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RESEARCH MEMORANDUM

A SIMPLE APPROXIMATE METHOD FOR OBTAINING SPANWISE LIFT DISTRIBUTIONS OVER SWEPT WINGS

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A SIMPLE APPROXIMATE METHOD FOR OBTAINING SPANWISE LIFT DISTRIBUTIONS OVER SWEPT WINGS

By Franklin W. Diederich

SUMMARY

It is shown how Schrenk's empirical method of estimating the lift distribution over straight wings can be adapted to swept wings by replacing the elliptical distribution by a new "ideal" distribution which varies with sweep. The application of the method is discussed in detail and several comparisons are made to show the agreement of the proposed method with more rigorous ones. It is shown how first-order compressibility corrections applicable to subcritical speeds may be included in this method.

INTRODUCTION

A great number of methods for estimating the lift distribution over unswept wings have been available for some time, most of them based on the concept of the lifting line. The accuracy of these rational methods is limited by the degree of closeness with which the lifting line represents the physical conditions, so that much of the time consumed in obtaining very accurate mathematical solutions for the lifting-line equation is unwarranted. However, the experience gained with solutions obtained by these more accurate methods has pointed the way to simplified empirical methods which do not involve the solution of any mathematical equations.

Glauert's suggestion (reference 1) that the shape of the lift distribution was "intermediate between that of the aerofoil (wing) and that of the ellipse" was followed up by Schrenk (reference 2). He decided that in the case of untwisted wings the lift distribution could be represented approximately by the arithmetic mean of an ideal distribution and the plan-form distribution of equal area, the ideal distribution being of elliptical shape. For twisted wings he resolved the lift distribution into an additional distribution to be obtained as described above and a basic distribution to be obtained by taking one-half of the distribution of the product of chord and angle of twist (measured from the angle of zero lift of the wing) and rounding off any sharp corners in that distribution. By means of several numerical comparisons he demonstrated that the results of his method checked those of the rational methods and experimental results with sufficient accuracy for many practical purposes. Further comparisons made in references 3 and 4 corroborate this observation.

The classical lifting-line methods do not apply to swept wings, however, so that Schrenk's approximate method, which is based on the results of lifting-line methods, must be expected to fail, as well. A number of rational lifting-surface methods have been developed to treat swept wings (for example, references 5 and 6), which are considerably more time-consuming than even the lifting-line methods. These lifting-surface methods have been used for a number of computations, and corroborative experiments have been carried out on a set of full-scale models (reference 7). The results provide a basis for a method similar to Schrenk's, which is outlined in the present report. The only difference consists of the fact that the ellipse is no longer considered as the ideal distribution for the additional lift distribution. A set of new ideal distributions which depend on sweep has been determined as the set of curves which, when averaged with the chord distribution, yields results which are closest to theoretical lifting-surface solutions for various plan forms and degrees of sweep. The theoretical solutions required were available in a number of published and unpublished reports for angles of sweep up to 45°; no ideal distributions have been derived for sweep angles larger than 45°. The distributions are independent of the aspect ratio within the range of practical aspect ratios, as in the case of the unswept wing. They can be used for estimating the additional loading (and, consequently, the total loading) over any swept wing with a degree of accuracy which is entirely adequate for many purposes.

SYMBOLS

A aspect ratio (b^2/S)

b span, feet

c chord, measured parallel to the plane of symmetry, feet

$\bar{c}$ average chord, feet (S/b)

C_L wing lift coefficient

c_l local lift coefficient

f ordinate of the "ideal" distribution curve

G Glauert-Prandtl correction $\left(1/\sqrt{1 - M^2}\right)$

$k\delta$ angular change in zero-lift direction of any section produced by flap displacement δ, radians

M Mach number

m slope of wing lift coefficient $\left(dC_L/d\alpha\right)$

m_o slope of section lift coefficient $\left(\left(dC_L/d\alpha\right)_{A=\infty}\right)$

