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COMPUTATION OF THE MEAN TANGENTIAL VELOCITY OF THE AIR LEAVING

THE BLADE TIPS OF A CENTRIFUGAL SUPERCHARGER

By W. Byron Brown

Aircraft Engine Research Laboratory
Cleveland, Ohio

WASHINGTON

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RESTRICTED BULLETIN

COMPUTATION OF THE MEAN TANGENTIAL VELOCITY OF THE AIR LEAVING THE BLADE TIPS OF A CENTRIFUGAL SUPERCHARGER

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SUMMARY

The mean tangential velocity of the air leaving the blade tips of a centrifugal supercharger was computed by the conventional turbine equation, which uses the actual increase in total enthalpy, and by an equation that uses the isentropic increase in total enthalpy. These computed velocities are compared with experimental values observed with three vaneless diffusers having diameters of 20, 24, and 27 inches. Tip speeds of 800, 1000, and 1200 feet per second were tested with each diffuser.

The mean tangential velocities given by the turbine equation averaged about 2 percent higher than the observed values, whereas those computed by the isentropic equation averaged about 15 percent lower than the observed values. When a correction derived from the measurements of windage losses was applied to the turbine equation to allow for the frictional torque of the fixed shrouds, the computed values agreed with the observed values within 1 percent.

INTRODUCTION

In diffuser design, estimates of air velocity at the impeller exit have been made by using the turbine equation that expresses the increase in total enthalpy in terms of the tip speed and the mean tangential air velocity. The use of the isentropic increase in total enthalpy instead of the actual increase in total enthalpy has been suggested by J. E. Ellor of the British Air Commission, Packard Motor Car Company. Inasmuch as these two increases often differ by as much as 10 to 20 percent, it is important to determine which method results in the more nearly correct value.

A series of velocity measurements was therefore made at the Cleveland laboratory of the NACA with vaneless diffusers of three different diameters in which each diffuser was tested at three tip speeds in order to determine the tangential velocity of the air at the impeller tip. These measured mean tangential velocities were then compared with the values computed by the conventional turbine equation and with those computed by an equation using the isentropic increase in total enthalpy.

ANALYSIS

The following symbols are used in the analysis:

b	breadth of vaneless-diffuser passage, feet
c_p	specific heat at constant pressure, foot-poundals per pound per $^{\circ}F$
D	impeller diameter, feet
D_h	hydraulic diameter of flow passage, feet
f	friction coefficient
g	acceleration due to gravity, 32.17 feet per second per second
H	actual increase in total enthalpy per unit mass, foot-pounds per pound
M	angular momentum, feet^2/per second
n	rotational speed, revolutions per second
P	total pressure, inches of mercury absolute
P_e	total pressure at impeller exit, inches of mercury absolute
P_i	total pressure at impeller inlet, inches of mercury absolute
p_s	static pressure, inches of mercury absolute
Q	volume flow of air at impeller exit, cubic feet per second
R	Reynolds number
r	radius, feet

T_s static temperature, $^{\circ}R$

T_t total temperature, $^{\circ}R$

T_{t_i} total temperature at impeller inlet, $^{\circ}R$

U mean resultant air velocity, feet per second

U_r mean radial air velocity, feet per second

U_t mean tangential air velocity, feet per second

ΔU_t loss in mean tangential air velocity due to windage, feet per second

V tip speed, feet per second

W weight flow of air, pounds per second

α angle between mean flow and circumference, degrees

β constant dependent on number and diameter of blades and nature of impeller (open, half shrouded, or fully shrouded)

γ ratio of specific heats

μ viscosity, pounds per foot-second

ρ density, pounds per cubic foot

Conventional turbine equation and correction for frictional torque. - The turbine equation is

$$U_t = \frac{gH}{V} \tag{1}$$

if the air has no angular momentum at the impeller entrance. If the windage correction ΔU_t is used, then

$$U_t = \frac{gH}{V} - \Delta U_t \tag{2}$$

The equation for ΔU_t is derived as follows:

$$\frac{\Delta U_t}{U_t} = \frac{fhp}{hp} \tag{3}$$

where fhp is friction horsepower and hp is horsepower input to the air. In reference 1 the results are summarized in the equation

$$\text{fhp} = \frac{0.0421}{10^6}\left(\beta\rho V^3 D^2\right) \qquad (4)$$

By substitution of equation (4) in equation (3) and by use of the relations

$$hp = \frac{HW}{550}$$

$$U_t = \frac{gH}{V}$$

$$V = \pi Dn$$

$$W = Q\rho$$

there is obtained

$$\Delta U_t = 72.9 \times 10^{-6}\left(\frac{\beta g V}{Q/nD^3}\right) \qquad (5)$$

From reference 1

$$\beta = 1.65 + 7.10\,(0.984 - D)$$

Because D is 1.02 feet, β is 1.40 and

$$\Delta U_t = \frac{23.9}{10^4} \times \frac{V}{(Q/nD^3)}$$

where Q must be determined at the impeller exit.

