ARR No. 5A06

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED
August 1945 as
Advance Restricted Report 5A06

AN INVESTIGATION OF AIRCRAFT HEATERS

XVIII - A DESIGN MANUAL FOR EXHAUST GAS AND AIR HEAT EXCHANGERS

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AN INVESTIGATION OF AIRCRAFT HEATERS

XVIII - A DESIGN MANUAL FOR EXHAUST GAS AND AIR HEAT EXCHANGERS

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SUMMARY

Heat exchangers for the transfer of heat from one fluid to another fluid at a lower temperature are basic elements of modern airplane installations. Examples of such exchangers are intercoolers, oilcoolers, cabin-air heaters, exhaust gas heat exchangers for wing anti-icing systems, and the heated wing itself. The basic elements for the design of oilcoolers and intercoolers are covered by the report entitled "Design, Selection, and Installation of Aircraft Heat Exchangers" by George P. Wood and Maurice J. Brevoort. The following report is concerned with the elements of design of cabin-air heaters, wing anti-icing heat exchangers, and other exchangers in which the hot fluid consists of air (or the products of combustion of air and a hydrocarbon fuel), and the cold fluid is air.

This report, which summarizes a series of reports issued by the NACA under the title "An Investigation of Aircraft Heaters" (I to XXIII), is divided into four parts. The basic equations for the determination of the thermal resistances involved in heat exchanger design are presented in part I. Several examples of the application of these basic equations of part I to the prediction of the thermal performance of a number of heaters are presented in part II. Nonisothermal pressure drop is discussed in part III. However, isothermal pressure drop characteristics are not presented in detail since several readily available references cover this aspect of heater design. Also in part III the heat requirements of aircraft are discussed briefly; the equations used to correct heater performance to any altitude are presented; and the

equations required to predict the performance of a ram-operated heater-and-duct system at any airplane speed and altitude are summarized.

Part IV consists of an appendix in which the physical properties of air are given for a range of temperatures from -100° to 1600° F.

INTRODUCTION

The design of an exchanger to transfer heat from the products of combustion of a hydrocarbon fuel to air, with the two fluids separated by a metallic wall, presents a complex problem. A complete design of a heat exchanger system must include consideration of the following data:

1. Heat requirements of the system to which the hot air is being supplied. The air may go to a cabin, to a wing anti-icing system, to the carburetor air intake, and so forth. Such items as heat losses through cabin walls, air leakage, heat losses along the airfoil, and so forth, must be known in order to establish the heater output necessary to perform the desired task.*

2. The allowable pressure drop through the heater. This specification depends upon the rates of air flow desired through the heater, the available total-pressure difference across the duct system, the design of the duct work, and so forth. As experience is gained with heater installation, maximum allowable pressure drops can be specified. Care must be taken, in particular, to design the duct work leading to and from the heater as carefully as possible. For example, in many installations the bend leading the hot air from the heater is poorly designed because of space limitations so that the pressure loss across this element of the duct system **may be several times the total pressure drop across the heater itself.**

3. The heater design may be established once the heater output and allowable pressure drops are specified. This report, in particular, includes the consideration of the thermal aspects of the design. The pressure drop discussion is

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*Methods of making these calculations or references thereto are presented in this report.

not detailed because these data may be found in various readily available publications. (See references 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.) Certain pressure loss phenomena are, however, discussed in part III of the report.

In addition to the thermal and flow analyses of the heater such items as available space, heater weight, heater life, and so forth, must be given careful consideration before the final heater design is established.

4. Design of Ducting. The design of the duct work leading the hot air from the heater to the desired location should be considered simultaneously with the heater design. The advantage of a low pressure drop heater may be wholly invalidated by poorly designed duct work. As mentioned previously, isothermal pressure losses in duct work can be estimated from data available in the literature. A discussion of the nonisothermal performance of the ducting will be found in part III of this report.

Many of the equations presented in this report require further experimental verification. Research of this nature is being carried out now in the Mechanical Engineering Laboratories of the University of California.

It is suggested that, before using the information in this report, part I section A be read carefully, since basic concepts and definitions are outlined in this part.

This investigation, conducted at the Mechanical Engineering Laboratories of the University of California, was sponsored by and conducted with the financial assistance of the National Advisory Committee for Aeronautics.

I. SUMMARY OF HEAT TRANSFER EQUATIONS

A. GENERAL DISCUSSION OF HEAT TRANSFER MECHANISMS

Heat is transferred by three mechanisms: conduction, convection, and radiation.*

*Heat also may be transported by the process of evaporation or condensation, but this mechanism is usually termed "mass transfer," rather than "heat transfer."

Conduction may be defined as heat transfer through a body, unaccompanied by any appreciable motions of matter.

Convection may be of two types: free convection and forced convection.

1 Free convection is the phenomenon of heat transfer from a stationary body to a "stationary" fluid - that is, a fluid which is at rest except for the convection currents set up by the buoyant forces resulting from the heating (or cooling) of the fluid in immediate contact with the surface of the body. In the normal gas-air heater design, free convection is unimportant.

2. Forced convection may be defined as heat transfer from a body to a fluid which has a velocity relative to the body, the flow being set up by some external agency, such as a pump, a fan, or ram pressure.

Radiation concerns energy which travels in the form of electromagnetic waves. Light, radio waves, X-rays, radiant heat, and so forth, all are forms of radiation. Heat is transferred by radiation when radiant energy starts from a body excited thermally, travels across an intervening space, and is finally absorbed by another body.

In the heat exchangers to be discussed in this report, in which heat from the products of combustion of a fuel is transferred to air, all three mechanisms may occur simultaneously.

In figure 1 is shown a typical temperature distribution in the fluid streams of a gas-air heat exchanger. As the hot gas passes the heat exchanger surface, heat is transferred to it by forced convection. In addition, some of the gases in the products of combustion will transfer heat to the surface by the process of gaseous radiation.* The sum of the heat transferred by convection and radiation then must pass through the heat exchanger wall by conduction. Finally, the heat is transferred by the process of forced convection to the cold air moving along the wall. Some heat may be lost also by radiation from the heat transfer surface if it "sees" another surface at a lower temperature.

*The gaseous radiant heat transfer usually is less than 10 percent of the convective heat transfer and may be neglected in a practical design. (See reference 11.)

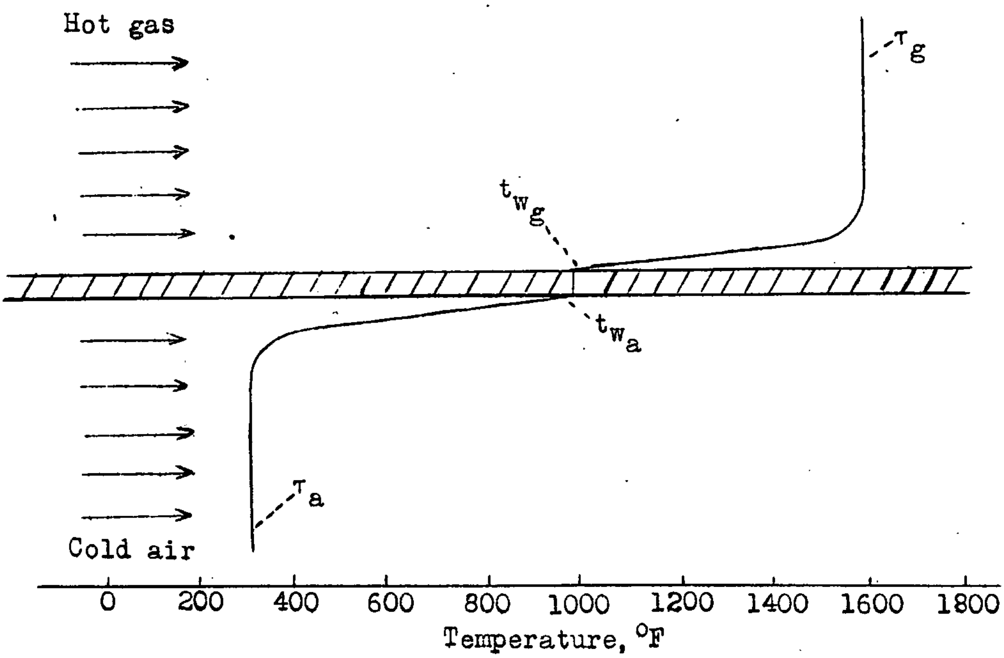

Figure 1.- Typical temperature distribution in the fluid streams of a gas-air heat exchanger.

1. Conduction.- The elementary equation for the unidirectional conduction of heat in the steady state through a solid, sometimes called Fourier's law, may be written in its simple form as:

$$q = -kA \frac{dt}{dx} \tag{1}$$

where

q rate of heat transfer by conduction, Btu/hr

k thermal conductivity of the solid, Btu/hr $ft^2 \left(\frac{^{\circ}F}{ft}\right)$

t temperature, which is independent of time, °F dt/dx temperature gradient in the solid, °F/ft (dt/dx is negative when the temperature decreases with increasing x)

x distance, measured in direction of heat flow, ft

A area, at any x, through which heat is flowing, measured in a plane perpendicular to direction of heat flow, ft^2

For the simple case of heat flow through a plane wall of thickness L, with temperature independent of time, and with flow in the x-direction only, equation (1) may be integrated:

$$\frac{q}{A}\int_0^L dx = -\int_{t_1}^{t_2} k\,dt$$

If k is independent of t and x,

$$q = \frac{Ak}{L}(t_1 - t_2) = \frac{kA\Delta t}{L} = \frac{\Delta t}{R} \qquad (2)$$

where

R = L/kA "thermal resistance" offered by a plane wall to heat transfer by conduction, °F/(Btu/hr)

Equation (1) also may be readily integrated for the case in which k varies with temperature in a simple manner, and for the case of unidirectional heat flow by conduction in cylinders, spheres, and other shapes for which A may be readily expressed algebraically as a function of x. (See reference 12, p. XIV-27.) In most heat exchanger designs equation (2) is sufficiently accurate.

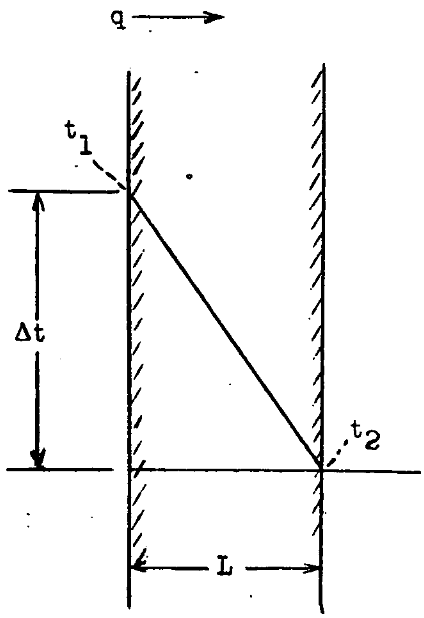

Figure 2.- Temperature distribution in wall of thickness L. k uniform; t_1, t_2 uniform.

2. <u>Convection</u>.- The rate of heat transfer by convection from a solid to a fluid is controlled by the conduction of heat through the fluid immediately in contact with the solid surface. The fluid directly in contact with the solid is at rest relative to the solid and thus the Fourier law of conduction can be written for the flow of heat from the body to the fluid:

$$\frac{q}{A} = -k_f \left(\frac{\partial \tau}{\partial y}\right)_{y=0} \tag{3}$$

where

$\frac{q}{A}$ rate of heat transfer per unit area by convection, $\frac{\text{Btu}}{\text{ft}^2\ \text{hr}}$

k_f thermal conductivity of the fluid, $\frac{\text{Btu}}{\text{hr ft}^2\ \frac{^\circ\text{F}}{\text{ft}}}$

$\left(\frac{\partial \tau}{\partial y}\right)_{y=0}$ temperature gradient in the fluid immediately in contact with the solid, °F/ft; the subscript $y=0$ refers to the surface of the solid; the partial differential form is used to emphasize that a particular point on the surface of the body is being considered.*

The temperature gradient $\left(\frac{\partial \tau}{\partial y}\right)_{y=0}$ is controlled by the type of flow existing near the surface of the body and by the physical properties of the fluid.

Although equation (3) is strictly the basic equation for convective heat transfer, engineers for many years have used the well-known "Newton's law of cooling" to express heat transfer by convection. This is usually written as:

$$q = f_c A(t_s - \tau_\infty) = \frac{(t_s - \tau_\infty)}{R} \tag{3a}$$

where

q rate of heat transfer by convection, Btu/hr

A heat transfer area, ft^2

t_s surface temperature of solid, °F

τ_∞ temperature of fluid far from the body, °F

f_c unit thermal convective conductance (sometimes called the film transfer factor or heat transfer coefficient), Btu/hr ft^2 °F

$R = \frac{1}{f_c A}$ thermal resistance to convective heat transfer, between the temperature t_s and τ_∞, °F/(Btu/hr)

Equation (3) <u>defines</u> the unit thermal conductance f_c.

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*Throughout this report the lower case (roman) letter t is used to signify the temperature of the solid, and the Greek letter τ to signify the temperature of the fluid. Capital T signifies absolute temperatures of either the fluid or the solid.

The unit thermal convective conductance f_c is usually determined experimentally by measuring q, A, t_s, and T_∞ and is correlated for use by designers by means of various dimensionless moduli to be mentioned.

It is instructive to note that equations (3) and (3a) are both expressions for heat transfer by convection. By equating the two, a basic definition of f_c can be established. Thus:

$$f_c = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{y=0}}{(t_s - T_\infty)} \tag{3b}$$

It is convenient to express equation (3b) in dimensionless form:

$$\frac{f_c\, l}{k_f} = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0}}{\frac{(t_s - T_\infty)}{l}} \tag{3c}$$

where l is a significant dimension in the system which fixes the geometry of the solid object from which convection is occurring. This point is discussed further.

The dimensionless modulus $\frac{f_c\, l}{k_f}$ is called the Nusselt number in heat transfer work. It is noted that the Nusselt number is equal to the temperature gradient in the fluid immediately in contact with the solid, divided by the ratio $\frac{(t_s - T_\infty)}{l}$. From this strict definition of the Nusselt number several conclusions can be drawn.

1. Since the temperature gradient $\left(\frac{\partial T}{\partial y}\right)_{y=0}$ varies over the surface of a solid, f_c, and thus the Nusselt number, will also vary from point to point. Most heat transfer experimenters to date have contented themselves with measurement of the <u>average</u> Nusselt number. In this report an effort is made to emphasize the point variation of the Nusselt number.

2. Any sharp change in temperature along the surface of the solid will produce sharp changes in f_c and thus the Nusselt number.

3. The Nusselt number will be a function of the factors which determine the temperature gradient in the fluid immediately in contact with the solid.

It has been shown analytically and has also been demonstrated experimentally that the temperature gradient $\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is mainly a function of the flow conditions which exist next to the object from which heat is being lost by convection. (See reference 13.) It is well known that in steady state forced convection the flow conditions are characterized by the Reynolds number of the flow system and the shape (including the roughness) of the object. The Reynolds number may be written as:

$$Re = \frac{(u\gamma)\,l}{\mu g} = \left(\frac{W}{A}\right)\frac{l}{3600\,\mu g} = \frac{G\,l}{3600\,\mu g} \qquad (4)$$

where:

l a significant dimension of object over which the fluid is flowing, ft

(The choice of this dimension depends upon the geometrical system being considered. Thus at the entrance to a pipe, the significant dimension is the distance from the entrance of the pipe to the point under consideration; once the entrance section has been traversed, however, the significant dimension becomes the pipe diameter. It is obvious that the significant dimension always must be specified when the Reynolds number is stated.)

μ absolute viscosity of fluid, lb sec/ft^2

u velocity of fluid, ft/sec

γ weight density of fluid, lb/ft^3

(Since in many cases, u, μ, and γ vary from point to point in the fluid stream, the statement of the Reynolds number must be accompanied by a designation of the manner by which the magnitudes of u, μ, and γ were established in the calculations of Re.)

g gravitational force per unit of mass, 32.2 lb $/\left(\frac{\text{lb sec}^2}{\text{ft}}\right)$

W fluid flow rate, lb/hr

A cross-sectional area through which fluid is flowing, ft^2

G weight rate of flow per unit cross-sectional area (W/A), lb/hr ft^2

The Reynolds number is representative of the ratio of acceleration forces to viscous forces in the fluid stream and therefore is a nondimensional parameter. In particular, small* magnitudes of the Reynolds number signify viscous flow, in which fluid particles flow parallel to each other with practically no mixing. Large* magnitudes of the Reynolds number indicate turbulent flow, during which appreciable mixing of the fluid occurs due to the eddies and vortices resulting from the instability of the turbulent motion. Because of the large magnitudes of the Reynolds number utilized in normal heater operation, except for the viscous flow near the leading edges of airfoil sections, cylinders, and flat plates, turbulent flow will be assumed to exist in all heater designs described in the remainder of the report.

In addition to the Reynolds number and the geometry of the system, which establish the flow pattern (velocity distribution), the temperature gradient $\left(\frac{\partial T}{\partial y}\right)_{y=0}$ and thus the Nusselt number is also a function of certain properties of the fluid, such as its thermal conductivity, viscosity, and specific heat. It has been shown both by theory and experi-

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*The terms "small" and "large" should be regarded in a relative sense, since the numerical magnitude of the critical Reynolds number associated with changes from viscous to turbulent motion depends upon the geometrical arrangement over which the flow is taking place, A nominal magnitude for the critical Re (based on pipe diameter D) for flow inside pipes is 2000. For flow across cylinders, however, the motion on the front side of the cylinder may be laminar for Re (based on pipe diameter) as high as 10^5. The flow in the wake behind the cylinder, on the other hand, may become turbulent for Re as low as 1800. (See reference 14, pp. 418 and 423.) (Viscous eddies may form behind a cylinder for Re as low as 1.0.)

ment that these fluid properties enter the problem as a dimensionless group called the Prandtl number. The Prandtl number can be expressed as:

$Pr = \frac{3600\ \mu\ c_p\ g}{k}$ Prandtl number (dimensionless)

μ absolute viscosity of fluid, lb sec/ft^2

c_p heat capacity of fluid, Btu/lb $^{\circ}$F

g gravitational force per unit mass, 32.2 lb$\Big/\left(\frac{\text{lb sec}^2}{\text{ft}}\right)$

k thermal conductivity of the fluid, Btu/hr ft$^2\left(\frac{^{\circ}F}{ft}\right)$

Thus at each point along the surface of the solid there may be written:

$$Nu_x = f\ (Re_x,\ Pr_x) \tag{5}$$

where the subscript x refers to the point values of Nu and Re.

Usually, in the literature, the average Nu is given rather than the point values, thus

$$Nu_{av} = f\ (Re_{av},\ Pr_{av}) \tag{5a}$$

In numerous cases, for many experimental data, equations (5) and (5a) can be expressed as a simple power function, which will apply in a limited range of the variables. For example, for turbulent flow in smooth, long pipes (the effect of the variation of f_c at entrance being negligible), it has been found that the following relation allows the prediction of the average f_c to be made successfully:*

*This equation, discussed in reference 15, is for all practical purposes equivalent to that presented by McAdams. (See reference 16, p. 168 of 2d ed.)

$$\frac{f_c D}{k} = 0.022 \left(\frac{GD}{3600\ \mu g}\right)^{0.8} \left(\frac{3600\ \mu\ c_p\ g}{k}\right)^{0.333} \qquad (6)$$

The form of equation (6) is very useful in that it applies to any fluid flowing through long tubes in turbulent motion. Equation (6), however, is unnecessarily clumsy to handle when a particular fluid is under consideration. Since this report is concerned mainly with heat exchangers in which air is the fluid, equation (6) may be greatly simplified by expressing each property of air* appearing in the equation as a function of temperature, and combining the resulting functions into a power function of the absolute temperature. (See reference 15.) This method has been followed throughout this report, so that, although all the equations presented are based on generalized forms such as those expressed by equation (5), the final form involves only the absolute temperature, significant dimensions, and the weight rates per unit area. For example, by the application of the method just discussed, equation (6) for air is reduced to the for

$$f_c = 5.4 \times 10^{-4}\ \frac{T^{0.3}\ G^{0.8}}{D^{0.2}} \qquad (7)^{**}$$

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*The properties of air also may be used for exhaust gases with fair accuracy. Reference 17 shows a very small difference between the viscosity of exhaust gases and air. The thermal conductivity of exhaust gases is not well known at high temperatures. The heat capacity of exhaust gases may be calculated if the composition of the gas is known. (See reference 18.)

**In earlier reports of this series the exponent of T in several equations was given as 0.296. In order to avoid giving the impression of undue accuracy, this exponent has been changed to 0.3. For each equation, then, the coefficient of T has been changed also in order to give the same numerical value as before. In other words, the exponent of T has been slightly increased and the coefficient has been slightly decreased, so that the result has not changed.

where

f_c unit thermal convective conductance

T mixed-mean absolute temperature of fluid, $^{\circ}R$

G weight rate per unit cross-sectional area, lb/ft^2 hr

D tube diameter, ft

This form of equation is very easy to use and yields results which agree with the more general form (equation (6)) within about 2 percent.

3. Radiation.- When a body is heated, it emits radiant energy at a rate dependent upon its absolute temperature. A Planckian radiator - that is, a body that absorbs all the radiant energy incident upon it (sometimes called a black body) - emits radiant energy at a rate proportional to the fourth power of its absolute temperature T_1. The rate of radiant energy transfer from a Planckian radiator radiating to evacuated space at absolute zero is (reference 16, ch. III of 2d ed.)

$$q_r = \sigma A_r \left(\frac{T_1}{100}\right)^4 \tag{8}$$

where

q_r rate of heat transfer by radiation, Btu/hr

σ Stefan-Boltzman radiation constant

$0.173 \text{ Btu/hr} \left(\frac{^{\circ}R}{100}\right)^4 ft^2$

A_r area of body,* ft^2

If the Planckian radiator, instead of radiating to space at absolute zero, radiates to surroundings (also Planckian, at a temperature T_2) which completely surround the body, the net rate of radiant energy transfer is given by

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*The area of the radiating body effective in radiation may not be equal to the total surface area of the body if the surface of the body has "re-entrant angles." (See pt. I, sec. G for a further discussion of this point.)

$$q_r = \sigma A_r \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right] \qquad (9)$$

No actual substance meets the specifications of a Planckian radiator, but as a first approximation (reference 16, ch. III of 2d ed.) some substances may be considered "gray" - that is, they may be defined as absorbing the same fraction of the radiant energy incident upon them at all wavelengths. The rate of heat transfer from a small "gray body" at temperature T_1 to a Planckian body which completely surrounds it at temperature T_2, is

$$q_r = \sigma A_r \epsilon_1 \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right] \qquad (10)$$

where ϵ_1 is the emissivity of the radiating gray body (always less than unity). If the body at the temperature T_2 is not a Planckian radiator and if the two bodies possess a given geometrical relationship to each other, the rate of heat transfer is given by

$$q_r = \sigma A_r F_{AE} \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right] \qquad (11)$$

where F_{AE} is a modulus which modifies the equation for the net radiation between Planckian radiators to account for the emissivities and relative geometry of the two bodies. (See reference 16, pp. 54-60 of 1st ed.; also references 19 and 20.) Several values* for F_{AE} are given in part I, section G.

In many engineering applications, a body at temperature t_1, $^{\circ}F$ (T_1, $^{\circ}R$) loses heat by radiation to the surroundings at a temperature t_2, $^{\circ}F$ (T_2, $^{\circ}R$) and at the same time loses heat by convection to a surrounding gas at a temperature τ_a. In order to simplify this problem, an equivalent unit thermal conductance for radiation f_r can be defined by an equation similar to that used for the definition of unit thermal conductances for convection. Thus

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*A mechanical integrator (reference 21) can be used as an aid in determining the shape modulus F_A.

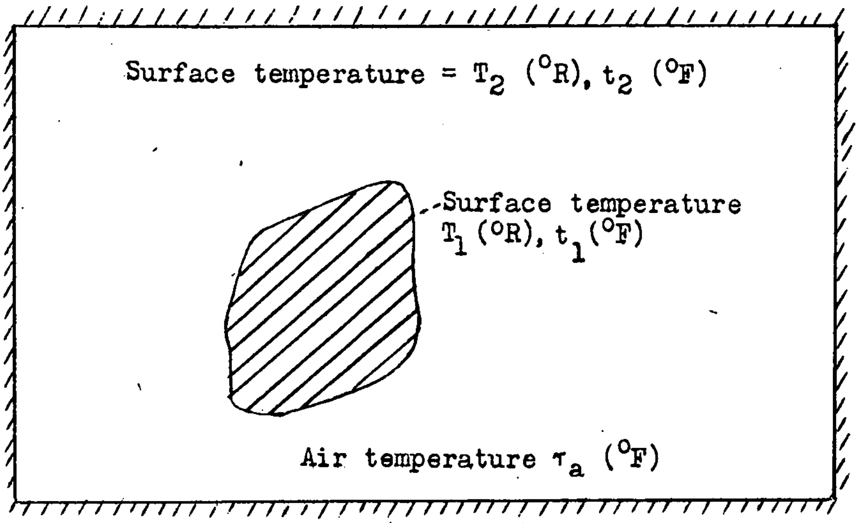

Figure 3.- Combined radiation and convection.

$$q_r = f_r A \; (t_1 - \tau_a) \tag{12a}$$

where A is the total heat transfer surface area of the radiating body. After the rate of radiant heat transfer is written in this form, the radiant and convective heat transfer rates may be added as shown:

$$q_T = q_r + q_c = (f_r + f_c) \; A \; (t_1 - \tau_a) \tag{12b}$$

The use of the equivalent conductance f_r reduces the radiant heat transfer equations to the form of Newton's law of cooling.* In order that the radiant heat rate given by equation (12a) be equal to that expressed by the more fundamental equation (11), the equivalent unit conductance for radiation f_r must be defined as

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*It should be noted that if the body gains heat by convection and loses heat by radiation (or vice-versa) which is often the case, the unit conductances f_c and f_r in equation (12b) will be of opposite sign. In particular, if a body is gaining as much heat by convection as it is losing by radiation, so that $q_T = 0$, the two unit conductances f_r and f_c are equal and opposite in sign. (See Example, fig. 25.)

$$f_r = \frac{\sigma A_r F_{AE}\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{A\ (t_1 - \tau_a)} \qquad (13)^*$$

Note that $T_1 = (t_1 + 460)$

The equivalent conductance f_r therefore is a function of

1. The emissivities of the radiating surfaces

2. The relative geometry of the radiating surfaces

3. The temperatures of the radiating surfaces and of the surrounding fluid

In the preceding equations the following symbols were utilized:

A total surface area of heat transfer, ft^2

A_r effective surface area of the radiating body,** ft^2

f_c unit thermal conductance for convection, Btu/hr ft^2 ^oF

f_r equivalent unit thermal conductance for radiation, Btu/hr ft^2 ^oF

F_{AE} combined shape and emissivity modulus defined by equation (8) (sometimes set equal to $F_A \times F_E$), dimensionless

F_A shape modulus to account for relative geometry of surfaces exchanging heat by radiation (dimensionless)

F_E emissivity modulus to account for emission characteristics of surfaces (dimensionless)

q heat transfer rate, Btu/hr

t_1 temperature of surface 1, ^oF

T_1 absolute temperature of surface 1, ^oR

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*The usual definition of f_r does not involve the fluid temperature τ_a. (See reference 16, p. 63 of 2d ed.)

**See pt. I, sec. G for a discussion of this term.

t_2 temperature of surface 2, $^{\circ}F$

T_2 absolute temperature of surface 2, $^{\circ}R$

σ Stefan-Boltzmann constant, 0.173 Btu/hr $\left(\frac{^{\circ}R}{100}\right)^4$ ft^2

τ_a temperature of ambient fluid, $^{\circ}F$

4. Combined heat transfer mechanisms.- In the previous sections the three mechanisms of heat transfer have been considered separately. In practice, however, two or more of the mechanisms usually occur simultaneously. The transfer of heat from one gas to another through a metallic surface illustrates this point. On the hot gas side heat is transferred by convection and radiation (reference 11) from the hot gas to the wall surface. The total rate of heat transfer from the hot gas to the wall surface is given by:

$$q_T = f_{c_g} A \left(\tau_g - t_{w_g}\right) + f_{r_g} A \left(\tau_g - t_{w_g}\right)$$

or

$$q_T = \left(f_{c_g} + f_{r_g}\right) A \left(\tau_g - t_{w_g}\right)$$

Since the steady state exists, the same rate of heat transfer occurs through the metallic wall by conduction. Thus

$$q_T = \frac{kA}{L} \left(t_{w_g} - t_{w_a}\right)$$

After passing through the metallic wall, the thermal energy is transferred to the cold air by convection, assuming the radiant heat transfer to the air to be negligible.* Thus:

*Radiant heat transfer to the cold air may be promoted by suspending metal plates in the air stream in view of the hot heat exchanger surface. Heat is transferred to these plates from the hot surface by radiation; then heat is transferred from the plates to the air stream by convection. These "irradiated convectors" may have an appreciable effect on the heat exchanger output but usually at the expense of greatly increased frictional pressure drop. (See example in pt. I, sec. G for details.)

$$q_T = f_{c_a} A \left(t_{w_a} - \tau_a\right)$$

By eliminating the intermediate temperatures, t_{w_g} and t_{w_a}, the following equation is obtained

$$q_T = \frac{\tau_g - \tau_a}{\frac{1}{\left(f_{c_g} + f_{r_g}\right) A} + \frac{L}{kA} + \frac{1}{f_{c_a} A}} \qquad (14)$$

The same result could have been readily obtained by noting that unidirectional thermal and direct current electrical circuits are analogous and the corresponding equations for the rate of heat flow and current are similar for the steady state. (See reference 22.) If the rate of heat flow is designated as a thermal current and the temperature differences act as potentials, the following analogous relations may be written for the steady unidirectional state.

$$i = \frac{\Delta E}{R} \text{ (electrical)}$$

$$q = \frac{\Delta t}{\left(\frac{L}{kA}\right)} \text{ (conduction)}$$

$$q = \frac{\Delta t}{\left(\frac{1}{f_c A}\right)} \text{ (convection)}$$

$$q = \frac{\Delta t}{\left(\frac{1}{f_r A}\right)} \text{ (radiation)}$$

Thus, $\left(\frac{L}{kA}\right)$, $\left(\frac{1}{f_c A}\right)$, $\left(\frac{1}{f_r A}\right)$ may be termed "thermal resistances."

The mechanism of heat flow from the hot to the cold gas then can be visualized as analogous to the flow of current in a simple electrical direct-current circuit.

Then, because

$$i = \frac{E_1 - E_4}{\dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} + R_3 + R_4}$$

then, by analogy,

$$q = \frac{\tau_g - \tau_a}{\dfrac{1}{\left(f_{r_g} + f_{c_g}\right)A} + \dfrac{L}{kA} + \dfrac{1}{f_c\ A}}$$

Many problems in steady heat flow can be readily analyzed by the technique presented in the foregoing paragraph.

The term

$$\left[\frac{1}{\dfrac{1}{\left(f_{r_g} + f_{c_g}\right)A} + \dfrac{L}{kA} + \dfrac{1}{f_{c_a}\ A}}\right] \tag{15}$$

is called the over-all conductance UA of the thermal system.

If the temperature at either surface of the metal is desired, inspection of figure 4 reveals, from the thermal circuit, that

$$\frac{t_{w_a} - \tau_a}{\tau_g - \tau_a} = \frac{\dfrac{1}{f_{c_a}\ A}}{\dfrac{1}{UA}}$$

or

$$t_{w_a} = \left(\frac{UA}{f_{c_a}\ A}\right)(\tau_g - \tau_a) + \tau_a \tag{16}$$

Then:

$$\frac{t_{w_g} - t_{w_a}}{\tau_g - \tau_a} = \frac{\dfrac{L}{kA}}{\dfrac{1}{UA}}$$

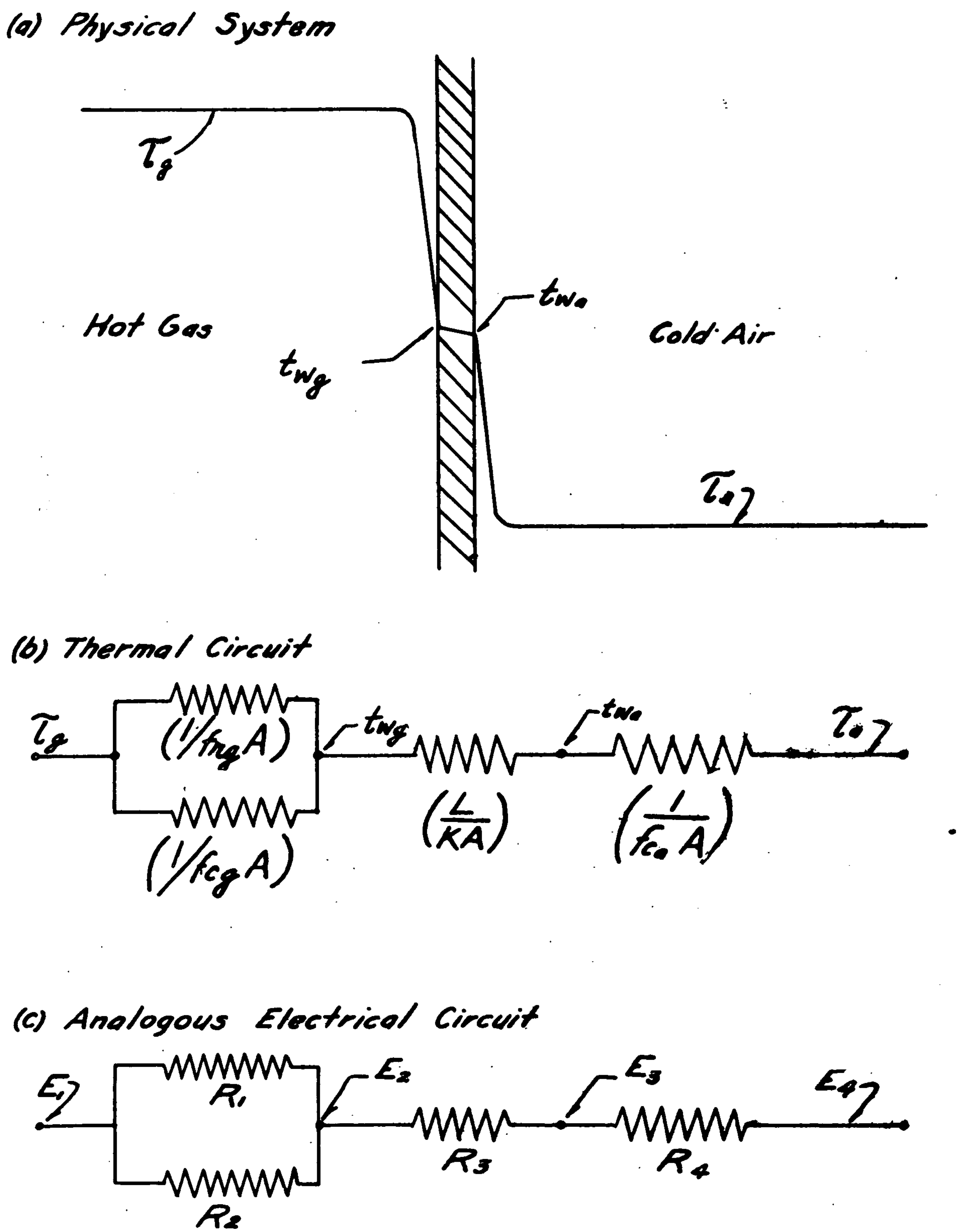

Fig. 4 - Thermal Circuit

or

$$t_{w_g} = t_{w_a} + \left(\frac{UA}{\frac{kA}{L}}\right) (\tau_g - \tau_a)$$

Thus the temperature of the metallic surface may be estimated readily once the unit thermal conductances, the over-all thermal conductance, and the two gas temperatures are known.* It is evident from equation (16) that if the ratio (UA/f_cA) is small, then the metal temperature will be almost equal to the air temperature τ_a.

In many cases the values of the thermal conductances (f_c and U) vary throughout the exchanger. In order to obtain the temperature of the exchanger surface, the <u>local</u> values of the thermal conductances at the point in question should be used in the foregoing equations.

The thermal conductances f_c may be adjusted to yield the proper metal temperatures by changing the fluid velocity, and so forth. The effect of these changes, of course, can be calculated by means of the foregoing equations.

In the foregoing equations the symbols have the following significance:

q rate of heat transfer, Btu/hr

q_T total (convective and radiation) heat transfer rate, Btu/hr

f_{c_g} unit thermal conductance for convection on the hot-gas side of the heat transfer surface, Btu/hr ft² °F

f_{r_g} equivalent unit thermal conductance for radiation from certain constituents of the hot exhaust gases to the heat transfer surface (defined by equation (13)) Btu/hr ft² °F. (See reference 11.)

f_{c_a} unit thermal conductance for convection on the cold-air side of the heat transfer surface, Btu/hr ft² °F

A surface area of heat transfer, ft²

τ_g mean temperature of hot gases, °F

*In many cases conduction of heat along the metal surface will have an important effect on the metal temperatures. Calculations considering the effect of convection to the metal surfaces and also conduction along the metal can be made using the "Southwell relaxation method." (See reference 81.)

t_{w_g} temperature of heat transfer surface in contact with hot gas, ^{o}F

t_{w_a} temperature of heat transfer surface in contact with cold air, ^{o}F

τ_a mixed-mean temperature of cold air, ^{o}F

k thermal conductivity of metallic heat transfer surface, Btu/hr ft^2 $\left(\frac{^{o}F}{ft}\right)$

L thickness of heat transfer surface, ft

E electromotive force, volts

i current, amperes

R electrical resistance, ohms

R_1, R_2, R_3, R_4 electrical resistances (shown in fig. 4), ohms

E_1, E_2, E_3, E_4 voltages (shown in fig. 4), volts

Mean Temperature Difference

The rate of heat transfer between the hot and cold gas can be written as:

$$q = UA\ (\tau_g - \tau_a)$$

In the analysis presented in the last paragraph the temperature difference $(\tau_g - \tau_a)$ has been assumed constant. In actual exchangers the temperature difference between hot and cold gas varies throughout the exchanger. A discussion of the proper mean temperature difference to be utilized for such cases is presented in part II of this report.

If the over-all conductance UA and the mean temperature difference $(\tau_g - \tau_a)_m$ are known, the thermal performance of the heater can be established.

The remainder of this section presents equations for the determination of the unit thermal conductances for the most common flow systems met in practice. Combination of the unit thermal conductances for any heater in the form of thermal resistances, will allow the evaluation of the over-all conductance UA and thus the heater performance.

B. FORCED CONVECTION ALONG PLATES

General

When a fluid flows along a fixed solid boundary, the fluid in immediate contact with the surface attains zero velocity. For fluids with relatively low viscosity, such as air, the velocity will change from zero to its free stream value within a thin layer of fluid next to the wall, called the "boundary layer." (Reference 14, p. 50.) The thickness of the boundary layer at any point on the body depends upon the kinematic viscosity of the fluid, the velocity in the free stream, and the geometry of the body.