$\bar{m}_o$ average slope of section lift coefficient

S wing area, square feet

y lateral ordinate, feet

α local angle of attack measured from a common reference, radians

$\bar{\alpha}$ average local angle of attack measured from the common reference (assumed to represent the zero-lift angle of the wing), radians

α_o local angle of attack measured from the zero-lift angle of the wing, radians

Λ angle of sweepback at the quarter-chord line (negative values of Λ indicate sweepforward), degrees

λ taper ratio $\left(c_{tip}/c_{root}\right)$

Subscripts:

a_1 additional lift distribution (for $C_L = 1$)

b basic lift distribution (for $C_L = 0$)

c compressible flow

e equivalent (in incompressible flow)

s equivalent unswept wing

DESCRIPTION OF THE METHOD

Additional Lift Distribution

The additional lift distribution is obtained from the relation

$$\frac{cc_l}{\bar{c}C_L} = \frac{1}{\bar{c}}\left(cc_{l_{a_1}}\right) = \frac{1}{2}\left(\frac{c}{\bar{c}}\frac{m_o}{\bar{m}_o} + f\right) \qquad (1)$$

where f, which replaces the elliptic distribution $\frac{4}{\pi}\sqrt{1-\left(\frac{y}{b/2}\right)^2}$ of references 2, 3, and 4, is to be read from figure 1 for the given sweep angle Λ, measured to the quarter-chord line. The value of $\bar{m}_o$ is obtained from the definition

$$\bar{m}_o = \frac{2}{S}\int_0^{b/2} m_o c \, dy \tag{2}$$

by graphical or numerical means, as described in references 3 and 4.

Wings of variable sweep, such as the M, W, and pterodactyl types, have been proposed from time to time. It appears that the lift distribution over such a wing cannot be estimated very accurately by any simple means. One way of obtaining a useful approximation, however, appears to be the use of an ideal distribution f corresponding to the average sweep angle $\bar{\Lambda}$ defined by

$$\tan\bar{\Lambda} = \frac{2}{S}\int_0^{b/2} \tan\Lambda c \, dy \tag{3}$$

Basic Lift Distribution

The basic lift distribution is less affected by sweep than the additional lift distribution. Furthermore, it usually constitutes a small part of the total lift distribution. Consequently, no essential deviation from Schrenk's method appears to be warranted. The basic lift distribution is obtained from the relation

$$\frac{1}{\bar{c}}\left(cc_{l_b}\right) = \frac{m_o}{2}\frac{c}{\bar{c}}\frac{m}{\bar{m}_o}\alpha_o \tag{4}$$

where

$$\alpha_o = \alpha - \bar{\alpha} \tag{4a}$$

and

$$\bar{\alpha} = \frac{2}{S}\int_0^{b/2} \alpha c \, dy \tag{4b}$$

All sharp corners are rounded off in accordance with the suggestions of references 3 and 4 in such a manner as to yield a total lift equal to zero. The factor $m/\bar{m}_o$ has been introduced because comparisons of basic lift distributions estimated by Schrenk's method with those calculated by lifting-surface methods indicate that much better agreement between the two can be obtained if Schrenk's results are multiplied by the factor $m/\bar{m}_o$; this applies to both swept and unswept wings.

Subcritical Compressibility Corrections

Analyses of linearized three-dimensional compressible flows (for example, reference 8) indicate that the lift of a wing in a compressible flow can be obtained by finding the lift of an equivalent wing in incompressible flow and multiplying it by the Glauert-Prandtl correction G. The aspect ratio and the sweep of the equivalent wing are related to those of the actual wing as follows:

$$A_e = \frac{A}{G} \tag{5}$$

$$\tan \Lambda_e = G \tan \Lambda \tag{5a}$$

The taper ratio is the same for both wings.

