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TECHNICAL MEMORANDUM 1353

SOME PROBLEMS OF THE THEORY OF CREEP

By Y. N. Rabotnov

Translation

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Section 1. The term creep of metals is applied to the phenomenon in which, at temperatures beyond a certain limit, the metal subjected to a load slowly undergoes deformation with time. For the case of steel, the creep phenomenon must be taken into account at temperatures above 400° C. Very slow deformations for a prolonged period are cumulative and lead either to inadmissible changes in the dimensions of a structural part or to its failure.

It is important to note that failure due to creep occurs for very small strains considerably less than those in static rupture.

In the design of steam power units, boilers and turbines, creep is a basic factor which determines the choice of the admissible stresses. On account of the extreme urgency of the problem, the creep phenomenon has claimed the widest attention of metallurgists, physicists, and to a lesser extent, technicians.

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At the same time, however, the theory of creep constitutes part of the mechanics of dense media and the mechanical formulation of the problem may be given as the following:

A body is subjected to the action of a given system of forces, or initial displacements are prescribed on its surface. It is required to find the stress distribution in the body and the changes of its deformations with time.

Such a statement of the problem immediately raises the following question: What tests should be set up in order that the mechanical characteristics of creep (certain constants or functions) may be determined? Is it sufficient for this purpose to make use of the generally accepted methods of testing, or is it necessary to supplement them?

For the solution of the problem of creep as thus formulated, a mechanical theory of creep is required. Such theory, at the present state of knowledge of the physics of the process, must necessarily bear

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an external, over-all character based, not on the investigation of microprocesses, but on the results of mechanical tests. The existing physical theories are as yet far from providing a quantitative description of the process in all its complication.

The usual method of creep tests is that of obtaining the strains for a constant load. For small deformations it may be assumed that to a constant load there corresponds a constant stress.

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The results of tests are generally represented in the form of creep curves (fig. 1), the time being laid off on the axis of abscissas and the strain ϵ on the axis of ordinates.

The intercept ϵ_0 represents elastic deformation if the stress σ does not exceed the elastic limit of the material. Often in place of the total strain there is laid off the plastic strain

$$p = \epsilon - \sigma/E$$

Many attempts have been made to give an analytical expression for the creep curves. The different equations proposed may be divided into two groups:

$$1.\ p = S(\sigma)T(t) \tag{1.1}$$

$$2.\ p = g(\sigma)\theta(t) + S(\sigma)t \qquad (\theta(\infty) = 1) \tag{1.2}$$

In writing equation (1.2), the essential assumption is that the rate of creep tends towards a constant value with time, that is, the creep curve has an asymptote. The function $g(\sigma)$ is the intercept ϵ' on figure 1. For the function $S(\sigma)$, the following equations have been proposed:

$$S(\sigma) = A\sigma^n \qquad \text{(Bailey)}$$

$$S(\sigma) = ve^{\sigma/\mu} \qquad \text{(Ludwik)}$$

$$S(\sigma) = v(e^{\sigma/\mu} - 1) \qquad \text{(Soderberg)}$$

$$S(\sigma) = 2v \text{ sh } \frac{c}{\mu} \qquad \text{(Nadai)}$$

$$S(\sigma) = b\sigma e^{\sigma/\mu} \qquad \text{(Oding)}$$

For the function $T(t)$, the best equation is apparently the power equation

$$T(t) = t^m \quad (0 < m < 1)$$

If equations of type (1.2) are used, the function $\theta(t)$ assumes the following forms:

$$\theta(t) = 1 - e^{-\beta t} \qquad \text{(Mac Vetty)}$$

$$\theta(t) = \frac{t}{a + t} \qquad \text{(Oding)}$$

For $g(\sigma)$, it has not as yet been possible to establish a law.

The series of creep curves for different values of σ give the representation of the functional relations between the three variables σ, ϵ, and t. If ϵ and σ are laid off on the coordinate axes, a series of curves is obtained, which is shown in figure 2 and characterized by different values of t. Figure 2 refers to the test data of Robinson which are unique in that these tests extended for 100,000 hours (from March 27, 1931 to October 8, 1942). We worked over a very large number of the tests in the series by this method and obtained in all cases, with an accuracy not exceeding the limits of the experimental accuracy, affinely related curves in the $\sigma\epsilon$ plane.

On the basis of this result, the following formula describing the law of creep for constant stress is proposed:

$$\varphi(\epsilon) = \left[1 + G(t)\right] \sigma \tag{1.3}$$

For the function $G(t)$, good results are generally given by the following expression:

$$G(t) = \frac{\chi}{1 - \alpha} t^{1-\alpha}$$

where the coefficient α fluctuates about the value 0.7, while the coefficient χ changes very strongly for different materials.