<u>Turbine equation using isentropic increase in total enthalpy.</u> The equation proposed by J. E. Ellor is

$$U_t V = c_p\, T_{t_i}\left[\left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad (6)$$

from which U_t is easily computed.

Determination of observed resultant velocity. - At each station in the vaneless diffuser, the resultant air velocity U was determined by means of the equation

$$U = 110.30 \sqrt{\frac{T_t}{1 + 1/Y}} \tag{7}$$

where $Y = \left(\frac{P}{p_s}\right)^{\frac{\gamma-1}{\gamma}} - 1$

Because of the continuity equation, the radial air velocity U_r can be obtained from

$$U_r = \frac{WT_s}{2\pi rb \times 1.322\ p_s} \tag{8}$$

The static temperature T_s is found from the total temperature T_t by the relation

$$T_s = T_t - \frac{U^2}{12,165} \tag{9}$$

Then

$$U_t = \sqrt{U^2 - U_r^2} \tag{10}$$

Thus U_t is known for certain stations several inches from the impeller tip.

At the station nearest the impeller tip (6.5-in. radius), values of static pressure p_s and total pressure P are unreliable because of the unsteady flow near the diffuser entrance. At the stations farther from the impeller tip (8.0, 9.5, and 11.5-in. radii), the observed values of P and p_s are usually trustworthy. Each of these values is accordingly reduced to the impeller-tip radius by adding to the measured angular momentum the loss between the impeller tip and the measuring station. This loss is given by the following equation derived in reference 2:

$$\frac{dM}{M} = \frac{-f\ dr}{b \sin \alpha} \tag{11}$$

Values of the friction coefficient f are found from the equation given by McAdams (reference 3):

$$1/\sqrt{f} = 3.2 \log_{10} (R \sqrt{f}\ ) + 1.2 \tag{12}$$

The Reynolds number R is computed by

$$R = \frac{U\rho D_h}{\mu}$$

$$= \frac{W}{\mu\pi r \sin \alpha}$$

where $\sin \alpha = U_r/U$.

APPARATUS AND TESTS

The supercharger impeller used with the vaneless diffusers was a conventional design centrifugal impeller of 12.23-inch diameter. Three vaneless diffusers of the constant radial-area type were tested. The first diffuser had a diameter of 27 inches and was modified by cutting the diameter down to 24 inches and then to 20 inches. The shape of the air passage is shown in figure 1.

The static pressure was measured by wall taps at radii of 8.0, 9.5, and 11.5 inches. Total pressures were measured by survey tubes located at the same radii but not at the same azimuth as the wall tubes. (See fig. 1.) At each survey station, readings were taken at four points across the channel width b: b/8, 3b/8, 5b/8, and 7b/8 from the front wall. The average of these readings was taken as the average total pressure for the section.

The total temperatures were measured only at the last station in the outlet pipe as one of the standard over-all measurements. Unpublished tests by the NACA have shown that, if heat transfer is kept small by lagging the air ducts, the total temperature is unchanged from the impeller exit to the outlet pipe. The value measured at the outlet pipe was therefore used for all stations beyond the impeller exit.

The over-all measurements were made in accordance with the standard procedure of rating and testing centrifugal superchargers given in reference 4. Photographs and a description of the test rig are presented in reference 5.

RESULTS AND DISCUSSION

The data obtained from the tests of the three diffusers were inserted in the conventional turbine equation (1), the corrected turbine equation (2), and the isentropic equation (6) to obtain the tangential air velocity U_t. Because the absolute pressures were measured within ±0.05 inch of mercury and because the differences on which the velocities depend were small (1 to 5 in.), the experimental error of a single observation was large. For this reason, 63 test points were observed and the method of least squares was used to average the results. The ratios of the computed tangential velocity $U_{t_{comp}}$ to the observed velocity $U_{t_{obs}}$ are shown in the following table:

Equation	$U_{t_{comp}}/U_{t_{obs}}$
Conventional turbine (1)	1.017 ±0.005
Corrected conventional turbine (2)	.993 ±.004
Isentropic (6)	.853 ±.005

The error caused by neglecting the windage is approximately 2 percent. The isentropic equation gives values about 15 percent lower than the experimental results, which is not surprising in view of the fact that it is based on the assumption of reversible compression in the impeller. The considerable loss in static pressure in the impeller affects equation (6) but not the momentum equation on which equations (1) and (2) are based.