This procedure results in lift distributions over unswept wings which are unaffected by compressibility. The lift distributions over swept wings, however, are modified by compressibility in the same manner as they are by increases in the angle of sweep in incompressible flow. Specifically, the ideal distribution f for a wing in compressible flow is to be chosen for the angle Λ_e rather than Λ.

Similarly, the lift slope of a wing in compressible flow may be obtained from that of an equivalent wing in incompressible flow. An unpublished analysis which takes account of the effect of sweep on downwash in incompressible flow in a rational but approximate manner has been performed by Mr. Thomas A. Toll. According to this analysis, the lift slope of a swept wing in incompressible flow may be estimated from the relation

$$m = \frac{(A + 2) \cos\Lambda}{A + 2 \cos \Lambda} m_s \tag{6}$$

where the lift slope m_s is to be taken for an unswept wing of the same aspect ratio and taper ratio. If the effects of compressibility are accounted for in the manner outlined previously, equation (6) is modified as follows:

$$m = \frac{(A_e + 2) \cos \Lambda_e}{A_e + 2 \cos \Lambda_e} m_{s_c} \tag{7}$$

The lift slope m_{s_c} is to be taken for an unswept wing of the actual aspect ratio and taper ratio operating at the actual free-stream Mach number. It is approximately equal to G times the slope m_s of a wing with the actual taper ratio and the aspect ratio A_e.

These corrections are based on the linearized equations of flow and consequently become invalid as the free-stream velocity approaches either that of sound or that corresponding to the critical Mach number of the wing. A more detailed analysis must be made if either a Mach number of about 0.9 or the critical Mach number of the wing (whichever is lower) is exceeded.

DISCUSSION

In considering the reliability of the method outlined in this report, it must be kept in mind that the method is based on results obtained by means of potential-flow lifting-surface methods and consequently does not take account of boundary-layer effects. Both this method and the more refined methods are useful because they provide first-order estimates of the load distribution at low and and medium angles of attack; they also provide bases for empirical corrections for boundary-layer effects where these effects are of particular importance, that is, for large angles of attack and sweep. Since, however, the boundary-layer effects depend on Reynolds number, Mach number, the section properties, and the plan-form parameters (which include sweep), the experimental information now available is entirely insufficient for the determination of a set of boundary-layer corrections. When more experimental information is available and such a set of corrections is obtained, it will apply to the approximate methods as well as to the rational potential-flow methods of estimating the load distribution.

Figure 2 shows the agreement of the additional lift distributions obtained for different wing configurations by the empirical method of this report with those calculated by Falkner's method (reference 5) and those measured on full-scale models (reference 7). It appears that, while the results estimated by the two methods differ slightly from each other, both agree equally well with the experimental results, at least at the relatively low angles of attack used in the tests. Figure 2(d), which shows the agreement of the approximate method with Falkner's in the case of an untapered sweptback wing, indicates that the approximation is valid even for large taper ratios. No experimental results appear to be available for comparison.

Figure 2(e) has been drawn for a pterodactyl wing in an attempt to show how accurate a result may be obtained for wings of variable sweep by the approximate method. The ideal distribution f for the pterodactyl wing shown in the figure was chosen for an average sweep angle $\bar{\Lambda} = 25^{\circ}$. The agreement of the distribution obtained in this case with that calculated by Falkner's method is adequate for most structural purposes.

The agreement between the results obtained by Schrenk's method, the lifting-line theory (reference 9), and Weissinger's method (reference 6) for flapped and twisted untapered wings of aspect ratio 5 is shown in figures 3(a) and 3(b), respectively. In fairing the distributions for Schrenk's method equal areas have been subtracted at the tip and at or near the root. It appears that the results of Schrenk's method are slightly lower than those of lifting-line theory and that they would have to be reduced about 30 percent in order to agree with those of Weissinger's method. Application of the factor $m/\bar{m}_o$ would reduce the value by 32 percent; this fact was the basis for including the factor in equation (4).