If, for example, a flat plate is placed with its surface parallel to the direction of flow, as the fluid comes in contact with the surface of the plate it is brought to rest. At the leading edge of the plate the boundary layer is very thin, since only the fluid in immediate contact with the plate has been brought to rest, while the remaining fluid flows on with the free-stream velocity u_∞. As the fluid proceeds along the plate the tangential stresses set up by the solid boundary cause more and more of the fluid to be retarded, and thus the thickness of the boundary layer continually increases. The growth of this boundary layer is illustrated in figure 5, in which the velocity distribution in the fluid stream at various points along the plate are shown, the vertical scale being greatly enlarged for clarity.

For this so-called laminar boundary layer, existing near the leading edge of the plate, it has been shown that the thickness of the boundary layer, the frictional drag on the plate (reference 14, p. 50), and the unit thermal conductance at any point on the plate (reference 14, p. 623) are all functions of the square root of the Reynolds number for the plate, $\left(Re = \frac{u_\infty x \gamma}{\mu g}\right)$. At a certain point along the surface the fluid in the boundary layer becomes turbulent, and, from this point on (fig. 6), the thickness of the boundary layer,

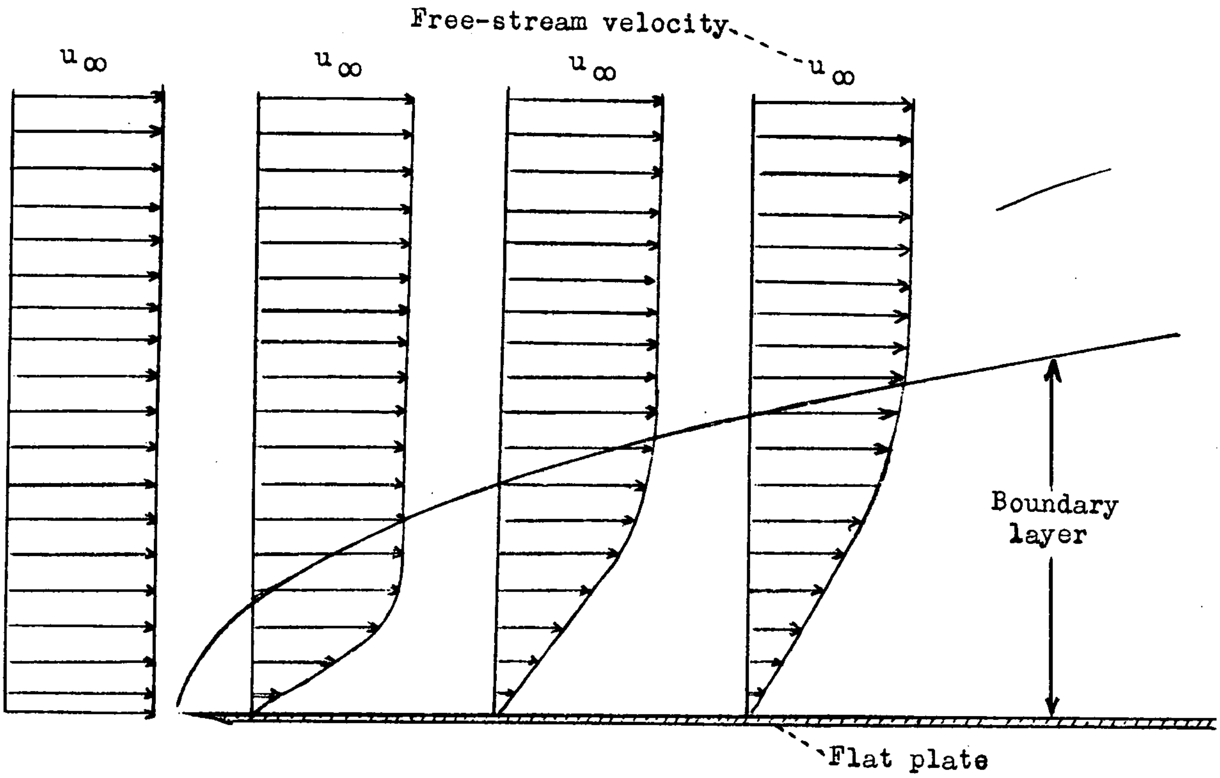

Figure 5.- Velocity distribution in laminar boundary layer.

the drag (reference 14, p. 362), and the unit thermal conductances are functions of the one-fifth power of the Reynolds number for the plate (reference 23) $\left(Re = \frac{u_\infty x \gamma}{\mu g}\right)$.

A number of analytical solutions for forced convection along plates have been presented in the literature (reference 14, p. 623), and several investigators have obtained experimental data on this flow system (reference 23). On the basis of this work, the following equations were derived. (See reference 24.) These equations allow the determination of the unit thermal conductance for a flat plate along which air or exhaust gas is flowing, at any altitude pressure and for a temperature range of -60° to 1600° F.

Laminar Boundary Layer

(a) Point unit conductance at any point x feet from the leading edge of flat plate:

$$f_{c_x} = 0.0562\ T_f^{0.50} \left(\frac{u_\infty \gamma}{x}\right)^{0.50} \tag{17}$$

(b) Average unit conductance in length l (laminar sublayer from $x = 0$ to $x = l$):

$$f_c = \frac{1}{l}\int_0^l f_{c_x}\,dx = 0.112\ T_f^{0.50} \left(\frac{u_\infty \gamma}{l}\right)^{0.50} \tag{18}$$

Turbulent Boundary Layer

(a) Point unit conductance at any point x from leading edge of flat plate:

$$f_{c_x} = 0.51\ T_f^{0.3}\ \frac{(u_\infty \gamma)^{0.8}}{x^{0.2}} \tag{19}$$

(b) Average unit conductance in length (turbulent boundary layer for $x = 0$ to $x = l$):

$$f_c = \frac{1}{l}\int_0^l f_{c_x}\,dx = 0.64\ T_f^{0.3}\ \frac{(u_\infty \gamma)^{0.8}}{l^{0.20}} \tag{20}$$

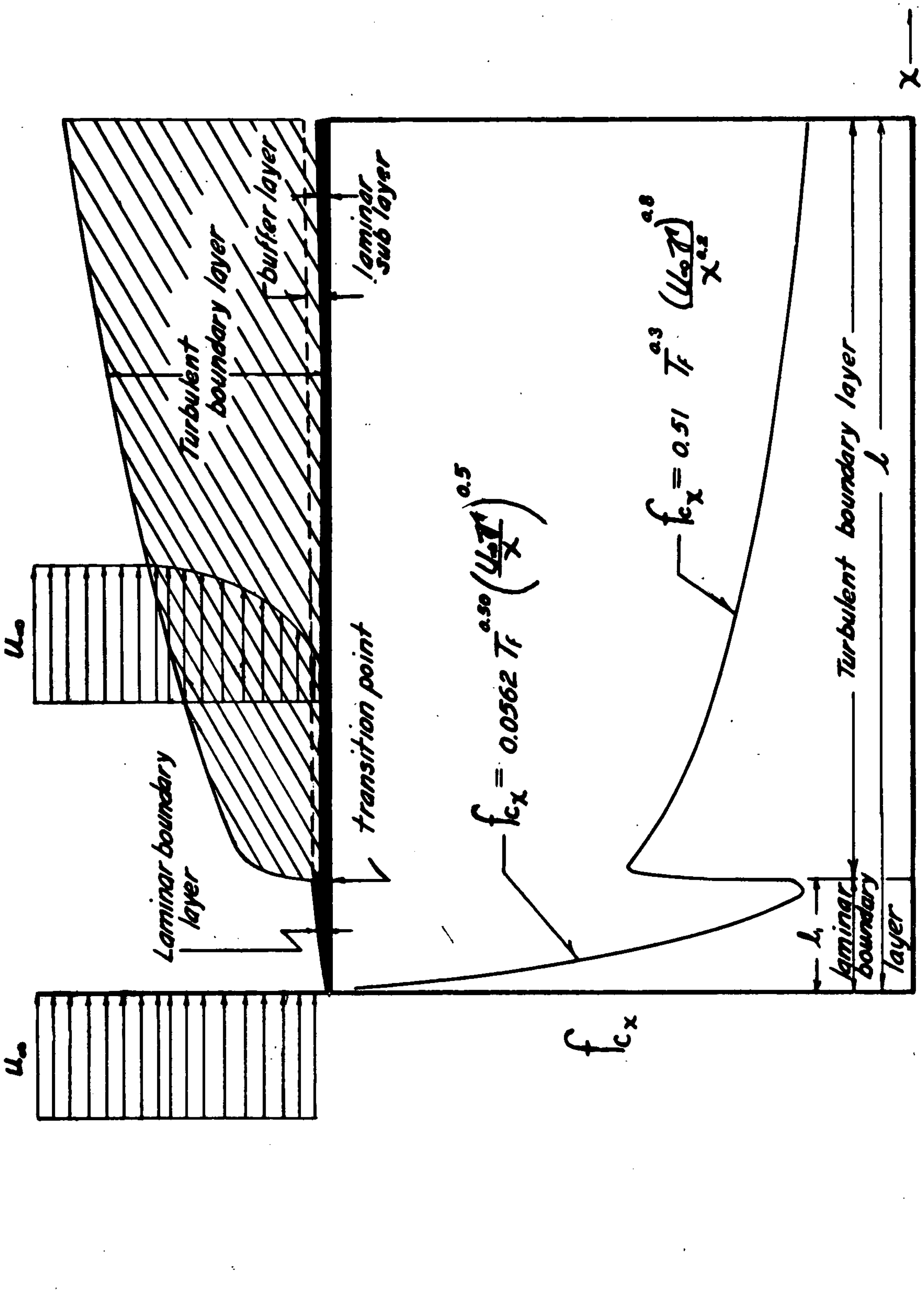

Fig. 6 - Flow Phenomena and Variation in Point Unit Thermal Conductance Along Flat Plate.

If a laminar boundary layer exists on part of the plate (fig. 6) and a turbulent boundary layer over the remainder, the average unit conductance in the length may be obtained by integration of the point unit conductance. Thus

$$f_c = \frac{1}{l}\left[\int_{0\,(\text{laminar})}^{l_1} f_{c_x}\,dx + \int_{l_1\,(\text{turbulent})}^{l} f_{c_x}\,dx\right] \tag{21}$$

In the foregoing equations,

f_{c_x} point unit conductance at a distance x from leading edge of flat plate, Btu/hr ft² °F

f_c average unit conductance of plate in length l, Btu/hr ft² °F

T_f arithmetic average of plate temperature and free air stream temperature, °R

u_∞ free air stream velocity, ft/sec

γ density of air at temperature T_f, and prevalent pressure, lb/ft³

x distance from leading edge of plate, ft

l_1 distance from leading edge of plate to point at which laminar boundary layer becomes turbulent, ft (See references 14, p. 361; and 23.)

l length of plate under consideration, ft

Example

The transition from a laminar boundary layer to a turbulent boundary layer along a flat plate 12 inches in length is postulated to occur at a magnitude of the point Reynolds number for the plate equal to 50,000. Plate temperature is 200° F; air temperature, 30° F; free stream air velocity, 100 feet per second; atmospheric pressure, 14.7 pounds per square inch absolute. (a) Calculate the variation of f_{c_x} along the plate. (b) What is the average f_c for the plate?

(a) Transition point

The Reynolds number at transition from the laminar to the turbulent boundary layer is:

$$Re = \frac{u_\infty l_1 \gamma}{\mu g} = 50,000$$

From the data:

$$T_f = 460 + \frac{200 + 30}{2} = 575^\circ \text{ R}$$

$$\mu_{T_f} = 0.406 \times 10^{-6} \frac{\text{lb sec}}{\text{ft}^2} \quad \text{(See fig. 42.)}$$

$$\gamma = \frac{P}{RT} = \frac{14.7 \times 144}{53.3 \times 575} = 0.0692 \frac{\text{lb}}{\text{ft}^3}$$

Thus,

$$l_1 = \frac{50000 \times 0.406 \times 10^{-6} \times 32.2}{100 \times 0.0692}$$

$$l_1 = 0.0943 \text{ ft} = 1.13 \text{ in.}$$

(b) Point unit conductance

A laminar boundary layer exists for the first 1.13 inches of the plate. Thus:

$$f_{c_x} = 0.0562 \, T_f^{0.50} \left(\frac{u_\infty \gamma}{x}\right)^{0.5} \quad \text{(equation (17))}$$

$$f_{c_x} = 0.0562 \times 575^{0.50} \left(\frac{100 \times 0.0692}{x}\right)^{0.5} = \frac{3.56}{x^{1/2}} \frac{\text{Btu}}{\text{hr ft}^2 \, {}^\circ\text{F}}$$

On the remainder of the plate there is a turbulent boundary layer. Thus for $0.0943 < x < 1.0$ feet.

$$f_{c_x} = 0.51 \, T_f^{0.3} \frac{(u_\infty \gamma)^{0.8}}{x^{0.2}} \quad \text{(equation (19))}$$

$$f_{c_x} = 0.51 \, (575)^{0.3} \frac{(100 \times 0.0692)^{0.8}}{x^{0.2}} = \frac{16.2}{x^{0.2}} \frac{\text{Btu}}{\text{hr ft}^2 \, {}^\circ\text{F}} \quad (0.0943 < x < 1 \text{ ft})$$

A plot of the variation of f_{c_x} with length is shown in figure 7.

(c) Average f_c.

The average f_c is obtained as follows:

$$f_c = \frac{1}{l}\left[\int_{0\,(\text{laminar})}^{l_1} f_{c_x}\,dx + \int_{l_1\,(\text{turbulent})}^{l} f_{c_x}\,dx\right]$$

$$= \frac{1}{1.0}\left[\int_{0}^{0.0943} \frac{3.56}{x^{0.5}}\,dx + \int_{0.0943}^{1.00} \frac{16.2}{x^{0.2}}\,dx\right] = 19.3\ \frac{\text{Btu}}{\text{hr ft}^2\ {}^{\circ}\text{F}}$$

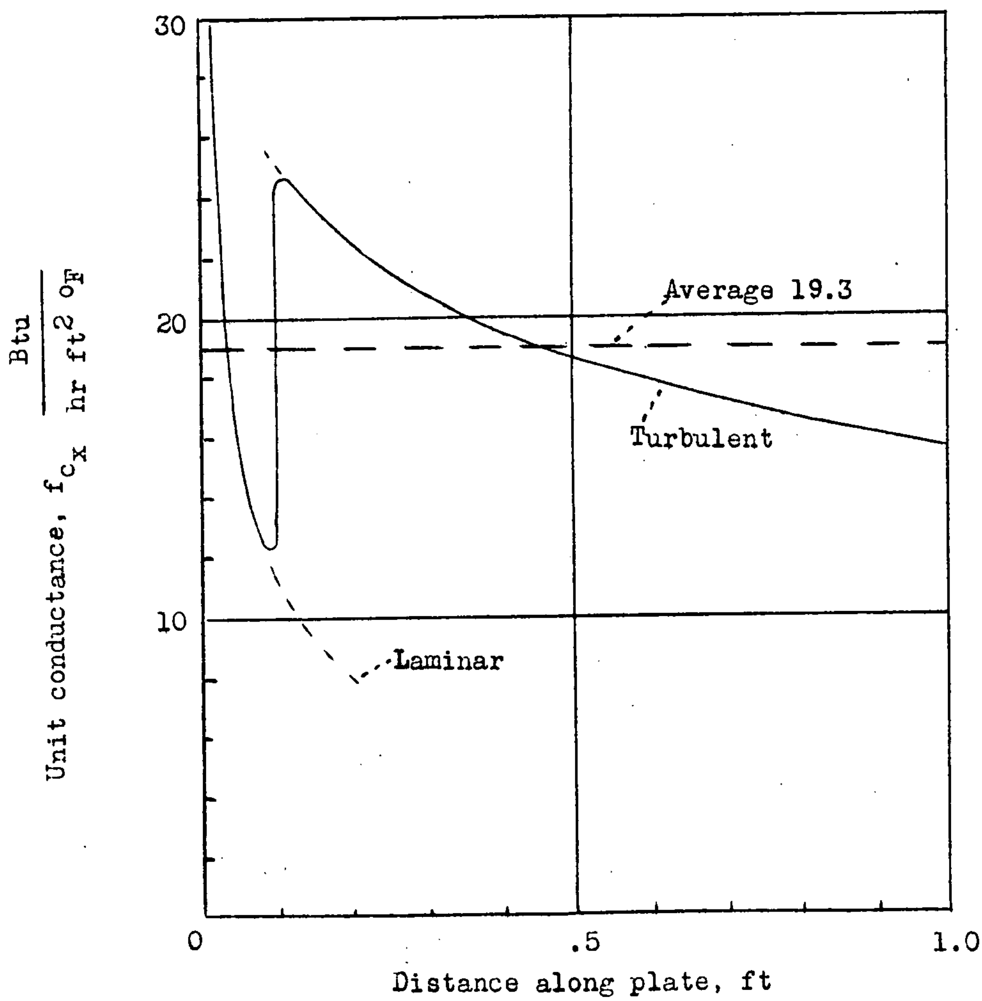

Figure 7.- Variation of unit thermal conductance along plate.

C. FORCED CONVECTION INSIDE TUBES

1. Turbulent Flow in Tubes

General.- As air in turbulent motion with a uniform velocity enters a tube, the fluid immediately adjacent to the tube wall is brought to rest. For a short distance along the tube wall a so-called laminar boundary layer is formed. (See pt. I, sec. B.) If the turbulence in the entering fluid stream is sufficiently great, the laminar boundary layer will quickly change into a turbulent boundary layer. The turbulent boundary layer will rapidly increase in thickness until it fills the whole of the pipe. From this point on, the velocity profile across the pipe remains unchanged for isothermal flow. (See reference 14, p. 360.)

As the flow pattern changes from a laminar boundary layer to a turbulent boundary layer, and finally to the fully developed turbulent velocity distribution, the fluid in immediate contact with the tube wall will maintain its viscous motion, although the fluid further from the wall will flow turbulently. (See reference 25.) This viscous layer of fluid (initially the laminar boundary layer) is called the "laminar sublayer" in the region in which the fully developed turbulent velocity distribution has been established. The thickness of this laminar sublayer may be readily estimated. (See references 26 and 27.) Next to the laminar sublayer a "buffer" layer may be visualized in which both viscous and turbulent forces play an important part. The remainder of the flow system may be considered as a completely turbulent "core" in which viscous forces are unimportant, and turbulent forces predominate. For a fluid the viscosity of which does not change appreciably with temperature, the thicknesses of the laminar sublayer and buffer layer are independent of distance along the tube (in the region in which the velocity distribution has been established) and depend only upon the Reynolds number, based on the tube diameter and the mean velocity of flow. Figure 8 illustrates these various phenomena.*

---

*A very high point unit thermal conductance exists at the entrance to a tube even when a hydrodynamic calming section precedes the heating section, due to the very large temperature gradient existing at the fluid-wall interface at the beginning of the heating section. Latzko in reference 28 considers this case as well as the case in which the fluid enters the tube with uniform velocity. The simplified equations presented in this section, however, apply more precisely to the latter case.

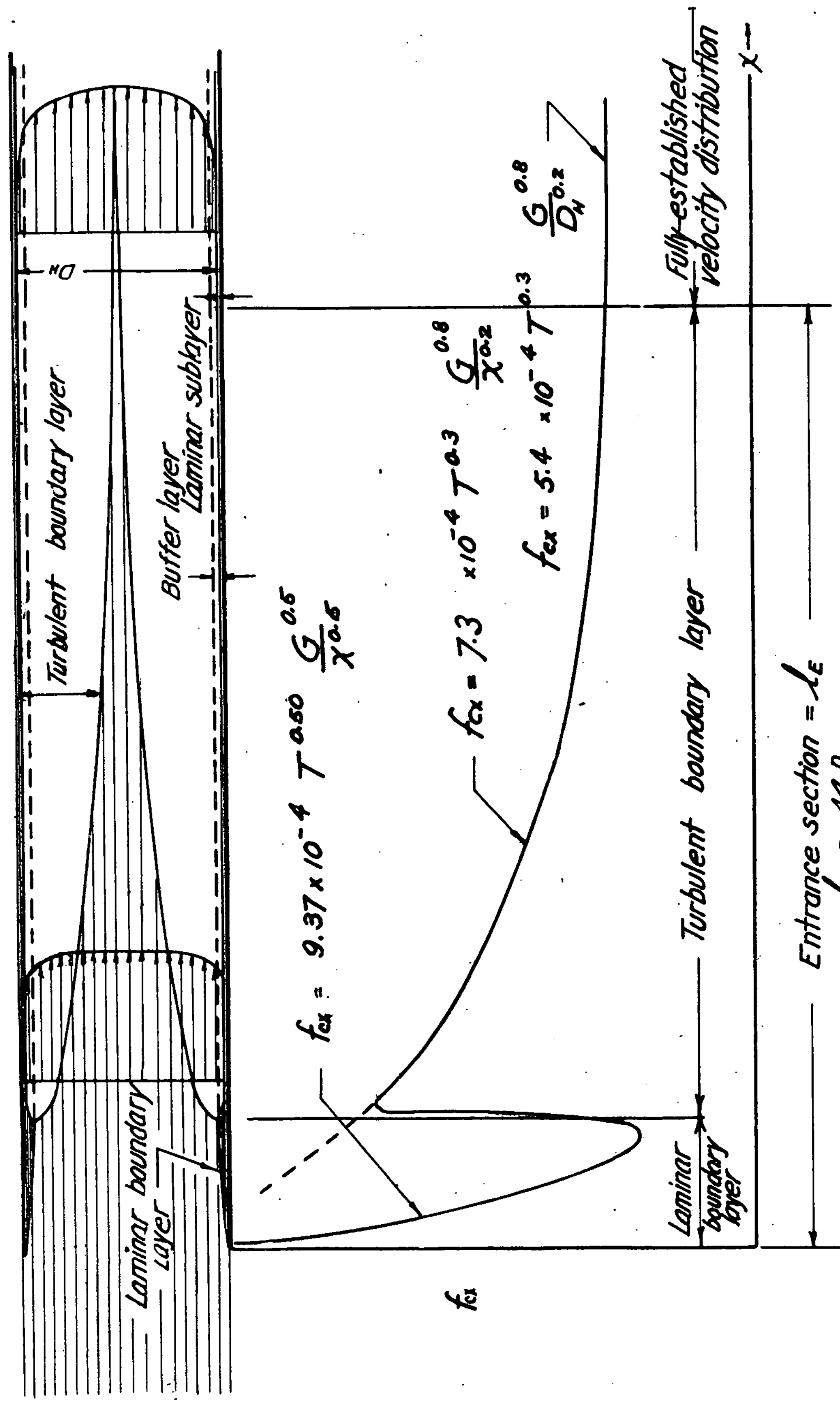

Fig. 8. Flow Phenomena and Variation in Point Unit Conductance at the Entrance to a Straight Duc

All the equations for the entrance section shown in figure 8 are approximations which are based on the postulate that the entrance section of the pipe may be considered to behave like a flat plate placed parallel to the direction of air flow.* (See pt. I, sec. B.) The equations for the unit conductance in the portion of the pipe in which the velocity distribution is established are close approximations (reference 15) to analytical equations which were derived by a consideration of the thermal resistance of the laminar sublayer, the buffer layer, and the turbulent core (reference 26).

In most heater designs, experience has shown that the laminar boundary layer occupies a negligible portion of the entrance section. If the location of the transition point from the laminar to the turbulent boundary layer can be assumed to exist at the very beginning of the entrance section, the laminar boundary layer can be entirely neglected.

Summary of Equations:

Range of Application

Fluids: air or exhaust gases in smooth tubes

Temperature range: -60° to 1600° F

Pressure range: pressure at any altitude

For short tubes either the point of transition from laminar to turbulent boundary layer must be known, or as an approximation, a turbulent boundary layer can be postulated to exist from the point $x = 0$.

*The length of the "entrance section" - i.e., the section at the entrance to the tube in which the velocity and temperature distribution have not yet attained their fully developed form - has been evaluated in this section by noting the distance from the tube entrance, at which the point unit conductance in the entrance section equaled that for the remainder of the tube. This procedure (reference 29) yields the entrance length $l = 4.4\ D_H$. More precise calculations by Latzko (reference 28) show that the entrance length is a function of the Reynolds number (based on pipe diameter and mean velocity). Latzko gives as the entrance length $l = 0.693\ D_H\ Re^{1/4}$. At a magnitude of Re = 10,000 this equation yields $l = 6.93\ D_H$, which checks roughly the magnitudes of the approximate method.

For long tubes the minimum $Re = \frac{GD}{3600\ \mu g} = 10,000$ (higher than 2000 to insure fully developed turbulence).

1. Short Tubes $\left(\frac{l}{D_H} < 4.4\right)$

(a) Point unit conductance (Turbulent Boundary Layer) (reference 29)

$$f_{c_x} = 7.3 \times 10^{-4} \frac{{T_f}^{0.3}\ G^{0.8}}{x^{0.2}} \qquad (22)$$

(b) Average unit conductance in length l (Turbulent Boundary Layer)

$$f_c = \frac{1}{l}\int_0^l f_{c_x}\,dx = 9.1 \times 10^{-4} \frac{{T_f}^{0.3}\ G^{0.8}}{l^{0.2}} \qquad (23)$$

2. Long Tubes $\left(\frac{l}{D_H} > 4.4\right)$

(a) Point unit conductance beyond the entrance length

$$f_{c_x} = 5.4 \times 10^{-4}\ T^{0.3} \frac{G^{0.8}}{{D_H}^{0.2}} \qquad (24)$$

(b) Average unit conductance (includes entrance effect)

$$f_c = \frac{1}{l}\int_0^l f_{c_x}\,dx = 5.4 \times 10^{-4} \frac{T^{0.3}\ G^{0.8}}{{D_H}^{0.2}}\left(1 + 1.1\ \frac{D_H}{l}\right) \qquad (25)$$

where

f_{c_x} point unit thermal conductance, x feet from entrance of tube, Btu/hr ft^2 $^{\circ}$F

f_c average unit conductance in length l of tube, Btu/hr ft^2 °F

x distance from entrance of tube, ft

l tube length, ft

D_H hydraulic diameter of tube, equal to

$$\frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}} = \frac{4A}{P}, \text{ ft}$$

A cross-sectional area of tube, ft^2

P wetted perimeter of tube, ft

G W/A = weight rate of air per unit cross-sectional area, lb/hr ft^2

W weight rate of air, lb/hr

T arithmetic average of the mixed-mean temperature of the air entering and leaving the tube, °R

T_f arithmetic average of air temperature and tube wall temperature, °R

Example - Short Duct

Air at an average temperature of 300° F flows through a tube with the dimensions given in figure 9 at a rate of 116 pounds per hour. The tube wall temperature is 800° F.

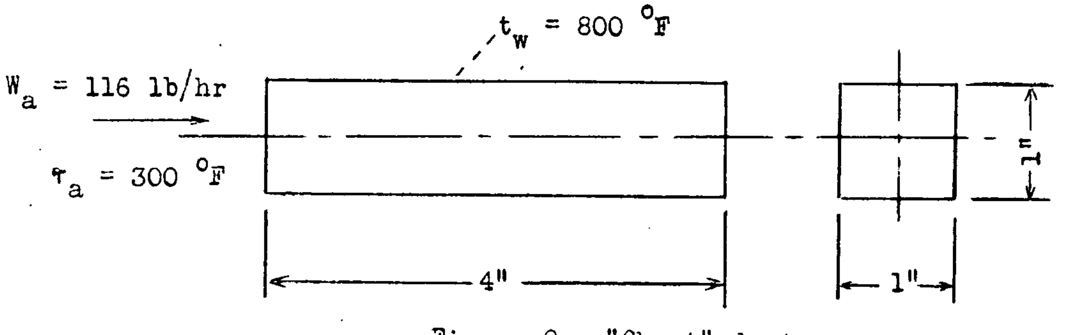

Figure 9.- "Short" duct.

P wetted perimeter = 4 in. = 0.333 ft

A cross-sectional area = 1 sq in. = 0.00695 ft^2

D_H hydraulic diameter = $\frac{4 \times 0.00695}{0.333} = 0.0833$ ft

l $\frac{4}{12} = 0.333$ ft

l/D_H $\frac{0.333}{0.0833} = 4.0$ (short tube)

G $\frac{116}{0.00695} = 16,700 \ \frac{lb}{hr\ ft^2}$

T_f $\frac{800 + 300}{2} + 460 = 1010^\circ$ R

(a) What is the average unit conductance along the tube?

(b) What is the variation of unit conductance along the tube, assuming a turbulent boundary layer starting at $x = 0$?

(a) Check Reynolds number (based on tube hydraulic diameter)

$$Re = \frac{G\ D_H}{3600\ \mu g} = \frac{16700 \times 0.0833}{0.50 \times 10^{-6} \times 32.2 \times 3600} = 24,000$$

The viscosity of air at 300° F was obtained from figure 42. Since Re is quite high, assumption of a turbulent boundary layer from $x = 0$ is probably valid.

From equation (23)

$$f_c = 9.1 \times 10^{-4}\ T_f^{0.3}\ \frac{G^{0.8}}{l^{0.2}}$$

$$f_c = 9.1 \times 10^{-4}\ \frac{(1010)^{0.3}\ (16700)^{0.8}}{(0.333)^{0.2}}$$

$$f_c = 21.6\ \frac{Btu}{hr\ ft^2\ {}^\circ F}$$

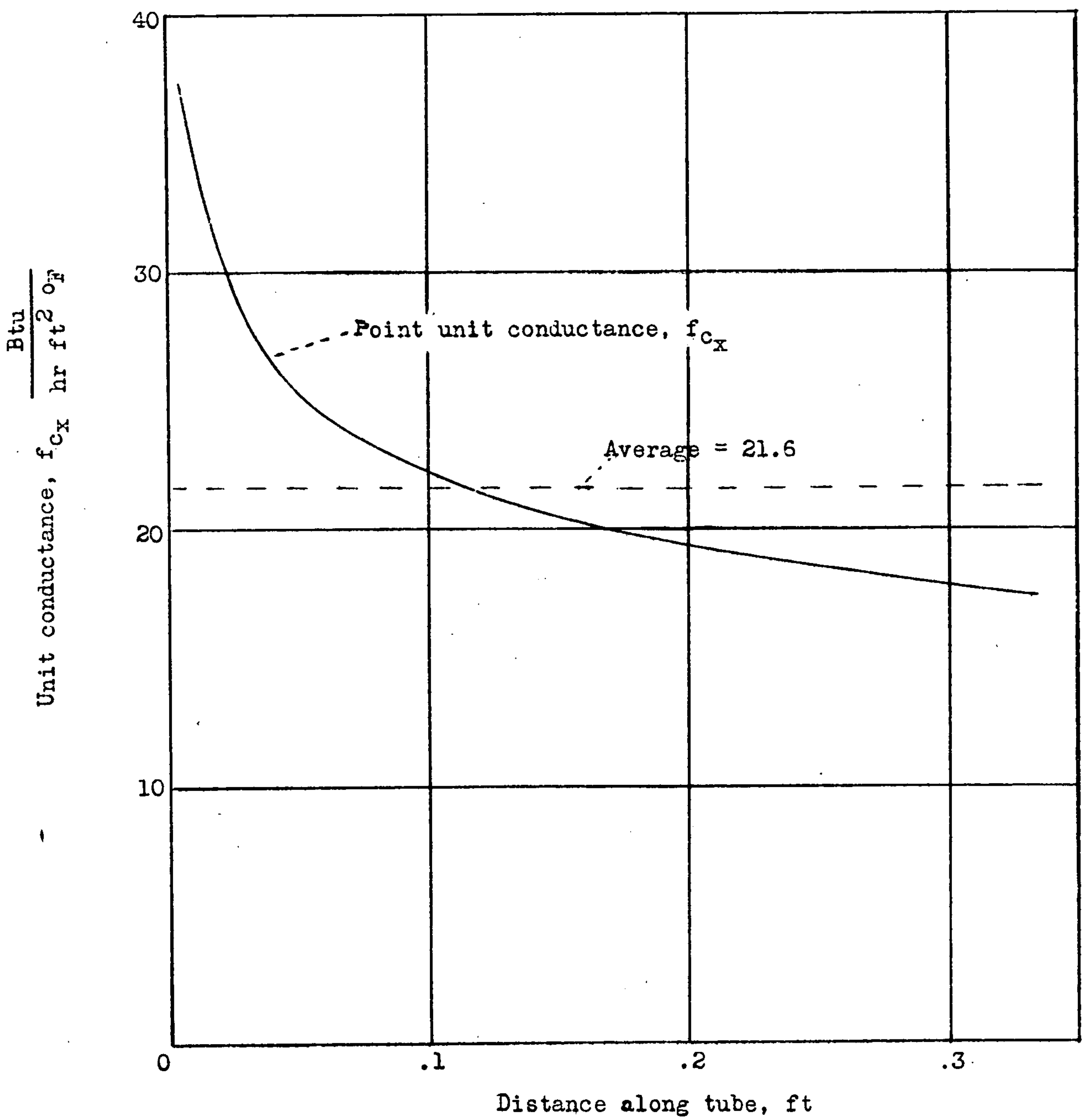

Figure 10.- Variation of unit thermal conductance along short duct.

(b) Variation of the point unit conductance along the tube.

From equation (22)

$$f_{c_x} = 7.3 \times 10^{-4} \frac{T_f^{0.3} G^{0.8}}{x^{0.2}} = \frac{13.9}{x^{0.2}} \text{ Btu/hr ft}^2 \text{ }^\circ\text{F} \quad (x \text{ in ft})$$

A plot of f_{c_x} versus x is shown in figure 10.

Example - Long Tube

What is the average unit thermal conductance for the following system (including entrance effect)?

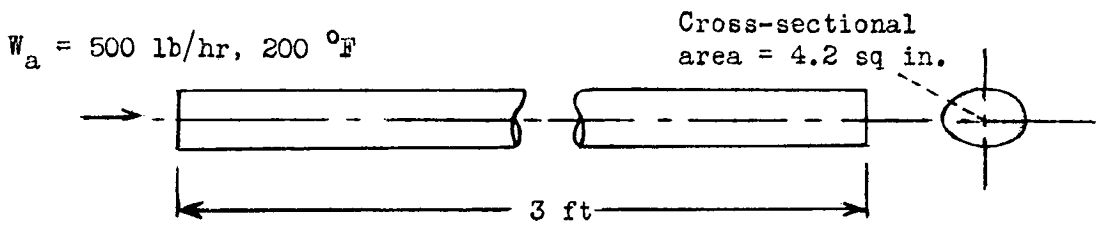

Figure 11.- "Long" tube.

l length of tube = 3.0 ft

A cross-sectional area = 4.2 sq in. = 0.0292 ft²

P wetted perimeter = 8.4 in. = 0.70 ft

$$D_H = \frac{4A}{P} = \frac{4 \times 0.0292}{0.70} = 0.167 \text{ ft}$$

$$l/D_H = \frac{3}{0.167} = 18 \text{ (long tube)}$$

T average fluid temperature = 200 + 460 = 660° R

W rate of air flow = 500 lb/hr

$$G \quad \text{weight rate per unit area} = \frac{500}{0.0292} = 17{,}100 \text{ lb/hr ft}^2$$

The Reynolds number for the flow is:

$$Re = \frac{G\ D}{3600\ \mu g} = 54,800$$

Thus equation (25) is applicable and,

$$f_c = 5.4 \times 10^{-4}\ T^{0.3}\ \frac{G^{0.8}}{{D_H}^{0.2}}\left(1 + 1.1\ \frac{D_H}{l}\right)$$

$$= 5.4 \times 10^{-4} \times (660)^{0.3} \times \frac{(17100)^{0.8}}{(0.167)^{0.2}}\left(1 + 1.1\ \frac{0.167}{3.0}\right) = 13.9\ \frac{\text{Btu}}{\text{hr}\ {}^{\circ}\text{F ft}^2}$$

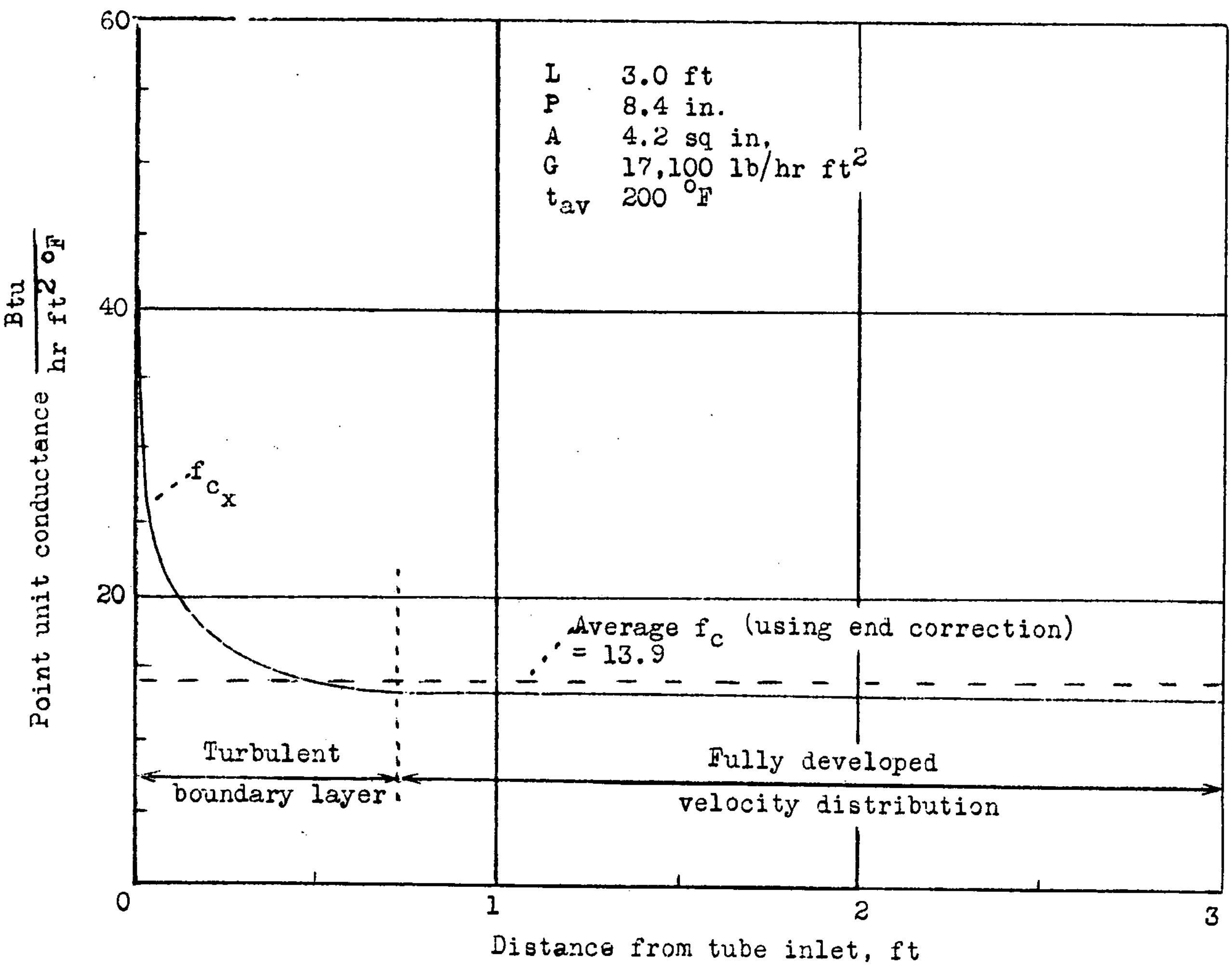

Figure 12.- Variation of unit conductance along a long tube.