The scatter in transferring all the points on to the curve $t = 0$ (fig. 2) is found to be not greater than the scatter of the modulus of elasticity of the specimens on which the given series of tests was conducted.

Section 2. The mechanical theory of creep for the case of a single axis must establish such relation between ϵ and σ, containing the time or time operators, which would permit predicting the course of the process varying with time. In particular, a second extreme case of the one-dimensional problem is the problem of relaxation. The latter is the process of decrease in stress in a rod the length of which remains constant. Test data on relaxation are very meager, only those data being of value which, together with the curves of relaxation, give the curves of creep for the same material. The only reliable data in the general literature are those published by Davis in 1943 on copper. 2653

The various mechanical theories of creep existing at the present time may be divided into three groups:

(1) Theory of constant rate: Assuming the existence of an asymptotic curve of creep, the curve is replaced by a straight line parallel to the asymptote and intercepting the segment $\epsilon_0 = \epsilon/E$ on the axis of ordinates. Then

$$\dot{p} = S(\sigma) \tag{2.1}$$

The theory of constant rate assumes this as the true relation for any conditions. In particular, for ϵ = constant, $\dot{p} = -\dot{\sigma}/E$, and from equation (2.1) there is readily obtained the law of relaxation which grossly contradicts test data, since the neglect of the primary creep (the curvilinear part of the curve ϵt) is not permissible in the problem of relaxation. In the case of the nonuniform stress state, equation (2.1) leads to very great difficulties and even the problem of the pure bending of a rod of rectangular cross section is not solvable.

The greater number of authors employing this theory take still another step and neglect the elastic deformation. The fundamental equation is then the following:

$$\dot{\epsilon} = S(\sigma) \tag{2.2}$$

Equation (2.2) is generally put at the basis of the theory of secondary creep widely applied in technical computations. Below shall be given another basis for this theory which considerably generalizes it. For the present we may note that from the point of view of equation (2.2) the problem of relaxation has no significance.

(2) Theory of aging: The theory of aging postulates the existence of a definite relation between σ, ϵ, and t:

$$f(\sigma, \epsilon, t) = 0 \tag{2.3}$$

or between $\dot{p}$, σ, t

$$F(\dot{p}, \sigma, t) = 0 \tag{2.4}$$

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It is easily shown that any theory, the fundamental equation of which contains the time explicitly, is contradictory. The physical law must be invariant relative to a time origin. Applying the theory of aging to the successive loading and unloading may yield absurd results.

The theory of aging in the form of equation (2.3), however, gives for smoothly varying loads satisfactory agreement with experiment. The methods of computation based on it are relatively simple and at the same time permit taking into account all the characteristic experimental curves which may be obtained in tests. Hence, one of the variants of the aging theory can be recommended as a technical method of computing structural parts working under the conditions of high temperatures. This point of view has been developed by us in a paper presented at the session of the Soviet Academy of Sciences in March 1948.

(3) Theory of strain hardening: This theory postulates the existence of an unvarying relation among the rate of plastic deformation, its magnitude, and the magnitude of the stress:

$$\Phi(\dot{p}, p, \sigma) = 0 \tag{2.5}$$

Methods exist for the graphical construction of the relaxation curve by a given family of creep curves on the basis of hypothesis (2.5). The analytical formulation of this theory, the choice of the functional relations for which it is possible to integrate equation (2.5), is found to be very difficult.

Section 3. The theories of creep enumerated in section 2 are not capable of explaining a number of phenomena observed during experiment. There is first of all the case for the strain-hardening effect. A specimen initially strained by a large force evidences creep to a considerably less extent than a specimen not initially strained. If the specimen was tested under stress σ_1 and σ_1 is decreased to σ_2, the creep practically vanishes. If, however, the stress σ_2 was initially imposed, the creep for this stress may be very marked. There are no serious investigations of this problem in the literature. The published data undoubtedly give a qualitative account of the phenomenon but the quantitative aspect still awaits investigation.

The second effect is the so-called reverse creep. If a specimen is subjected to a creep test at constant load and the load is then removed, the specimen immediately shortens by the amount of the elastic elongation but the process does not stop there. In the course of time the specimen continues to shorten, returning in this way a part of its residual deformation. This phenomenon of the type of elastic aftereffect has been subjected to a careful experimental study but not one of the previously enumerated theories provides an explanation for it.