It appears that there is comparatively little difference between the distributions for the three different values of sweep, although the usual trends are noted, in that the tendency of both sweepback and sweepforward is to reduce the ordinates of the lift-distribution curve for a given angle of attack and the tendency of sweepback is to shift the load outboard, whereas that of sweepforward is to shift the load inboard. Inasmuch as these differences are smaller than those between the estimated distribution and the calculated distribution for zero sweep, and inasmuch as the basic lift distribution usually forms only a small part of the total distribution, it appears that little gain could be had by attempting to obtain a closer estimate for it.

It might be expected that of the family of curves presented in figure 1 the one for zero sweep would be an ellipse. Actually, there is a slight deviation, since the family of curves is based on lifting-surface theory, whereas Schrenk's adoption of the ellipse is based on lifting-line theory. Experimental results for the total lift of wings with zero sweep agree more closely with those of lifting-surface theory than those of lifting-line theory (reference 6), and, while few corresponding comparisons appear to have been made for the lift distributions on wings of zero sweep, the same relative accuracy may be expected. A lifting-line calculation has been made for the wing of figure 2(a) by Multhopp's lifting-line method (reference 9) and it appears that Falkner's distribution is in slightly better agreement with experimental values than the lifting-line distribution.

The fact that the ideal distribution curves do not pass through zero at the wing tip results from the consideration that in order to yield zero lift at the tip the positive value of the tip chord requires an equal negative value of the ideal distribution to be averaged with it. Since the tip chords vary for different plan forms, it has been found to be expedient to draw the distributions only up to the point where they reach zero. If in using them any sharp corners are obtained in the lift distribution, they may be faired by eye. The areas under the ideal distribution curves have been so adjusted as to give an area of 1.000 under the curves for $\frac{cc_l}{\bar{c}C_L}$ plotted against $\frac{y}{b/2}$, with an allowance made for the fairing that may normally be required.

The term "ideal" applied to the distributions of figure 1 has been carried over from reference 2 and is actually somewhat of a misnomer in the case of swept wings since, by Munk's stagger theorem, the elliptic distribution always causes the least induced drag, regardless of sweep. The term is not intended to imply that the distributions of figure 1 constitute desirable lift distributions.

CONCLUDING REMARKS

It has been demonstrated that the lift distribution over swept wings can be estimated with adequate accuracy for most practical purposes by means of simple extensions to Schrenk's method. These extensions consist primarily of using a new set of ideal lift-distribution curves which depend on sweep instead of the ellipse in estimating the additional lift distribution. The shape of the basic lift-distribution curves can be estimated by Schrenk's method without any modification for all practical sweep angles, but the magnitude should be multiplied by the ratio of the wing lift slope to the average section lift slope for all cases, including the straight wing.

The results of this empirical method are in as good agreement with experimental results as those furnished by the rational lifting-surface methods for all practical sweep angles and for tapered wings. For wings with sweep angles which vary along the span the agreement between the empirical and rational results is not as good, but may be adequate for structural purposes.

Insufficient experimental information exists at this time to provide a basis for empirical corrections to account for boundary-layer effects; this shortcoming affects the results of both the empirical and the rational methods. However, at low and medium angles of attack no correction is required for most purposes.

In order to estimate the shape of the distribution curve for compressible flows the ideal-distribution curve must be chosen for an increased sweep angle. A first-order estimate of the magnitude of the curve or, for that matter, the lift slope is given by equation (7).

Both these corrections are applicable only to subsonic and subcritical speeds.