A plot of the variation of f_{c_x} along the tube length is shown in figure 12. The value of f_{c_x} at the entrance to the tube was obtained by means of equation (22). Because of the small effect of the entrance section, magnitudes of the mean f_c and the point f_{c_x} are practically equal over most of the tube length.

2. Viscous Flow in Tubes

General.- When a fluid flows through a tube with a Reynolds number less than 2000 (based on tube diameter and mean velocity), the rate of heat transfer from the fluid to the tube walls is very low. The poor heat transfer is a direct result of the mechanism of viscous flow,* for in this type of fluid motion the particles of fluid tend to follow parallel paths with little or no mixing of adjacent layers of the fluid. Viscous flow is rarely encountered in aircraft heater design, but in some special applications, such as heating the interior of the leading edge of a wing by hot air, viscous motion of the fluid may result at low rates of air flow.

The determination of the rates of heat transfer during viscous motion is complicated by the fact that, due to the very slow motion of the fluid through the heat transfer passages, appreciable free convection heat transfer may be superimposed on the heat transfer by viscous forced convection. In references 31 and 32 a method is outlined for the approximate determination of the rates of heat transfer due to combined free and forced convection in the region of viscous flow,** when the velocity distribution at the entrance to the tube is parabolic and the tube temperature is uniform. Based on this work, the following approximate expression was derived,*** for short vertical circular tubes in which the free and forced convection velocities are in the same direction:

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*See references 16, pp. 189,199 of 2d ed.; 30, 31, and 32 for data on viscous forced convection.

**Reference 16, pp. 189,199 of 2d ed. gives a summary of average unit thermal conductances in viscous flow in tubes and ducts.

***Equation (25a) has been verified experimentally for several liquids with a positive coefficient of thermal expansion flowing vertically upward while being heated, and for water being cooled while flowing vertically downward (reference 31). (The viscosity of all the fluids used decreases with increased temperature.)

$$\frac{f_{c_x} D}{k} = 1.16 \sqrt[3]{31.0 + \frac{Wc_p}{k\ x} + 0.090 \left(\frac{Gr\ Pr\ D}{x}\right)^{3/4}} \qquad (25a)$$

where

f_{c_x} point unit thermal conductance along the tube starting from $x = 0$, Btu/hr ft^2 °F

D inner diameter of tube, ft

k thermal conductivity of the fluid, Btu/hr ft^2 (°F/ft)

W rate of flow of fluid, lb/hr

c_p heat capacity of fluid, Btu/lb °F

x distance from entrance of tube, ft

Gr Grashof number (see reference 32 for details)

Pr Prandtl number

A similar equation for heat transfer from two infinite vertical parallel plates (uniform in temperature) to a fluid being heated while flowing vertically upward (between the plates) in viscous motion with a parabolic velocity distribution at the entrance to the heating section, is also presented in reference 31. Based on this work, the following equation may be written, which may be applied to the analysis of heat transfer in narrow rectangular ducts (fig. 12a).

$$\frac{f_{c_x}\ \delta}{k} = 0.98 \sqrt[3]{59 + \frac{Wc_p}{kx}\frac{\delta}{B} + 0.153 \left(\frac{Gr\ Pr\ \delta}{x}\right)^{3/4}} \qquad (25b)$$

where all terms are as defined for equation (25a) except

δ smallest distance between sides of ducts, ft

B breadth of duct, ft

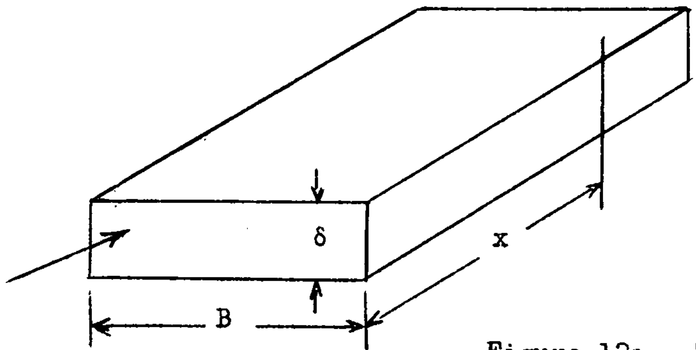


Figure 12a.- Diagram of narrow rectangular duct.

Simplified Equations

For use in design of air heat exchangers the following further approximations of equations (25a) and (25b) are presented:

For circular tubes:

$$f_{c_x} = 3.65 \frac{k}{D} \sqrt[3]{1 + \left[\frac{0.38\ W + 3500\ D^3\ \gamma^2\ \Delta t}{x}\right]} \qquad (25c)$$

For flat ducts:

$$f_{c_x} = 3.80 \frac{k}{\delta} \sqrt[3]{1 + \left[\frac{0.20\ W\left(\frac{\delta}{B}\right) + 3000\ \delta^3\ \gamma^2\ \Delta t}{x}\right]} \qquad (25d)$$

where

γ average weight density of air flowing through tube, lb/ft^3

Δt average temperature difference between air and tube walls, $^\circ$F

Equation (25c) is applicable for air flowing in round (or nearly round) tubes when the Reynolds number for the air flow is less than about 2000. Equation (25d) is applicable to air flowing viscously in flat ducts. The effect of free convection on the rate of heat transfer is approximately accounted for in the equations by the term involving the temperature difference Δt; the effect of free convection usually is small. The equations are most accurate for altitude pressures and for a temperature near 300° F, but may be used for higher temperatures (1000° F) for approximate values of f_{c_x}. Although equations (25c) and (25d) are based on an analysis of vertical tubes and plates with uniform temperatures and with a parabolic velocity distribution of the fluid at the entrance to the heating section, they may be used as a rough approximation for tubes and flat ducts with other orientations and with other entrance conditions. The equations are least accurate when the velocities due to free convection are in the opposite direction than those due to forced convection.

Example:

Air flows through a 1-inch inside diameter round tube 1.5 feet long under the following conditions:

Mean-air temperature	300° F
Air rate	7 lb/hr
Mean tube surface temperature . .	40° F
Absolute air pressure	10 lb/in²

Determine the variation of f_{c_x} with tube length:

(a) Calculation of the Reynolds number

$$Re = \frac{4}{\pi} \frac{W}{3600\, D\mu g}$$

$$= \frac{4 \times 7}{3.14 \times 3600 \times \frac{1}{12} \times 0.498 \times 10^{-6} \times 32.2} = 1850$$

Equation (25c) is applicable, because $Re < 2000$.

(b) Fluid Properties (appendix A, Properties of Air)

Air density $\gamma = \frac{P}{RT} = \frac{10 \times 144}{53.3 \times 760} = 0.0355 \frac{\text{lb}}{\text{ft}^3}$

Thermal conductivity $k = 2.05 \times 10^{-2} \dfrac{\text{Btu}}{\text{hr ft}^2 \left(\frac{^\circ\text{F}}{\text{ft}}\right)}$

Then from equation (25b)

$$f_{c_x} = 3.65 \times \frac{0.0205}{1/12} \sqrt[3]{1 + \left[\left(\frac{0.38 \times 7}{x}\right) + \left(\frac{3500 \times 0.0355^2 \ (300 - 40)}{12^3 \ x}\right) \right]}$$

$$= 0.875 \sqrt[3]{1 + \frac{3.3}{x}}$$

A plot of f_{c_x} versus x is shown in figure 12b.

x (ft)	$\frac{3.3}{x}$	$1 + \left(\frac{3.3}{x}\right)$	f_{c_x} (Btu/hr ft² °F)
0.05	66	67	3.55
.1	33	34	2.83
.2	16.5	17.5	2.26
.4	8.25	9.25	1.83
.8	4.13	5.13	1.51
1.5	2.2	3.2	1.29

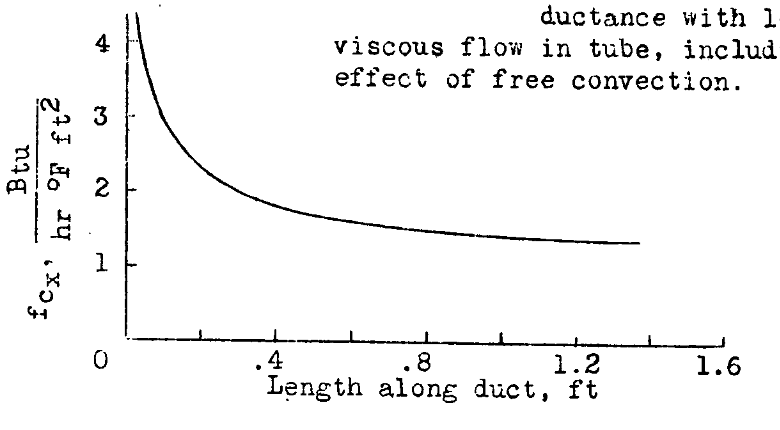


Figure 12b.- Variation of unit conductance with length for viscous flow in tube, including the effect of free convection.

The average f_c is obtained by integrating under the curve of f_{c_x} versus x and dividing by l. Thus, for round tubes,

$$f_{c_{av}} = 3.65 \frac{K}{D} \int_0^l \sqrt[3]{1 + \frac{c}{x}} \frac{dx}{l}$$

where

$$c = [0.38 W + 3500 D^3 \gamma^2 \Delta t]$$

To simplify the evaluation of $f_{c_{av}}$ for round tubes and ducts, a plot of

$$\lambda = \int_0^l \sqrt[3]{1 + \frac{c}{x}} \frac{dx}{l}$$

is given in figure 12c as a function of l/c.

Example: In the previous example, what is the average f_c along the tube?

$$c = 3.3$$

$$l = 1.5 \text{ ft}$$

$$\sqrt{\frac{l}{c}} = \sqrt{0.455} = 0.675$$

so

$$\lambda = 2.05 \quad \text{(fig. 12c)}$$

Thus

$$f_{c_{av}} = \frac{3.65 K}{D} \times \lambda = \frac{3.65 \times 0.0205}{\frac{1}{12}} \times 2.05 = 1.84 \frac{\text{Btu}}{\text{hr ft}^2 \text{ }^\circ\text{F}}$$

Transition Region:

As has been discussed, the laminar heat transfer equations presented in this section are applicable only below a Reynolds number of 2000, and the turbulent heat transfer equations are applicable only above a Reynolds number of 10,000. The region between Re = 2000 and Re = 10,000 cannot be described simply by either the laminar of turbulent equations, since in this region the fluid flows viscously in the initial portion of the tube and then breaks

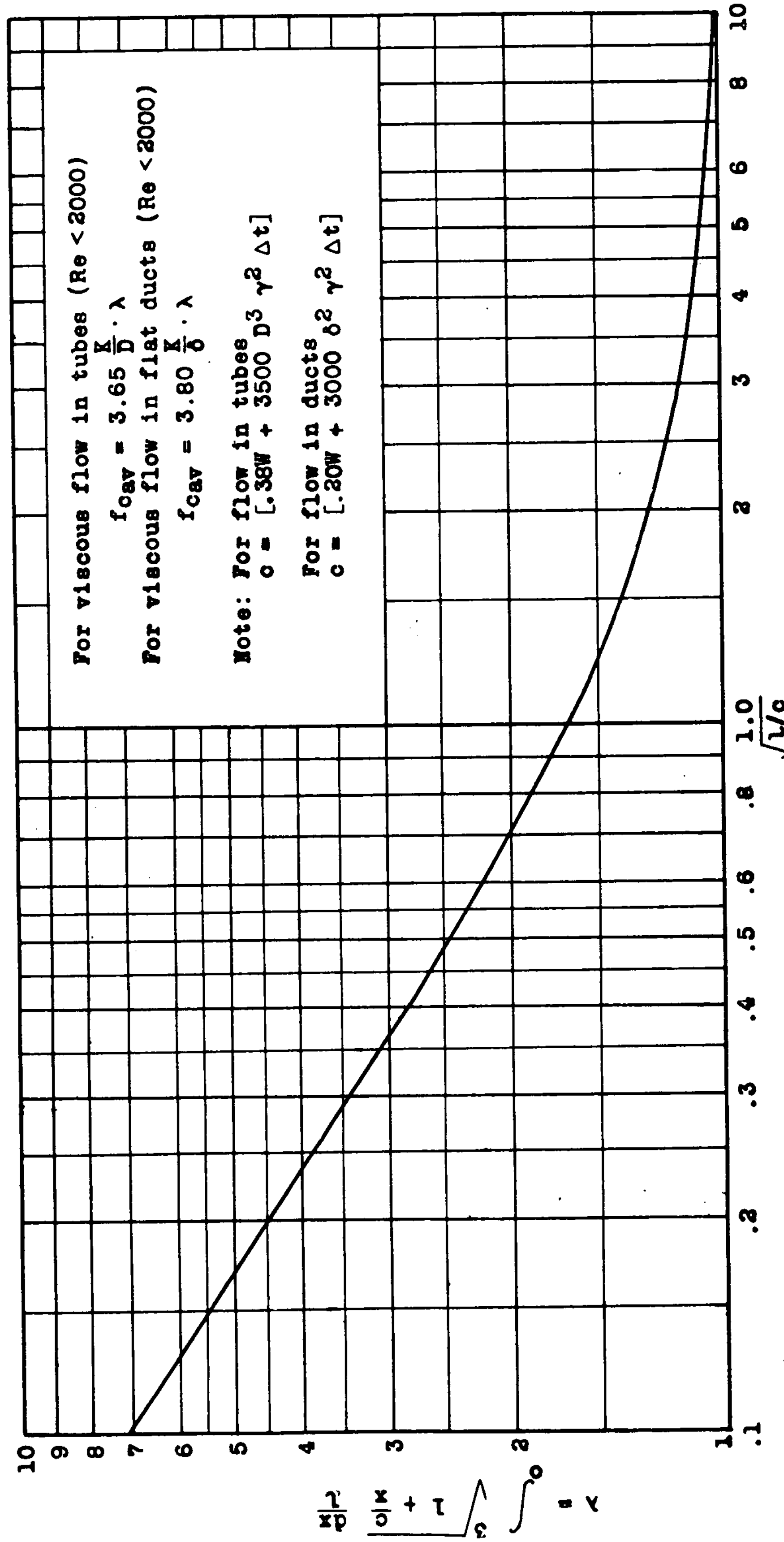


Figure 12c.- Function λ for use in viscous flow heat transfer equations to obtain average value of f_c.

into turbulent flow in the remainder. Figure 12d represents a typical variation of the point unit conductance along a tube in which air is flowing with a Reynolds number between 2000 and 10,000. The point of transition from viscous to turbulent flow must be known before the unit conductance in the transition region can be determined.

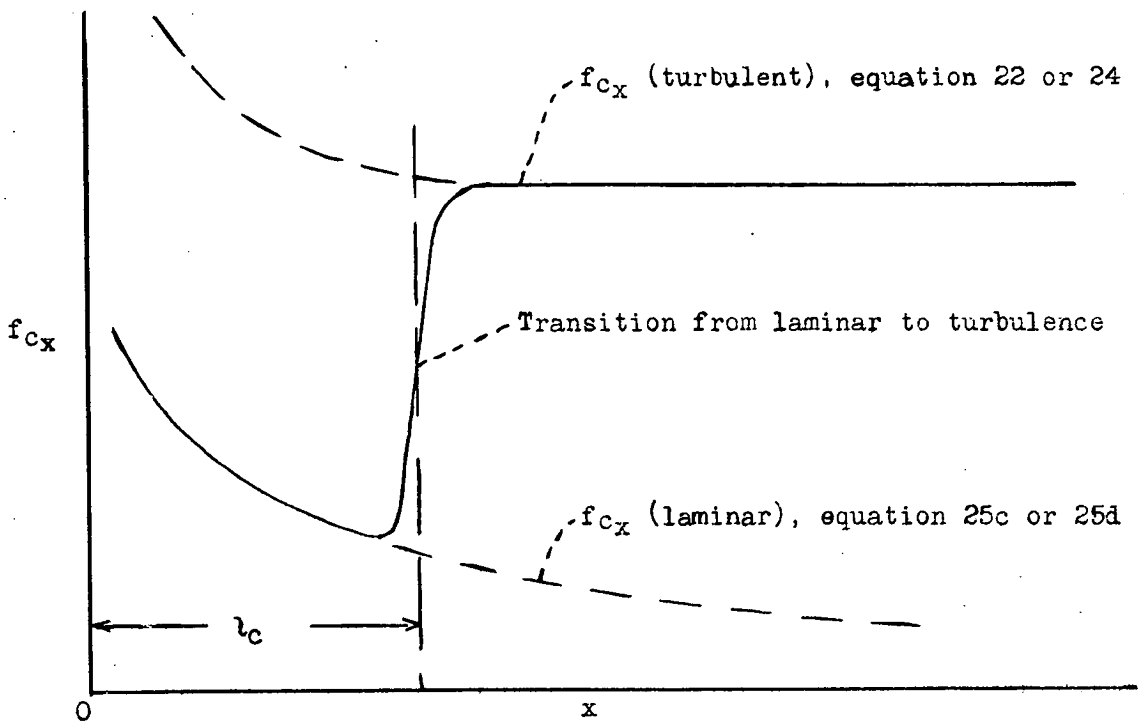


Figure 12d.- Typical distribution of f_{c_x} in duct for fluid flowing with Reynolds number between 2,000 and 10,000.

Some evidence exists that at each magnitude of the Reynolds number Re (based on tube diameter) transition may take place at definite magnitudes of (Re x c/D), but until this evidence is investigated further, the evaluation of the unit conductance in the transition region can be only a rough approximation.

D. FORCED CONVECTION ACROSS TUBES

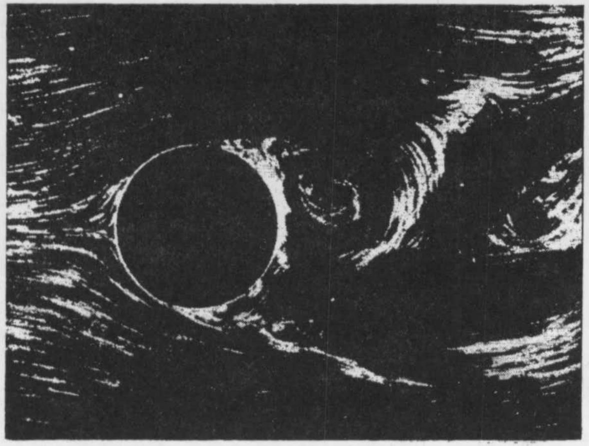
General

The flow across a single cylinder can be visualized as shown in figure 13. At low magnitudes of the Reynolds number (based on the outer diameter of the cylinder and the velocity in the free stream) a laminar boundary layer, starting at the front stagnation point, exists over the greatest portion of the cylinder. Because of the unfavorable pressure gradient existing around the cylinder (reference 14, p. 56), separation takes place about halfway between the front and rear stagnation points, as shown in figure 13.

The rate of heat transfer, therefore, is controlled by a laminar boundary layer on the forward portion of the cylinder (reference 14, p. 631), and on the rear portion by the turbulence existing in the wake of the cylinder The point values of f_c, measured by Schmidt and Wenner (reference 33) on the surface of a right circular cylinder, are shown in figure 13. The data reveal that the minimum rate of heat transfer exists on the sides of the cylinder. Because of the turbulence, at high values of the Reynolds number the unit conductance at the rear stagnation point may exceed that at the forward stagnation point.

In the discussion of heat transfer from flat plates (pt. I, sec. B) it was noted that for the laminar boundary layer, the unit thermal conductance varied with the 0.5 power or the product of velocity and density of the fluid $(u_\infty \gamma)^{0.5}$. For turbulent conditions, the unit conductance varied with the 0.8 power of this product. It is reasonable, therefore, that for flow across cylinders the unit thermal conductance should vary with a power of $(u_\infty \gamma)$ between 0.5 and 0.8. Experimentally, for a range of Reynolds numbers (based on tube diameter) between 1000 and 50,000, the unit conductance has been found to vary with about the 0.6 power of the product. (See reference 16, p. 222 of 2d ed.)

When several tubes are placed one behind the other, the turbulence produced by the first cylinder will increase the heat transfer from the second, and so forth. Thus it will be found that the rate of heat transfer from several banks of tubes is higher than for the same considered individually. (See reference 8.)



FLOW ABOUT A CYLINDER (reference 34)
Note viscous flow on forward half of cylinder and turbulent wake on rear half.

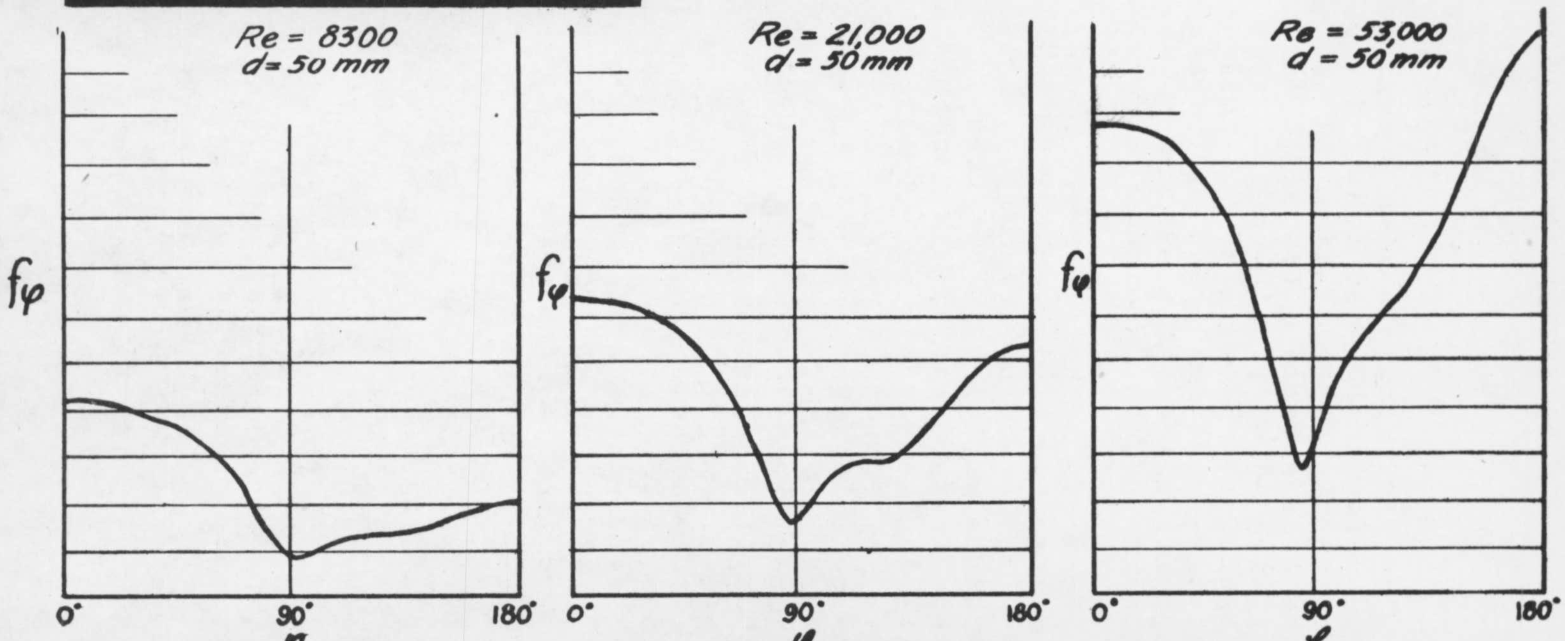


Fig 13 - Flow Phenomena and Variation of the Point Unit Thermal Conductance for Flow Across Cylinders

A summary of the equations to be used for heat transfer for air or exhaust gases flowing across single cylinders and tube banks, based on the data of references 8; 14, p. 631; 16, p. 222 of 2d ed.; and 33 is presented. These equations are applicable at any altitude pressure and for a temperature range of -60° to 1600° F.

Point Unit Conductances

(a) Unit conductances at front stagnation point of cylinder

$$f_{c_0} = 0.194\ T_f^{0.49}\left(\frac{u_\infty \gamma}{D}\right)^{0.50} \qquad (26)$$

(b) Point unit conductance on front portion of cylinder at any angle φ from 0° to 90°. The point of separation is assumed to be located at $\varphi = 90°$. (See reference 24.)

$$f_{c_\varphi} = 0.194\ T_f^{0.49}\left(\frac{u_\infty \gamma}{D}\right)^{0.5}\left(1 - \left|\frac{\varphi}{90}\right|^3\right) \qquad (27)$$

Average Unit Conductances

(a) Single Cylinder*

$$f_c = 0.211\ T_f^{0.43}\ \frac{(u_\infty \gamma)^{0.6}}{D^{0.4}} \qquad (28)$$

*Based on reference 16, p. 222 of 2d ed. for a range of Reynolds numbers of 1000 to 50,000.

(b) Banks of Tubes* (in line)

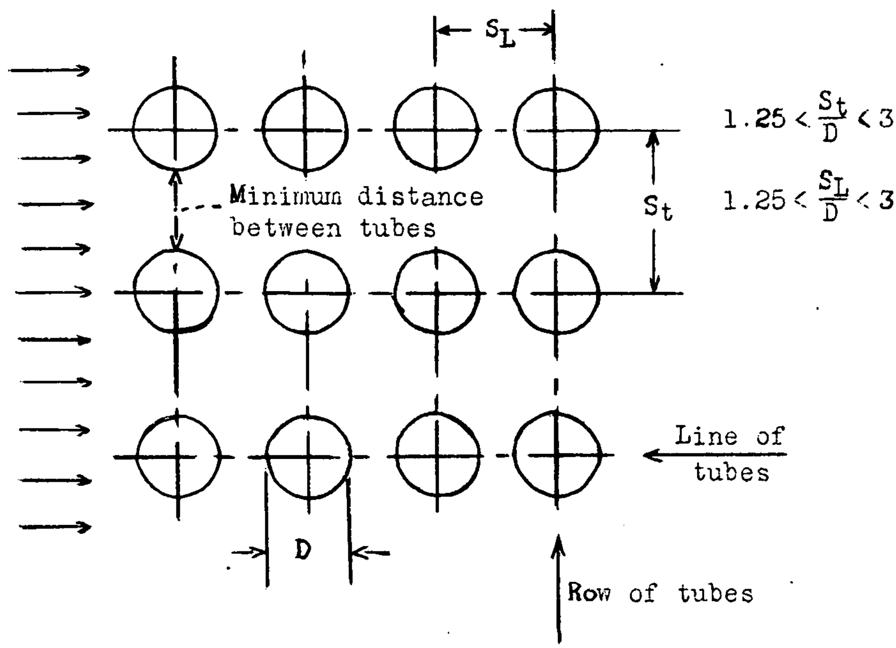


Figure 14.- Typical in-line tube bank. Number of rows, 4. Maximum G_0 based on minimum distance between tubes as shown in the figure.

$$f_c = 14.5 \times 10^{-4} F_a T_f^{0.43} \frac{G_0^{0.6}}{D^{0.4}} \tag{29}$$

*Based on the data in reference 8 for a Reynolds number (based on the tube diameter) of 20,000. For Re of 20,000 or greater, the tube arrangement apparently has little effect on f_c and equation (28) will yield results well within 10 percent of those in reference 8. For Re less than 20,000, however, the tube arrangement becomes more important, f_c becoming lower as S_L is decreased for a fixed value of S_t. In most aircraft heater designs Re is in the vicinity of 20,000. If a design is contemplated with Re less than 15,000, p. 590 of reference 8 should be consulted. Note that the definition of F_a in reference 8 differs from that in this report.

TABLE I.- TUBE ARRANGEMENT MODULUS FOR IN-LINE TUBE BANKS

Number of rows	1	2	3	4	5	6	7	8	9	10 or more
F_a	1.00	1.10	1.17	1.24	1.29	1.34	1.37	1.40	1.42	1.43

(c) Banks of Tubes* (staggered)

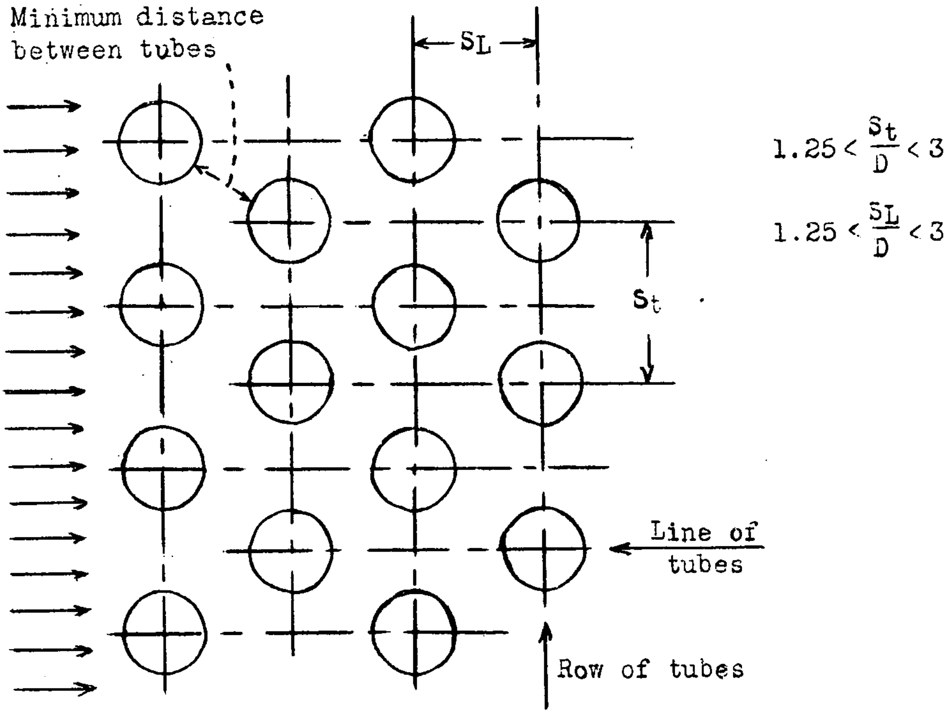


Figure 15.- Typical staggered tube bank. Number of rows, 4. Maximum G_o based on minimum distance between tubes. The position of the minimum distance varies with tube bank geometry.

$$f_c = 14.5 \times 10^{-4} \; F_a \; T_f^{\;0.43} \; \frac{G_o^{\;0.6}}{D^{0.4}} \tag{29a}$$

*Based on the data in reference 8, for a Reynolds number of 20,000.

TABLE II.- TUBE ARRANGEMENT MODULUS FOR STAGGERED TUBE BANKS

Number of rows	1	2	3	4	5	6	7	8	9	10 or more
F_a	1.00	1.11	1.23	1.31	1.39	1.45	1.48	1.51	1.53	1.54

In the foregoing equations,

f_{c_o} unit thermal conductance at stagnation point of cylinder, Btu/hr ft^2 $^\circ$F

f_{c_φ} point unit conductance at any angle φ from stagnation point (up to point of separation, $0 < \varphi < 90^\circ$), Btu/hr ft^2 $^\circ$F

f_c average unit conductance for any number of rows of cylinders, Btu/hr ft^2 $^\circ$F

φ angle from forward stagnation point, degrees. (φ must be less than 90°.)

T_f arithmetic average of tube wall absolute temperature and mixed-mean absolute air temperature, $^\circ$R

u_x velocity in free air stream, ft/sec

γ density of air at temperature T_f, lb/ft^3

D outer diameter of tube, ft

G_o maximum weight rate per unit area for flow past tube banks (W/A), lb/ft^2 hr

W weight rate of air, lb/hr

A minimum free area in flow path of air through tube banks, ft^2

F_a "tube arrangement modulus" for in-line and staggered tube banks, which is mainly a function of the number of banks of tubes in the direction of air flow. (See table below each equation.) The magnitude of the modulus F_a reveals the increase in the unit thermal conductance for multi-row banks with reference to single-row banks, due to the increased turbulence in the air stream.

Example

What is the average unit conductance for the following bank of tubes?

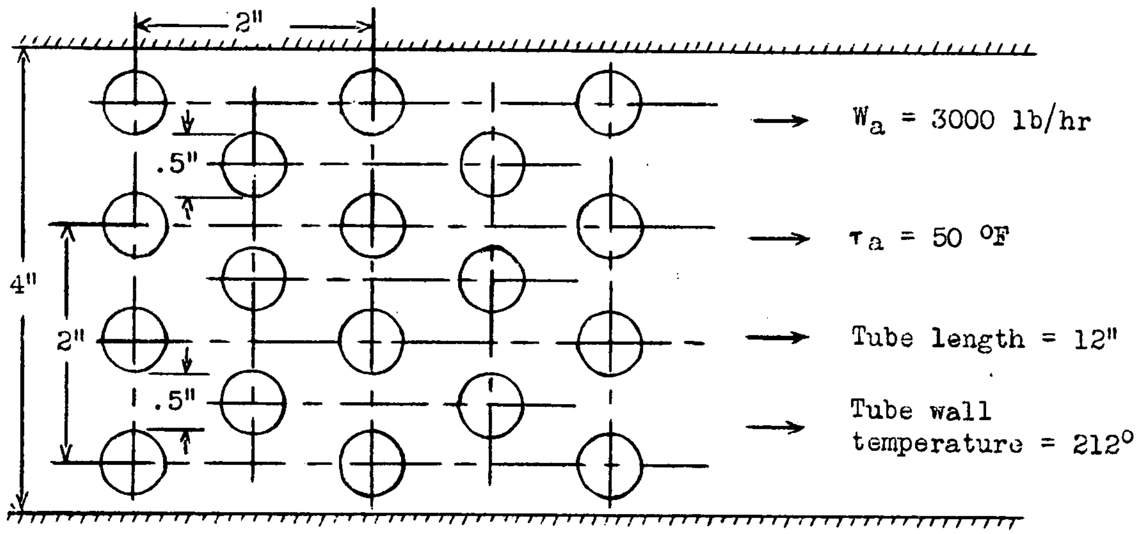


Figure 16.- Staggered bank of tubes.

$$\text{Minimum free area} = (4 \times 12) - \left(4 \times \frac{1}{2} \times 12\right) = 24 \text{ sq in.}$$

$$= 0.168 \text{ ft}^2$$

$$G_o = \frac{3000}{0.168} = 18{,}000 \text{ lb/hr ft}^2$$

$$F_a = 1.39 \text{ (table II, for staggered tube banks)}$$

$$\text{Average "film" temperature} = T_f = \left(\frac{212 + 50}{2}\right) + 460 = 591^\circ \text{ R}$$

$$D = 0.5 \text{ in.} = 0.0417 \text{ ft}$$

Thus, the average unit conductance for the tube banks is:

$$f_c = 14.5 \times 10^{-4} \times F_a \times T_f^{0.43} \times \frac{G_o^{0.6}}{D^{0.4}}$$

$$= 14.5 \times 10^{-4} \times 1.39 \times (591)^{0.43} \frac{(18000)^{0.6}}{(0.0417)^{0.4}}$$

$$= 40.0 \text{ Btu/hr ft}^2 \text{ }^{\circ}\text{F}$$

E. FORCED CONVECTION ALONG AIRFOIL SHAPES

General

As air flows over an airfoil, a laminar boundary layer is initiated at the stagnation point and increases in thickness as the fluid proceeds. (See reference 14, p. 466.) At some point along the airfoil, depending largely upon the turbulence in the free air stream and on the airfoil design and surface finish, the laminar boundary layer will change into a turbulent boundary layer. (See references 14, p. 482; 35 and 36.) The growth of both the laminar and turbulent boundary layers is greatly affected by the pressure gradient along the airfoil. Except for the uncertainty concerning the point of transition between the laminar and turbulent boundary layers, it is a relatively simple task to predict the boundary-layer behavior around the airfoil. (See references 14, p. 156; 37, and 38.) An analytical prediction of the unit thermal conductance at a surface along which there exists a variable pressure gradient is, however, extremely difficult. See references 39, 40, and 41 for presentation of several analytical techniques.* An approximate solution is suggested in the following paragraphs. (See reference 24.)

Along Leading Edge

The boundary-layer behavior near the stagnation point approaches that which occurs around a cylinder with a radius equal to the "equivalent radius** of curvature" of the airfoil leading edge. There is a difference in the behavior of

---

*A comparison of these techniques is presented in a summary report soon to be published.

**See example under sec, E.

the boundary layer over the actual airfoil as compared to that over the equivalent cylinder, of course, owing to the difference in the pressure distributions in the two cases. As a first approximation, however, the leading edge of an airfoil may be considered as the front half of a right circular cylinder and the equations already presented for this case may be applied directly to evaluate the unit thermal conductance at the airfoil leading edge. (See pt. I. sec. D.)