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The theory of creep proposed by us represents an extension of the theory of elastic heredity of Volterra to plastic deformation. In the same way as the elastic heredity develops about a straight line $\sigma = E\epsilon$ in the $\sigma\epsilon$ plane, the plastic heredity, or creep, develops about a certain curve in this plane. It will be inconvenient, in what follows, to lay off as usual the value of the stress σ on the ordinate axis and we shall therefore take a certain fictituous plane $\varphi\epsilon$ and a curve $\varphi = \varphi(\epsilon)$ in this plane (fig. 4).

The curve $\varphi(\epsilon)$ represents the ideal curve of strain with the exclusion of the time factor (actually never realized). For active processes, that is, those accompanied by motion along this curve upward, the fundamental law is written in the following manner:

$$\varphi(\epsilon) = (1 + K^*)\ \sigma \tag{3.1}$$

where K^* is the integral operator of Volterra, that is,

$$K^*\sigma = \int_0^t K(t - \tau)\ \sigma(\tau)\ d\tau$$

Here and in what follows, use will be made of the notations and results of our previous paper (ref. 1).

The increase in the stress σ is no longer a criterion of the activity of the process as in the theory of plasticity but such criterion is given by the increase in ϵ.

For unloading processes, it is necessary in the left-hand side of equation (3.1) to introduce in place of $\varphi(\epsilon)$ the value of the ordinate of the linear unloading AB expressed as a function of ϵ:

$$E\ (\epsilon - \epsilon') + \varphi' = (1 + K^*)\ \sigma \tag{3.2}$$

If Γ^* is the solving operator,

$$\frac{1}{1 + K^*} = 1 - \Gamma^*$$

there follows from equation (3.1)

$$\sigma = (1 - \Gamma^*)\,\varphi(\epsilon)$$

Setting

$$K^* \cdot 1 = G(t) \quad \text{and} \quad \Gamma^* \cdot 1 = R(t)$$

the law of creep will then be

$$\varphi(\epsilon) = [1 + G(t)]\ \sigma_0 \tag{3.3}$$

and the law of relaxation

$$\sigma = [1 - R(t)]\,\varphi(\epsilon_0) \tag{3.4}$$

Equation (3.3) entirely agrees with that obtained from tests, formula (1.3). The expression given in section 1 for $G(t)$ shows that the operator K^* possesses a singularity of the type of the Abel operator. In the simplest case we may assume

$$K^* = \chi I^*_{-\alpha}$$

where $I^*_{-\alpha}$ is an operator with kernel $(t - \tau)^{-\alpha}/\Gamma(1-\alpha)$. Introducing the Э-operators employed in the previously cited paper (ref. 1) yields a very close agreement with test results, since the Э-operator contains an additional constant. [NACA Reviewer's Note: The Э-operator is an operator with the kernel $\text{Э}_\alpha(\beta, t - \tau) = (t - \tau)^\alpha \sum_0^\infty \frac{\beta^n (t - \tau)^{n(1+\alpha)}}{\Gamma[(n + 1)(1 + \alpha)]}$. The additional constant referred to is β.]

If, as is usually the actual case, the body is acted upon by constant loads or the displacement of its points is maintained by constant stresses, it is necessary in actual computation to deal with the two functions $G(t)$ and $R(t)$ which may be determined from tests forgetting about their origin from the kernel and as resolvents of the same integral equation.

This variant of the technical theory of creep is a forward step as compared with the aging theory which retains equation (3.3) both for creep and for relaxation.

The phenomenon of reverse creep (fig. 5) will now be considered. A stress σ is applied to the specimen and it undergoes the instantaneous deformation σ/E (point A); in the course of time T, creep occurs along the curve AB; at the instant T the stress is removed and at the same time there is removed the elastic deformation $BC = OA = \sigma/E$. In the time elapsed θ after the unloading there is also removed the deformation ϵ_r (point D). This process will be followed in the plane $\varphi\epsilon$ (fig. 6).

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The instantaneously applied load corresponds to the motion along the curve from point O to point A; during time T the strain increases and the function $\varphi(\epsilon)$ up to point B; the creep process is described by equation (3.1). At point B

$$\varphi(\epsilon_B) = [1 + G(T)]\,\sigma \tag{3.5}$$

With instantaneous unloading we drop to point C on the segment BC = AO, the reverse creep corresponding to the motion along this line up to the point D at instant θ. [NACA Reviewer's Note: It is also of interest to examine the effect of an instantaneous increase in load occurring after a period of constant-stress creep. If ϵ_o is the strain at the end of the creep period and $\delta\epsilon$ is an instantaneously imposed strain increment, then, according to equation (3.1), there is an instantaneous stress increment given by

$$\delta\sigma = \varphi(\epsilon_o + \delta\epsilon) - \varphi(\epsilon_o) \tag{a}$$