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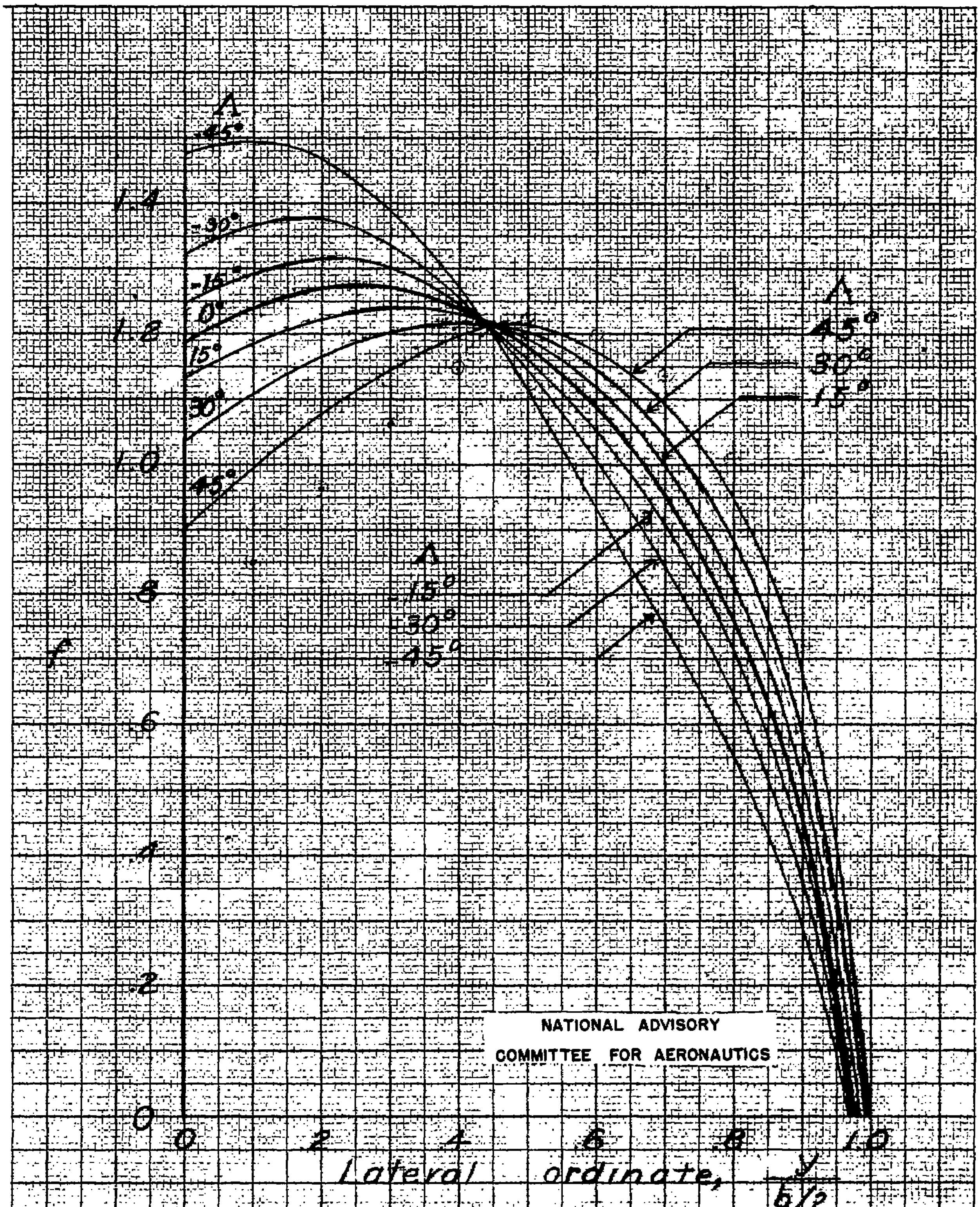


Figure 1 - Ideal distribution curves for swept wings.