Along Remainder of Airfoil

As a first approximation the remainder of the airfoil, other than the leading edge, may be considered as an equivalent flat plate. The velocity along this equivalent flat plate is assumed to vary in the same manner as the velocity at the edge of the boundary layer of the airfoil. This velocity variation is easily obtained from the pressure distribution about the airfoil.* The origin of the equivalent

---

*In order to obtain the velocity u_1 at the edge of the boundary layer the following equations are utilized (reference 42):

From the Bernoulli equation, neglecting differences in elevation,

$$\frac{P_1}{\gamma} + \frac{u_1^2}{2g} = \frac{P_\infty}{\gamma} + \frac{u_\infty^2}{2g} \qquad (30)$$

where

P_1 static pressure at any point on the airfoil, lb/ft^2

γ density of air in free air stream, lb/cu ft

u_1 velocity at edge of boundary layer, ft/sec

P_∞ free stream static pressure, lb/ft^2

u_∞ free-stream velocity, ft/sec

g gravitational force per unit mass, 32.2 $\text{lb}\Big/\left(\frac{\text{lb sec}^2}{\text{ft}}\right)$

Thus

$$u_1 = u_\infty\sqrt{1 - \frac{2g(P_1 - P_\infty)}{u_\infty^2\gamma}} \qquad (31)$$

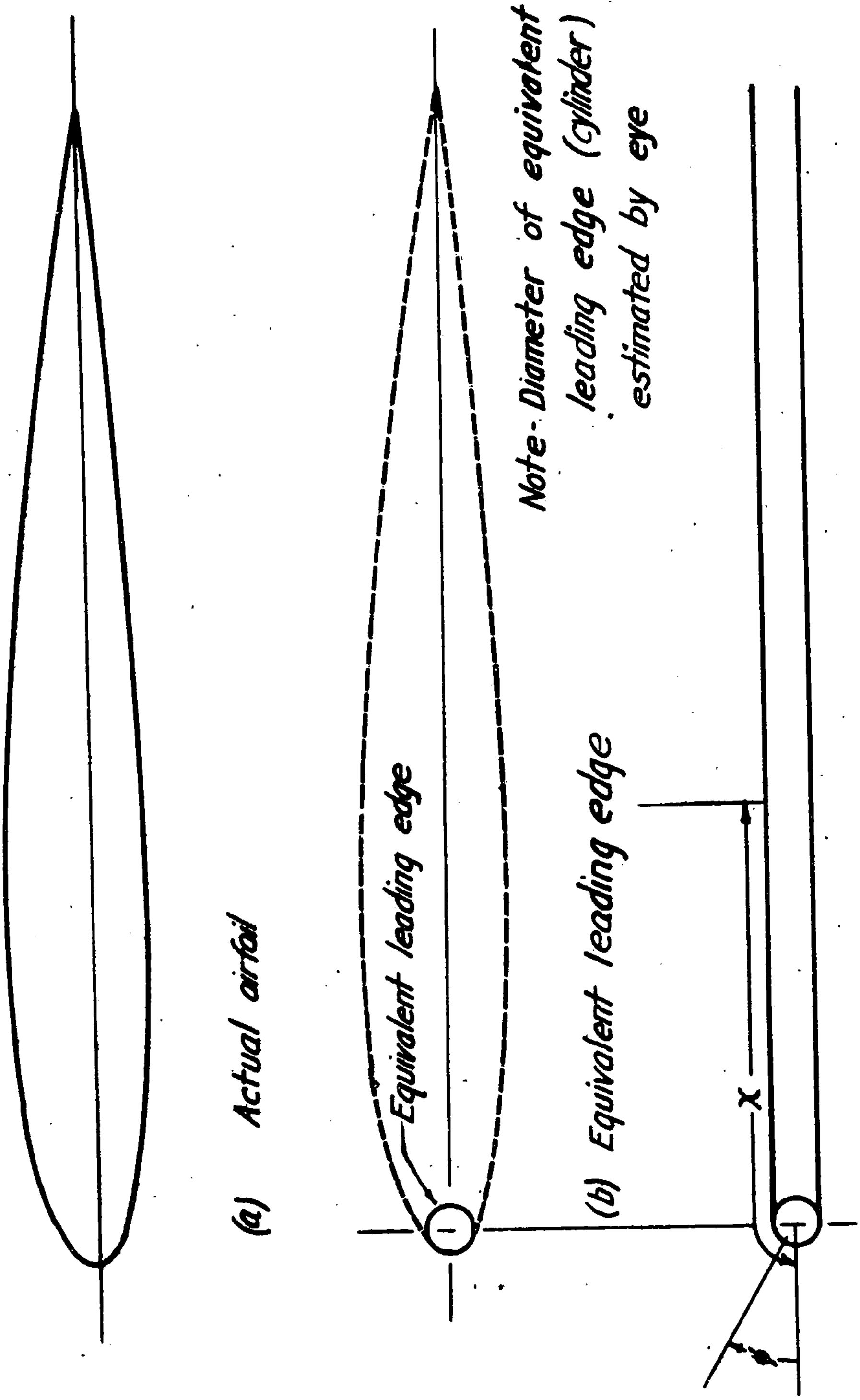


Fig. 17.- Sketch of Equivalent Airfoil.

flat plate is assumed to be the stagnation point of the airfoil. The unit thermal conductance at any point x feet from the stagnation point then may be calculated from the equations for the point unit conductance along a flat plate presented in an earlier section. (See pt. I, sec. B.) The exact point of transition from a laminar boundary layer to a turbulent boundary layer still remains in doubt.

Recapitulation

As shown in figure 17, in order to determine approximately the variation of unit conductance over an actual airfoil, an equivalent airfoil made up of a cylinder and flat plate may be visualized. Application to the equivalent airfoil of the equations for the variation of unit conductance over cylinders and along flat plates then will yield the desired estimate of the point unit conductance. The actual velocity variations over the airfoil, based on the pressure distribution, are to be utilized in calculating the point unit conductance.*

Summary of Equations

(a) Leading Edge (angle φ measured from stagnation point)

$$f_{c_\varphi} = 0.194\ T_f^{0.49}\left(\frac{u_\infty\gamma}{D}\right)^{0.50}\left(1-\left|\frac{\varphi}{90}\right|^3\right) \tag{32}$$

(b) Laminar Boundary Layer (beyond leading edge) (x = 0 at stagnation point)

$$f_{c_x} = 0.0562\ T_f^{0.50}\left(\frac{u_1\gamma}{x}\right)^{0.50} \tag{33}$$

(c) Turbulent Boundary Layer (beyond leading edge) (x = 0 at stagnation point)

$$f_{c_x} = 0.51\ T_f^{0.3}\ \frac{(u_1\gamma)^{0.8}}{x^{0.2}} \tag{34}$$

*See preceding footnote.

where

f_{c_φ} point unit conductance at leading edge of airfoil $(0 < \varphi < 90^\circ)$ Btu/hr ft^2 $^\circ$F

T_f arithmetic average of temperature of airfoil surface and free stream air temperature, $^\circ$R

u_∞ free stream air velocity, ft/sec

γ weight density of air at temperature T_f and pressure at given altitude, lb/ft^3

D diameter of "equivalent" cylinder at leading edge of airfoil, ft

f_{c_x} point unit thermal conductance at point x beyond leading edge, Btu/hr ft^2 $^\circ$F

u_1 velocity at edge of boundary layer at any point x along airfoil (obtained from static pressure distribution over airfoil), ft/sec

x distance along airfoil surface measured from stagnation point, ft

Example

Calculate the point unit conductance along an NACA 23012 airfoil with a chord of 6 feet for the following conditions:

True airspeed	300 mph
Altitude	15,000 ft
Wing surface temperature	32° F
Air temperature	5° F
Angle of attack	8°

A sketch of the airfoil is shown in figure 17. The equivalent airfoil also is shown in this figure. By eye, the equivalent cylinder which best fitted the leading edge was found to have a diameter of approximately 0.282 foot. The following equation now may be applied:

(a) Leading Edge

$$f_{c_\varphi} = 0.194\, T_f^{0.49} \left(\frac{u_\infty \gamma}{D}\right)^{0.5} \left[1 - \left|\frac{\varphi}{90}\right|^3\right] \qquad \text{(a)}$$

(b) Laminar Boundary Layer

$$f_{c_x} = 0.0562\ T_f^{0.50} \left(\frac{u_1 \gamma}{x}\right)^{0.5} \qquad \text{(b)}$$

(c) Turbulent Boundary Layer

$$f_{c_x} = 0.51\ T_f^{0.3} \frac{(u_1 \gamma)^{0.8}}{x^{0.2}} \qquad \text{(c)}$$

The mean temperature $T_f = \frac{5 + 32}{2} + 460^\circ = 478^\circ$ R

The air density γ at 15,000-foot altitude is approximately 0.048 pounds per cubic foot. (See appendix B.) The velocity u_∞ equals 300 miles per hour equals 440 feet per second. Thus the foregoing equations reduce to

$$f_{c_\varphi} = 34.4 \left[1 - \left|\frac{\varphi}{90}\right|^3\right] \qquad \text{(Leading Edge)} \qquad \text{(d)}$$

$$f_{c_x} = 0.270 \left(\frac{u_1}{x}\right)^{1/2} \qquad \text{(Laminar Boundary Layer)} \qquad \text{(e)}$$

$$f_{c_x} = 0.286 \left(\frac{u_1}{x^{0.25}}\right)^{0.8} \qquad \text{(Turbulent Boundary Layer)} \qquad \text{(f)}$$

In order to evaluate the velocity u_1, at the edge of the boundary layer, equation (30) is utilized. Thus:

$$u_1 = u_\infty \sqrt{1 - \frac{2g(P_1 - P_\infty)}{\gamma u_\infty^2}} \qquad \text{(g)}$$

The pressure distribution about the NACA 23012 airfoil, obtained from reference 43, is shown in figure 18. From this curve and equation (g) the velocity u_1 at any point x on the lower and upper surface of the airfoil can be readily determined. Substitution of these magnitudes of u_1 in equations (e) and (f) allows the prediction of f_c at any x for either the laminar or turbulent boundary layers. The resulting values of f_{c_x} are plotted in figure 19.

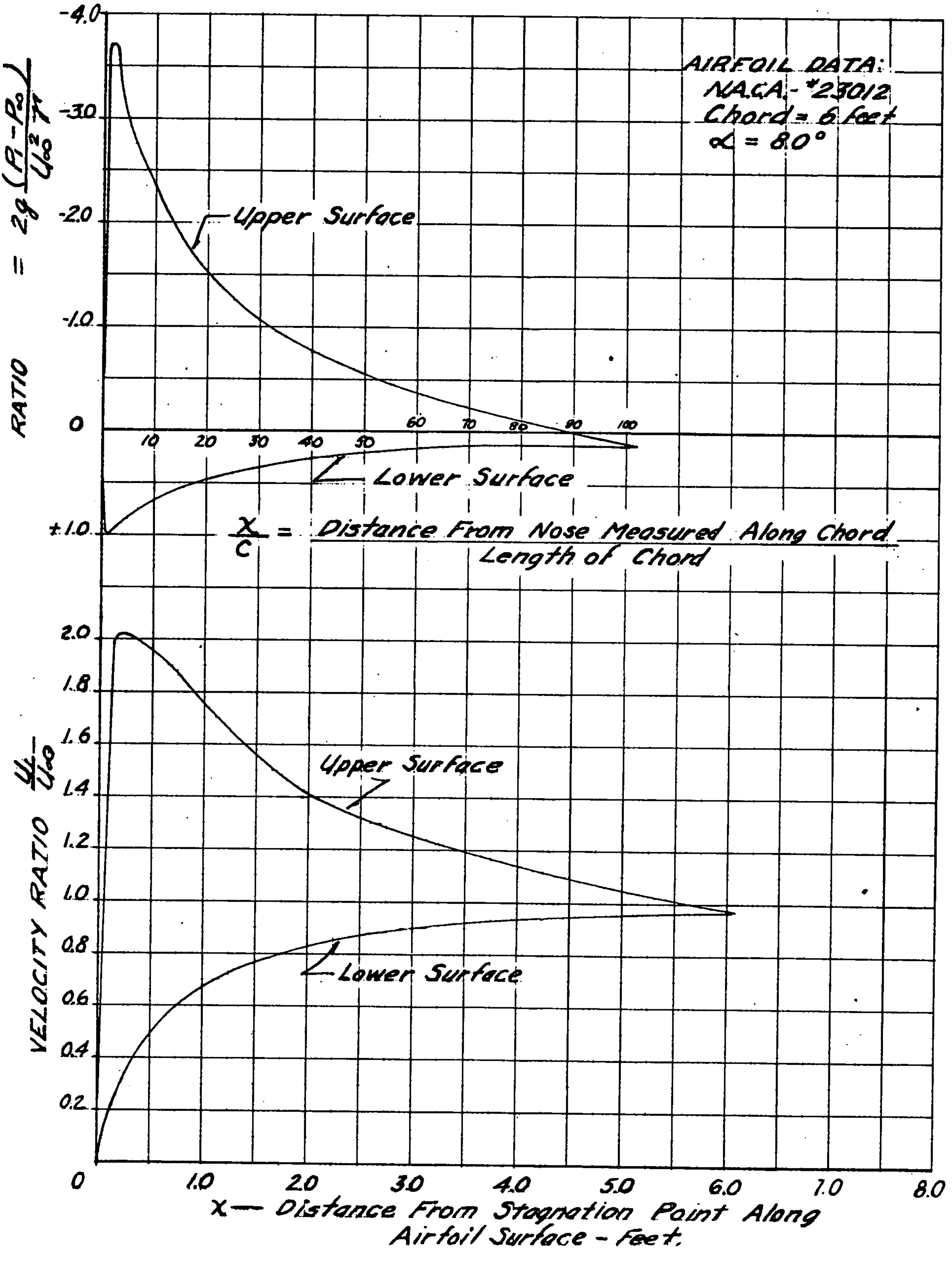


Fig. 18.- Data for N.A.C.A. #23012 Airfoil.

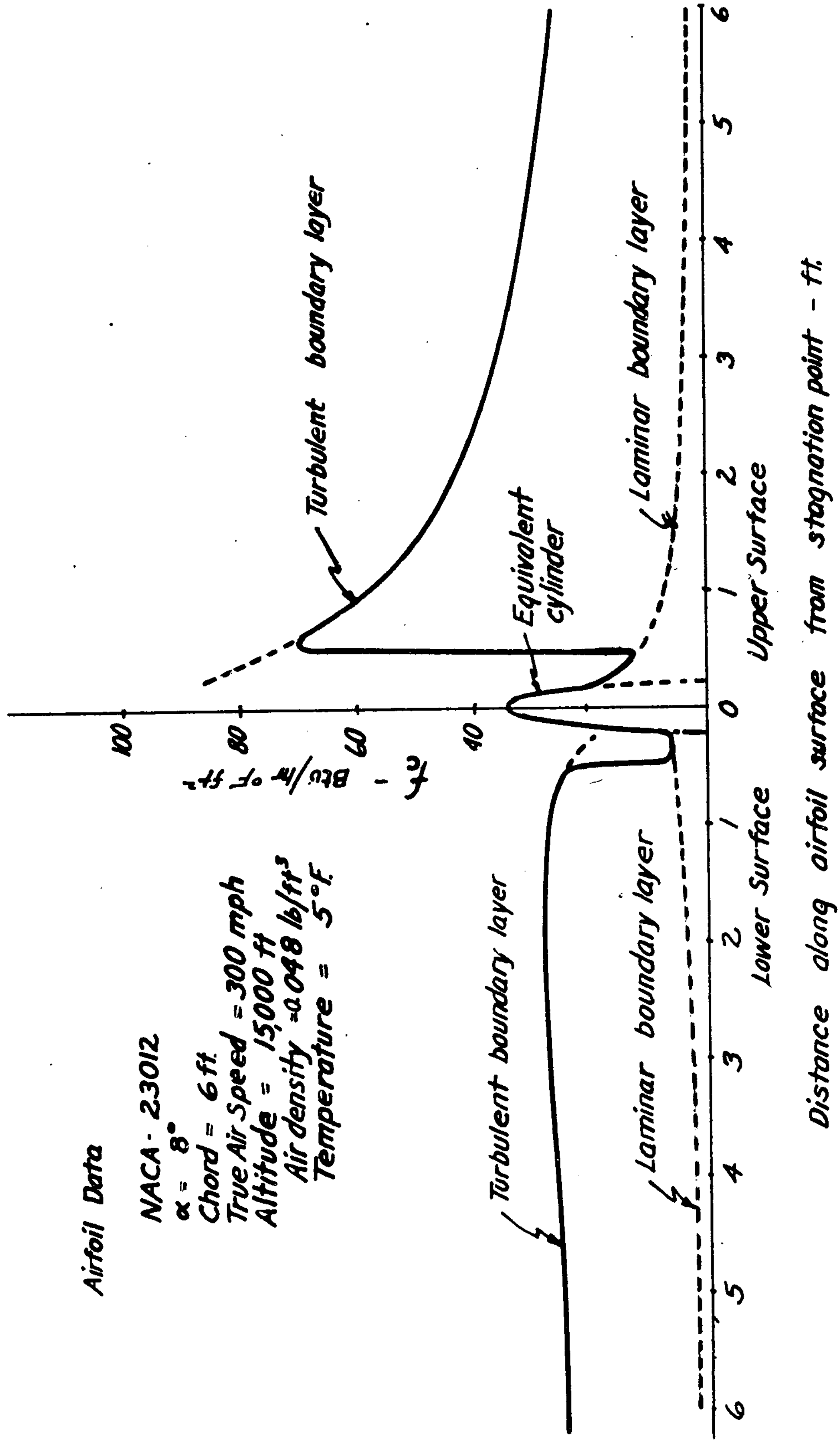


Fig. 19 - UNIT THERMAL CONDUCTANCE ALONG AN AIRFOIL

The point of transition between laminar and turbulent boundary layers is not well known, and this lack of knowledge limits the effectiveness of the prediction. In figure 19 the point of transition was arbitrarily chosen at $x = 0.5$ foot on both the upper and lower surfaces in order to show how f_c may vary under certain conditions. In any actual design, information concerning the location of transition must be available in order to establish the variation of f_c with x.

F. FINNED SURFACES

General

Fins, placed on a heat exchange surface, increase the rate of heat transfer by increasing the "effective" heat transfer area. When fins are used, they should be placed on the side of the exchange surface on which the thermal resistance between the fluid and the surface is highest. Fins will have a negligible effect on the over-all conductance if placed on the side of the heat exchange surface having the lowest thermal resistance. (See reference 44.)

Simplified equations for the performance of fins are shown. The general equation for the effective total conductance of the finned surface is (reference 45):

$$(fA)_e = \left[n\sqrt{f_F PkA}\ \tanh\sqrt{\frac{f_F PL^2}{kA}} \right] + f_u A_u \qquad (35)$$

where

$(fA)_e$ effective total conductance for finned surface, (Btu/hr)/°F temperature difference between base of fin and fluid flowing over fins

n number of fins

f_F unit thermal conductance along fin, Btu/hr ft^2 °F. The magnitude of unit conductance f_F for fins may be determined by use of data in part I, sections A, B, C, D, E, and G of this report. When radiant and convective heat transfer occur simultaneously to the fin, $f_F = (f_c + f_r)$ (Pt. I, sec. A.)

L length of fin projecting into fluid stream, ft

A cross-sectional area of fin perpendicular to direction of heat flow along fin, ft^2

P perimeter of fin measured parallel to base of fin, ft

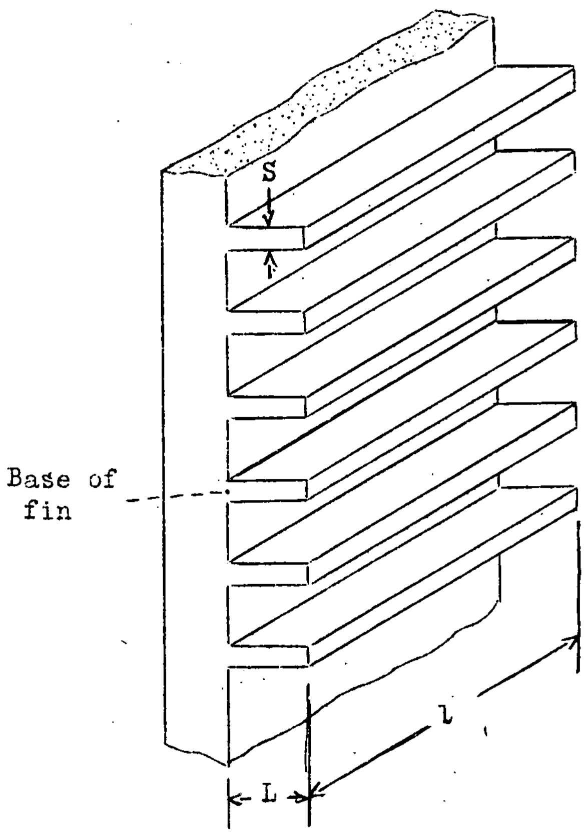
k thermal conductivity of fin material, Btu/hr ft^2 (°F/ft)

f_u unit thermal conductance along unfinned surface, Btu/hr ft^2 °F. The unit thermal conductance f_u may be determined by use of the data in part I, sections A, B, C, D, E, and G of this report.

A_u surface area not covered by fins, ft^2

Three particular forms are often met in practice:

1. Rectangular Fins



Fin perimeter, $P = 2(s+l) \cong 2l$

Fin cross-sectional area, $A = sl$

Figure 20.- Rectangular fins.

Thus from equation (35)

$$(fA)_e = \left[n l \sqrt{2skf_F} \tanh \sqrt{\frac{2f_F L^2}{ks}} \right] + f_u A_u \qquad (36)$$

where

s thickness of fin, ft

l length of fin, as shown in figure 20, ft

2. Cylindrical Fins*

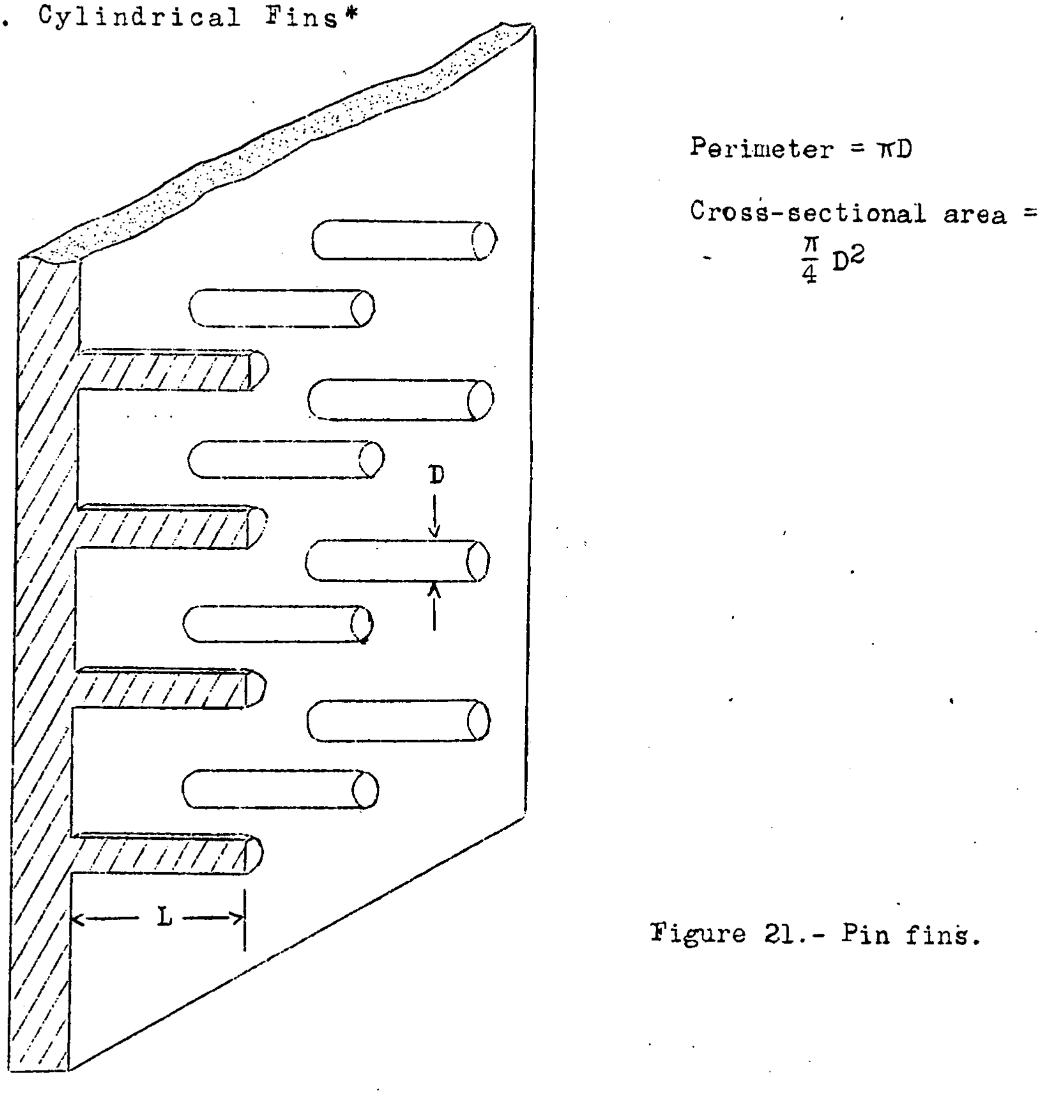


Figure 21.- Pin fins.

Thus, from equation (35):

$$(fA)_e = \left[\frac{\pi n D}{2} \sqrt{Dkf_F} \tanh \sqrt{\frac{4f_F L^2}{kD}} \right] + f_u A_u \qquad (37)$$

where

D outer diameter of fins, ft

3. Annular Fins **(reference 46)**

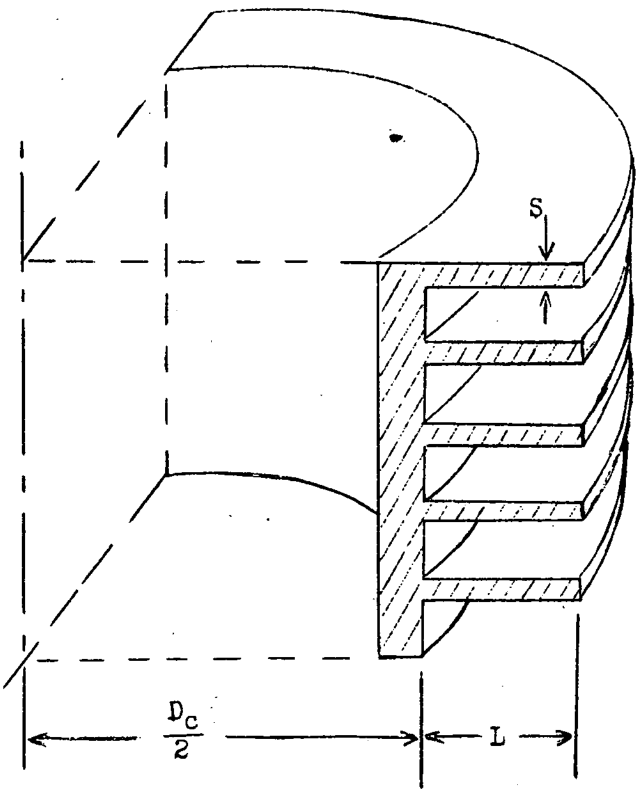


Figure 22.- Annular fins.

$$(fA)_e = \left[\pi D_c n \sqrt{2f_F ks} \left(1 + \frac{L}{D_c}\right) \tanh \sqrt{\frac{2f_F L^2}{ks}} \right] + f_u A_u \qquad (38)$$

where

D_c outer diameter of cylinder to which fins are attached, ft

Example

Determine the effective total conductance $(fA)_e$ for the following aluminum finned surface.

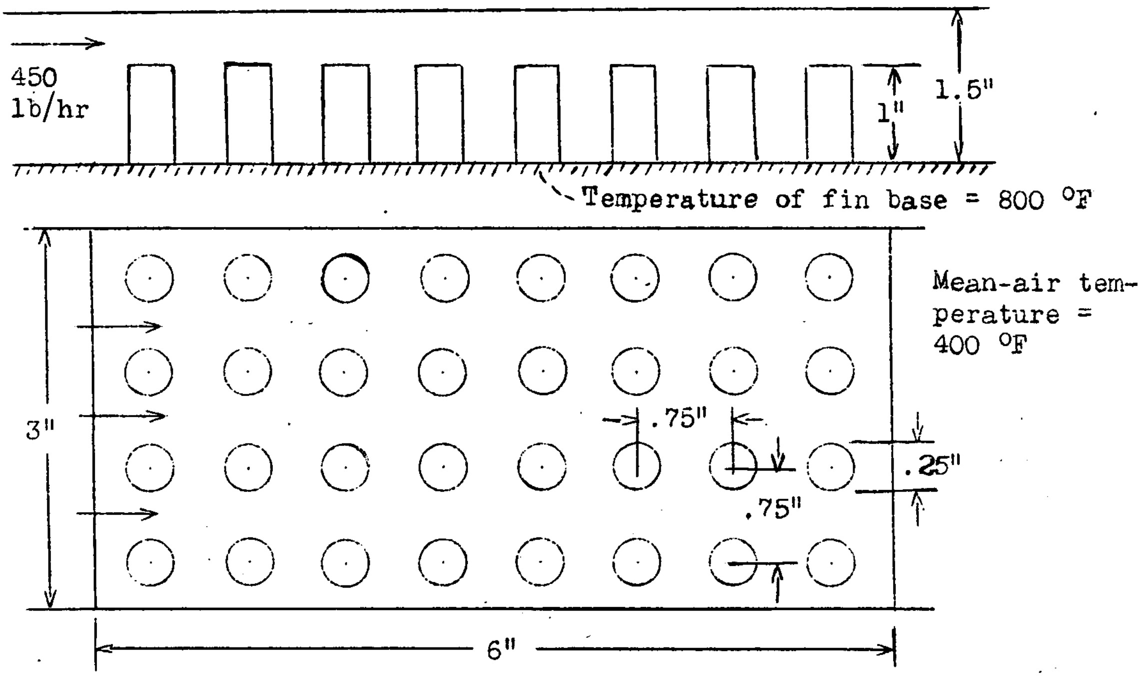


Figure 23.- Pin fins.

$$(fA)_e = \left[\frac{n\pi D}{2}\sqrt{Dkf_F}\ \tanh\sqrt{\frac{4f_F L^2}{kD}}\right] + f_u A_u$$

Inspection of the figure reveals that a reasonable procedure for the evaluation of f_F is to consider the fins as a bank of tubes. The equations presented in part I, section D for tube banks then may be utilized to predict an average f_F.

$$f_F = 14.5 \times 10^{-4}\ F_a\ T_f^{0.43}\ \frac{G_o^{0.6}}{D^{0.4}}$$

From table I

$$F_a = 1.40$$

The film temperature, $T_f = \frac{400 + 800}{2} + 460 = 1060^\circ$ R (neglecting temperature drop along fin as a first approximation*). The minimum cross-sectional area for air flow is:

$$A = \frac{1.5 \times 3 - 4 \times 0.25 \times 1}{144} = 0.0243 \text{ ft}^2$$

Maximum weight rate per unit area

$$G_0 = \frac{450}{0.0243} = 18,500 \frac{\text{lb}}{\text{hr ft}^2}$$

$$D = 0.25 \text{ in.} = 0.0208 \text{ ft}$$

Thus

$$f_F = 14.5 \times 10^{-4} \times 1.40 \times (1060)^{0.43} \times \frac{(18500)^{0.6}}{(0.0208)^{0.4}} = 69.4 \frac{\text{Btu}}{\text{hr ft}^2 \ {}^\circ\text{F}}$$

The exact evaluation of f_u, the unit thermal conductance along the unfinned area, is difficult. As a first approximation the f_u may be calculated by considering the unfinned surface as a section of a short tube (pt. I, sec. C). Then:

$$f_a = 9.1 \times 10^{-4}\ T^{0.3} \left(\frac{G^{0.8}}{l^{0.2}}\right)$$

Fluid temperature = $T = 400 + 460 = 860^\circ$ R

Weight rate per unit area = G = 18,500 lb/hr ft^2

Tube length = l = 0.50 ft

$$f_u = 9.1 \times 10^{-4} \frac{(860)^{0.3}\ (18500)^{0.8}}{(0.50)^{0.2}} = 20.7 \frac{\text{Btu}}{\text{hr ft}^2 \ {}^\circ\text{F}}$$

The unfinned area, $A_u = \frac{3 \times 6}{144} - \frac{\pi}{4} \times 32 \times \frac{0.25^2}{144} = 0.114 \text{ ft}^2$

Number of fins, n = 32

*See reference 12, ch. II and reference 16, p. 232 of 2d ed. for temperature distribution along fin.

Diameter of fins $D = 0.25$ in. $= 0.0208$ ft

Thermal conductivity of fin material, $k = 133 \dfrac{\text{Btu}}{\text{hr ft}^2 \left(\frac{^\circ\text{F}}{\text{ft}}\right)}$

Length of fin = 1 in. = 0.0833 ft

The term $\sqrt{\dfrac{4f_F L^2}{kD}} = \sqrt{\dfrac{4 \times 69.4 \times 0.0833^2}{133 \times 0.0208}} = 0.835$ (dimensionless)

Then

$$(fA)_e = \left[\frac{32.2 \times 3.14 \times 0.0208}{2}\sqrt{0.0208 \times 133 \times 69.4}\right] \tanh 0.835 + 20.7 \times 0.114$$

$$= 14.5 \tanh 0.835 + 2.36$$

$$= 9.90 + 2.36 = 12.26 \text{ Btu/hr } ^\circ\text{F}$$

The fins, in this example, increase the effectiveness of the heat transfer surface appreciably. The heat transfer from the ends and radiant heat transfer between the fins are neglected here. A method of accounting for the heat transfer from the fin ends is presented in reference 47.

The heat transfer rate, as computed herein, is slightly overestimated because the mean velocity is not uniform across the section. See references 48 and 49 for experimental data concerning heat transfer, and velocity and temperature distribution between longitudinal fins.

If the rate of heat flow from the finned surface is desired, the value of $(fA)_e$ must be multiplied by the mean temperature difference between the fin base and the air flowing over the fins. The temperature distribution along the fin need not be considered.

G. RADIATION

General

In an earlier section of this report (pt. I, sec. A) it was pointed out that heat transfer by radiation usually accompanies convective heat transfer, the two processes acting in parallel. The radiant heat transfer, since it varies with the fourth power of the absolute temperature of the radiating body, becomes of great importance when heat transfer from bodies at high temperatures is being studied. In some heater designs therefore, since the heat transfer surfaces may attain temperatures in the neighborhood of 1000° F, radiant heat transfer may play an important role.

The heat transfer by radiation between two surfaces with temperatures T_1 and T_2 may be calculated from equation (11)

$$q_r = 0.173\ A_r F_{AE} \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4 \right] \qquad (39)$$

The modulus F_{AE}, as discussed under part I, section A, modifies the equation for radiation between Planckian radiators (black bodies) to account for the emissivities and geometry of the radiating surfaces. The modulus F_{AE} is a function of the areas of the surfaces (A_1 and A_2), the corresponding emissivities ϵ_1 and ϵ_2 of the two surfaces and the relative geometry of the system. Values of F_{AE} for the most common systems met in practice are given in table III. Other values for more complex systems may be found in references 16, pp. 54-60 of 1st ed.; 19; and 20.

TABLE III.- VALUES OF MODULUS F_{AE}

System	Modulus F_{AE}	Area to be used in equation (8)
1. Two infinite parallel plates	$\dfrac{1}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1}$	$A_r = A_1$ or A_2
2. Completely enclosed small body (1 refers to enclosed body)	1	$A_r = A_1$
3. Concentric spheres or infinite concentric cylinders (1 refers to enclosed body)	$\dfrac{1}{\dfrac{1}{\epsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)}$	$A_r = A_1$

In table III,

A_1 surface area of smaller body, ft^2

A_2 surface area of surrounding body, ft^2

ϵ_1 emissivity of smaller body (dimensionless)

ϵ_2 emissivity of surrounding body (dimensionless)

Values of surface emissivities ϵ for various materials may be found in references 16, pp. 54-60 of 1st ed.; 19, 50, and 51.

In equation (8), A_1 and A_2 are the surface areas of the smaller and larger bodies, respectively, as long as the surfaces do not contain re-entrant angles. In cases where the surfaces contain re-entrant angles, the area A_1 or A_2 no longer signifies the surface area, and equation (8) is not exactly applicable. As an approximation, however, an effective "projected" area can be used for A_1 or A_2. For example, if the radiation between the surfaces in figure 24 is required, the effective projected area can be used in equation (8) rather than the actual area of the inner surface. The effective emissivity of the surface A_r is higher than the actual emissivity of the metal because of the indentations. For an exact treatment of this case see reference 19.

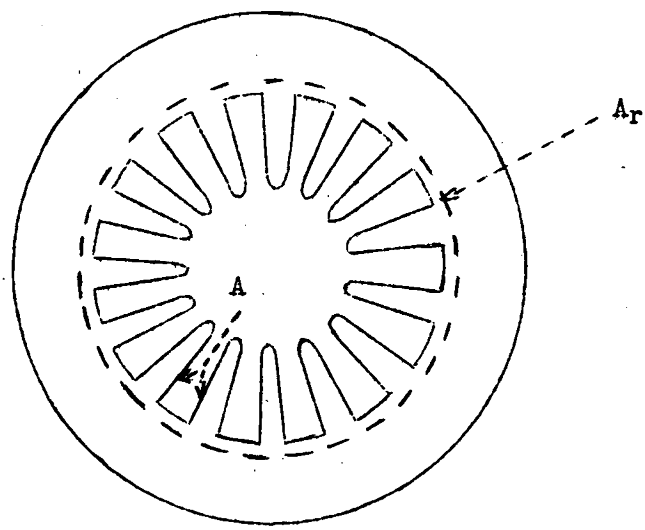


Figure 24.- Effective area for radiation.

In some designs, the space between the radiant surfaces contains substances such as water vapor, CO_2, hydrocarbon vapors or other athermanous gases. These gases absorb and emit radiant energy in certain wavelength bands and thus the net exchange of heat between the surfaces is altered because of the characteristics of these constituents. An estimation of this effect may be obtained by use of the data presented in reference 11.

If either of the radiant surfaces is also transferring heat by convection, equation (12b) containing the equivalent unit conductance for radiation f_r must be used.

$$q = (f_c + f_r)\ A\ (t_1 - \tau_a) \tag{12b}$$

where

$$f_r = \frac{0.173\ A_r\ F_{AE}\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{A\ (t_1 - \tau_a)} \tag{13}$$

The calculation of the total heat transfer (convection plus radiation) is thus straight-forward when the surface temperatures T_1 and T_2, and the gas temperature τ_a are known. In most problems of a heater design or the prediction of its thermal performance, however, these temperatures are not all known. The magnitudes of T_1 and T_2 may be estimated by means of a heat balance on the particular surface - that is, by a consideration of the temperatures of the fluid on either side of the surface and the corresponding thermal resistances. Because these thermal resistances $\frac{1}{(f_c + f_r)\ A}$ are functions of the equivalent conductances for radiation f_r which, in turn, are functions of the unknown temperatures T_1 and T_2, the determination of these temperatures must be performed by trial and error. When these temperatures are found, the total heat rate may be calculated.

In most heaters the radiant heat transfer will be found to be small compared to the convective transfer. In some instances, however, by special design, the heat transfer rate of an exhaust gas and air heat exchanger may be increased appreciably through the use of "irradiated convectors," which are placed in the ventilating air stream in "view" of the surfaces heated by the hot gases. All the heat absorbed through radiation by these surfaces is then transferred to the air by convection. The following example illustrates their effectiveness.

Example

For the system pictured in figure 25, where an irradiated convector is placed between the heated surfaces, which are sides of two exhaust gas passages of an exhaust gas to air heat exchanger, what percent increase in heat transfer rate will result by the use of the irradiated convector?

For this problem: let $\tau_g = 1500^\circ$ F and $\tau_a = 200^\circ$ F, the absorption of the radiation between surfaces 1 and 2 by any intervening athermanous gases is neglected, the fluid rates and dimensions of passages are such as to yield the unit thermal conductances* $f_{c_g} = 20$ and $f_{c_a} = 15$ Btu/hr ft^2 $^\circ$F For this case the modulus $F_{AE} = \dfrac{1}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1}$, The emissivity of surface 1 will be taken to be 0.8 and that of surface 2 to be 0.9, so that F_{AE} has the magnitude 0.735.