This result may be at variance with the facts, for recent experimental evidence, including tests on the propagation of plastic waves in bars subject to creep, suggests that after some creep has taken place materials behave elastically for small instantaneous increments of stress, that is,

$$\delta\sigma = E\,\delta\epsilon \tag{b}$$

Only for sufficiently small ϵ_o would equation (a) reduce to equation (b).] Here equation (3.2) must be used:

$$E\ (\epsilon - \epsilon_B) + \varphi_B = \left[G(T + \theta) - G(\theta)\right]\ \sigma \qquad (3.6)$$

[NACA Reviewer's Note: The coefficient σ in equation (3.6) was incorrectly omitted in the original.]

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It is seen from figure 6 that

$$\epsilon_r = \epsilon_B - \frac{\sigma}{E} - \epsilon$$

Making use of equations (3.5) and (3.6) yields

$$\epsilon_r = \frac{\sigma}{E}\left\{G(T) + G(\theta) - G(T + \theta)\right\} \qquad (3.7)$$

The linear dependence of the reverse creep on the stress and the symmetry of its dependence on T and θ is well confirmed by experiment (ref. 2).

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An important particular case of the relation (3.1) is obtained when $\varphi(\epsilon) = a\epsilon_\alpha^\beta$ and $K^* = \chi I^*_{-\alpha}$. The law of creep is then

$$a\epsilon_\alpha^\beta = \left[1 + \frac{\chi}{\Gamma(2 - \alpha)}\ t^{1 - \alpha}\right]\ \sigma$$

For large t, the second term in the brackets is the dominating one and

$$\epsilon = A\sigma^n t^m \qquad (3.8)$$

where

$$n = \frac{1}{\beta}, \qquad m = \frac{1-\alpha}{\beta}$$

If $1 - \alpha = \beta$, formula (3.8) gives, in the limit, a constant creep rate.

Section 4. The application of the theory presented in the preceding section to the problem of pure bending is considered herein. For simplicity, we restrict ourselves to the case of a rod of rectangular cross section, since we are concerned with the theoretical side of the problem. Assuming the hypothesis of plane sections, set

$$\epsilon = z/\rho$$

where ρ is the radius of curvature of the bent axis, and z the coordinate measured from the neutral axis and varying from $-h$ to $+h$.

By equation (3.1)

$$(1 + K^*)\ \sigma = \varphi\left(\frac{z}{\rho}\right) \tag{4.1}$$

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Multiplying by bzdz, where b is the width of the section, we integrate from $z = 0$ to $z = h$, and multiply the result by 2 (the function $\varphi(\epsilon)$ is analytically determined only for positive values of the argument; it is continued as an odd function in the region of negative values). Since the time and space operators are interchangeable,

$$(1 + K^*)\ M = 2b \int_0^h \varphi\left(\frac{z}{\rho}\right)\ zdz$$

where M is the bending moment. Introducing the notation

$$\frac{1}{x^2} \int_0^x \varphi(x)\ xdx = \phi(x)$$

yields

$$(1 + K^*)\ M = 2bh^2\ \phi\left(\frac{h}{\rho}\right) \tag{4.2}$$

The graph for the function $\phi(h/\rho)$ can be easily constructed. It is thus always possible to find the magnitude of the curvature from the given moment M for a given instant of time and then, by solving integral equation (4.1), to determine the stress distribution.

If the function $\varphi(\epsilon)$ is a power function, the problem is considerably simplified. Let $\varphi(x) = ax^{\beta}$. Then

$$\phi(x) = \frac{a}{2 + \beta}\ x^{\beta}$$

Dividing equation (4.1) by (4.2) yields

$$\frac{(1 + K^*)\ \sigma}{(1 + K^*)\ M} = \frac{2 + \beta}{2bh^2}\left(\frac{z}{h}\right)^\beta$$

whence

$$\sigma = \frac{2 + \beta}{2bh^2}\left(\frac{z}{h}\right)^\beta M \tag{4.3}$$

The distribution of the stresses is found to be independent of the time. We thus have in a certain sense a secondary creep although the rate of deformation is not constant as is clear from equation (4.2).

The result, equation (4.3), was obtained by Davis (ref. 3) from entirely different considerations. He made use of the theory of strain hardening, using equation (2.4) in the form

$$\dot{p} = Bp^k\sigma^t$$

As he found difficulty in accurately solving the problem as thus posed, Davis identified the plastic deformation with the total deformation and arrived at a creep law of the type (3.8) from which he readily obtained equation (4.3) in the case of bending. The experimental verification conducted by Davis well confirms the theoretical result.