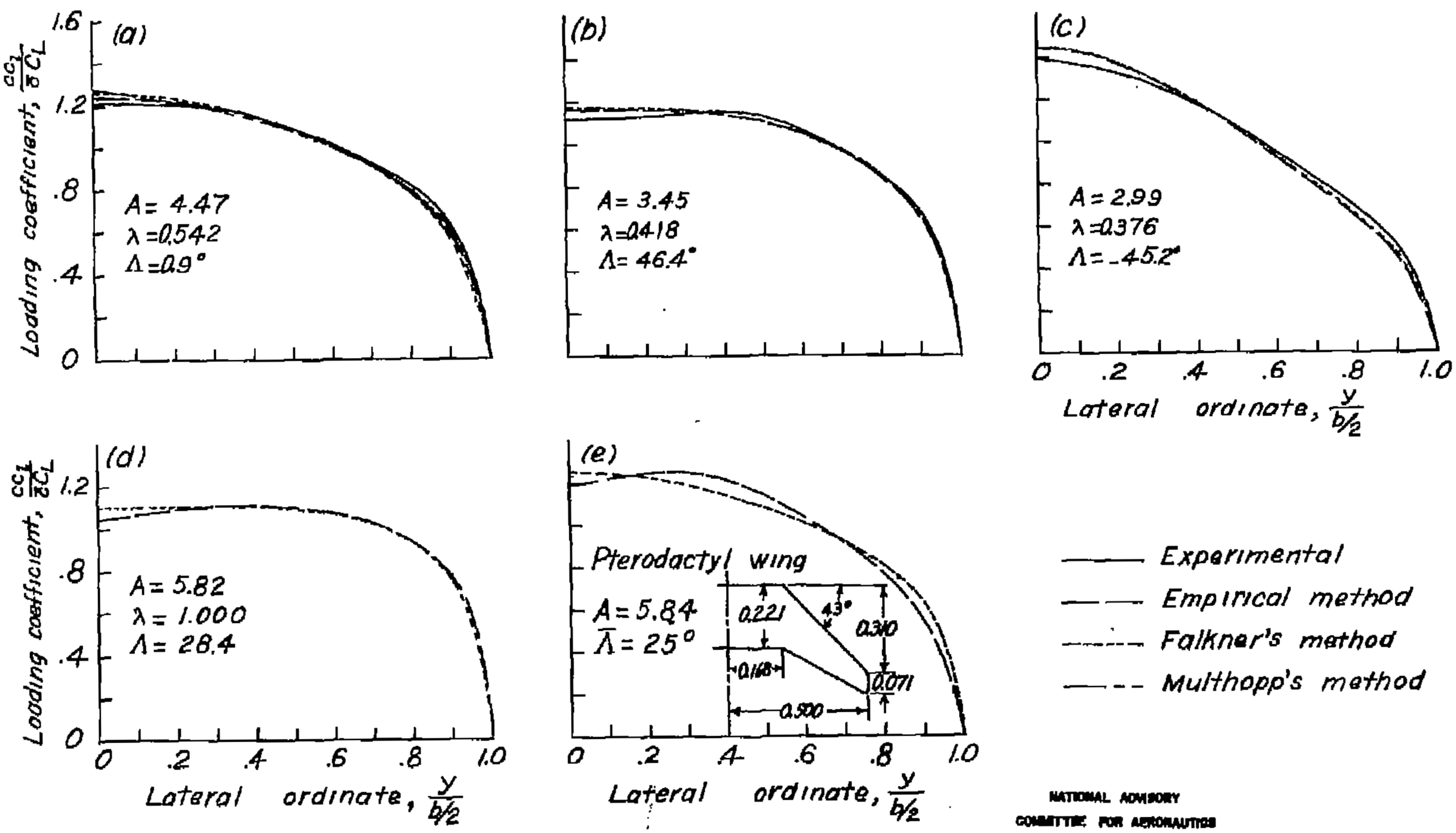
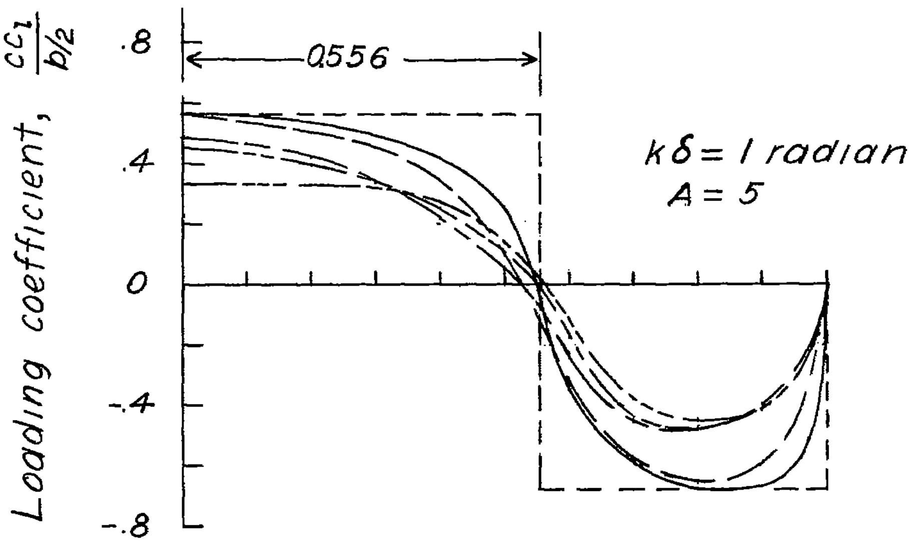
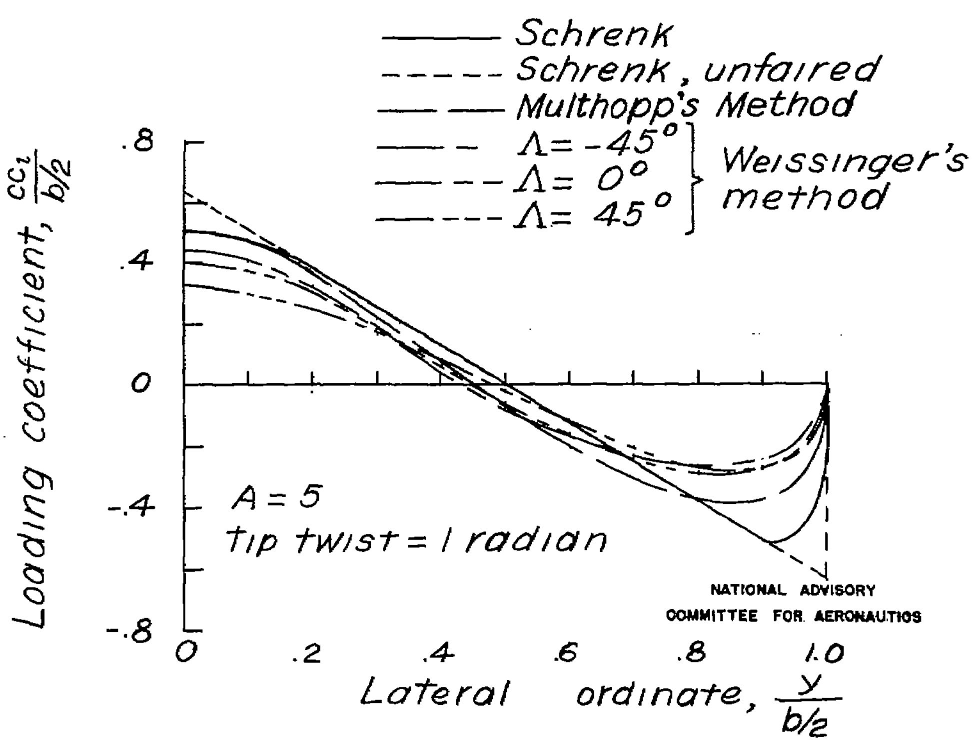


Figure 2.- Comparison of additional lift distributions for swept wings.



(a) Wing with 55.6 % span flap.



(b) Wing with linear twist.

Figure 3.- Comparison of basic lift distributions for untapered swept wings.