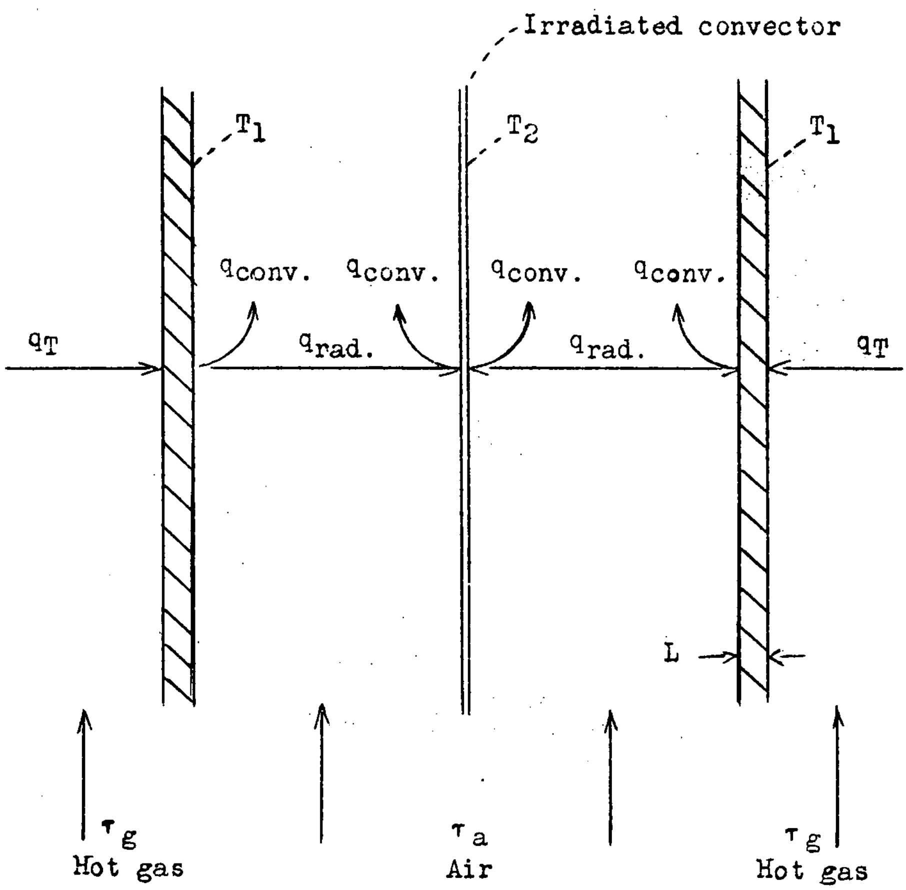


Figure 25.- Irradiated convector between two heated surfaces.

*The values of f_c may be obtained from the equations in pt. I, sec. B.

A heat rate balance on surface 1 gives the equation:

Heat gained by surface 1 from hot gas = Heat lost by surface 1 to air by convection and to plate 2 by radiation.

$$q_T = f_{c_g} A (\tau_g - t_1) = (f_{c_a} + f_{r_1}) A (t_1 - \tau_a) \qquad \text{(a)}$$

where

$$f_r = \frac{0.173 \; A_r F_{AE} \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{A (t_1 - \tau_a)} \qquad \text{(b)}$$

A similar equation for surface 2 is;

Heat gained by surface 2 by radiation from surface 1 = Heat lost by surface 1 to air by convection.

That is,

$$f_{r_2} A (t_2 - \tau_a) = f_{c_a} A (t_2 - \tau_a), \quad \text{or} \quad f_{r_2} = f_{c_a} \qquad \text{(c)}$$

Where

$$f_{r_2} = \frac{0.173 \; A_r \; F_{AE} \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{A (t_2 - \tau_a)} \qquad \text{(d)}$$

$$T = t + 460^\circ \text{ and, for this case, } A_r = A$$

In equations (a), (b), and (c) all the variables, except the surface temperatures T_1 and T_2, are known. The simultaneous solution of these expressions, therefore, will yield T_1 and T_2. However, the form of the equations is such as to necessitate a trial-and-error solution. As a first approximation, a value of T_1 may be found from equation (a) by setting $f_{r_1} = 0$. This magnitude would be the surface temperature if the radiant heat rate were zero. This first value of T_1 may be substituted into equations (c) and (d) in order to find an approximate value of T_2. With these first values of T_1 and T_2 the first approximation to the unit thermal conductance for radiation f_{r_1} may be found from equation (b). When this value of f_{r_1} is substituted into equation (a), a better value of T_1 then can be calculated. As before, a new value of T_2 may be found from equations (c) and (d). By repeating this procedure, values of T_1 and T_2 which satisfy the equations (a), (b), (c), and (d) are determined. The temperatures were found to be, $t_1 = 855^\circ$ F and $t_2 = 410^\circ$ F. Thus:

$$f_{r_1} = \frac{0.173\ A \times 0.735 \left[\left(\frac{855 + 460}{100}\right)^4 - \left(\frac{410 + 460}{100}\right)^4 \right]}{A\ (855 - 200)}$$

$$= 4.7\ \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}}$$

The over-all unit thermal conductance when the irradiated convector is inserted between the two heated surfaces would be

$$\frac{1}{(UA)_{rad}} = \frac{1}{f_{c_g}A} + \frac{1}{(f_{c_a} + f_{r_1})A}$$

The corresponding value without the radiation plate present is

$$\frac{1}{UA} = \frac{1}{f_{c_g}A} + \frac{1}{f_{c_a}A}$$

The ratio of these two over-all conductances is

$$\frac{(UA)_{rad}}{UA} = \frac{\frac{1}{f_{c_g}A} + \frac{1}{f_{c_a}A}}{\frac{1}{f_{c_g}A} + \frac{1}{\left(f_{c_a} + f_{r_1}\right)A}} = \frac{\frac{1}{20} + \frac{1}{15}}{\frac{1}{20} + \frac{1}{15 + 4.7}} = 1.17$$

The addition of the irradiated convector thus increased the heat rate by 17 percent.

The reduction of the hydraulic diameter for fluid flow D in this case would have approximately doubled the static pressure drop due to skin friction.

II. SINGLE PASS HEAT EXCHANGERS

The previous section presented the basic equation for the determination of the unit thermal conductances in heat exchangers utilizing as working fluids air and products of combustion of hydrocarbon fuels. The following section combines these basic equations for the analysis and design of such heat exchangers.

A. MEAN TEMPERATURE DIFFERENCE AND HEAT EXCHANGER EFFECTIVENESS

Single-pass heat exchangers usually may be classified according to the mode of flow of the hot and cold fluids passing through them, as follows:

1. Parallelflow, in which the two fluids flowing along the heat transfer surface move in the same direction.

2. Counterflow, in which the two fluids flowing along the heat transfer surface move in opposite direction.

3. Crossflow, in which the two fluids flowing along the heat transfer surface move at right angles to each other. Two cases of this type of heat exchanger are presented in this report. In the first case each fluid is unmixed as it passes through the exchanger, and therefore the temperature of the fluid leaving the heater section is not uniform, being hotter on one side than on the other. (See fig. 28.) A flat-plate type heater (see fig. 36) approximates this type of exchanger. In the second case one of the fluids is unmixed and the other fluid is perfectly mixed as it flows through the exchanger. The temperature for the mixed fluid will be uniform across the section and will vary only in the direction of flow. An example of this type of heater is an unbaffled tube-bank crossflow type exchanger. (See fig. 37.) In the example given, the air is mixed and the gas is unmixed.

Many exchangers utilized in practice do not fall exactly into any of the preceding classifications, but are a combination of these; usually, however, one form of flow is sufficiently predominant to permit classification.

The manner in which the temperature of the two fluids passing through the exchanger varies with distance along the heat transfer surface depends on whether the exchanger is parallelflow, counterflow, or crossflow. Typical temperature variations for these flow types are shown in figures 26, 27, and 28.

It is evident that a constant temperature difference between the two gases does not exist. The effective temperature difference which determines the rate of heat transfer between the two fluids is a function of the terminal temperature differences (that is, the difference in temperatures between the two fluids entering the exchanger $(T_{g_1} - T_{a_1})$ and the difference in temperature between the two fluids leaving the exchanger $(T_{g_2} - T_{a_2})$). For the parallelflow and counterflow exchangers the effective temperature difference is the familiar log-mean temperature difference. (See reference 12, ch. XIV; also figs. 26 and 27.) For the crossflow exchangers the effective temperature difference is a more complex function of the terminal temperature differences.* (See references 1, p. 35; 54, 55, and 56.) Charts for the determination of the effective temperature differences for the three types of exchangers are shown in figures 29, 30, and 31.

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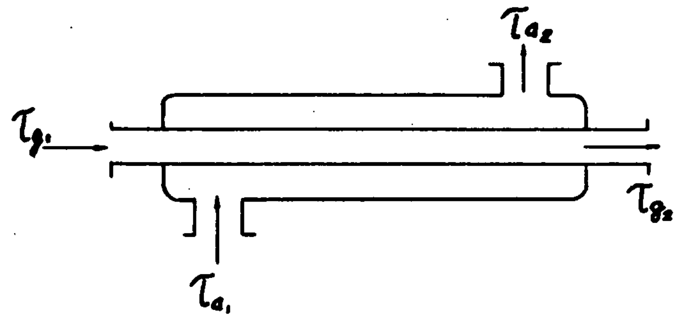
*The log-mean temperature differences and the effective temperature difference for crossflow are based on the following postulates:

1. The over-all unit thermal conductance U must be uniform throughout the exchanger. However, as seen in pt. I, secs. B, C, D, and E, the unit conductances usually will vary somewhat throughout the exchanger. Utilization of an average value of the over-all conductance usually allows the use of the log-mean or effective temperature differences without excessive error. (An exact evaluation of the rate of heat transfer is extremely complex when the conductances vary throughout the heat exchanger, Reference 52 presents an evaluation for the simple case in which the over-all conductance varies linearly with temperature.

2. The relation between rate of heat transfer and temperature change of the fluid must be linear. This statement implies

(Continued on p. 80)

PARALLEL FLOW
(See Fig. 29-Also)



$$\Delta\tau_{m_p} = \frac{(\tau_{g_1}-\tau_{a_1})-(\tau_{g_2}-\tau_{a_2})}{\ln\frac{(\tau_{g_1}-\tau_{a_1})}{(\tau_{g_2}-\tau_{a_2})}}$$

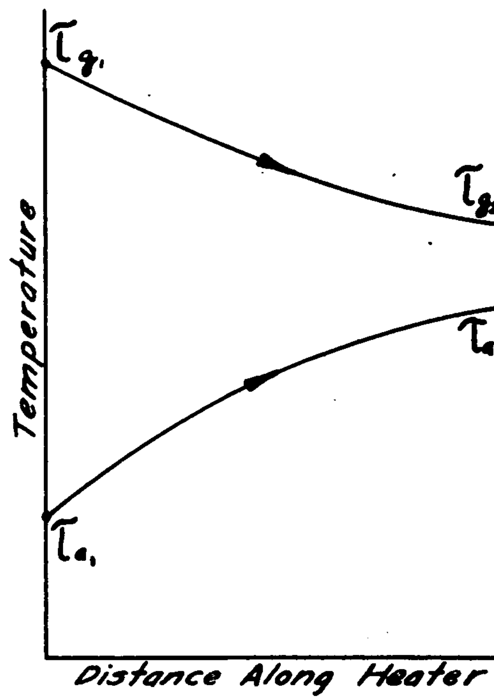
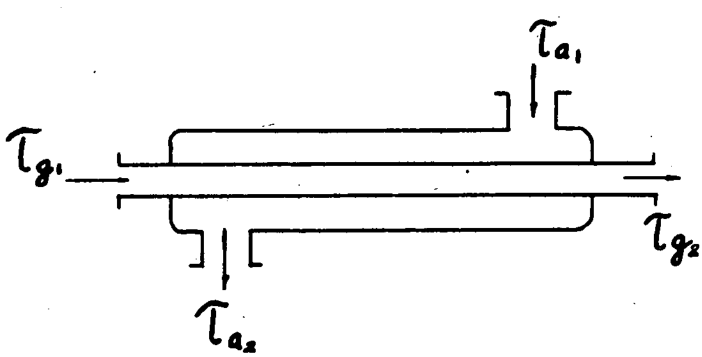


Fig. 26 - Temperature Variation in Parallel-Flow Heat Exchanger.

COUNTER FLOW
(See Fig. 30 - Also)



$$\Delta\tau_{m_c} = \frac{(\tau_{g_1}-\tau_{a_2})-(\tau_{g_2}-\tau_{a_1})}{\ln\frac{(\tau_{g_1}-\tau_{a_2})}{(\tau_{g_2}-\tau_{a_1})}}$$

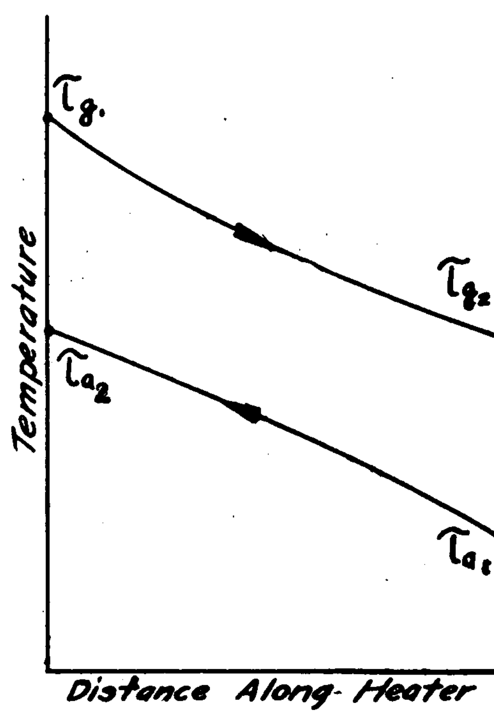
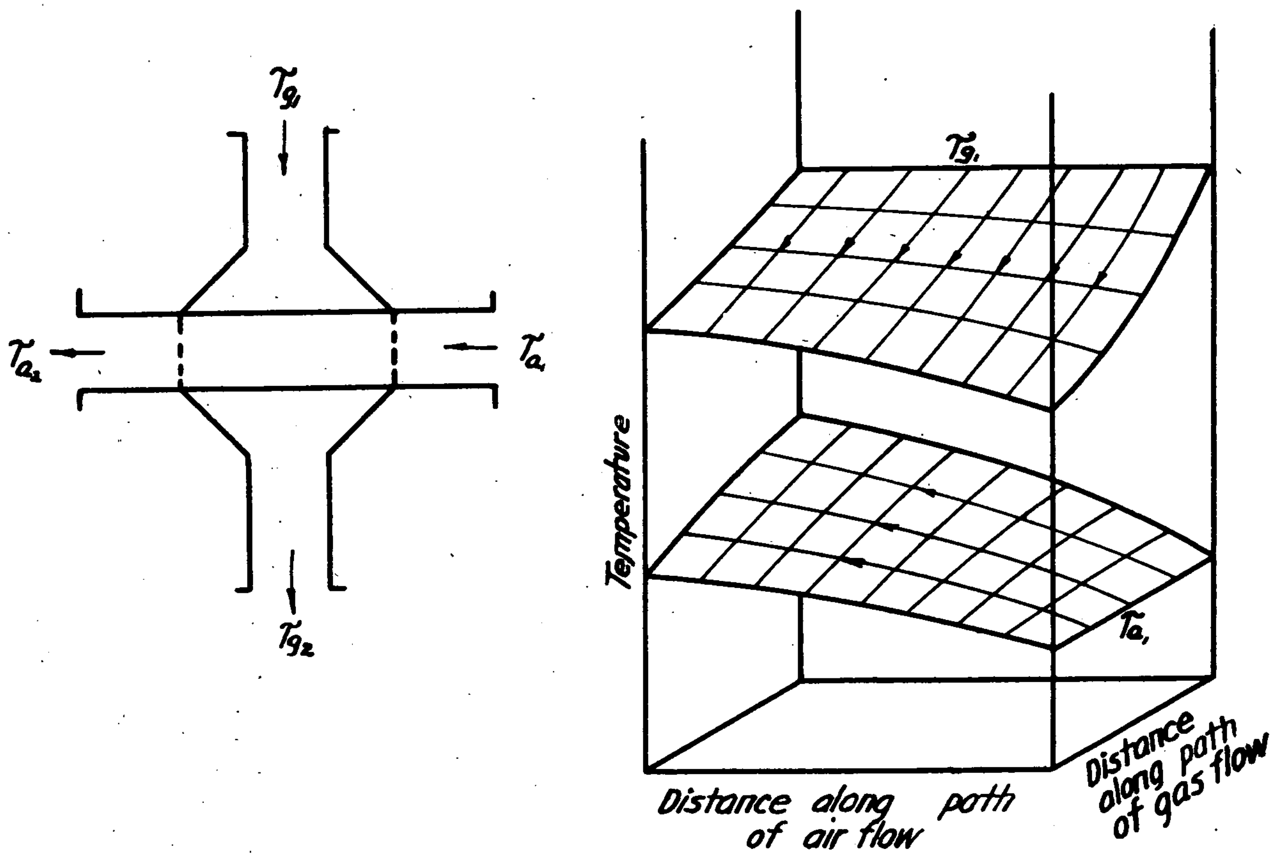


Fig. 27 - Temperature Variation in Counter-Flow Heat Exchanger

Cross Flow



$$\frac{\Delta t_{mx}}{T_{g_1}-T_{a_1}} = f\left(\frac{T_{a_2}-T_{a_1}}{T_{g_1}-T_{a_1}}\ ,\ \frac{T_{g_1}-T_{g_2}}{T_{g_1}-T_{a_1}}\right)$$

(See Fig. 31)

Fig. 28 - Temperature Variation in Crossflow Exchanger
Neither Fluid Mixed

Thermal Output of Exchanger

The thermal output of a heat exchanger in which no heat is lost to the surroundings* may be expressed as:

	$q = (UA)\ \Delta\tau_{mp}$	Parallelflow	(41)
Form I	$q = (UA)\ \Delta\tau_{mc}$	Counterflow	(42)
	$q = (UA)\ \Delta\tau_{mx}$	Crossflow	(43)
	$q = W_a\ c_{p_a}(\tau_{g_1} - \tau_{a_1})\ \Phi_p$	Parallelflow	(44)
Form II	$q = W_a\ c_{p_a}(\tau_{g_1} - \tau_{a_1})\ \Phi_c$	Counterflow	(45)
(Reference 52)	$q = W_a\ c_{p_a}(\tau_{g_1} - \tau_{a_1})\ \Phi_x$	Crossflow	(46)

Form I will be found convenient when all the terminal temperature differences are known, and is employed generally when designing an exchanger to given specifications. Figures 29, 30, and 31 allow the graphical evaluation of the effective mean temperature differences.

Form II is convenient when the temperatures of the fluids entering the exchangers and the over-all conductance UA are known, but the temperatures of the fluids leaving the exchanger are not known. Form II is particularly useful in calculating the performance of a given heat exchanger at various gas and air rates. The two forms will yield exactly the same results. Charts for the determination of the heater effectiveness Φ are shown in figures 32, 33, and 34.

(Continued from p. 77)

(a) An adiabatic exchanger or one which loses to the surroundings a constant fraction of the heat transferred (reference 53)

(b) Constant heat capacities

(c) No change in phase (condensation, etc.) during any portion of the fluid's path through the exchanger. If the change in phase takes place over all of the fluid's path, however, the temperature usually will remain constant and the log-mean can be utilized.

***See references 53 and 57 for the case in which the heater loses heat to the surroundings.**

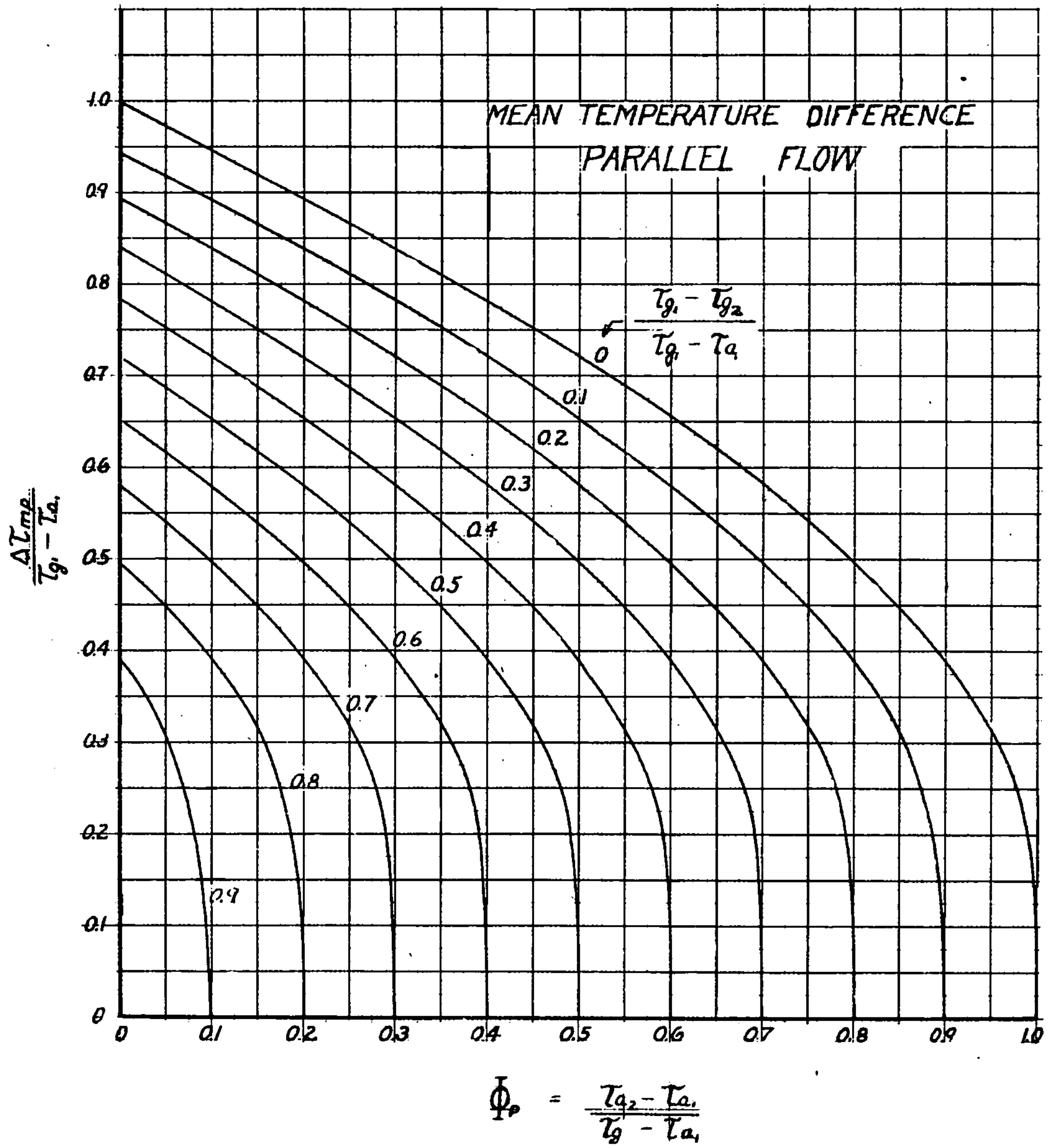


Fig. 29. - Mean Temperature Difference for Parallel Flow.

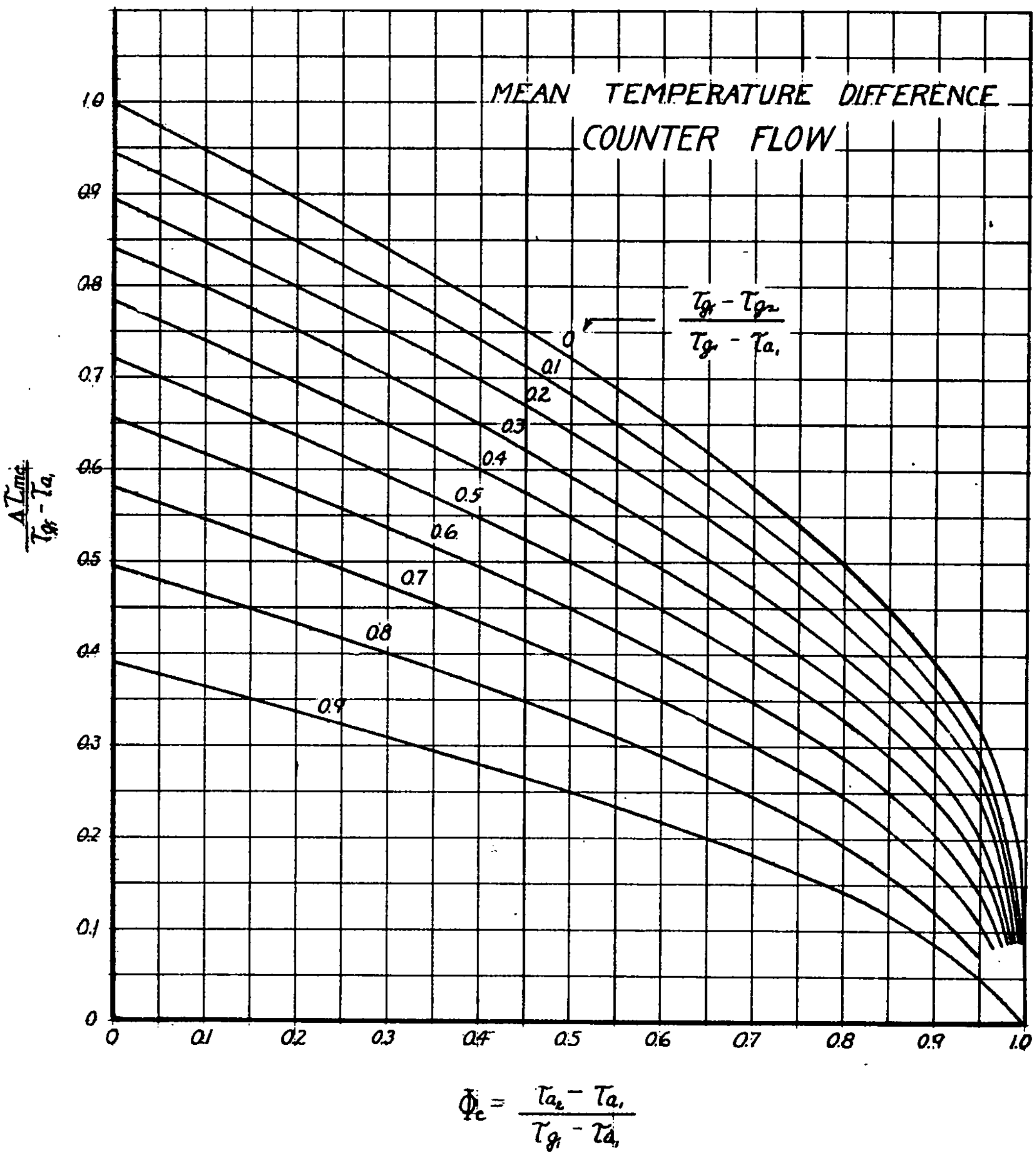


Fig. 30. - Mean Temperature Difference fur Counter Flow.

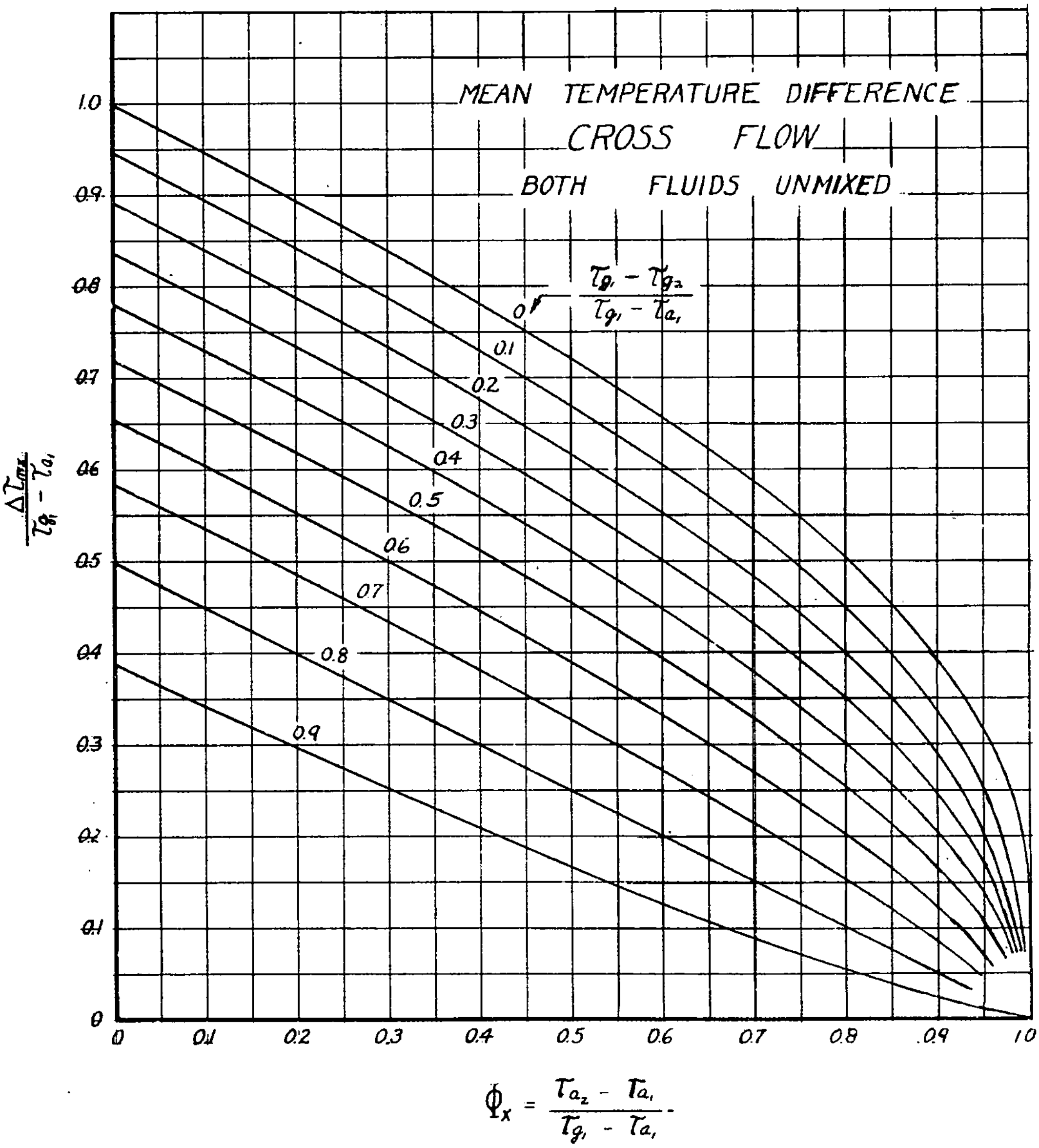


Fig. 31a. - Mean Temperature Difference for Cross Flow.

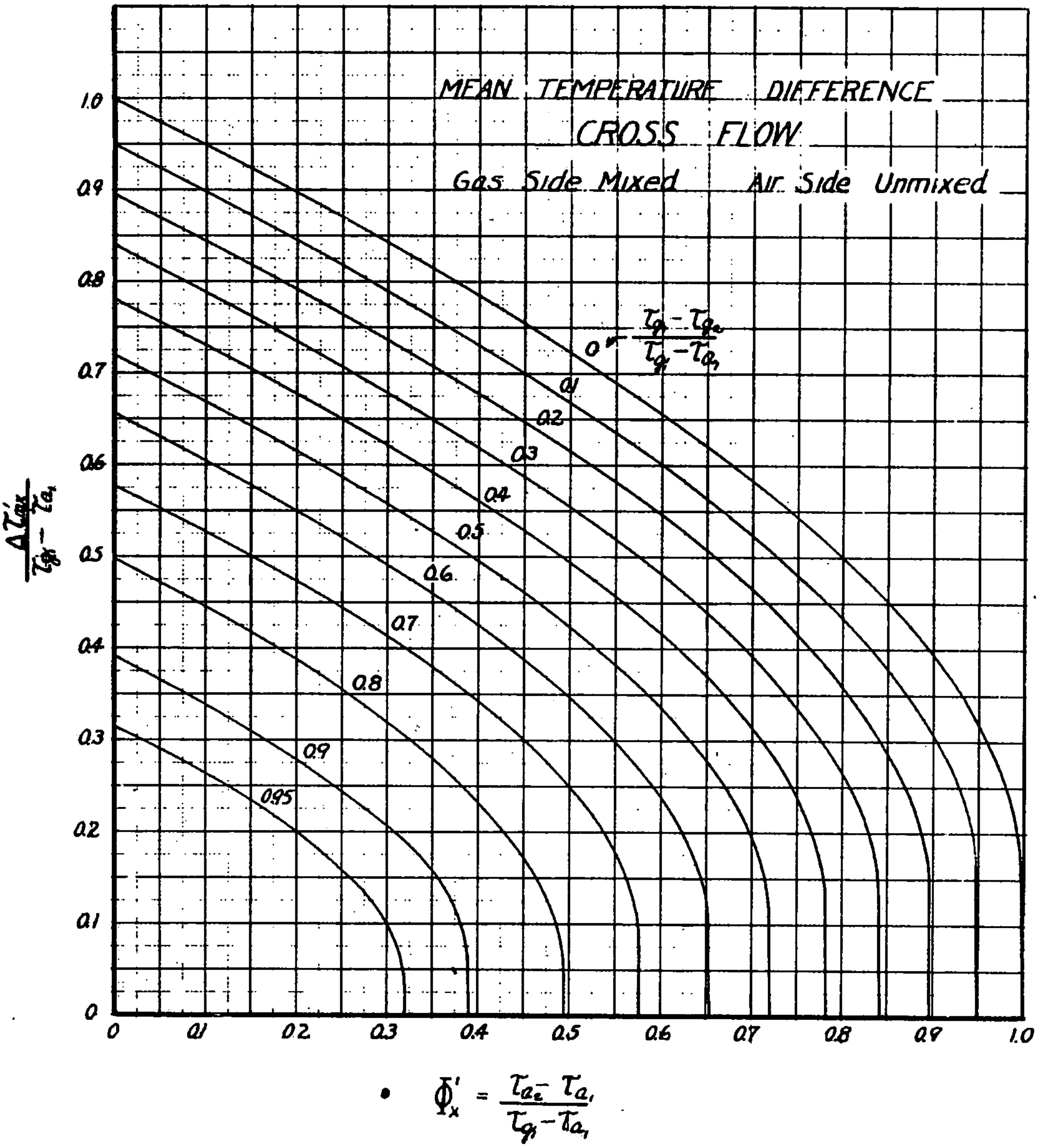


Fig. 31b.- Mean Temperature Difference for Cross Flow – Gas Side Mixed, Air Side Unmixed.

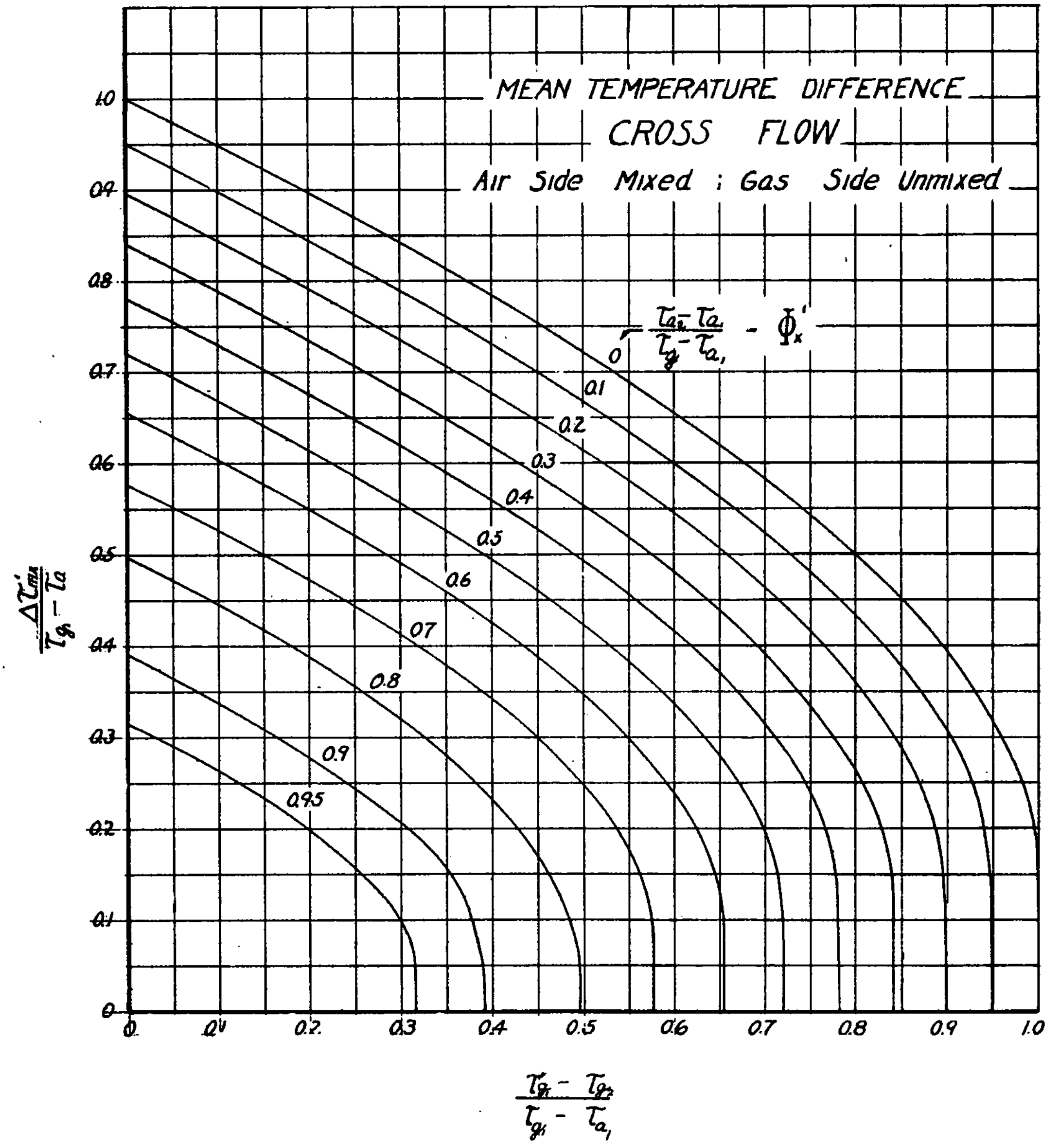


Fig. 31c - Mean Temperature Difference for Cross Flow Air Side Mixed — Gas Side Unmixed.

