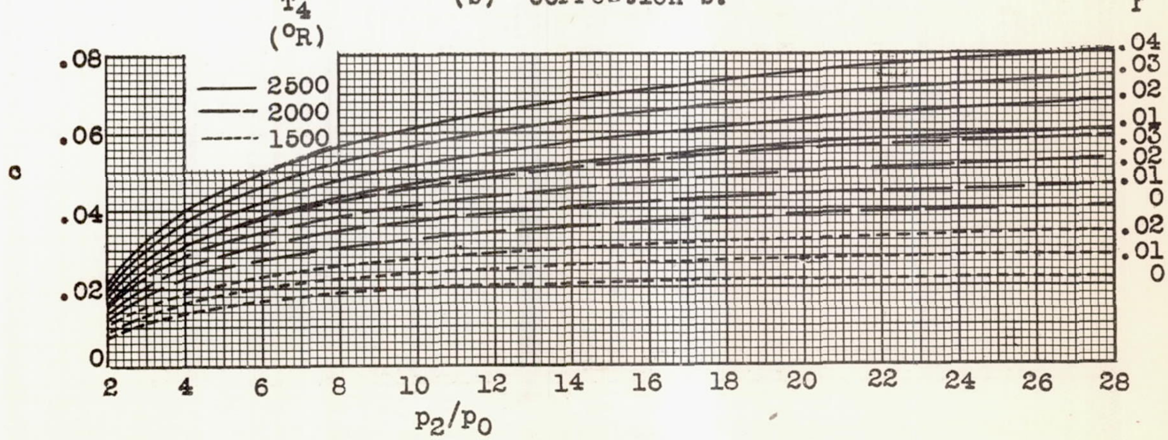
Figure 6. - Chart for determining the fuel-air ratio for various values of rise in total temperature across the combustion chamber and combustion chamber outlet total temperature. (h = 18,900 Btu/lb) (A $21\frac{3}{4}$-in. by $31\frac{1}{2}$-in. print of this chart is enclosed with the report.)



(a) Correction a.



(b) Correction b.



(c) Correction c.

Figure 7. - Chart for determining the factor ϵ. ($\epsilon = 1\text{-a-b+c}$).