Section 5. The generalization of any of the theories of creep to the three-dimensional case may be effected if there are taken as a basis the equations of the theory of small elasto-plastic deformations:

$$\sigma_x - \sigma = \frac{2\sigma_i}{3\epsilon_i}\epsilon_x, \quad \tau_{xy} = \frac{\sigma_i}{3\epsilon_i}\epsilon_{xy} \tag{5.1}$$

The change in volume at high temperatures may be neglected with greater justification than for normal temperatures since, starting from 400°, the Poisson coefficient for steel assumes a value of the order of 0.45 to 0.47.

The intensity of the stresses and the strains which are determined, following A. A. Ilyushin, as

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)}$$

$$\epsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_x - \epsilon_y)^2 + (\epsilon_y - \epsilon_z)^2 + (\epsilon_z - \epsilon_x)^2 + \frac{3}{2}\left(\epsilon_{xy}^2 + \epsilon_{yz}^2 + \epsilon_{zx}^2\right)}$$

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are assumed to be connected with each other by the same relations as the stress and strain in the one-dimensional problem.

By the theory herein,

$$\varphi(\epsilon_i) = (1 + K^*)\,\sigma_i \tag{5.2}$$

The problem of a pipe under the action of an internal pressure is very simply solved on the assumption of the absence of axial deformation. Let σ_r and σ_θ be the radial and transverse stresses, $\epsilon_r = du/dr$ and $\epsilon_\theta = u/r$ the corresponding strains expressed in terms of the radial displacement u, b and a the external and the internal radius of the pipe, and q the internal applied pressure. Since $\epsilon_x = 0$, there follows from the condition of incompressibility

$$\frac{du}{dr} + \frac{u}{r} = 0$$

whence

$$\epsilon_r = -\epsilon_0 = \frac{\sqrt{3}}{2}\frac{ea^2}{r^2}, \qquad \epsilon_i = \frac{ea^2}{r^2}$$

where e is an as yet undetermined function of time. Equations (5.1) give [NACA Reviewer's Note: In the following three equations, an error in sign appearing in the original has been corrected.]

$$\sigma_r - \sigma = \frac{1}{\sqrt{3}}\,\sigma_i$$

$$\sigma_\theta - \sigma = -\frac{1}{\sqrt{3}}\,\sigma_i$$

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whence

$$\sigma_r - \sigma_o = \frac{2}{\sqrt{3}} \sigma_i \tag{5.3}$$

Substituting (5.3) in the equation of equilibrium

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_o}{r} = 0$$

yields

$$\frac{d\sigma_r}{dr} = \frac{2}{\sqrt{3}} \frac{\sigma_i}{r} \tag{5.4}$$

Multiplying by $(1 + K^*)$, integrating from $r = a$, and noting that

$$(1 + K^*)\ \sigma_i = \varphi(\epsilon_i) = \varphi\left(\frac{ea^2}{r^2}\right)$$

yield

$$(1 + K^*)(\sigma_r + q) = \frac{2}{\sqrt{3}} \int_a^r \varphi\left(\frac{ea^2}{r^2}\right) \frac{dr}{r}$$

Setting

$$\int_o^x \varphi(x)\ \frac{dx}{x} = \Phi(x)$$

gives

$$(1 + K^*)(\sigma_r + q) = \frac{1}{\sqrt{3}} \left[\Phi(e) - \Phi\left(e\ \frac{a^2}{r^2}\right)\right] \tag{5.5}$$

For $r = b$, $\sigma_r = 0$. From this condition, the integral equation for determining the function of time e is obtained:

$$(1 + K^*)q = \frac{1}{\sqrt{3}} \left[\Phi(e) - \Phi\left(e\ \frac{a^2}{b^2}\right)\right] \tag{5.6}$$

The case where $\varphi(x) = ax^{\beta}$ here too leads to considerable simplification; in this case

$$\Phi(x) = \frac{1}{\beta} x^{\beta}$$

[NACA Reviewer's Note: The parameter a in ax^{β} is evidently being taken as 1.]