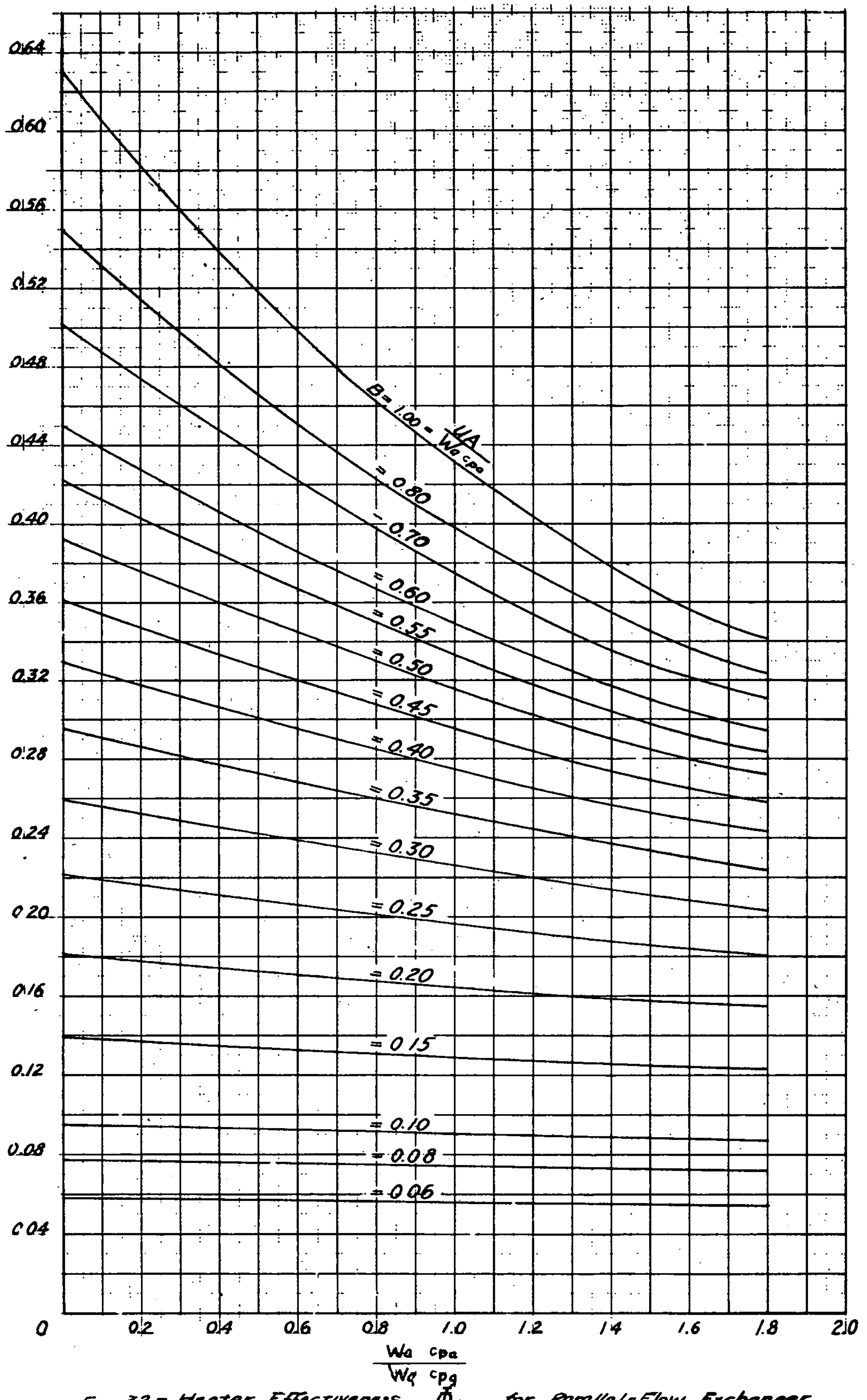


Fig. 32 - Heater Effectiveness Φ_p, for Parallel-Flow Exchanger.

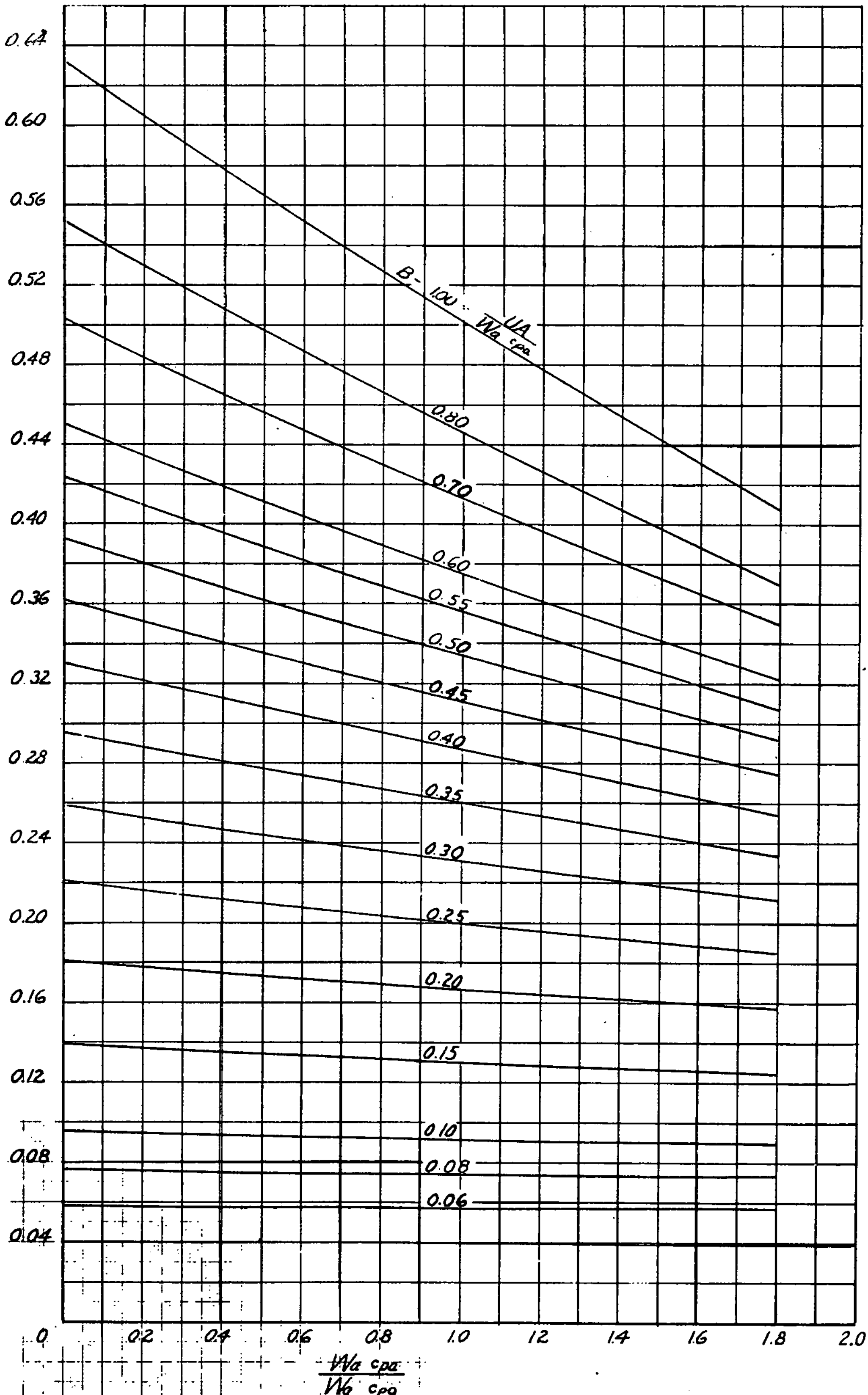


Fig. 33 - Heater Effectiveness Φ_c, for Counter-Flow Exchanger.

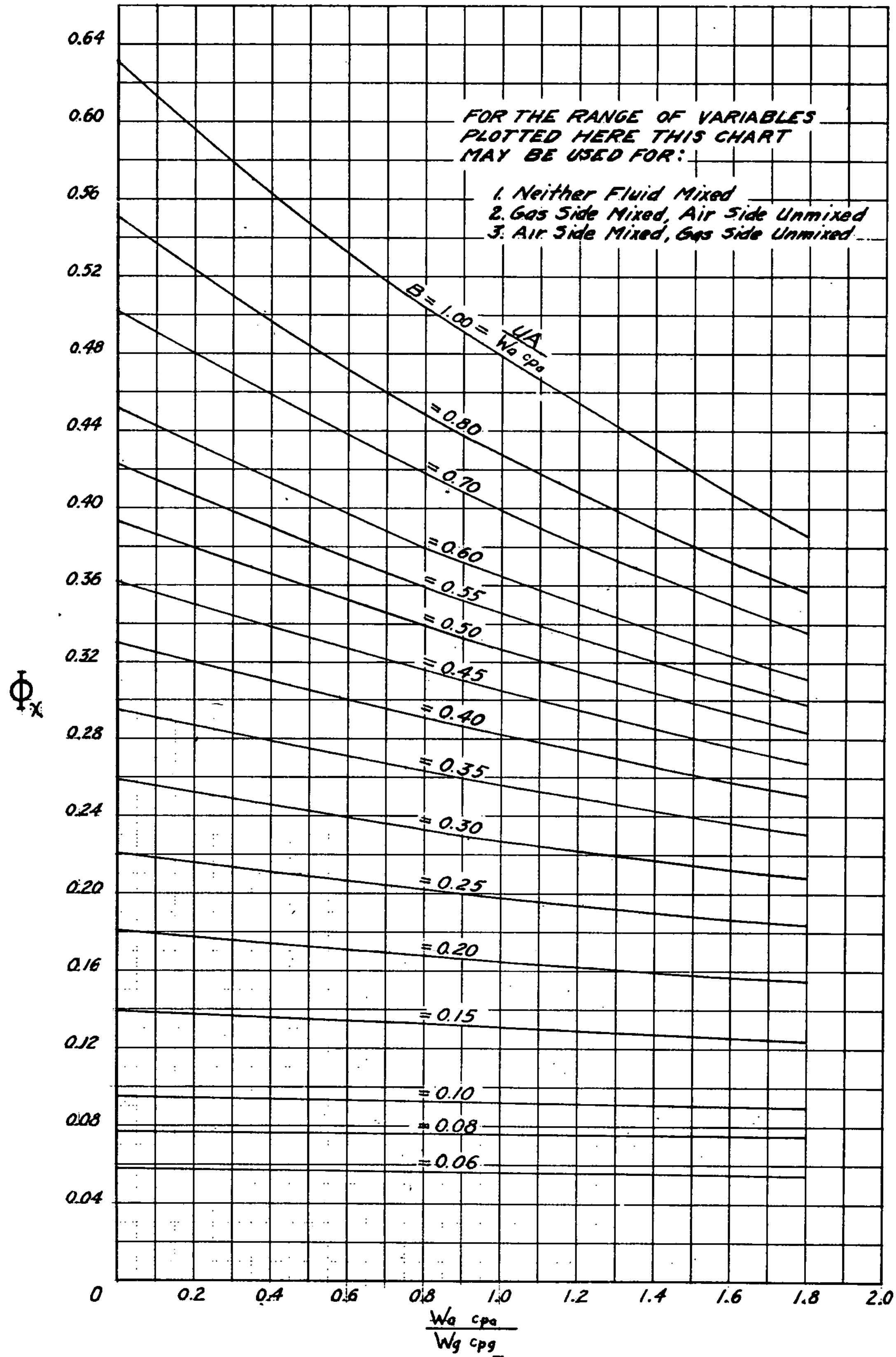


Fig. 34. - Heater Effectiveness Φ_x, for Cross-Flow Exchanger.

In the equations for the heater output presented in equations (41) to (46), the nomenclature is as follows:

q thermal output of heater, Btu/hr

$\Delta\tau_{mp}$ effective mean temperature difference for parallelflow, shown in figure 29. For parallelflow $\Delta\tau_{mp}$ is the log-mean temperature difference as defined in figure 26, °F

$\Delta\tau_{mc}$ effective mean temperature difference for counterflow, shown in figure 30. For counterflow $\Delta\tau_{mc}$ is the log-mean temperature difference as defined in figure 27, °F

$\Delta\tau_{mx}$, $\Delta\tau_{mx}'$ effective mean temperature difference for crossflow, shown in figure 31. For crossflow $\Delta\tau_{mx}$ is a somewhat complex function of **terminal temperature differences, °F (See references 1, p. 35; and 54, 55, and 56.)**

τ_{a_1} mixed-mean temperature of cold air entering exchanger, °F

τ_{a_2} mixed-mean temperature of hot air leaving exchanger, °F

τ_{g_1} mixed-mean temperature of hot gases entering exchanger, °F

τ_{g_2} mixed-mean temperature of hot gases leaving exchanger, °F

W_a air rate, lb/hr

W_g hot gas rate, lb/hr

c_{p_a} heat capacity of air at arithmetic mean mixed temperature, Btu/lb °F

c_{p_g} heat capacity of gas at arithmetic mean mixed temperature, Btu/lb °F

Φ_p effectiveness of parallelflow heat exchanger - that is, $\Phi_p = \dfrac{\tau_{a_2} - \tau_{a_1}}{\tau_{g_1} - \tau_{a_1}}$ - temperature rise of cold fluid divided by difference between hot gas and cold air temperatures at entrance to heat exchanger. (See fig. 32.)

Φ_c effectiveness of counterflow heat exchanger - that is, $\Phi_c = \frac{T_{a_2} - T_{a_1}}{T_{g_1} - T_{a_1}}$ - temperature rise of cold fluid divided by difference between hot gas and cold air temperatures at entrance to heat exchanger. (See fig. 33.)

Φ_x, Φ_x' effectiveness of crossflow heat exchanger - that is, $\Phi_x = \frac{T_{a_2} - T_{a_1}}{T_{g_1} - T_{a_1}}$ - temperature rise of cold fluid divided by difference between hot gas and cold air temperatures at entrance to heat exchanger. (See fig. 34.)

UA over-all thermal conductance, (Btu/hr °F) which is defined by:

$$\frac{1}{UA} = \frac{1}{(fA)_a} + \frac{L}{kA} + \frac{1}{(fA)_g} \tag{47}$$

where

$(fA)_a$, $(fA)_g$ total conductance on the air and hot gas side, respectively (Btu/hr °F)

The total conductance equals the product of the unit conductance $(f_c + f_r)$ in Btu/hr ft^2 °F, and the area of the heat transfer surface in ft^2, for unfinned heat exchangers. For finned heat exchanger, $(fA)_a$, $(fA)_g$ are the equivalent total conductance on the air and gas sides, respectively. (See example of finned exchanger calculation, pt. II, sec. 4.)

L thickness of heat transfer surface material measured in direction of heat flow, ft

k thermal conductivity of heat transfer surface material, Btu/hr ft^2 $\left(\frac{^\circ F}{ft}\right)$. (In the usual heat exchanger the term L/kA is negligible compared with the other two thermal resistances.)

From the equation for the thermal output of the heater it is noted that two variables affect this output more than the others: namely, the effective mean temperature difference and the over-all thermal conductance. The mean temperature difference is usually fixed by design conditions, and may be readily determined from figures 29, 30, and 31. Then, to obtain a given heater output, a magnitude of UA equal to $q/\Delta\tau_m$ must be provided for by the design.

Example:

A crossflow type heater in which neither fluid is mixed is to be designed to raise the temperature of 3000 pounds of air per hour from 10° to 400° F. The temperature of the hot gases available for heating the air is 1600° F, and the hot gas rate is 6000 pounds per hour. What must be the over-all conductance of the heater, assuming no heat loss from the heat exchanger to the surroundings?

If no heat is lost to the surroundings, the heat gained by the air is lost by the hot gas. Thus

$$W_a c_{p_a} (\tau_{a_2} - \tau_{a_1}) = W_g c_{p_g} (\tau_{g_1} - \tau_{g_2})$$

The heat capacity of air at an average temperature of 205° F = 0.241 Btu/lb °F. (See appendix, pt. IV, sec. A.) The heat capacity of the hot gas* at an approximate temperature of 1500° F is 0.277 Btu/lb °F. Thus the temperature of the hot gas leaving the exchanger is

$$\tau_{g_2} = \tau_{g_1} - \frac{W_a c_{p_a}}{W_g c_{p_g}} (\tau_{a_2} - \tau_{a_1})$$

$$= 1600 - \frac{3000 \times 0.241}{6000 \times 0.277} (400 - 10) = 1430^\circ \text{ F}$$

*The heat capacity of exhaust gases may be calculated from the data given in appendix A for the pure components of the mixture. The value 0.277 was taken to be that for pure air for this example.

From figure 31a, the mean temperature difference ΔT_{mx} may be readily obtained. The parameters necessary for the use of the curves are

$$\Phi_x = \frac{T_{a_2} - T_{a_1}}{T_{g_1} - T_{a_1}} = \frac{400 - 10}{1600 - 10} = 0.245$$

and

$$\frac{T_{g_1} - T_{g_2}}{T_{g_1} - T_{a_1}} = \frac{1600 - 1430}{1600 - 10} = 0.106$$

At the intersection of $\Phi_x = 0.245$ and $\frac{T_{g_1} - T_{g_2}}{T_{g_1} - T_{a_1}} = 0.106$ the value of $\frac{\Delta T_{mx}}{T_{g_1} - T_{a_1}} = 0.820$ is obtained. Thus the effective mean temperature difference between the hot gases and the air is:

$$\Delta T_{mx} = 0.820 \times (1600 - 10) = 1300^{\circ}\ F$$

The output of the heater is to be

$$q = W_a\, c_{p_a} (T_{a_2} - T_{a_1})$$

$$= 3000 \times 0.241 \times 390 = 282{,}000\ \text{Btu/hr}$$

Thus the necessary over-all conductance UA is given by

$$UA = \frac{q}{\Delta T_{mx}} = \frac{282000}{1300} = 217\ \frac{\text{Btu}}{\text{hr}\ {}^{\circ}\text{F}}$$

A large number of heaters may be designed all of which will have a given value of UA, since any of the variables which control the over-all conductance can be adjusted at will. The complete design of a heat exchanger for a given application, however, involves a series of compromises which can be made only by the designer in each special instance. Thus, in addition to the thermal output of the heater at given air and gas rates, items such as allowable maximum pressure drop through the heater, manufacturing facilities and techniques, space, weight, life requirements, and so forth, have an important role in the final choice of a heater design. A complete consideration of all these items cannot be presented in this part, but methods for evaluating the thermal conductance UA for several given heater types will be outlined. These examples will illustrate the application to heater design of the basic equations given in the first part of this report.*

The thermal performance of most aircraft heat exchangers has been predicted within about 20 percent by means of the expressions contained in this report, and the predicted performances are usually on the conservative side. Comparisons of measured and predicted heater performance for various types of heaters are presented in graphical form in references 29, 45, 58, 59, 60, 61, 62, 63, 64, 65, and 66. Reasons for discrepancies between measured and predicted results are discussed in the texts of these reports.

B. EXAMPLES

Example 1 - "Fluted" Type Heat Exchanger

Calculate the performance of the parallel flow "fluted" type heater shown in figure 35 for the following operating conditions:

W_a ventilating air rate = 3000 lb/hr

W_g hot gas rate = 5000 lb/hr

T_{g_1} temperature of hot gases entering exchanger = 1600° F

T_{a_1} temperature of cold air entering exchanger = 10° F

As a first approximation to the heater performance, the actual geometry of the ends of the heater can be neglected and all the calculations based on the dimensions at the center of the heater as shown in figure 35. The length of the heater is measured between the midpoints of the tapered sections.

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*Radiant heat transfer is postulated to be zero in the four examples which follow. In actual heaters the radiant heat transfer is a small part of the total unless special irradiated convectors are used. (See example in pt. I, sec. G.)

The flow systems for the air and gas sides then reduce to flow in a straight duct.

The following pertinent dimensions can be evaluated from the data of figure 35.

L length of duct = 1.17 ft

D_{H_a} hydraulic diameter on air side = $\frac{4 A_a}{P_a}$ = 0.0577 ft

D_{H_g} hydraulic diameter on gas side = $\frac{4 A_g}{P_g}$ = 0.0620 ft

A heat transfer area = 1.17 × 12.4 = 14.5 ft^2

Ratio $\frac{L}{D_{H_a}} = \frac{1.17}{0.0577} = 20.3$

The unit conductance for the heater on both the gas and air sides can be based on the data for flow in long ducts. (pt. I, sec. C). (The Reynolds number for the air and gas flows will be found to be well over 10,000 and consequently equation (25) may be utilized to evaluate f_{c_a} and f_{c_g}. The equations are:

For the air side

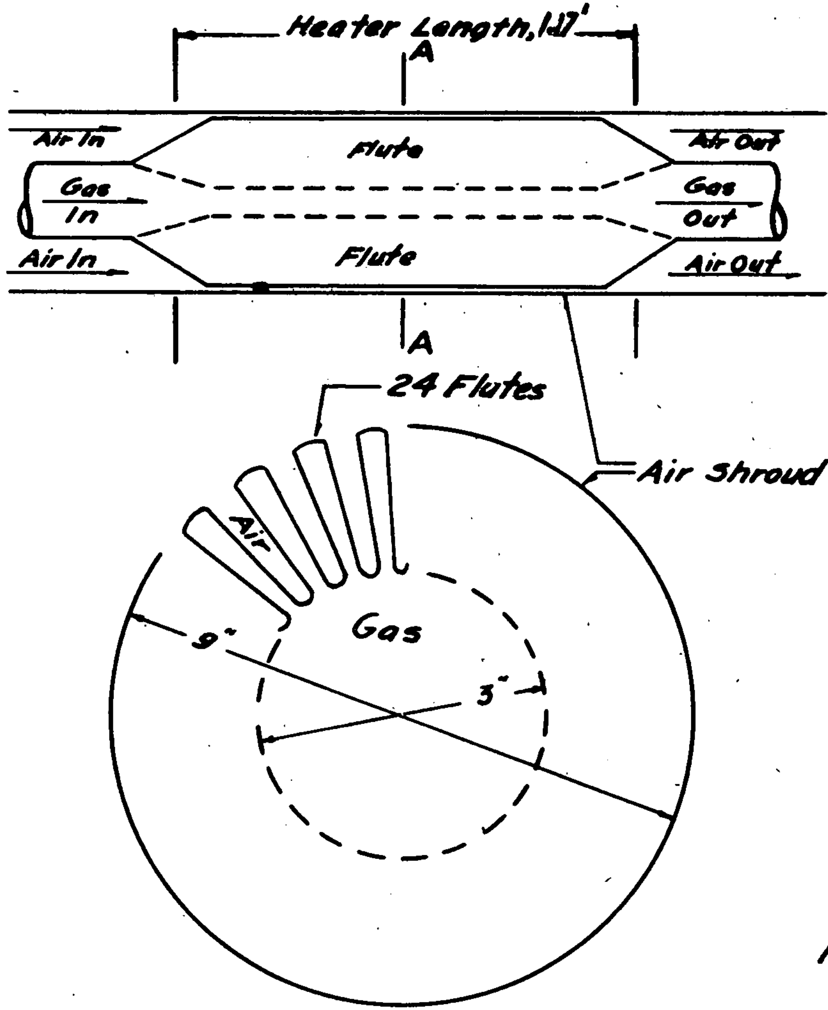
$$f_{c_a} = 5.4 \times 10^{-4} \frac{T_a^{0.3} G_a^{0.8}}{D_{H_a}^{0.2}} \left[1 + 1.1 \frac{D_{H_a}}{L}\right] \qquad (25)$$

and for the gas side,

$$f_{c_g} = 5.4 \times 10^{-4} \frac{T_g^{0.3} G_g^{0.8}}{D_{H_g}^{0.2}} \left[1 + 1.1 \frac{D_{H_g}}{L}\right] \qquad (25)$$

The weight rates per unit area, G_a, G_g, may be readily calculated. The temperatures T_a and T_g must be estimated and then checked with the final results, since the outlet gas and air temperatures are not known.

Reasonable values of T_a and T_g are as follows:



Enlarged Section at A-A

Dimensions At Section A-A

P_H = Perimeter of Heat Transfer Surface = 12.4 ft

P_a = Wetted Perimeter on Air Side (perimeter of surface contacting air) = 13.6 ft.

P_g = Wetted Perimeter on Gas Side = 13.6 ft.

A_a = Cross-Sectional Area of Air Passage = 0.196 ft^2

A_g = Cross-Sectional Area of Gas Passage = 0.211 ft^2

Fig. 35 - Fluted Type Heat Exchanger

$$T_a = 610^\circ \text{ R}$$

$$T_g = 1990^\circ \text{ R}$$

The weight rates per unit area are:

$$G_a = \frac{3000}{0.196} = 15{,}300 \ \frac{\text{lb}}{\text{hr ft}^2}$$

$$G_g = \frac{5000}{0.211} = 23{,}700 \ \frac{\text{lb}}{\text{hr ft}^2}$$

Thus the air side unit conductance is:

$$f_{c_a} = 5.4 \times 10^{-4} \times \frac{610^{0.3} \times 15300^{0.8}}{0.0577^{0.2}} \left[1 + 1.1 \times \frac{0.0577}{1.17}\right]$$

$$= 15.3 \text{ Btu/hr ft}^2 \ ^\circ\text{F}$$

The gas side unit conductance is:

$$f_{c_g} = 5.4 \times 10^{-4} \times \frac{1990^{0.3} \times 23700^{0.8}}{0.0620^{0.2}} \left[1 + 1.1 \times \frac{0.0620}{1.17}\right]$$

$$= 30.4 \text{ Btu/hr ft}^2 \ ^\circ\text{F}$$

The thermal resistance of the metallic wall is negligible. Thus:

$$\frac{1}{UA} = \frac{1}{A}\left(\frac{1}{f_{c_a}} + \frac{1}{f_{c_g}}\right) = \frac{1}{14.5}\left(\frac{1}{30.4} + \frac{1}{15.3}\right)$$

or

$$UA = 147 \text{ Btu/hr ft}^2$$

In order to determine the performance of the heater, figure 32 is utilized. The parameters necessary to determine the heater effectiveness Φ_p are:

$$B = \frac{UA}{c_{p_a} W_a} = \frac{147}{0.241 \times 3000} = 0.203$$

$$\frac{W_a c_{p_a}}{W_g c_{p_g}} = \frac{3000 \times 0.241}{5000 \times 0.277} = 0.522$$

From figure 32, the effectiveness Φ_p is

$$\Phi_p = 0.174$$

Thus the heater output is:

$$q = W_a c_{p_a} (T_{g_1} - T_{a_1}) \Phi_p$$

$$= 3000 \times 0.241 (1600 - 10) 0.174$$

$$= 200,000 \text{ Btu/hr}$$

A second approximation can be obtained by using the computed temperature and recalculating the convective conductances.

If the actual geometry of the ends of the heater has been considered in the calculation, due both to the change in heat transfer area and the variation in the unit conductance on the end surfaces, the heater output may be as much as 15 percent different than the amount calculated. To estimate the temperature of the metallic surfaces, equation (16) (pt. I, sec. A) may be utilized.

For test results on a fluted-type heater, see references 29, 59, and 65.

Example 2 - Flat-Plate Type Heater

Calculate the performance of the crossflow "flat-plate" type heater in which both fluids are unmixed shown in figure 36 for the following operating conditions:

W_a ventilating air rate = 3000 lb/hr

W_g' hot gas rate = 5000 lb/hr

T_{g_1} temperature of hot gas entering exchanger = 1600° F

T_{a_1} temperature of air entering exchanger = 10° F

As a first approximation to the heater performance, the actual geometry of the ends of the heater can be neglected and all the calculations based on the dimensions at the center of the heater as shown in figure 36. The flow systems for the air and gas streams are similar to flow in straight ducts.

The following dimensions may be evaluated from the data of figure 36.

L_a length of air duct = 0.583 ft

L_g length of gas duct = 1.13 ft

D_{H_a} hydraulic diameter of air duct $= \frac{4\ A_a}{P_a} = 0.0427$ ft

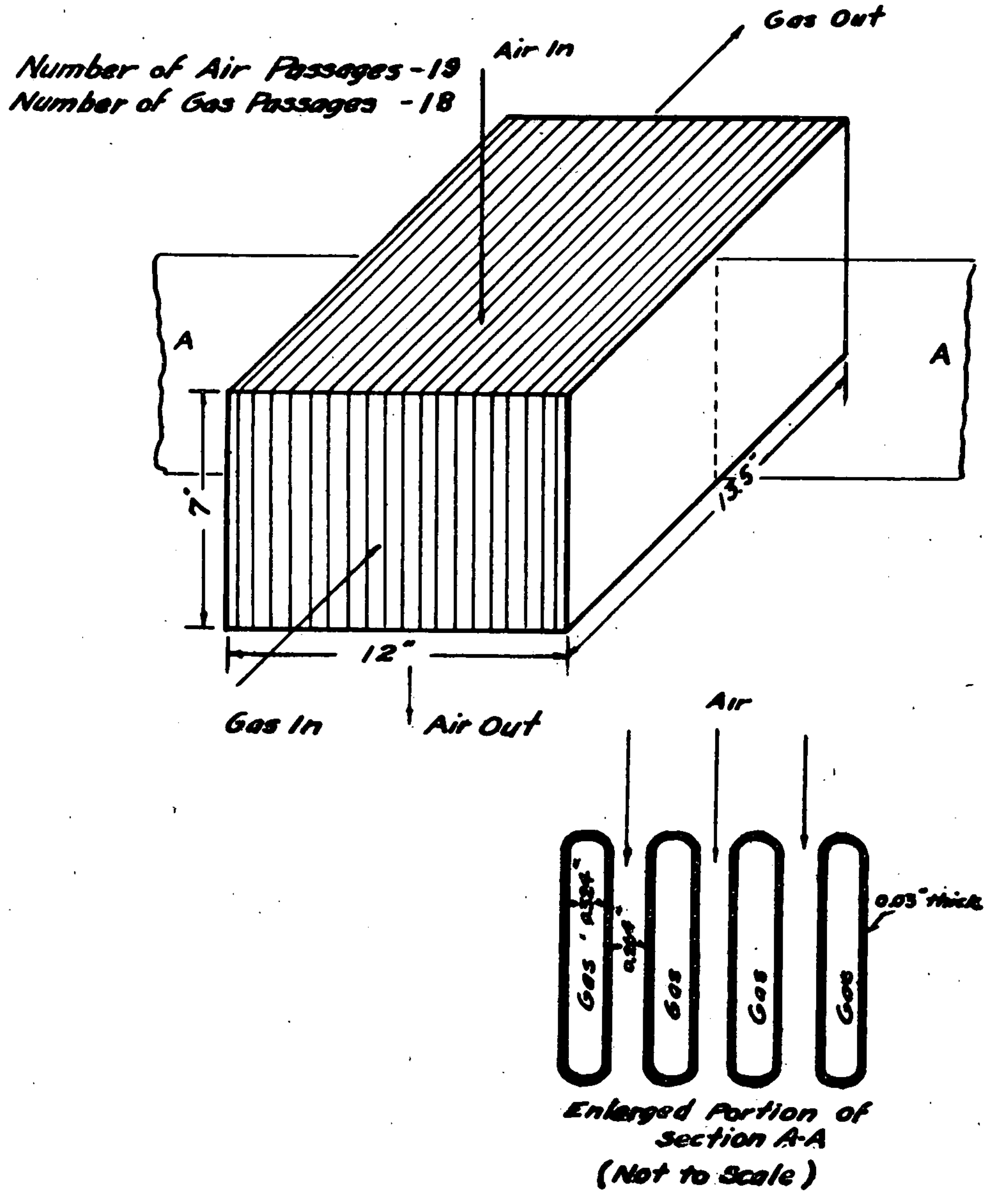
D_{H_g} hydraulic diameter of gas duct $= \frac{4\ A_g}{P_g} = 0.0516$ ft

A heat transfer area = 23.6 ft^2

Ratio $\frac{L_a}{D_{H_a}} = \frac{0.583}{0.0427} = 13.7$ (long tube, see pt. I, sec. C)

The unit conductances for both the air and gas sides may be evaluated from the equation for flow in long ducts. The Reynolds number for both the air and the gas sides will be found to be between 5000 and 10,000, and therefore the equations in part I, section C are not exactly applicable but are used here because better equations are not available. Thus, for the air side, from equation (25)

$$f_{c_a} = 5.4 \times 10^{-4} \frac{T_a^{0.3}\ G_a^{0.8}}{D_{H_a}^{0.2}} \left[1 + 1.1 \frac{D_{H_a}}{L_a}\right] \qquad (25)$$



Dimensions at Section A-A

P_a = Wetted Perimeter on Air Side	2.3 ft.
P_g = Wetted Perimeter on Gas Side	1.22 ft.
A_a = Cross-Sectional Area of Air Passage (per passage)	0.0246 ft²
A_g = Cross-Sectional Area of Gas Passage (per passage)	0.0158 ft²
Heat Transfer Area	23.6 ft²

Fig. 36.- Flat Plate Type Heat Exchanger.

and for the gas side

$$f_{c_g} = 5.4 \times 10^{-4} \frac{T_g^{0.3} \, G_g^{0.8}}{D_{H_g}^{0.2}} \left[1 + 1.1 \frac{D_{H_g}}{L_g} \right] \qquad (25)$$

The weight rates per unit area can be readily calculated. The temperatures T_a and T_g must be estimated and then checked with the final results, since the exit temperatures of the gas and the air are not known. Reasonable values of T_a and T_g are as follows:

$$T_a = 610^\circ \text{ R}$$

$$T_g = 1990^\circ \text{ R}$$

The weight rates per unit area are:

$$G_a = \frac{3000}{19 \times 0.0246} = 6420 \frac{\text{lb}}{\text{hr ft}^2}$$

$$G_g = \frac{5000}{18 \times 0.0158} = 17{,}550 \frac{\text{lb}}{\text{hr ft}^2}$$

The air side unit conductance is:

$$f_{c_a} = 5.4 \times 10^{-4} \times \frac{610^{0.3} \times 6420^{0.8}}{0.0427^{0.2}} \left[1 + 1.1 \times \frac{0.0427}{0.583} \right]$$

$$= 8.30 \text{ Btu/hr ft}^2 \text{ }^\circ\text{F}$$

The gas side unit conductance is:

$$f_{c_g} = 5.4 \times 10^{-4} \times \frac{1990^{0.3} \times 17550^{0.8}}{0.0516^{0.2}} \left[1 + 1.1 \times \frac{0.0516}{1.13} \right]$$

$$= 24.5 \text{ Btu/hr ft}^2 \text{ }^\circ\text{F}$$

The thermal resistance of the metallic wall is negligible. Thus

$$\frac{1}{UA} = \frac{1}{f_{c_g} A} + \frac{1}{f_{c_a} A} = \frac{1}{23.6}\left(\frac{1}{8.30} + \frac{1}{24.5}\right)$$

or

$$UA = 146 \text{ Btu/hr } ^{\circ}\text{F}$$

In order to determine the performance of the heater, figure 34 is utilized. The parameters necessary to determine the heater effectiveness Φ_x are:

$$\frac{W_a\ c_{p_a}}{W_g\ c_{p_g}} = \frac{3000 \times 0.241}{5000 \times 0.277} = 0.522$$

$$\frac{UA}{W_a\ c_{p_a}} = \frac{146}{3000 \times 0.241} = 0.202$$

From figure 34 the effectiveness Φ_x is:

$$\Phi_x = 0.172$$

Thus the heater output is:

$$q = W_a\ c_{p_a} (\tau_{g_1} - \tau_{a_1})\ \Phi_x = 3000 \times 0.241\ (1600 - 10)\ 0.172$$

$$= 197{,}000 \text{ Btu/hr}$$

If the ends of the heater had been considered in the calculation, the heater output probably would be about 10 percent higher than the value calculated.

A great improvement in heater output can be obtained from the heater analyzed if the weight rate per unit area G_a is increased. As noted in the calculations the thermal resistance in the air side is much greater than that on the gas side. Thus, a great improvement in heater output will result upon increasing the weight rate per unit area on the air side. This increase is readily accomplished by decreasing the cross section of the air passages. It is clear, however, that the decrease in air passage area increases the isothermal total pressure drop across the air side of the heater. The proper size of the air passage is then a compromise between effective heat transfer and an allowable pressure drop. The metallic surface temperatures also should be calculated in order to insure safe metal working temperatures (equation (16), pt. I, sec. A).

See reference 66 for details of measurement and prediction of results in a flat-plate type heater.

Example 3 - "Tube-Bank" Type Exchanger

Calculate the thermal performance of the crossflow "tube-bank" type of heat exchanger shown in figure 37 for the following operating conditions.

W_a ventilating air rate = 3000 lb/hr

W_g hot gas rate = 5000 lb/hr

T_{g_1} temperature of hot gases entering exchanger = 1600° F

T_{g_2} temperature of cold air entering exchanger = 10° F

The flow system on the hot gas side consists of flow in straight ducts; on the air side the flow is over a bank of staggered tubes.

The following pertinent dimensions can be evaluated from the data of figure 37.