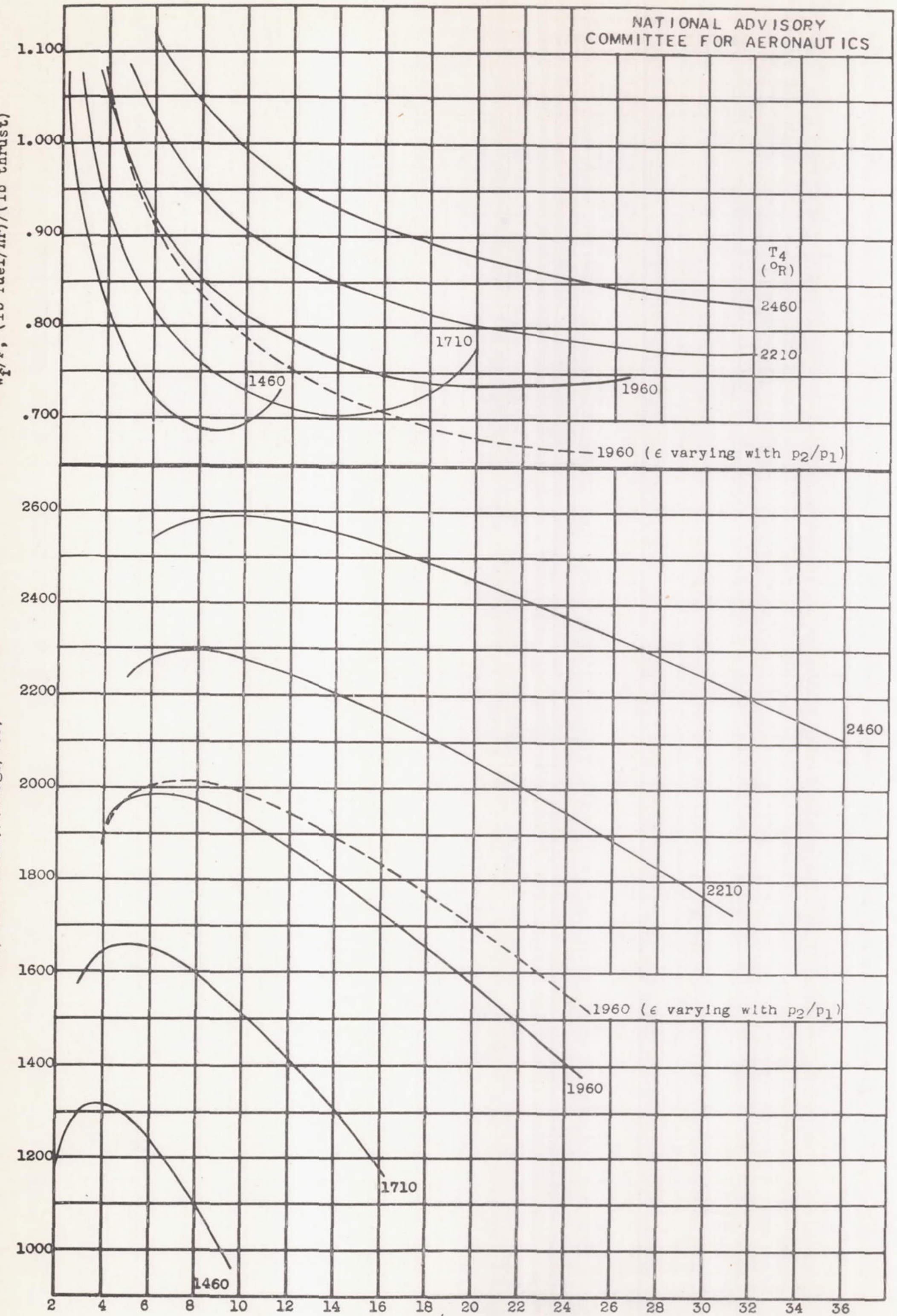