From equation (5.5),

$$(1 + K^*)(\sigma_r + q) = \frac{1}{\beta\sqrt{3}} e^{\beta} \left[1 - \frac{a^{2\beta}}{r^{2\beta}}\right]$$

$$(1 + K^*)q = \frac{1}{\beta\sqrt{3}} e^{\beta} \left[1 - \frac{a^{2\beta}}{b^{2\beta}}\right]$$

Dividing one equation by the other cancels the time function e on the right side. The integral operator $(1 + K^*)$ likewise cancels and a time-independent law for the stress distribution is obtained:

$$\sigma_r = q\left(\frac{a}{r}\right)^{2\beta} \frac{r^{2\beta} - b^{2\beta}}{b^{2\beta} - a^{2\beta}} \tag{5.7}$$

The velocities are not constant and they are found from equation (5.6) where, if q is given, the case reduces to quadratures and the successive determination of the function e from the graph. If e is given, the problem reduces to the solution of an integral equation for which it is necessary to know the resolvent of the kernel.

Section 6. Proceeding to the general case of the three-dimensional problem, we restrict ourselves to the consideration of those processes in which the stresses vary simultaneously in proportion to a parameter $\lambda(t)$, while the strains vary in proportion to a parameter $\mu(t)$. Then

$$\sigma_x = \lambda\bar{\sigma}_x \ . \ . \ . \ \epsilon_x = \mu\bar{\epsilon}_x$$

where $\bar{\sigma}_x$, . . . $\bar{\epsilon}_x$ . . . are functions only of the coordinates. [NACA Reviewer's Note: The bars which appear above σ_x and ϵ_x in this clause were incorrectly omitted in the original.] Equations (5.1) give

$$\lambda(\bar{\sigma}_x - \bar{\sigma}) = \frac{2}{3} \sigma_i \frac{\bar{\epsilon}_x}{\bar{\epsilon}_i}$$

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The stresses σ_i are connected with the strains by the relation (5.2).

If $\varphi(\epsilon_i)$ is a power function

$$\varphi(\epsilon_i) = a\epsilon_i^\beta = a\mu^\beta \overline{\epsilon}_i^\beta \tag{6.2}$$

Multiplying both sides of equation (6.1) by $1 + K^*$ gives

$$(1 + K^*)\lambda(\overline{\sigma}_x - \overline{\sigma}) = \frac{2}{3}\mu^\beta a \overline{\epsilon}_i^\beta \frac{\overline{\epsilon}_x}{\overline{\epsilon}_i} \tag{6.3}$$

[NACA Reviewer's Note: Errors in the subscripts on the right side of equation (6.3) appearing in the original have been corrected.]

Equations (6.3), of which only the first is written out, correspond to the integral equation

$$(1 + K^*)\lambda = \mu^\beta \tag{6.4}$$

and to a system of relations entirely agreeing with the equations of the theory of small elasto-plastic deformation:

$$\overline{\sigma}_x - \overline{\sigma} = \frac{2}{3}\frac{\overline{\sigma}_i}{\overline{\epsilon}_i}\overline{\epsilon}_x, \quad \overline{\tau}_{xy} = \frac{1}{3}\frac{\overline{\sigma}_i}{\overline{\epsilon}_i}\overline{\epsilon}_{xy}, \tag{6.5}$$

. . . .

where

$$\overline{\sigma}_i = a\overline{\epsilon}_i^\beta = \varphi(\overline{\epsilon}_i) \tag{6.6}$$

Systems (6.5) and (6.6) are systems of equations of the theory of secondary creep taken in a more general sense than the ordinary. The rates are now no longer constant but variable since the factors λ and μ which depend on the time are obtained from the integral equation (6.4).

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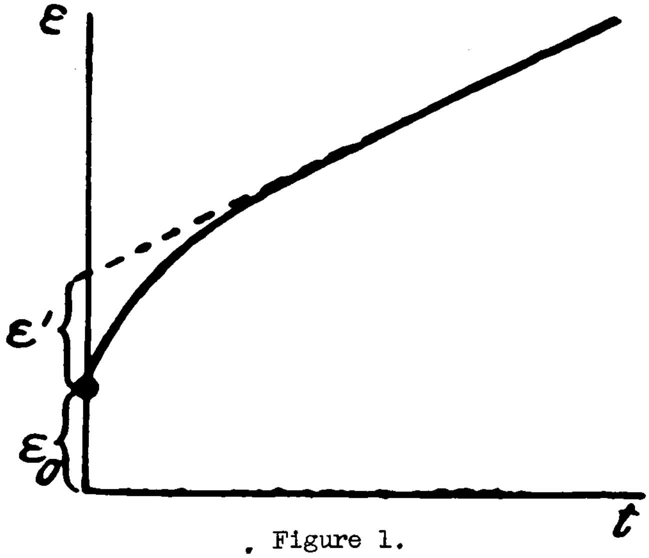

Figure 1.