L_g length of tube through which the hot gas flows = 1 ft

D_{H_g} hydraulic diameter on gas side = 0.0783 ft

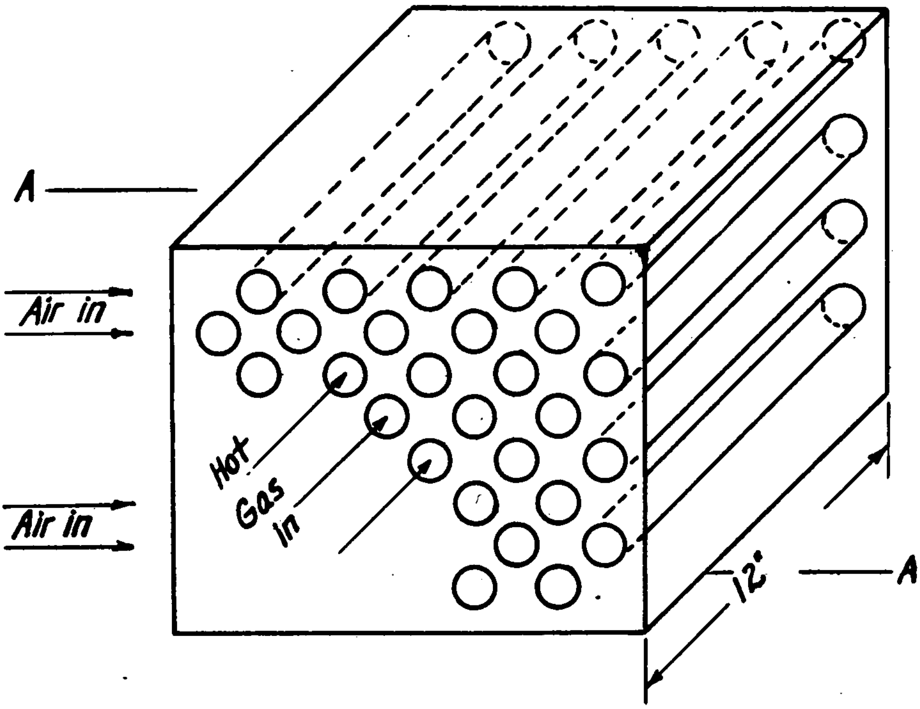
Ratio $\frac{L_g}{D_{H_g}} = 12.8$

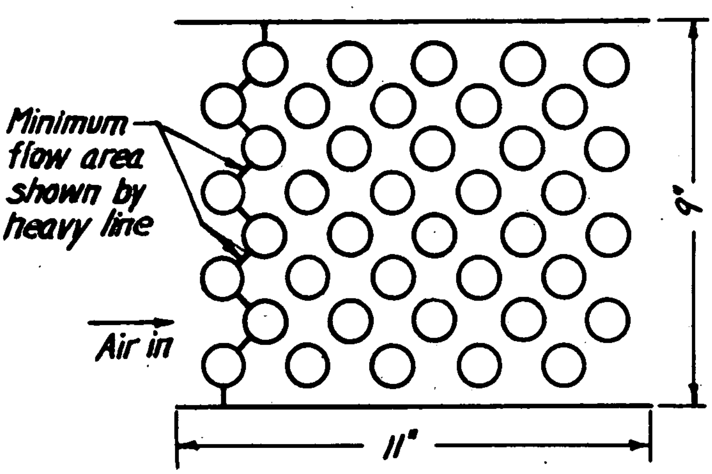
D_a outer diameter of tubes = 0.0833 ft

A_g heat transfer area on gas side = 9.82 ft^2

A_a heat transfer area on air side = 10.5 ft^2

Since the ratio of $\frac{L_g}{D_{H_g}} > 4.4$ and the Reynolds number for the gas side = 19,900, equation (25) (pt. I, sec. C) may be used





Section at AA

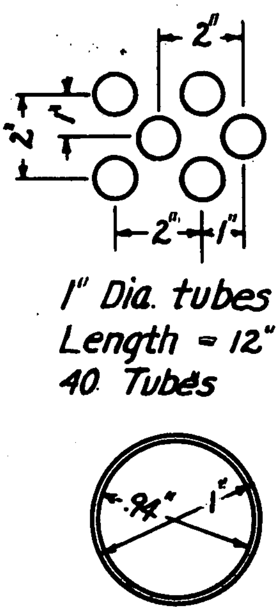


Fig. 37 Tube Bank Type Heat Exchanger

to calculate the unit thermal conductance on the gas side. The weight rate per unit area on the gas side is

$$G_g = \frac{5000}{40 \times 0.0481} = 26,000 \frac{\text{lb}}{\text{hr ft}^2}$$

The mean temperature of the hot gas is unknown, but a reasonable estimate is:

$$T_g = 1990^\circ \text{ R}$$

(This value must be checked with the final calculated magnitude.)

Then

$$f_{c_g} = 5.4 \times 10^{-4} \frac{T_g^{0.3} G_g^{0.8}}{D_{H_g}^{0.2}} \left(1 + 1.1 \frac{D_{H_g}}{L_g}\right)$$

$$f_{c_g} = 5.4 \times 10^{-4} \times \frac{1990^{0.3} \times 26000^{0.80}}{0.0783^{0.2}} \left(1 + 1.1 \frac{0.0783}{1}\right)$$

$$= 32.2 \text{ Btu/hr ft}^2 \text{ }^\circ\text{F}$$

The unit conductance for the air side may be calculated as follows: The Reynolds number for the flow over the tubes in the tube bank is found to be 15,800. Equation (29) presented in part I, section D is then applicable.

$$f_{c_a} = 14.5 \times 10^{-4} F_a T_f^{0.43} \frac{G_{o_a}^{0.6}}{D_a^{0.4}}$$

The tube bank has 10 rows of staggered tubes. Thus from table II, $F_a = 1.54$. The minimum area of flow for the air is shown in figure 37 and equals:

$$A = \left(\frac{1 + 7 \times 0.414}{144}\right) 12 = 0.325 \text{ ft}^2$$

Thus the maximum weight rate per unit area is

$$G_{o_a} = \frac{3000}{0.325} = 9230 \text{ lb/hr ft}^2$$

The approximate surface temperature of the tubes can be postulated to be midway between the mean gas and air temperatures. Thus the film temperature T_f can be calculated as follows:

$$T_f = \left(\frac{150 + 1530}{2} + 150\right) \frac{1}{2} + 460 = 955^{\circ} \text{ R}$$

Then

$$f_{c_a} = 14.5 \times 10^{-4} \times 1.54 \times \frac{955^{0.43} \times 9230^{0.6}}{0.0833^{0.4}} = 27.7 \text{ Btu/hr ft}^2 \, ^{\circ}\text{F}$$

(Since $f_{c_g} \cong f_{c_a}$, the assumption that the tube surface temperature is the arithmetic mean of the air and the hot gas temperatures is justified.) (See equation (16), pt. I, sec. A.)

The thermal resistance of the metallic wall is negligible, so that:

$$\frac{1}{UA} = \frac{1}{(f_c A)_a} + \frac{1}{(f_c A)_g} = \frac{1}{27.7 \times 10.5} + \frac{1}{32.2 \times 9.82}$$

or

$$UA = 153 \text{ Btu/hr } ^{\circ}\text{F}$$

The hot gas, which flows through the tubes, is unmixed; while the air passing over the tubes is mixed. The heater is therefore classified as a crossflow heater, gas side unmixed, air side mixed. (See pt. II, sec. A, item 3.) In order to determine the performance of the heater, figure 34 is utilized. The parameters necessary to determine the heater effectiveness Φ_x' are:

$$B = \frac{UA}{c_{p_a} W_a} = \frac{153}{0.241 \times 3000} = 0.210$$

$$\frac{W_a \, c_{p_a}}{W_g \, c_{p_g}} = \frac{3000 \times 0.241}{5000 \times 0.277} = 0.522$$

From figure 34 the effectiveness Φ_x' is

$$\Phi_x' = 0.180$$

Thus the heater output is:

$$q = W_a c_{p_a} (\tau_{g_1} - \tau_{a_1}) \Phi_x'$$

$$= 3000 \times 0.241 \times (1600 - 10) \times 0.180 = 207,000 \text{ Btu/hr}$$

A comparison of measurements and predictions of results on a tubular-type heater is given in reference 64.

Example 4 - Finned-Type Exchanger

Calculate the performance of the crossflow finned-type exchanger shown in figure 38 for the following operating conditions:

W_a ventilating air rate = 3000 lb/hr

W_g hot gas rate = 5000 lb/hr

τ_{g_1} temperature of hot gases entering the exchanger = 1000° F

τ_{a_1} temperature of air entering the exchanger = 10° F

The exchanger consists of an aluminum casting with longitudinal fins in the center along which the hot gases flow, and with circumferential fins on the outside, along which the air flows at right angles to the direction of the hot gases. The hot gases flow through a system consisting of a straight duct. As a first approximation the air flowing along the circumferential fins can be considered as flow in a curved duct, and as a further approximation the equation for the unit thermal conductance in straight ducts utilized to calculate f_{c_a}. The following pertinent dimensions now can be calculated:

A_a cross-sectional area for air flow (sec. A-A) =

$$\frac{(12 \times 1.45 - 40 \times 0.14 \times 1.25)(2)}{144} = 0.145 \text{ ft}^2$$

A_g cross-sectional area for gas flow (sec. B-B) =

$$\frac{\pi \times 36}{4 \times 144} - \frac{30 \times 1.30}{144} \times \frac{3}{16} = 0.145 \text{ ft}^2$$

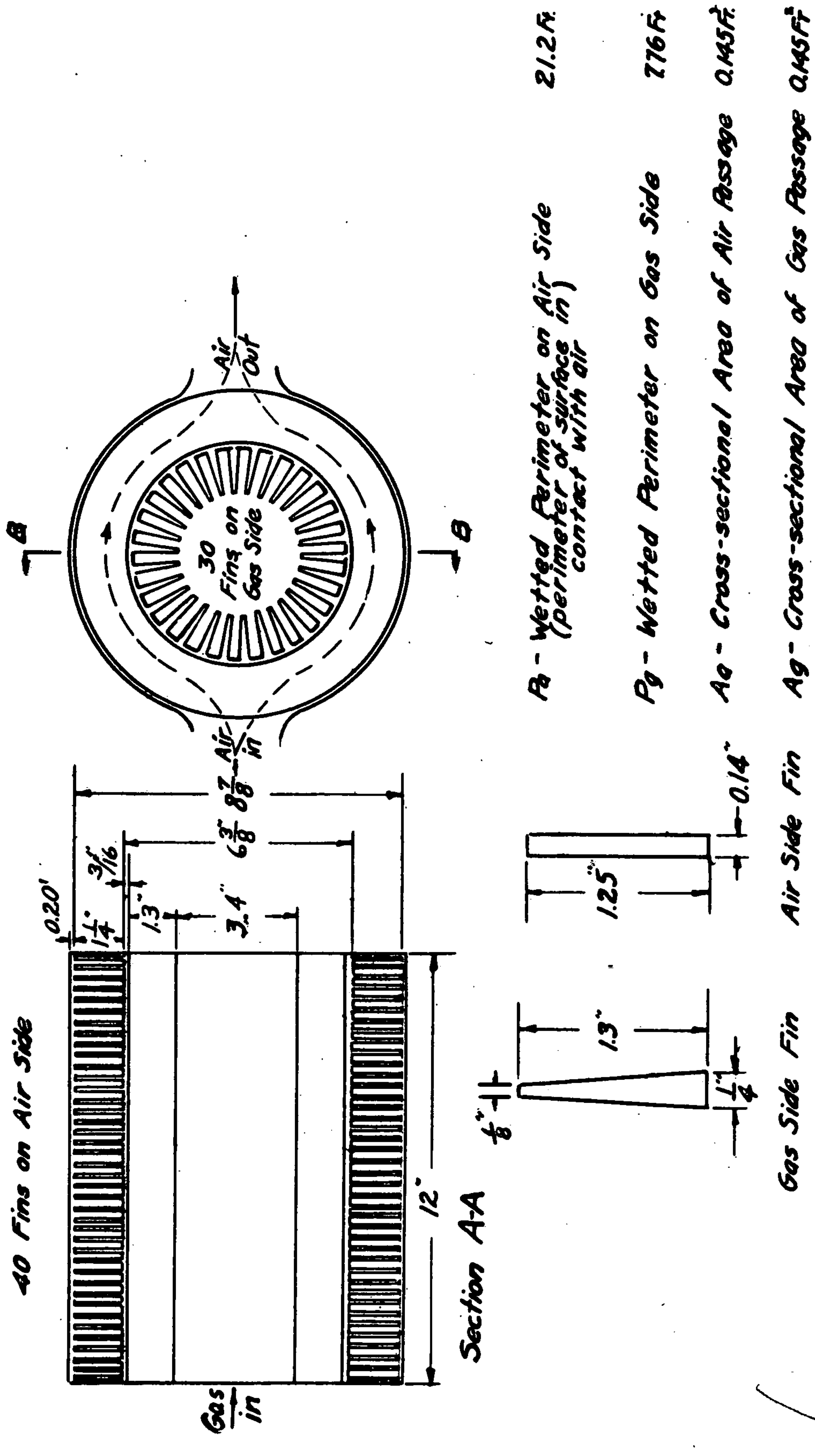


Fig. 38 - Finned-Type Heat Exchanger.

P_a wetted perimeter on air side (sec. A-A) =

$$\frac{(1.25 \times 2 \times 40 + 24 + 2.90)\ 2}{12} = 21.2 \text{ ft}$$

P_g wetted perimeter on gas side (sec. B-B) =

$$\frac{1.30 \times 30 \times 2 + \pi \times 6 - 30 \times \frac{1}{4} + 30 \times \frac{1}{8}}{12} = 7.76 \text{ ft}$$

Hydraulic diameter on air side

$$D_{H_a} = \frac{4\ A_a}{P_a} = \frac{4 \times 0.145}{21.2} = 0.0273 \text{ ft}$$

Hydraulic diameter on gas side

$$D_{H_g} = \frac{4\ A_g}{P_a} = \frac{4 \times 0.145}{7.76} = 0.0747 \text{ ft}$$

Length of duct on air side

$$L_a = \frac{7 \times \pi}{2 \times 12} = 0.916 \text{ ft}$$

Length of duct on gas side

$$L_g = 1 \text{ ft}$$

Determination of unit thermal conductances:

The Reynolds number on the air side = 10,000

The Reynolds number on the gas side = 23,700

Thus equation (25) (pt. I, sec. C) is applicable for the determination of the unit conductance on both the gas and the air side:

<u>Air Side</u>

$$f_{c_a} = 5.4 \times 10^{-4} \frac{T_a^{0.3}\ G_a^{0.8}}{D_{H_a}^{0.2}} \left(1 + 1.1 \frac{D_{H_a}}{L_a}\right)$$

The mean temperature T_a is not known, but a reasonable estimate is 550° R. This estimated magnitude must be checked with the final calculated results. The rate of flow per unit cross-sectional area G_a is:

$$G_a = \frac{3000}{0.145} = 20,800 \ \frac{\text{lb}}{\text{hr ft}^2}$$

Then

$$f_{c_g} = 5.4 \times 10^{-4} \times \frac{550^{0.3} \times 20800^{0.8}}{0.0273^{0.2}} \left(1 + 1.1 \ \frac{0.0273}{0.92}\right)$$

$$= 22.0 \text{ Btu/hr ft}^2 \ {}^{\circ}\text{F}$$

This unit conductance can be assumed to exist along both the fins and the unfinned area.

Gas Side

From equation (25)

$$f_{c_g} = 5.4 \times 10^{-4} \ \frac{T_g^{\ 0.3} \ G_g^{\ 0.8}}{D_{H_g}^{\ 0.2}} \left(1 + 1.1 \ \frac{D_{H_g}}{L_g}\right)$$

The mean temperature T_g is not known, but a reasonable estimate is 1420° R. This estimated magnitude must be checked with the final calculated results. The rate of flow per unit cross-sectional area G_a is

$$G_a = \frac{5000}{0.145} = 34,500 \ \frac{\text{lb}}{\text{hr ft}^2}$$

Then

$$f_{c_g} = 5.4 \times 10^{-4} \times \frac{1420^{0.3} \times 34500^{0.8}}{0.0747^{0.2}} \left(1 + 1.1 \ \frac{0.0747}{1.0}\right)$$

$$= 37.0 \text{ Btu/hr ft}^2 \ {}^{\circ}\text{F}$$

This magnitude of the unit conductance can be assumed to exist along both the fins and the unfinned area.

Effective Conductance of the Finned Surfaces

Air Side

The extended surface on the air side of the exchanger consists of circumferential fins. Thus equation (38) (pt. I, sec. F) is applicable for the evaluation of the effective thermal conductance on the air side of the heat exchanger surface.

$$(fA)_{ea} = \pi D_c n \sqrt{2f_F ks} \left(1 + \frac{L}{D_c}\right) \tanh \sqrt{\frac{2f_F L^2}{ks}} + f_u A_u$$

From figure 38 and the previous calculations:

D_c diameter of cylinder to which fins are attached = 0.531 ft

n number of fins = 40

f_F unit thermal conductance from fin to air = f_{c_a} = 22.0 Btu/hr ft^2 $^\circ$F

k thermal conductivity of fin material (aluminum) at an average temperature of 200° F = 120 Btu/hr ft^2 ($^\circ$F/ft)

s thickness of fin = 0.0117 ft

L length of fin perpendicular to cylinder = 0.104 ft

f_a unit thermal conductance from surface of cylinder to air, approximately equal to f_{c_a} = 22.0 Btu/hr ft^2 $^\circ$F

A_u area of cylinder not covered by fins = $\pi \times 0.531 \times 1 - 40 \times 0.0117$ = 1.20 ft^2

Substituting the foregoing values in the equation for $(fA)_{ea}$ yields:

$$(fA)_{ea} = \pi \times 0.531 \times 40 \sqrt{2 \times 22.0 \times 120 \times 0.0117} \left(1 + \frac{0.104}{0.531}\right) \tanh 0.582$$

$$= 1.20 \times 22.0$$

$$(fA)_{ea} = 329 + 26 = 355 \text{ Btu/hr ft}^2 \ ^\circ\text{F}$$

Gas Side

On the gas side of the exchanger the extended surface consists of rectangular fins parallel to the direction of gas flow. Thus equation (36) is applicable for the evaluation of $(fA)_{eg}$.

$$(fA)_{eg} = n l \sqrt{2skf_F} \tanh \sqrt{\frac{2f_F L^2}{ks}} + f_u A_u$$

From figure 38 and previous calculations:

L 0.108 ft

n number of fins, 30

l length of fins in direction of gas flow = 1 ft

s thickness of fins = 0.0156 in.

k thermal conductivity of aluminum at 600° F
= 140 Btu/hr ft² $\left(\frac{°F}{ft}\right)$

f_F unit thermal conductance from gas to fin = f_{c_g}
= 37.0 Btu/hr ft² °F

f_u unit thermal conductance from gas to part of surface not covered by fins = f_{c_g} = 37.0 Btu/hr ft² °F

A_u = area of cylinder not covered by fins = $\pi \times 0.5 - \frac{30 \times 0.25}{12} = 0.95$ ft²

Substituting the preceding magnitudes of the variables into the equation for $(fA)_{eg}$ yields:

$$(fA)_{eg} = 30 \times 1 \sqrt{2 \times 0.0156 \times 140 \times 37} \tanh 0.630 + 0.95 \times 37$$

$$(fA)_{eg} = 213 + 35 = 248 \text{ Btu/hr } °F$$

The over-all conductance of the heat exchanger may now be calculated:

$$\left(\frac{1}{UA}\right) = \frac{1}{(fA)_{ea}} + \frac{1}{(fA)_{eg}} = \frac{1}{355} + \frac{1}{248} = 0.00686$$

Thus

$$UA = 146 \text{ Btu/hr } ^{o}F$$

Because both gases flow between fins and are thereby prevented from mixing, the heater is a crossflow type heater with neither fluid mixed. (See pt. II, sec. A, item 3.) In order to determine the performance of the heater figure 34 is utilized. The parameters necessary to determine the heater effectiveness Φ_x are:

$$\frac{W_a\ c_{p_a}}{W_g\ c_{p_g}} = \frac{3000 \times 0.241}{5000 \times 0.263} = 0.550$$

$$\frac{UA}{W_a\ c_{p_a}} = \frac{146}{3000 \times 0.241} = 0.202$$

From figure 34 the effectiveness Φ_x is:

$$\Phi_x = 0.172$$

Thus the heater output is:

$$q = W_a\ c_{p_a} (\tau_{g_1} - \tau_{a_1})\ \Phi_x$$

$$q = 3000 \times 0.241\ (1000 - 10) \times 0.172$$

$$q = 123,000 \text{ Btu/hr}$$

It should be noted that although the effectiveness Φ_x of this heater is as high as those determined in the other examples, the heater output is smaller because of the lower temperature of the entering hot gas. For test data on finned exchangers, see references 58, 60, and 63.

III. HEATER PERFORMANCE IN FLIGHT

A. HEAT REQUIREMENTS

1. Cabin Heating

The heat requirements for cabin heating depend upon the heat loss through the cabin walls and the rate of air leakage into the cabin. (See reference 67.) In order to establish the heater requirements, a heat balance can be performed on the cabin of an airplane as shown in figure 39.

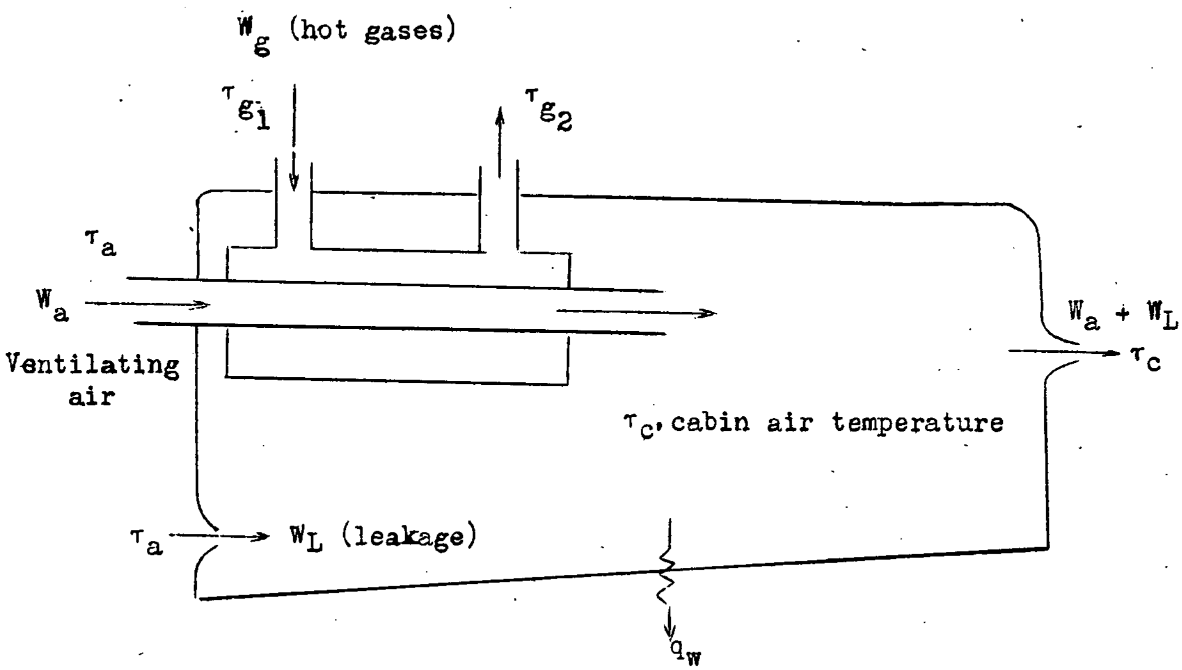


Figure 39.- Mass and thermal balance on airplane cabin.

Then:

$$W_g \, c_{p_g} \, (\tau_{g_1} - \tau_{g_2}) + c_{p_a} \, (W_a + W_L) \, (\tau_a - \tau_c) - q_w = 0$$

but

$$W_g \, c_{p_g} \, (\tau_{g_1} - \tau_{g_2}) = q_H \quad \text{(Heater output)}$$

Thus:

$$q_H = (W_a + W_L)\, c_{p_a}\, (\tau_c - \tau_a) + q_w$$

or

$$\frac{q_H}{\tau_c - \tau_a} = \left[W_L\, c_{p_a} + \frac{q_w}{(\tau_c - \tau_a)} \right] + W_a\, c_{p_a}$$

since q_w, the rate of heat flow through cabin walls equals $q_w = (UA)\,(\tau_c - \tau_a)$ the following relation may be written:

$$\frac{q_H}{(\tau_c - \tau_a)} = \text{(necessary heater output in (Btu/hr) per degree difference in temperature between cabin air and outside air)}$$

$$= \left[W_L\, c_{p_a} + UA \right] + W_a\, c_{p_a} \qquad (48)$$

or

$$= UA \left[1 + \frac{(W_L + W_a)}{UA}\, c_{p_a} \right] \qquad (49)$$

where

W_L rate of flow of leakage air into cabin, lb/hr

c_{p_a} unit heat capacity of air at constant pressure, Btu/lb °F

UA over-all thermal conductance between cabin air and outside air, Btu/hr °F

W_a ventilating air rate for a ram operated heater, lb/hr

The multiplier of UA in equation (49) is the ratio of the heater output required when there is air leakage (either through the heater W_a or through cracks W_L) to that which would be required if there was no leakage of any kind, The magnitude of this multiplier is a measure of the "tightness" of an airplane cabin.

The magnitudes of UA can be readily estimated by utilizing the equations presented in reference 67, or by installation of heat meters (references 68 and 69) on the cabin walls. Obviously UA can be greatly decreased by the use of insulation.

The leakage air rate W_L usually is unknown, but for a given airplane may be determined by measurement, for equation (48). By measuring in flight all other terms in the equation except W_L, this magnitude may be easily determined as a function of altitude, airplane speed, and so forth. Efforts should be made to reduce the leakage term as much as possible in order to reduce the thermal output of the heater required to maintain a given cabin air temperature.*

It is instructive to note that, particularly if the leakage air rate W_L is made negligible, it is advantageous to design a heater with a low ventilating air rate W_a, since the smaller W_a, the smaller the heater output required. If the leakage is large, however, decreasing W_a has a small effect on the necessary heater output.

2. Wing Anti-Icing

In the case of wing anti-icing, the conductances over the airfoil and in the air ducts may be estimated by means of the equations presented in this report. Valuable flight data on wing anti-icing systems will be found in the reports from the Ames laboratory. (See references 70, 71, 72, 73, 74, and 75.) A valuable report on wing anti-icing has been written recently by Myron Tribus. (See reference 82.)

B. CORRECTION OF THE PERFORMANCE OF HEAT EXCHANGERS TO ALTITUDE CONDITIONS

1. Thermal Performance

The thermal output of a heater, as discussed in part II, may be written as:

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*If the temperatures in the cabin are sufficiently uniform, it may be possible to determine both UA and W_L by direct flight measurement as follows: At one altitude and airplane speed measure W_a, τ_c, τ_a at two different values of q_H. This will allow equation (48) to be written twice with two unknowns, UA and W_L. Since these variables are practically unchanged in the two tests, they may be solved for. By performing these tests at various altitudes and airplane speeds the variation of UA and W_L with these variables may be determined.

$$q = W_a c_{p_a} (\tau_{g_1} - \tau_{a_1}) \Phi \tag{50}$$

where Φ, the effectiveness of the heater, a function of $\left(\frac{UA}{W_a c_{p_a}}\right)$ and $\left(\frac{W_a c_{p_a}}{W_g c_{p_g}}\right)$ is shown in figures 32, 33, and 34, for parallelflow, counterflow, and crossflow exchangers. Inspection of equation (50) reveals that for fixed values of $W_a c_{p_a}$ and $W_g c_{p_g}$, the heater output is proportional to the ratio of the entrance temperature difference and the heater effectiveness Φ. Thus, for fixed weight rates*

$$\frac{q_{alt}}{q_{lab}} = \frac{(\tau_{g_1} - \tau_{a_1})_{alt}}{(\tau_{g_1} - \tau_{a_1})_{lab}} \frac{\Phi_{alt}}{\Phi_{lab}} \tag{51}$$

The ratio of inlet temperature differences is self-explanatory. This ratio depends only on the temperatures existing during the laboratory test and during the actual operating conditions at altitude. The magnitude of the atmospheric pressure does not affect this ratio.

The second ratio is more complex, but, as shown in reference 58, for equal weight rates of gas and air it is independent of atmospheric pressure and slightly dependent upon the mean absolute temperature of the two gases. As shown in reference 58, for the maximum temperature variations met in practice, the ratio $\frac{\Phi_{alt}}{\Phi_{lab}}$ does not vary from unity more than 10 percent. Because of the complexity of evaluation of this ratio, it is suggested that for a first estimate the heater output can be corrected to any altitude with sufficient accuracy by multiplying by the temperature difference ratio only. Thus for fixed gas and air weight rates

$$\frac{q_{alt}}{q_{lab}} = \frac{(\tau_{g_1} - \tau_{a_1})_{alt}}{(\tau_{g_1} - \tau_{a_1})_{lab}} \tag{52}$$

---

*c_{p_a} and c_{p_g} vary slightly with temperature and are practically independent of pressure in the range under consideration.

where

q_{alt} heater output at any altitude for given values of air and gas rates, Btu/hr

q_{lab} heater output obtained in laboratory (or calculated), Btu/hr, for the same values of W_g, W_a as required for q_{alt}

$(\tau_{g_1}-\tau_{a_1})_{alt}$ difference between the temperature of hot gases and cold air entering exchanger at any altitude, ^oF

$(\tau_{g_1}-\tau_{a_1})_{lab}$ difference in temperature between hot gases and cold air entering heater, in laboratory test, ^oF

If a more precise correction is required, reference 58 should be consulted.

2. Pressure Drop across Heater

a. Isothermal.- The basic pressure drop measurement required to establish heater performance is the isothermal total-pressure drop across the heater. The isothermal total-pressure drop represents the loss due to frictional forces such as skin friction, sudden expansion, sudden contraction, and so forth, and is an irrecoverable loss. Pressure drops due to the acceleration of the fluid may be recovered.

The isothermal total-pressure drop may be obtained experimentally in two ways:

(1) By traversing the duct before and after the heater with a total-head tube during isothermal flow. The total-head loss (reference 7, p. 206 and reference 76) for a heater with circular inlet and outlet ducts then will be:

$$\Delta F_{a-b} = (P_a - P_b) + \frac{3600\,\gamma^2}{2g}\left[\frac{\int_0^{r_a} 2\pi\, v_a^3\, r dr - \int_0^{r_b} 2\pi\, v_b^3\, r dr}{W}\right] \qquad (53)$$

The integrals indicated must be evaluated graphically, utilizing the pitot tube data to establish v_a, v_b as a function of r. An approximation of these integrals may be obtained from measurements with a **pitot tube at appropriate** points across the pipe section. **(See "ten-point method" in reference 77.)**

In oquation (53)

ΔF_{a-b} isothermal frictional pressure loss between sections a and b, lb/ft^2

P_a static pressure at section a, lb/ft^2

P_b static pressure at section b, lb/ft^2

γ density of fluid, lb/ft^3

g gravitational force per unit mass = 32.2 $lb\Big/\left(\frac{lb\ sec^2}{ft}\right)$

v_a velocity of fluid at any radius of pipe r at section a, ft/sec

v_b velocity of fluid at any radius of pipe r at section b, ft/sec

r any radius, ft

r_a inside radius of pipe at section a, ft

r_b inside radius of pipe at section b, ft

W weight rate of fluid, lb/hr

(2) If the areas at a and b are equal and if the velocity distribution across the two sections is postulated to be the same, equation (53) reduces to:

ΔF_{a-b} (friction pressure loss)

$= P_a - P_b$ (static pressure drop)

Thus, as an approximation, the isothermal friction pressure loss across a heater may be obtained by the static pressure drop measured at sections of equal areas.

b. Nonisothermal.- Once the isothermal frictional pressure loss ΔF_{a-b} has been obtained for a series of weight rates, the nonisothermal static pressure drop for the same weight rates at any altitude may be readily calculated by means of the following equation (reference 16, p. 130 of 2d ed., and reference 78):

$$(P_a - P_b)_{non-iso} = \Delta F_{a-b}\left(\frac{T_a + T_b}{2\ T_{iso}}\right)^{1.13}\left(\frac{P_{iso}}{P}\right)$$

$$+ \left(\frac{W_{iso}}{3600}\right)^2 \frac{R\ T_a}{2g\ A_h^2\ P}\left[\left(\frac{A_h^2}{A_b^2} + 1\right)\frac{T_b}{T_a} - \left(\frac{A_h^2}{A_a^2} + 1\right)\right] \qquad (54)$$

where

P_a static pressure at section a, entrance to heater, lb/ft^2

P_b static pressure at section b, exit from heater, lb/ft^2

ΔF_{a-b} isothermal friction pressure loss through heater at weight rate W_{iso}, temperature T_{iso}, and pressure P_{iso}, lb/ft^2 (This friction pressure loss includes any irrecoverable losses from sudden contraction, expansion, etc., as well as skin friction.)

T_a temperature of fluid entering exchanger, ^oR

T_b temperature of fluid leaving exchanger, ^oR

T_{iso} temperature of fluid during isothermal pressure drop determination, ^oR

P_{iso} average pressure in heater during isothermal pressure drop determination, lb/ft^2 abs.

P average pressure in heater at any altitude, lb/ft^2 abs.

W_{iso} weight rate of fluid for which isothermal and nonisothermal pressure drops are being determined, lb/hr

R gas constant for air = 53.3, ft-lb/lb °R

g gravitational force per unit mass = 32.2 $lb/\left(\frac{lb\ sec^2}{ft}\right)$

A_h constant cross-sectional area of portion of heater in which heat transfer takes place, ft^2

A_a cross-sectional area of inlet duct to heater at which static pressure P_a is measured, ft^2

A_b cross-sectional area of heater outlet duct at which static pressure P_b is measured, ft^2

The first term on the right of the equal sign

$$\Delta F_{a-b} \left(\frac{T_a + T_b}{2T_{iso}}\right)^{1.13} \left(\frac{P_{iso}}{P}\right)$$

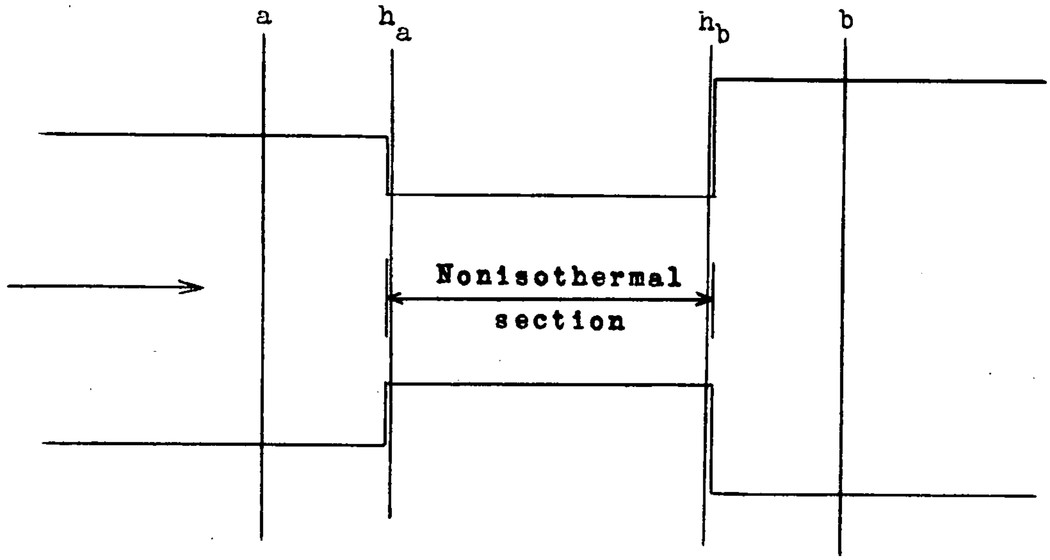
represents the irrecoverable, **nonisothermal** pressure loss due to friction, including sudden expansion, contraction, and so forth. This term, which also can be written

$$\left(\frac{P_{iso}}{P_{av}}\right)\left(\frac{T_{av}}{T_{iso}}\right)^{0.13} \Delta F_{a-b}$$

is used to predict the **nonisothermal** frictional pressure losses from the isothermal values.

The ratio $\left(\frac{T_{av}}{T_{iso}}\right)^{0.13}$ is an approximate correction for the change in friction factor ξ with change of Reynolds number (Reynolds number changes are caused by changes in absolute viscosity as the fluid is heated or cooled).

The ratio $\left(\frac{P_{iso}}{P_{av}}\right)$ is an approximate correction for the variation of pressure drop with changes of density caused by temperature and altitude effects. Because of the fact that an arithmetic average density is used, the correction (for a fluid being heated) is too high for the heater-entrance contraction losses and too low for the heater-exit expansion



Section	a	h_a	h_b	b
Area	A_a	A_h	A_h	A_b
Temperature (iso.)	T_{iso}	T_{iso}	T_{iso}	T_{iso}
Temperature (noniso.)	T_a	T_a	T_b	T_b
Mean velocity	u_a	u_{h_a}	u_{h_b}	u_b
Static pressure	p_a	p_{h_a}	p_{h_b}	p_b
Average absolute static pressure	P	P	P	P
Specific volume (noniso.)	V_a	V_a	V_b	V_b

Figure 39a.- Isothermal and nonisothermal flow conditions in heater.

losses. These errors* probably partially compensate one another. A slight error is made, also, when ΔF_{a-b} is multiplied by $\left(\frac{T_{av}}{T_{iso}}\right)^{0.13}$, because expansion and contraction losses should not be corrected for changes in viscosity with temperature.

The second term to the right of the equal sign of equation (54) represents a pressure drop due to the acceleration of the fluid, which results both from changes in cross-sectional areas and changes in density due to heating or cooling. This term

$$\left(\frac{W_{iso}}{3600}\right)^2 \frac{R\ T_a}{2g\ A_n^{\ 2}\ P} \left[\left(\frac{A_h^{\ 2}}{A_b^{\ 2}} + 1\right)\frac{T_b}{T_a} - \left(\frac{A_h^{\ 2}}{A_a^{\ 2}} + 1\right)\right]$$

may be written as

$$\left(\frac{1}{2}\rho_b\ u_b^{\ 2} - \frac{1}{2}\rho_a\ u_a^{\ 2}\right) + \left(\frac{1}{2}\rho_b\ u_{h_b}^{\ 2} - \frac{1}{2}\rho_a\ u_{h_a}^{\ 2}\right)$$

or more simply

$$(q_b - q_a) + (q_{h_b} - q_{h_a})$$

where q is used to denote the velocity pressure $\rho\ u^2/2$

Equation (54) may thus be rewritten as:

$$(P_a + q_a) - (P_b + q_b) = \Delta F_{a-b}\left(\frac{P_{iso}}{P_{av}}\right)\left(\frac{T_{av}}{T_{iso}}\right)^{0.13} + (q_{h_b} - q_{h_a}) \quad (54a)$$

---

*These errors are reduced by use of the following expression:

$$\Delta F_{a-b} = \left(\frac{P_{iso}}{P_a}\right)\Delta F_{a,iso} + \left(\frac{P_{iso}}{P_{av}}\right)\left(\frac{T_{av}}{T_{iso}}\right)^{0.13}\Delta F_{fric} + \left(\frac{P_{iso}}{P_b}\right)\Delta F_{b,iso}$$

in which the pressure loss through the heater shown in fig. 39a is divided into a contraction term, a friction term, and an expansion term.

The term $(P_a + q_a) - (P_b + q_b)$ is the difference in total pressures at points a and b as measured by an impact (pitot) tube. Since total pressure represents the energy available for pumping this fluid, equation (54a) probably has more physical significance than its equivalent, equation (54). It should be noted that the nonisothermal frictional (or irrecoverable) pressure loss between sections a and b (the term in equation (54a) involving ΔF_{a-b}) differs from the difference of total pressures by the term $(q_{h_b} - q_{h_a})$ which is the change in velocity pressure through the heater resulting from the heating or cooling of the fluid. The term $(q_{h_b} - q_{h_a})$ for air being heated represents a loss, which cannot be "recovered" except by cooling the fluid as it passes through the discharge duct.* It is not possible to recover the loss $q_{h_b} - q_{h_a}$ by diffusers and other mechanical means. Thus, if the discharge duct from a heater is adiabatic, the term $q_{h_b} - q_{h_a}$ in effect represents an irrecoverable loss which should be charged against the heater. If the fluid is cooled in the discharge duct, however, a gain in velocity pressure will result which can be less than, equal to, or greater than the loss $q_{h_b} - q_{h_a}$, depending on the amount of cooling; that is, cooling the fluid is equivalent to introducing a pump in the discharge duct. In practice, however, the term $q_{h_b} - q_{h_a}$ usually is small and can be neglected. It should be emphasized, however, that equation (54a) reveals that in nonisothermal flow the frictional pressure loss is not exactly equal to the total-pressure difference.