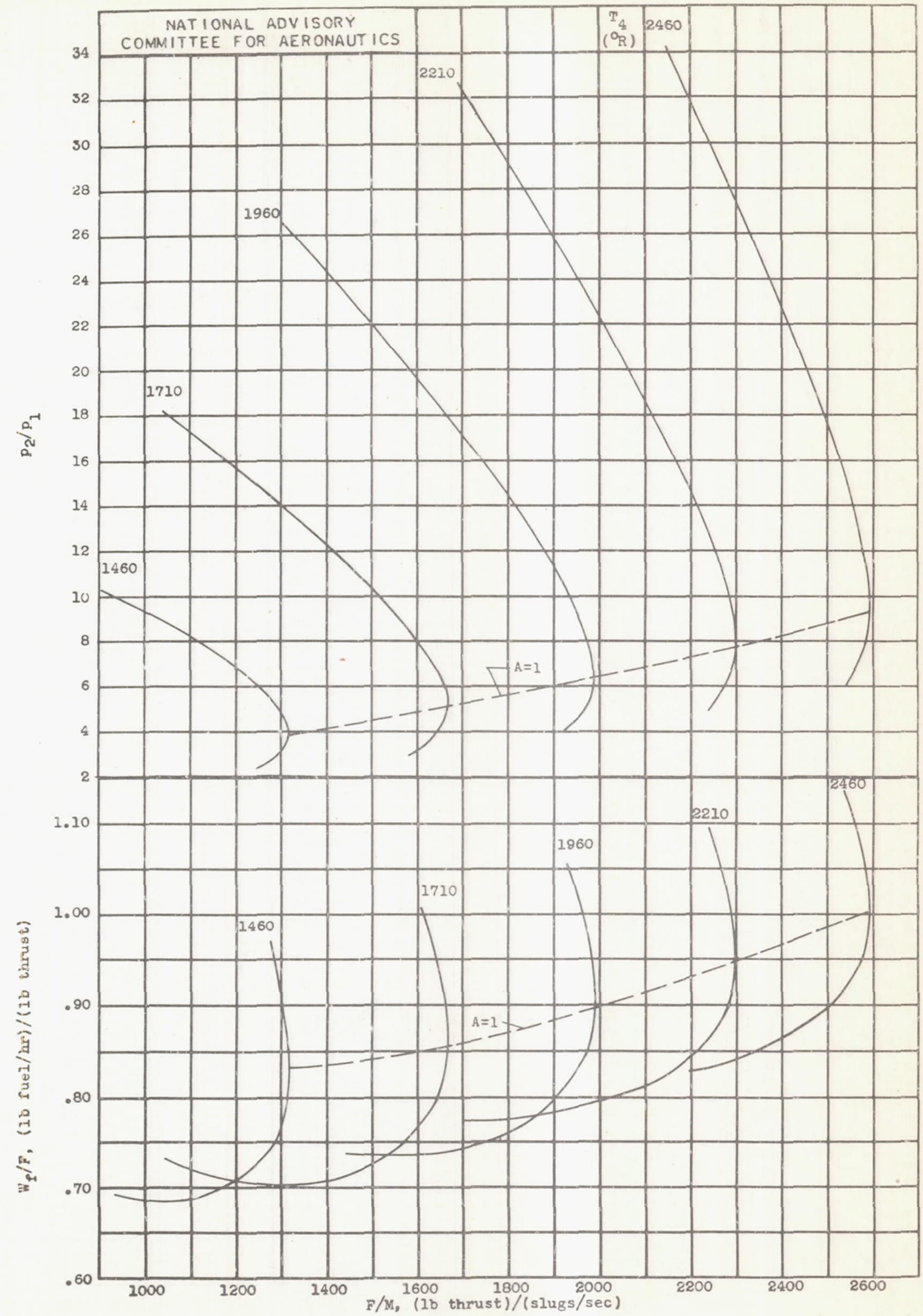