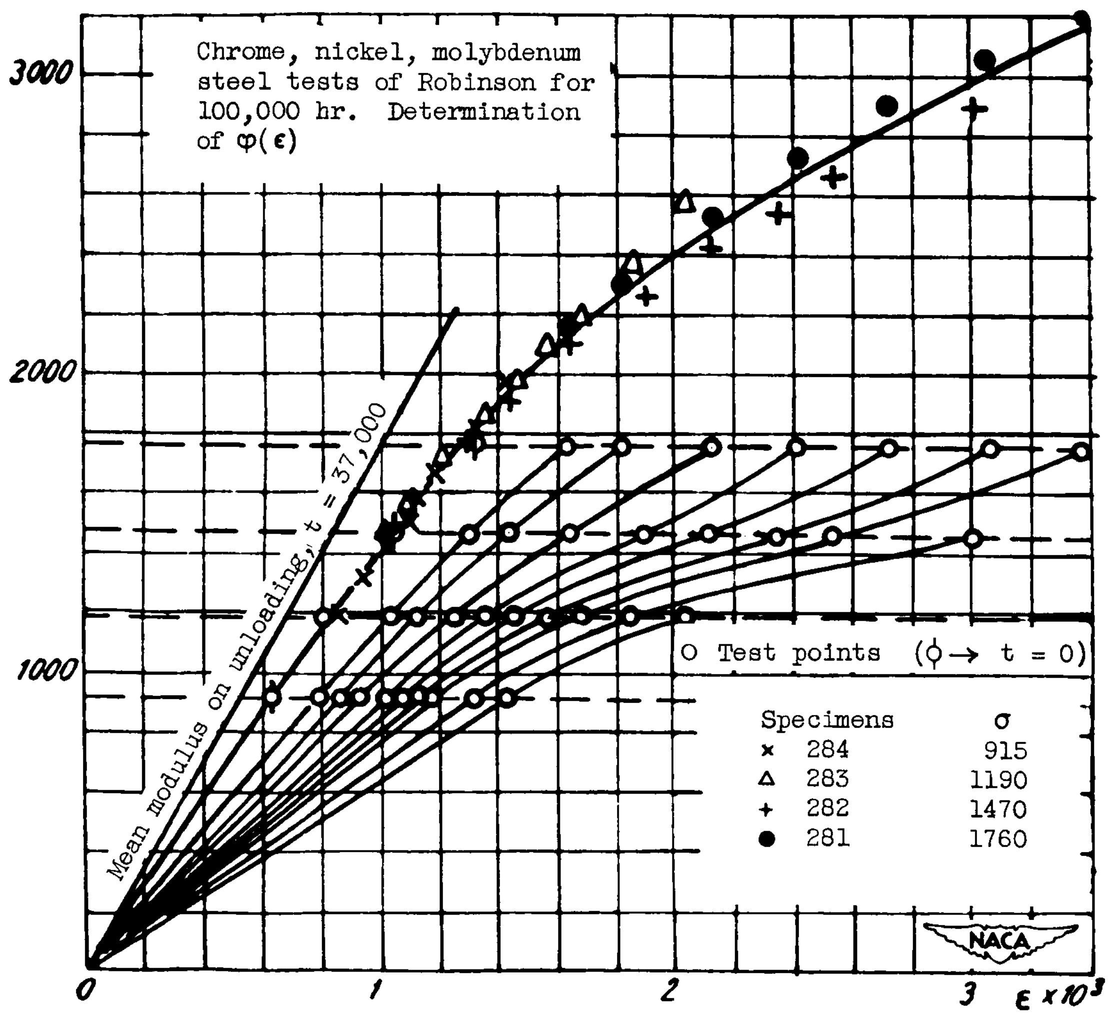

Figure 2.

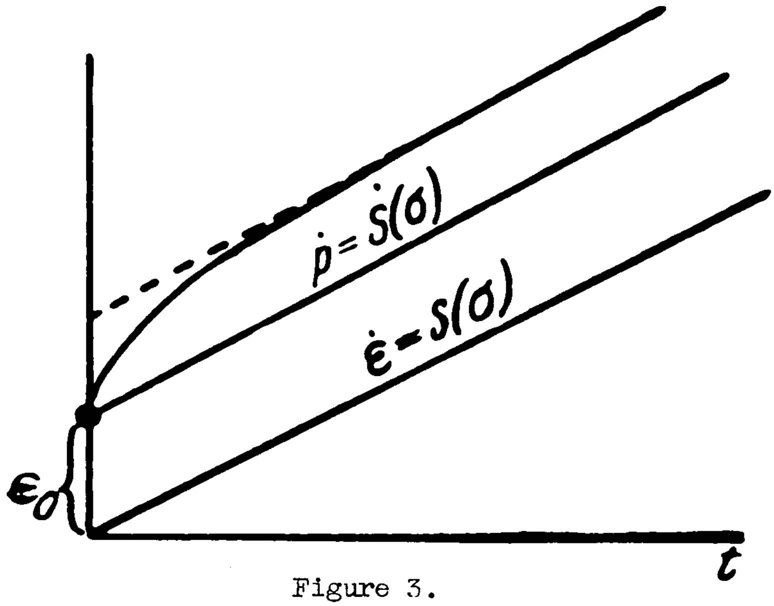

Figure 3.