When the conventional equation (1) was used, it was found that the points computed for light loads near surge were noticeably higher than those near the load of peak efficiency. This result is probably caused by the large amount of recirculated air at this condition. The value of actual increase in total enthalpy H undergoes a large increase without a corresponding effect on U_t. At full throttle, the other extreme, the value of H tends to be low. The points in figure 2 that show large departures from the curve represent these two extreme conditions.

Because the points computed by the isentropic equation (6) are based on total-pressure measurements near the impeller tips, they are more scattered than those computed by the turbine equation (1). Chaotic flow conditions at the impeller exit give rise to larger errors in the measurements taken there than in those taken farther from the center.

Neither an increase in tip speed from 800 to 1000 feet per second and then to 1200 feet per second nor successive reductions in diameter from 27 to 24 to 20 inches affected the results. Aside from extremely high and extremely low loads, the results were concordant for the entire range of tip speeds and diameters.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

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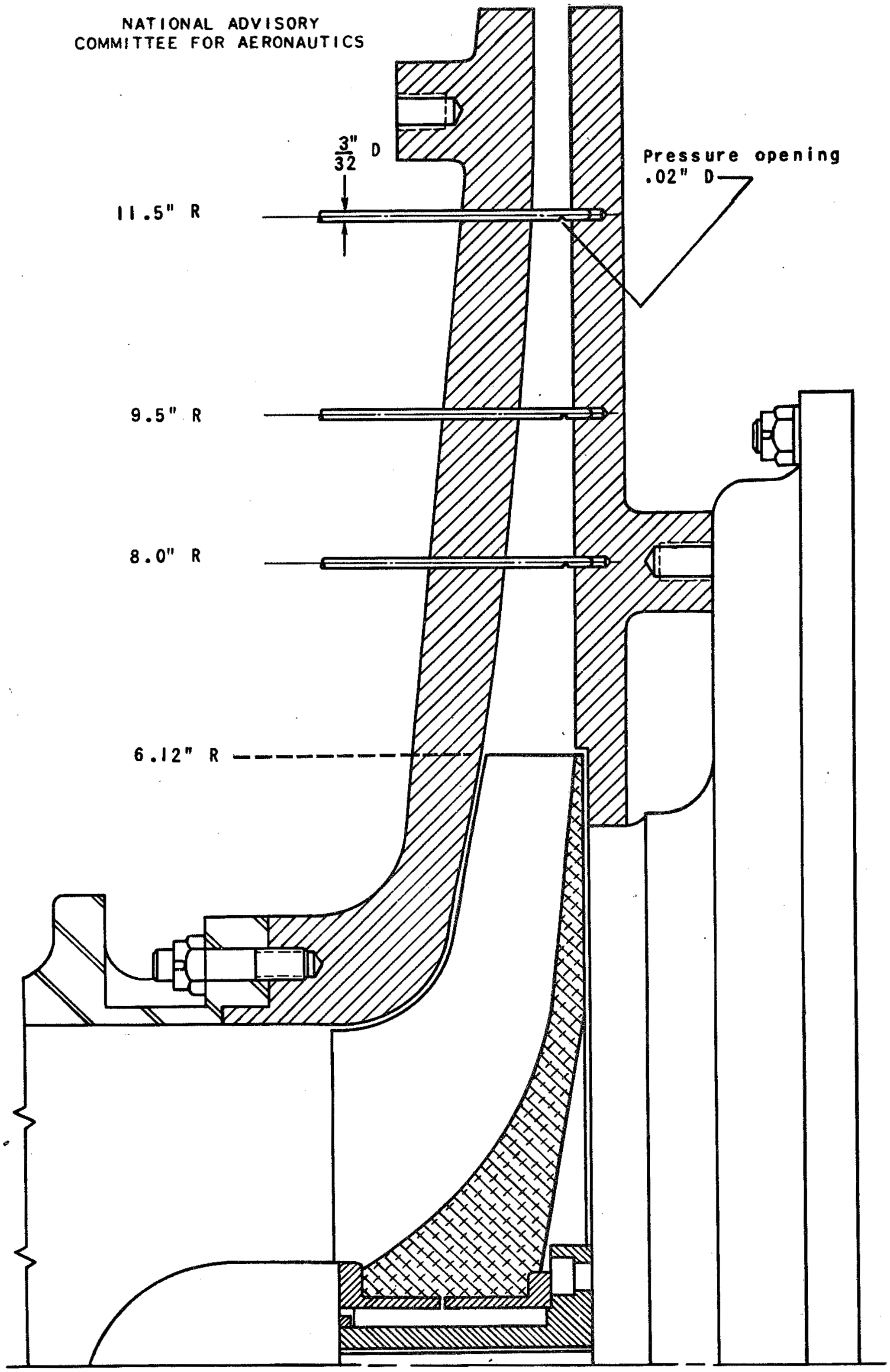

Figure 1. - Air-passage profile showing location of survey tube in vaneless diffusers.