Figure 8. - Fuel rate per unit thrust and thrust per unit mass rate of air flow for various compressor pressure ratios and combustion-chamber discharge temperatures for illustrative case. (V_0, 0 ft/sec; T_0, 519° R; η_c, 0.85; η_t, 0.90; η_f, 0.96; h, 18,900 Btu/lb; C_V, 0.97; ε, 1.00.)



(a) V_0, 0 feet per second; T_0, 519° R.

Figure 9. - Compressor pressure ratio and fuel rate per unit thrust for various thrusts per unit mass rate of air flow and combustion-chamber discharge temperatures for illustrative case. (η_c, 0.85; η_t, 0.90; η_f, 0.96; h, 18,900 Btu/lb; C_V, 0.97; ϵ, 1.00.)

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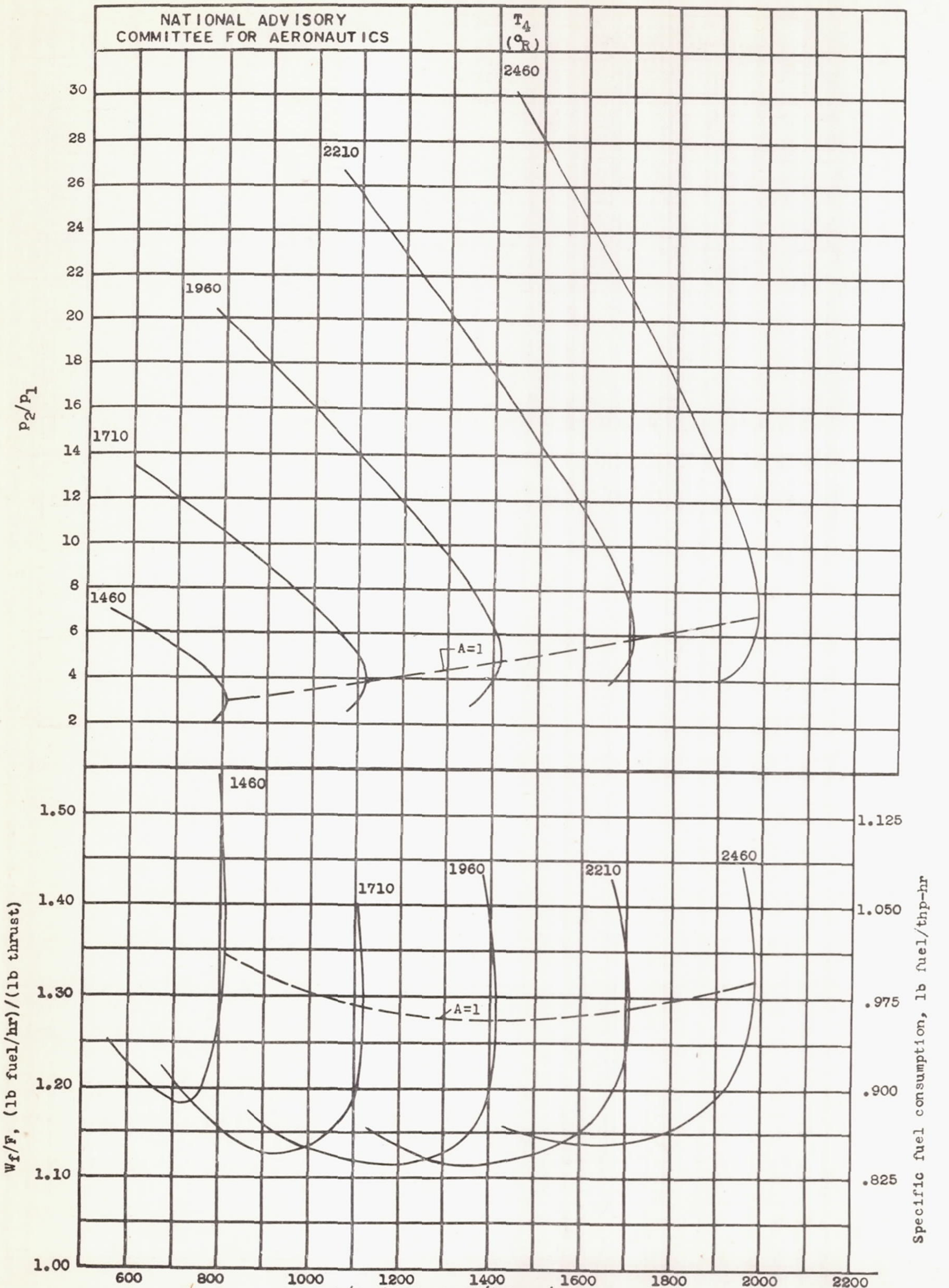
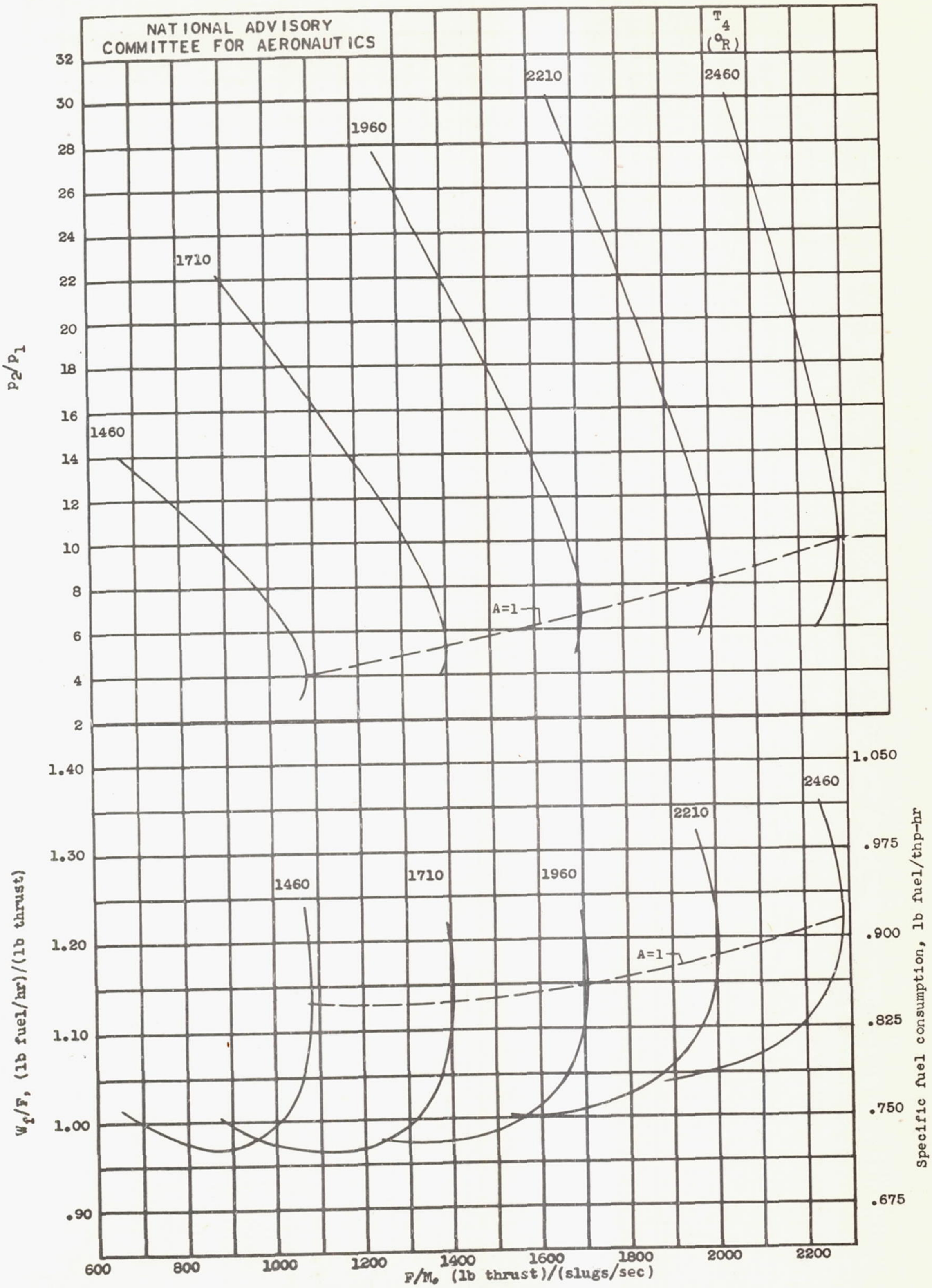