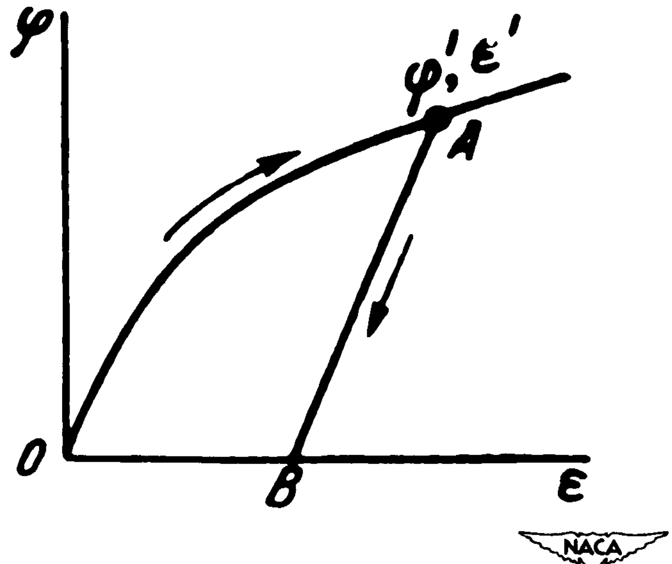

Figure 4.

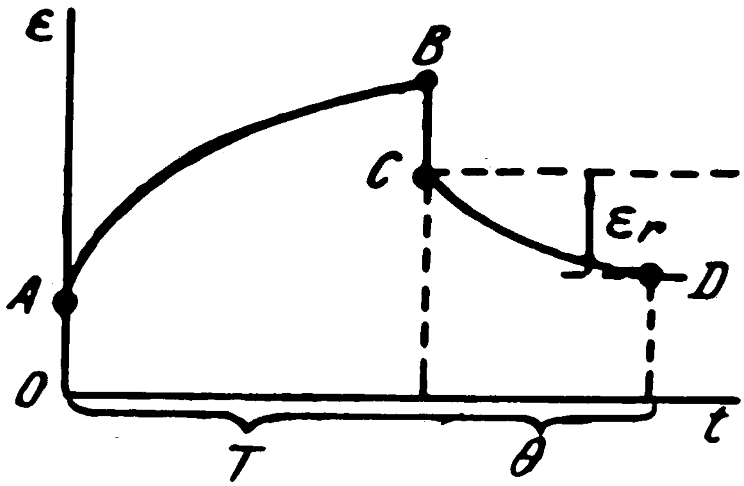

Figure 5.

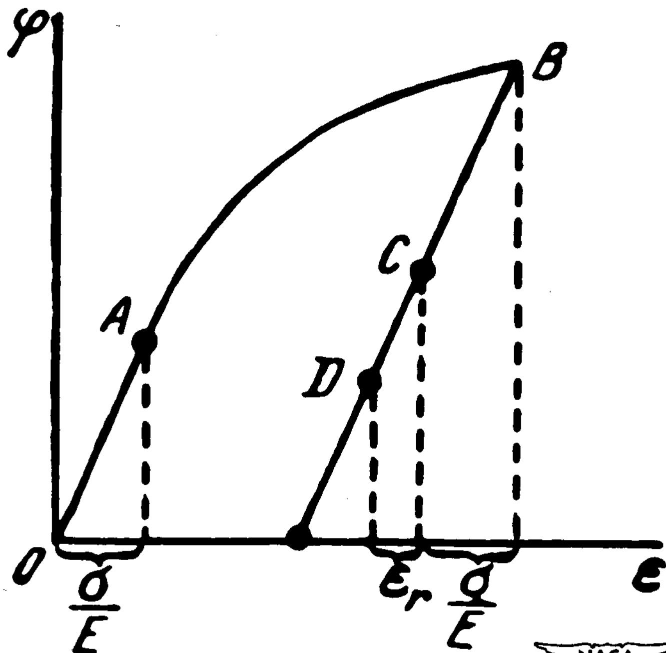

Figure 6.