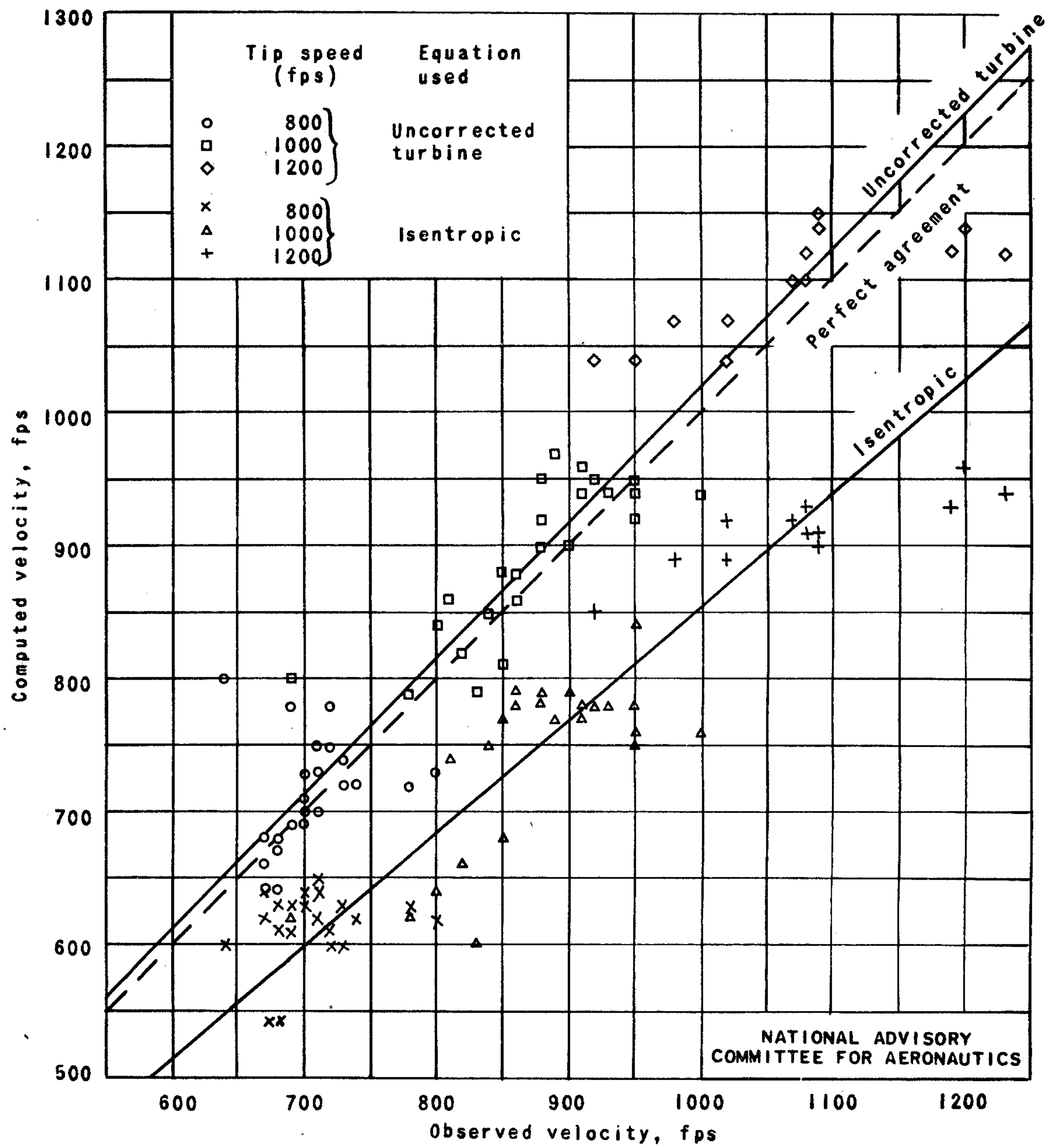

Figure 2. - Mean tangential air velocity V_t computed by the uncorrected turbine equation and by the isentropic equation.