(b) V_0, 733 feet per second; T_0, 519° R.

Figure 9. - Continued.



(c) V_0, 733 feet per second; T_0, 412° R.

Figure 9. - Concluded.

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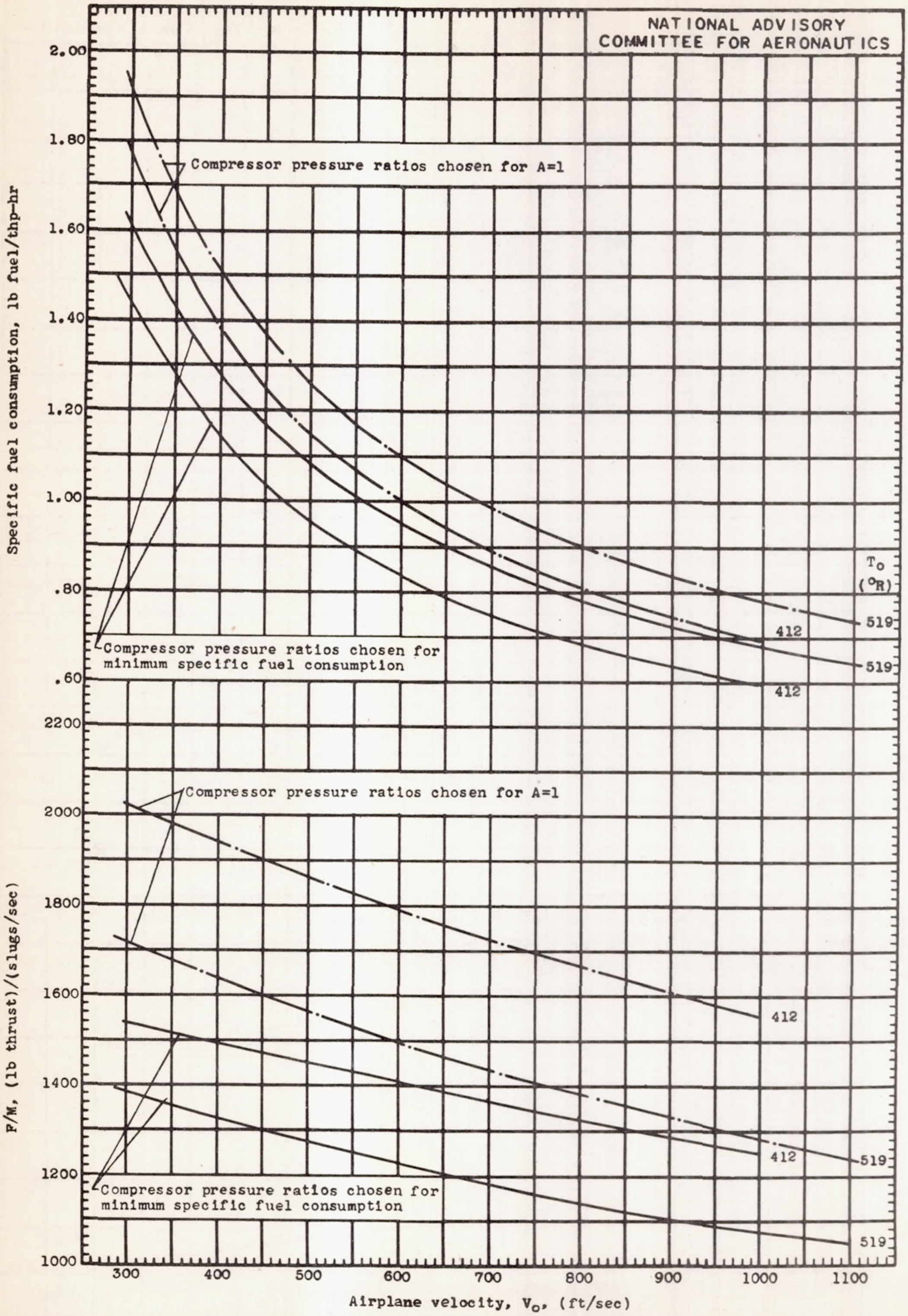
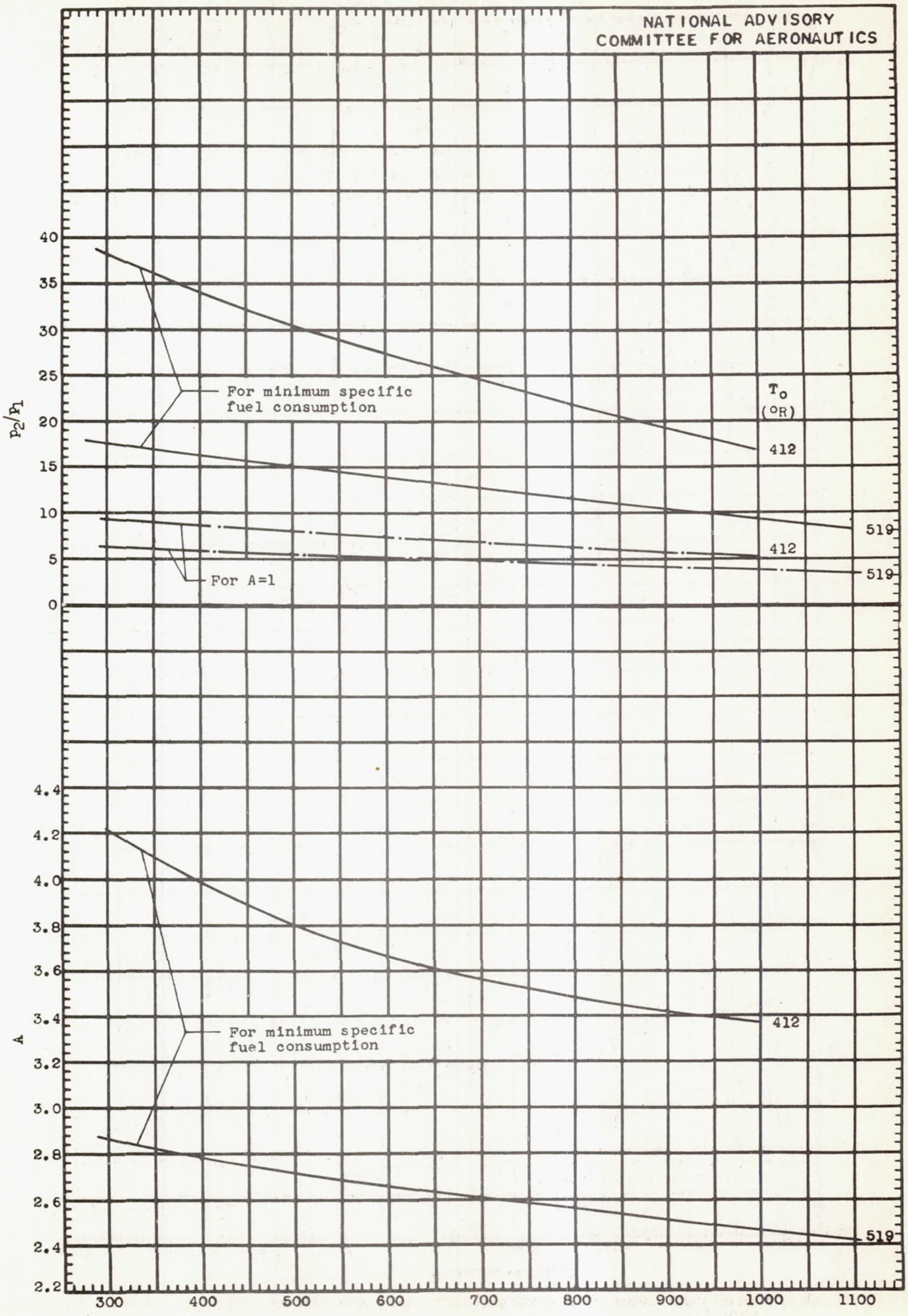


(a) Specific fuel consumption and thrust per unit air flow at various airplane velocities.

Figure 10.- Performance of jet-propulsion unit at conditions for minimum specific fuel consumption and for pressure ratios giving A=1 for illustrative case. (T_4, 1960° R: η_c, 0.85; η_t, 0.90; η_f, 0.96; C_v, 0.97; h, 18,900 Btu/lb; ϵ, 1.00.)



(b) Compressor pressure ratios and A at various airplane velocities.

Figure 10.- Concluded.