C. ALTITUDE PERFORMANCE OF HEATER AND DUCT SYSTEM

The analysis of the performance of a ram (or fan) operated heater and duct system (see fig. 40) as a function of true airplane speed and altitude is presented in detail in reference 78. The two basic equations necessary for the analysis are presented:

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*This is accomplished in a wing de-icing system.

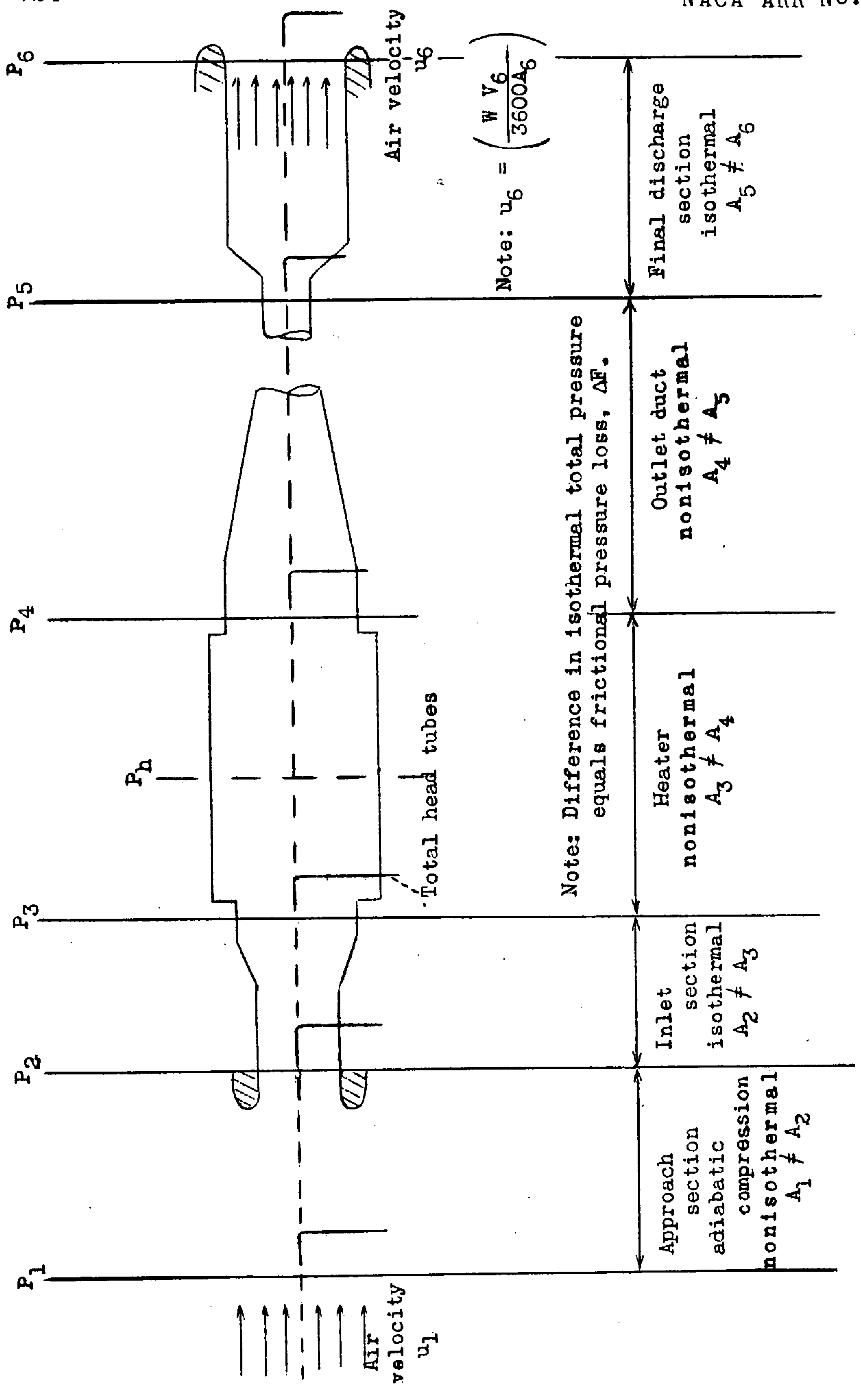


Figure 40.- Elements of heater and duct system.

$$\left(P_1 + \frac{u_1^2}{2g\ V_1}\right) - \left(P_6 + \frac{u_6^2}{2g\ V_6}\right)$$

$$= \left(\frac{W}{3600}\right)^2 \frac{R}{2g\ P_1} \left\{\frac{T_3}{A_3^2} + \frac{T_3}{A_h^2} \left[\left(\frac{A_h^2}{A_4^2} + 1\right)\frac{T_4}{T_3} - \left(\frac{A_h^2}{A_3^2} + 1\right)\right]\right.$$

$$\left. + \frac{2}{T_4 + T_5} \left[\left(\frac{T_5}{A_5}\right)^2 - \left(\frac{T_4}{A_4}\right)^2\right] - \frac{T_5}{A_5^2}\right\}$$

$$+ \left(\frac{W}{W_{iso}}\right)^n \left(\frac{P_{iso}}{P_1}\right) \left\{\Delta F_{1-2} \left(\frac{T_1 + T_2}{2\ T_{iso}}\right)^{1.13} + \Delta F_{2-3} \left(\frac{T_2}{T_{iso}}\right)^{1.13}\right.$$

$$+ \Delta F_{3-4} \left(\frac{T_3 + T_4}{2\ T_{iso}}\right)^{1.13} + \Delta F_{4-5} \left(\frac{T_4 + T_5}{2\ T_{iso}}\right)^{1.13}$$

$$\left. + \Delta F_{5-6} \left(\frac{T_5}{T_{iso}}\right)^{1.13}\right\} - \Delta P_{fan} \qquad (55)$$

where

ΔP_{fan} total pressure rise across a fan which may be placed in inlet duct, lb/ft²

n exponent to account for friction loss variation with W due to expansion and contraction and to skin friction (Its value will be between 1.75 and 2.00; nearer to 1.75 if the major loss is due to skin friction.* (May be obtained from a plot of W_{iso} versus isothermal frictional pressure loss through system.).)

P_1 absolute static pressure in free air stream before air scoop, lb/ft²

***See references 15, 29, 45, 58, 59, 60, 61, 62 and 63 for typical values of the exponent n.**

u_1 true air speed of airplane + air velocity produced by propeller (velocity of air stream relative to airplane, ahead of air scoop), ft/sec

V_1 specific volume of air in free air stream before air scoop, ft^3/lb

g gravitational force per unit mass

$$= 32.2 \text{ lb} \Big/ \left(\frac{\text{lb sec}^2}{\text{ft}}\right)$$

P_6 absolute static pressure at point of air discharge, lb/ft^2

u_6 velocity of air relative to airplane at point of air discharge, ft/sec

V_6 specific volume of air at point of air discharge, cu ft/lb

$\left(P_1 + \frac{u_1^2}{2g\ V_1}\right)$ total pressure in free air stream before scoop, lb/ft

W air flow through duct, lb/hr

W_{iso} air flow through duct for which isothermal total head losses, ΔF_{1-2}, ΔF_{2-3}, ΔF_{3-4}, ΔF_{4-5}, etc., were determined, lb/hr

R gas constant for air = 53.3 ft lb/lb $^{\circ}R$

T_1 absolute temperature of air in free air stream, $^{\circ}R$

T_2 absolute temperature of air, just inside air scoop, $^{\circ}R$

T_3 absolute temperature of air at entrance to heat exchanger, $^{\circ}R$

A_3 cross-sectional area at section 3-3, entrance to heat exchanger, ft^2

T_4 mixed-mean absolute temperature of air at section 4-4, exit of heat exchanger, $^{\circ}R$

t_4 mixed-mean temperature of air at section 4-4, exit of heat exchanger, °F

A_4 cross-sectional area at section 4-4, exit of heat exchanger, ft²

T_5 mixed-mean absolute temperature of air after passing through nonisothermal duct just before final discharge section, °R

A_5 cross-sectional area of duct at section 5-5, just before final discharge section, ft²

P_{iso} average static pressure in duct system during the isothermal total-pressure drop test, lb/ft²

ΔF_{1-2} frictional pressure loss between the free air stream and entrance to air scoop for isothermal conditions specified by P_{iso}, T_{iso}, W_{iso}, lb/ft²

ΔF_{2-3} frictional pressure loss between entrance of air scoop and entrance to heat exchanger, for isothermal conditions specified by P_{iso}, T_{iso}, W_{iso}, lb/sq ft*

ΔF_{3-4} frictional pressure loss across heat exchanger for isothermal conditions specified by P_{iso}, T_{iso}, and W_{iso}, lb/ft²

ΔF_{4-5} frictional pressure loss through all ducts after heat exchanger up to final discharge section for isothermal conditions specified by P_{iso}, T_{iso}, and W_{iso}. If desired, the pressure drop ΔF_{4-5} may be subdivided into any number of smaller components, each of which must be corrected to nonisothermal conditions by the methods outlined in equation (6) of reference 78, lb/sq ft

ΔF_{5-6} frictional pressure loss in isothermal discharge section for isothermal conditions specified by P_{iso}, T_{iso}, and W_{iso}, lb/sq ft

In equation (55) the terms on the left of the equal sign represent the difference in total pressure between the free air stream and the point of air discharge. The first term on the right of the equal sign represents the pressure

*ΔF can easily be measured at sea level by an isothermal pressure loss test on a mock-up of the air distribution system.

changes due to the acceleration of the air in the duct. owing both to changes in area and changes in specific volume. This term usually is quite small compared with the second. The second term represents the irrecoverable pressure loss due to the friction in the complete duct system. It should be noted that each isothermal frictional loss is corrected to the operating temperature by different temperature corrections, depending on the type of flow system represented by each separate ΔF. Thus, any complex flow system can be broken up into a series of systems, and the pressure drop through each corrected to nonisothermal conditions by the method outlined. The last term, ΔP_{fan} represents the total pressure rise across a fan which may be placed in (say) the inlet duct to augment the ram pressure. The pressure change due to the adiabatic compression of the air between the free stream and the scoop entrance, sections 1-2, is neglected in this equation. This pressure change is small for usual aircraft speeds, but the temperature rise may be appreciable for velocities in excess of 300 miles per hour and may be calculated from the equation (reference 78):

$$T_2 = T_1 + \frac{k-1}{R\,k}\left[\frac{u_1^2 - u_2^2}{2g}\right]$$

where:

k exponent for adiabatic compression in equation

$$P_1 V_1^k = P_2 V_2^k$$

R gas constant for air, 53.3 ft-lb/lb °R

In equation (55), for a given duct system for which the isothermal friction pressure loss ΔF_{1-2}, ΔF_{2-3}, ΔF_{3-4}, ΔF_{4-5}, and ΔF_{5-6} are known, the remaining unknowns are W and T_4. The fixing of the altitude, the airplane speed, and the heat loss from the duct establishes all other variables in the equation. Thus, for any altitude and airplane speed a curve of W versus T_4 can be drawn, which will reveal the rate of flow possible through the duct system for any temperature T_4.

The relative importance of the various portions of the duct system may be readily established, for the largest of the corrected pressure drop terms in equation (55), will be the term which controls the rate of air flow. If it becomes necessary to increase the rate of flow through the heater-duct system, attention should be focused on the largest term. By breaking up a complex duct system into a series of small units, the units causing a difficulty then may be readily isolated.

Having established the curve of W versus T_4 from a consideration of the pressure drop characteristics of the duct system (from equation (55)), the thermal performance of the heater must be utilized in order to establish the operating point of the heater-duct system. The thermal performance of the heater is used to establish a second curve of W versus T_4, which is fixed by the thermal output of the heater, since for any particular W and exhaust gas temperature, only one magnitude of T_4 is possible. The relation,

$$q_{alt} = W c_p (T_4 - T_3) \tag{56}$$

or

$$T_4 = \frac{q_{alt}}{W c_p} + T_3 \tag{57}$$

is utilized to obtain this second curve. The heater capacity q_{lab}, determined in the laboratory, must be corrected to altitude and temperature conditions by the method outlined in part III, section B-1. Temperature T_3 must include the temperature increase of the air due to compressibility between the free air stream and the scoop. The intersection of the curve of W versus T_4 obtained from the pressure drop characteristics of the heater-duct system (equation (55)), and the curve of W versus T_4 from equation (57), fixes the operating point of the system at the particular altitude and airplane speed under consideration, and allows the complete prediction of ventilating air rate, air temperature leaving the heater and heater output as a function of airplane speed and altitude.*

For convenience in calculating the approximate performance of the heater and duct system at various altitudes and airplane speeds, when flight data have been obtained at one altitude and airplane speed the following simplifications of equation (55) are presented:

***See reference 78 for a detailed example of such a prediction.**

1. The first term on the right of the equal sign, which represents acceleration pressure drop, is usually negligible in comparison with the other terms. Thus, as a very close approximation (omitting the fan and the corresponding ΔP_{fan}),

$$\left(P_1 + \frac{u_1^2}{2g\ V_1}\right) - \left(P_6 + \frac{u_6^2}{2g\ V_6}\right)$$

$$= \left(\frac{W}{W_{iso}}\right)^n \left(\frac{P_{iso}}{P}\right) \sum \Delta F_{a-b} \left(\frac{T_a + T_b}{2T_{iso}}\right)^{1.13} \qquad (58)$$

2. If the velocity u_6 is very small and the static pressure P_1 is nearly equal to P_6, which is the usual case for cabin heating, the equation reduces to:*

$$\frac{u_1^2}{2g\ V_1} = \left(\frac{W}{W_{iso}}\right)^n \left(\frac{P_{iso}}{P}\right) \sum \Delta F_{a-b} \left(\frac{T_a + T_b}{2T_{iso}}\right)^{1.13} \qquad (59)$$

The ratio $\frac{u_1^2}{2g\ V_1}$ is proportional to the square of the indicated airspeed u_i^2. If, as a rough approximation, the temperatures of the air passing through the duct are considered invariant with operating conditions, then the summation term is a constant for one duct-heater combination. Thus

$$W = k \left[(u_i)^2\ P \right]^{\frac{1}{n}} = k(u_i \sqrt{P})^{\frac{2}{n}} \qquad (60)$$

where k = constant. If the variation of the exponent n from the second power is neglected, the rate of ventilating air flow due to the ram

---

*If the velocity u_6 cannot be neglected, as may be the case in a wing anti-icing system, the term $\frac{u_6^2}{2g\ V_6}$ must be retained in the equation.

(when the velocity $u_6 = 0$) is approximately proportional to the first power of the indicated airspeed and the square root of the altitude pressure (when the velocity u_6 is small). If by test the rate of air flow through the heater and duct system is known at one altitude and indicated airspeed, the ventilating air rate at any other altitude and airspeed can be readily estimated by means of equation (60). Once the rate of air flow at various altitudes has been estimated, the temperature of the air leaving the exchanger may be calculated by means of the equation

$$q_{alt} = W_a c_{p_a} (T_4 - T_3) \tag{61}$$

where

q_{alt} heater output at weight rate W_a, corrected to altitude conditions, Btu/hr

W_a ventilating-air rate, lb/hr

c_{p_a} heat capacity of air, Btu/lb °F

T_3 absolute temperature of air entering exchanger, °R

T_4 absolute temperature of air leaving exchanger, °R

This method may be used to obtain an estimate of heater and duct performance. The method is approximate and reveals nothing of the pressure distribution along the duct system. For a more detailed and precise analysis which will reveal the pressure distribution, the graphical method presented in reference 78 must be utilized.

Referring again to equation (55), the following pertinent facts are noted. The terms of the form

$$\Delta F_{a-b} \left(\frac{W}{W_{iso}}\right)^n \left(\frac{P_{iso}}{P}\right) \left(\frac{T_a + T_b}{2T_{iso}}\right)^{1.13} \tag{62}$$

represent the irrecoverable friction pressure loss between two sections of the flow system, a and b, for nonisothermal flow. It should be noted particularly that the temperature multiplier $\left(\frac{T_a + T_b}{2T_{iso}}\right)^{1.13}$ is greatest for the air just leaving the heater, because, during normal operation, the air temperature is highest when just leaving the heater. Thus, special care must be taken to design the heater discharge section (or elbow) so as to make the isothermal friction pressure loss ΔF_{a-b} across this section as small as possible. If the isothermal friction pressure loss across this section is large, the nonisothermal friction pressure loss (due to the temperature multiplier) becomes excessive and may readily invalidate the advantages of a heater with low pressure drop.

University of California,
Berkeley, Calif., January 1944.

IV

APPENDIX A

PROPERTIES OF AIR*

Temperature (°F)	c_p (Btu/lb °F)	$\mu \times 10^9$ (lb sec/ft²)	k $\left(\text{Btu/hr ft}^2 \left(\frac{°F}{ft}\right)\right)$	Prandtl number $= \frac{\mu c_p}{k}$ (3600 g)
-100	0.2393	280	0.0104	0.743
0	.2398	343	.0130	.731
100	.2403	398	.0157	.706
200	.2412	449	.0182	.690
300	.2427	498	.0205	.682
400	.2449	542	.0228	.677
500	.2476	587	.0250	.672
600	.2505	630	.0272	.668
700	.2534	663	.0293	.666
800	.2566	699	.0314	.663
900	.2598	732	.0334	.660
1000	.2630	767	.0355	.658
1100	.2660	800	.0376	.655
1200	.2690	832	.0399	.652
1300	.2715	864	.0419	.650
1400	.2740	896	.0440	.648
1500	.2766	928	.0461	.646
1600	.2789	960	.0484	.643

*See references 17 and 79 for properties of other gases, N_2, O_2, CO, CO_2, and H_2.

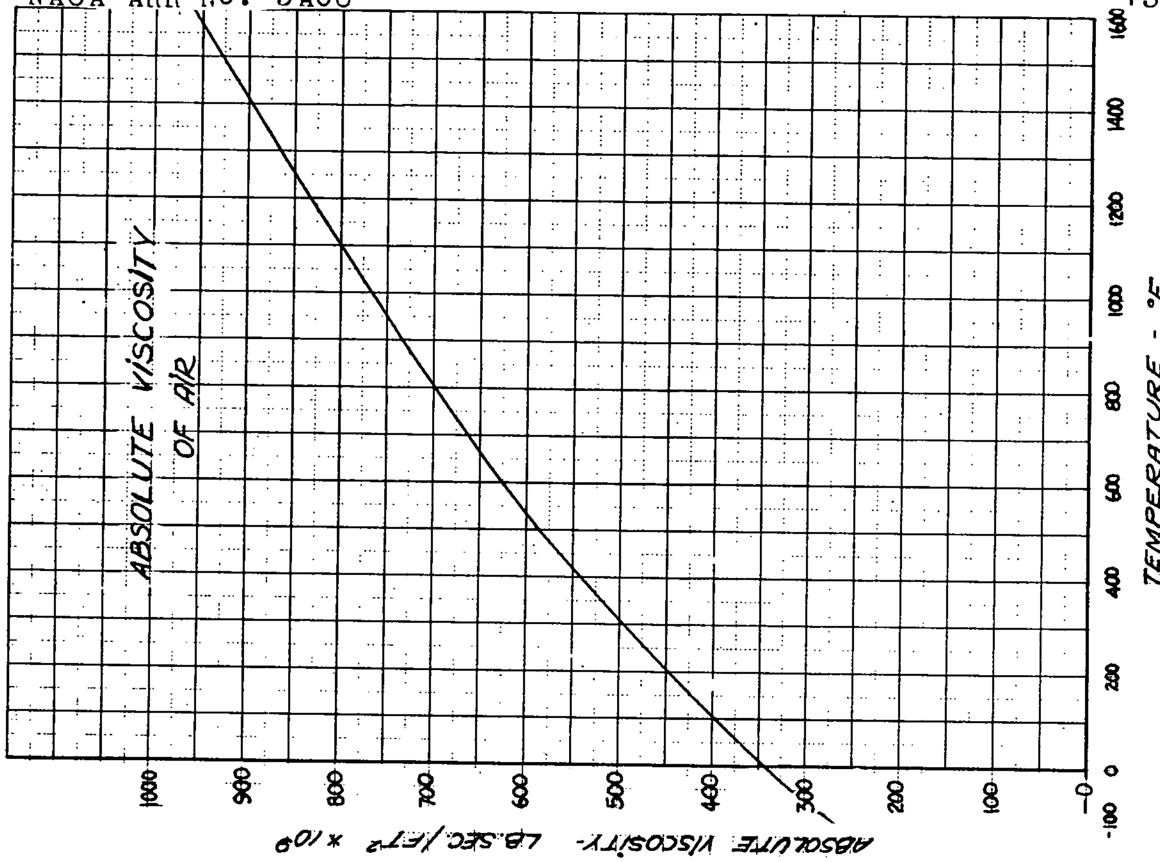

ABSOLUTE VISCOSITY OF AIR
ABSOLUTE VISCOSITY- LB.SEC./FT² × 10⁹
1000
900
800
700
600
500
400
300
200
100
0
TEMPERATURE - °F
-100
0
200
400
600
800
1000
1200
1400
1600

FIG. 42.

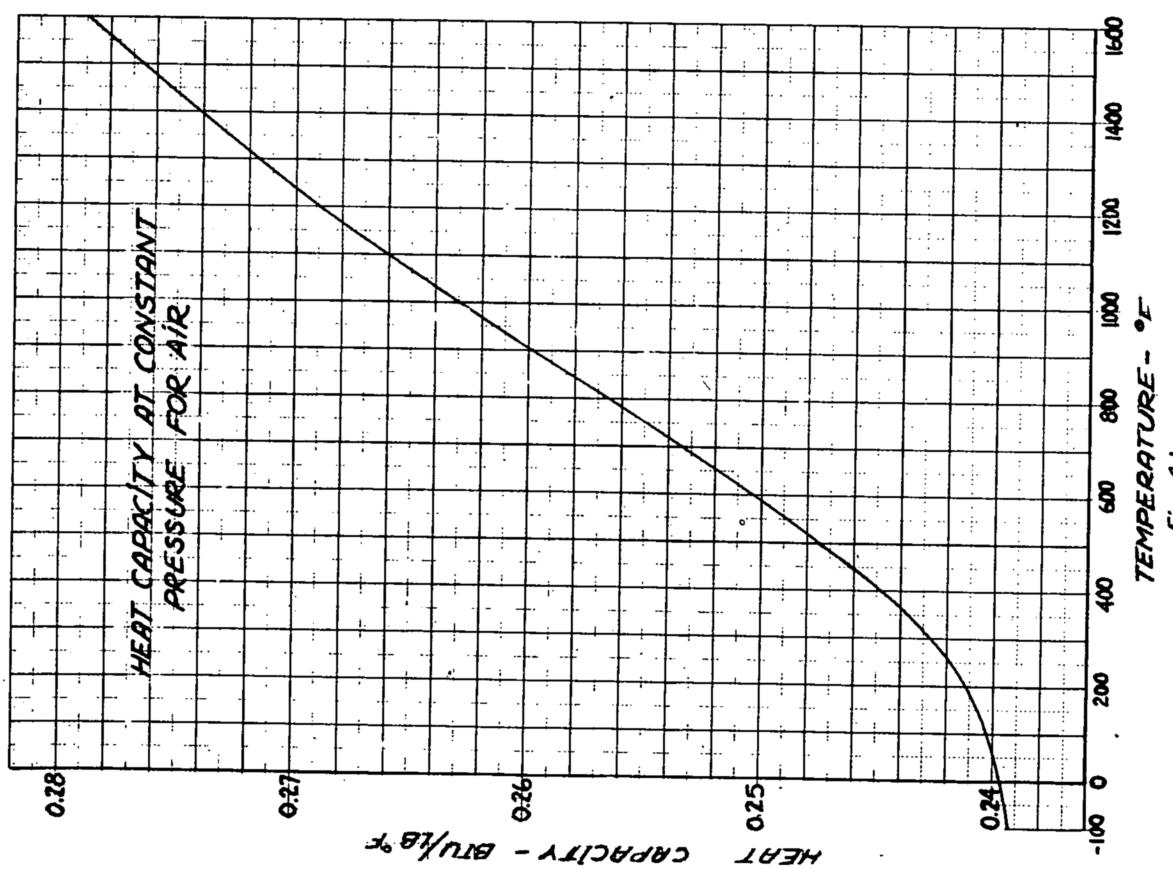

HEAT CAPACITY AT CONSTANT PRESSURE FOR AIR
HEAT CAPACITY - BTU/LB °F
0.28
0.27
0.26
0.25
0.24
TEMPERATURE - °F
-100
0
200
400
600
800
1000
1200
1400
1600

Fig 41

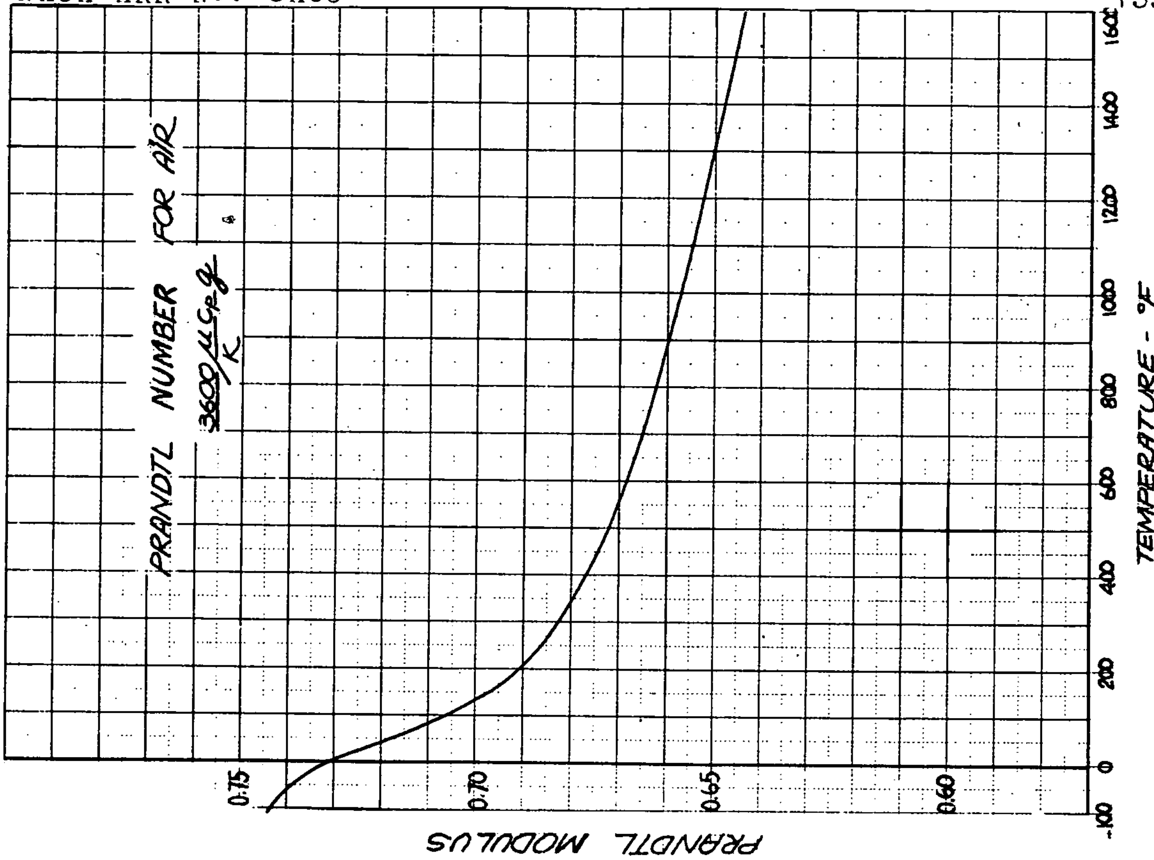

PRANDTL NUMBER FOR AIR
3600 μ Cp g / K
PRANDTL MODULUS
0.75
0.70
0.65
0.60
TEMPERATURE - °F
-100
0
200
400
600
800
1000
1200
1400
1600

Fig 4.4

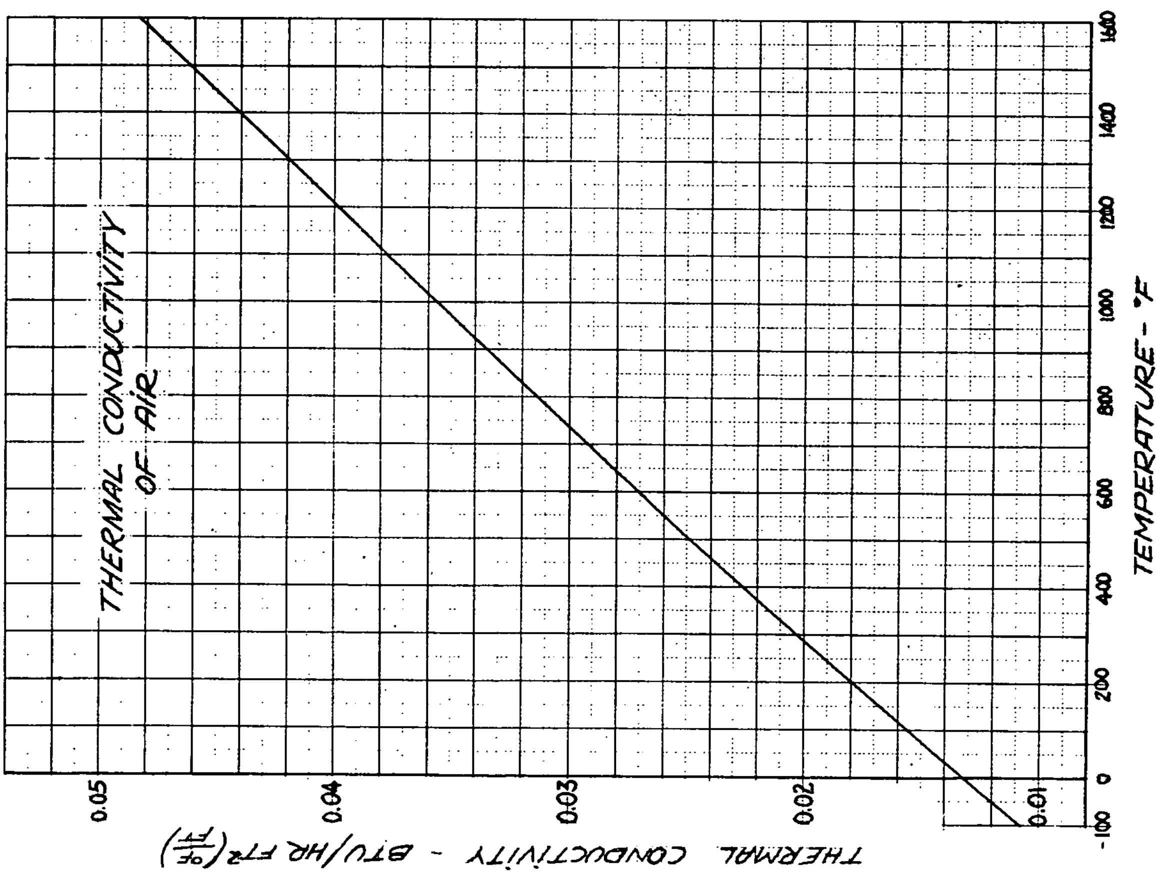

THERMAL CONDUCTIVITY OF AIR
THERMAL CONDUCTIVITY - BTU/HR FT2 (°F/FT)
0.05
0.04
0.03
0.02
0.01
TEMPERATURE - °F
-100
0
200
400
600
800
1000
1200
1400
1600

Fig 4.3.

APPENDIX B

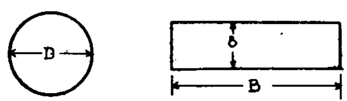
NACA STANDARD ATMOSPHERE DATA*

Altitude (ft)	Temperature (°F)	Pressure (in. Hg.)	Density (lb/ft³)	Density ratio
0	59.00	29.92	0.07651	1.0000
1,000	55.43	28.86	.07430	.9710
2,000	51.87	27.82	.07213	.9428
3,000	48.30	26.81	.07001	.9151
4,000	44.74	25.84	.06794	.8881
5,000	41.17	24.89	.06592	.8616
6,000	37.60	23.98	.06395	.8358
7,000	34.04	23.09	.06202	.8016
8,000	30.47	22.22	.06013	.7859
9,000	26.90	21.38	.05829	.7619
10,000	23.34	20.58	.05649	.7384
11,000	19.77	19.79	.05474	.7154
12,000	16.21	19.03	.05303	.6931
13,000	12.64	18.29	.05136	.6712
14,000	9.07	17.57	.04973	.6499
15,000	5.51	16.88	.04814	.6291
16,000	1.94	16.21	.04658	.6088
17,000	-1.63	15.56	.04507	.5891
18,000	-5.19	14.94	.04359	.5698
19,000	-8.76	14.33	.04216	.5509
20,000	-12.32	13.75	.04075	.5327
21,000	-15.89	13.18	.03938	.5148
22,000	-19.46	12.63	.03806	.4974
23,000	-23.02	12.10	.03676	.4805
24,000	-26.59	11.59	.03550	.4640
25,000	-30.15	11.10	.03427	.4480
26,000	-33.72	10.62	.03308	.4323
27,000	-37.29	10.16	.03192	.4171
28,000	-40.85	9.72	.03078	.4023
29,000	-44.42	9.29	.02968	.3869
30,000	-47.99	8.88	.02861	.3740
31,000	-51.55	8.48	.02757	.3603
32,000	-55.12	8.10	.02656	.3472
33,000	-58.68	7.73	.02558	.3343
34,000	-62.25	7.38	.02463	.3218
35,000	-65.82	7.04	.02369	.3098
36,000	-67.00	6.71	.02265	.2962
37,000	-67.00	6.39	.02160	.2824
38,000	-67.00	6.10	.02059	.2692
39,000	-67.00	5.81	.01963	.2566
40,000	-67.00	5.54	.01872	.2447
41,000	-67.00	5.28	.01785	.2332
42,000	-67.00	5.04	.01701	.2224
43,000	-67.00	4.80	.01622	.2120
44,000	-67.00	4.58	.01546	.2021
45,000	-67.00	4.36	.01474	.1926
46,000	-67.00	4.16	.01405	.1837
47,000	-67.00	3.97	.01339	.1751
48,000	-67.00	3.78	.01277	.1669
49,000	-67.00	3.60	.01217	.1591
50,000	-67.00	3.44	.01161	.1517

*See reference 30.

APPENDIX C

SUMMARY OF EQUATIONS

General form	Equations for air
Flat plate	
Laminar boundary layer	
$\frac{f_{c_x}}{3600\ u_\infty\ \gamma\ c_p}(Pr)^{\frac{2}{3}} = \frac{c_{f_x}}{2} = \frac{0.332}{\sqrt{Re_x}}$	$f_{c_x} = 0.0562\ T_f^{0.5}\left(\frac{u_\infty \gamma}{x}\right)^{0.5}$ $f_{c_{av}} = 0.112\ T_f^{0.5}\left(\frac{u_\infty \gamma}{l}\right)^{0.5}$
Turbulent boundary layer	
$\frac{f_{c_x}}{3600\ u_\infty\ \gamma\ c_p}(Pr)^{\frac{2}{3}} = \frac{c_{f_x}}{2} = \frac{0.0296}{Re_x^{0.2}}$	$f_{c_x} = 0.51\ T_f^{0.3}\frac{(u_\infty \gamma)^{0.8}}{x^{0.2}}$ $f_{c_{av}} = 0.64\ T_f^{0.3}\frac{(u_\infty \gamma)^{0.8}}{l^{0.2}}$
Pipes and ducts	
Turbulent entrance section $0 < x < 4.4\ D_H$ $\frac{f_{c_x}}{G\ c_p}(Pr)^{\frac{2}{3}} = \frac{0.0296}{Re_x^{0.2}}$	$f_{c_x} = 7.3 \times 10^{-4}\ T_f^{0.3}\frac{G^{0.8}}{x^{0.2}}$ $f_{c_{av}} = 9.1 \times 10^{-4}\ T_f^{0.3}\frac{G^{0.8}}{l^{0.2}}$
Beyond entrance section; turbulent flow $4.4\ D_H < x < \infty$ $\frac{f_{c_x}}{G\ c_p}(Pr)^{\frac{2}{3}} = \frac{\zeta}{8} = \frac{0.176}{8 \times Re_D^{0.2}}$	$f_{c_x} = 5.4 \times 10^{-4}\ T^{0.3}\frac{G^{0.8}}{D_H^{0.2}}$ $f_{c_{av}} = 5.4 \times 10^{-4}\ T^{0.3}\frac{G^{0.8}}{D_H^{0.2}}\left(1 + 1.1\frac{D_H}{l}\right)$
Laminar flow-parabolic velocity distribution at entrance	
(a) Round tubes $\frac{f_{c_x} D}{k} = 1.16\sqrt[3]{31 + \frac{W\ c_p}{kx}}$	$f_{c_x} = 3.65\frac{k}{D}\sqrt[3]{1 + 0.38\frac{W}{x}}$
(b) Rectangular ducts $\frac{f_{c_x}\delta}{k} = 0.98\sqrt[3]{59 + \frac{W\ c_p}{kx}\frac{\delta}{B}}$	$f_{c_x} = 3.80\frac{k}{\delta}\sqrt[3]{1 + 0.20\frac{W}{x}\frac{\delta}{B}}$
Flow across single cylinders	
1. Average over cylinder $Nu = 0.26\ Re^{0.6}\ Pr^{0.3}$	1. $f_{c_{av}} = 0.211\ T_f^{0.43}\frac{(u_\infty \gamma)^{0.6}}{D^{0.4}}$
2. Local value at stagnation point ($\varphi = 0^\circ$) $Nu = 1.14\ Re^{0.5}\ Pr^{0.4}$	2. $f_{c_{\varphi=0^\circ}} = 0.194\ T_f^{0.49}\left(\frac{u_\infty \gamma}{D}\right)^{0.5}$
3. Local value along front half of cylinder ($0 < \varphi < 90^\circ$) $Nu = 1.14\ Re^{0.5}\ Pr^{0.4}\left[1 - \left\lvert\frac{\varphi}{90^\circ}\right\rvert^3\right]$	3. $f_{c_\varphi} = 0.194\ T_f^{0.49}\left(\frac{u_\infty \gamma}{D}\right)^{0.5}\left[1 - \left\lvert\frac{\varphi}{90^\circ}\right\rvert^3\right]$
Flow across tube banks	
$Nu = 0.244\ F_a\ Re^{0.6}\ Pr^{\frac{1}{3}}$	$f_{c_{av}} = 14.5 \times 10^{-4}\ T_f^{0.43}\ F_a\frac{G_o^{0.6}}{D^{0.4}}$

Values of F_a

Number of tubes	1	2	3	4	5	6	7	8	9	10
In-line	1.00	1.10	1.17	1.24	1.29	1.34	1.37	1.40	1.42	1.43
Staggered	1.00	1.11	1.23	1.31	1.39	1.45	1.48	1.51	1.53	1.54

For $Re = 20{,}000$ and $1.25 < \frac{S_t}{D} < 3.0$

$1.25 < \frac{S_L}{D} < 3.0$

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