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# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1273

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By S. N. Rzhevkin

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At the present time, considerable literature exits on resonance sound absorbers (references 1 to 8). In a particular case, a resonance sound absorber represents a system of resonators formed by one or several rigid perforated sheets (fig. 1) placed behind one another at certain distances $L_1$ and $L_2$ from the immovable wall A; the sound-absorbing properties are conditioned by the suitable choice of porous material (fabric or net) placed in the openings $\sigma_1$ and $\sigma_2$. For a normal incidence of sound, in the case where the openings of the resonators are arranged over a straight network, the presence of partitions between the sheets of the resonators is immaterial in computing the reflection and absorption of sound (evident from considerations of symmetry). For oblique incidence, the behavior of the system is essentially different for the cases with and without screens. The systems of resonators for absorbing sound and that possess partitions (compartments) are termed "resonance sound absorbers" whereas systems without partitions are termed "lamellar resonance sound absorbers". The computation of both systems in the aforementioned papers and in a number of unpublished papers has been carried out to a point of practical application and the properties of systems of resonance sound absorbers have been so thoroughly studied theoretically and experimentally that they can be fully recommended for application and for replacing the usual porous sound absorbers that have a number of defects.

In the theoretical computation, the front sheet of the resonance system as well as the intermediate sheets are assumed to be immovable. For the practical realization of the lamellar resonance system, the sheets may be attached without sufficient rigidity or, in general, may be constructed in the form of freely hanging perforated screens that may then carry out oscillations by the action of sound waves. In the case of systems with compartments, the importance of sympathetic vibrations of the resonator front wall and the perforated sheets should be considered. Such phenomenon may produce

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a marked effect when very light sheets are used. The theory and its experimental verification for layers of fabric was given by Wintergerst (references 1 and 15); Maliuzhinets (reference 9) made use of analogous considerations in obtaining more accurate computations of the sound absorption of a freely suspended perforated sheet. The present work is an attempt to carry out a more detailed investigation of the problem with the object of studying the effect of sympathetic vibration of the resonator front wall on sound absorption. The investigation is restricted to the case of a single-sheet resonance system for normal incidence of sound.

## 1. IMPEDANCE OF RESONATOR WITH YIELDING WALL

The computation of the sound absorption of a resonance system with the resonator openings arranged over a rectangular network may be replaced by the computation for a single resonator placed at the end of a pipe with rigid walls having a cross-sectional area $\Sigma$ equal to the area of the wall associated with one resonator. A plane sinusoidal sound wave of circular frequency $\omega$ with amplitude of the sound pressure $p_0$ falls on the resonator (fig. 2). The sound wave is assumed to act on the air layer at the opening $\sigma$ of the resonator with the force $\sigma p_0 e^{j\omega t}$ and on the front wall with the force $(\Sigma - \sigma)p_0 e^{j\omega t}$. The front wall of the resonator is represented by a plate fixed at the edges and fixed to the partitions that separate the cells from each other (fig. 3(a)). Under the action of sound waves, the plate will bend and undergo vibrations; the amplitude of the vibrations depends on the frequency $\omega$ and on the following parameters: mass $m_2$, friction, and elasticity of the plate. The plate may be replaced by a certain equivalent piston with area $\gamma\Sigma$ (fig. 3(b)) and mass $\beta m_2$ (where $\beta$ and $\gamma$ are numbers less than one) so that the amplitude of vibration of the piston is equal to the amplitude at the center of the plate and the corresponding frequencies are the same. A similar problem was solved for the circular plate (reference 10). For the frequency below resonance, $\gamma = 1/3$ and $\beta = 1/5$ are obtained. For a rectangular plate, the problem has not been solved but $\beta$ and $\gamma$ do not differ too greatly from their values for the round plate so that the same values may be assumed for initial computations.

In those cases where the series of neighboring resonators are not separated from each other by walls but are placed in a general compartment at the edges of which the front wall (or sheet) is fixed to a supporting frame, it may be assumed that in the region $\Sigma$ of one resonator (one opening), the wall moves as a piston; that $\beta = \gamma = 1$ may also be assumed.

The velocity of the air layer, the "plug", in the opening of the resonator is denoted by $\dot{\xi}_1$ (the volume velocity by $\dot{X}_1$); the velocity at the center of the plate by $\dot{\xi}_2$ (the volume velocity by $\dot{X}_2$); the volume of the resonator by $v = \Sigma L$, where $L$ is the depth of the resonator; the mass of the air plug in the throat of the resonator by $m_1'$; and the associated mass during the vibrations of the air plug relative to the wall by $m_1''$. The total mass of the vibrating air at the throat is given as $m_1 = m_1' + m_1''$. The friction during the vibrations of the air plug at the resonator opening is designated $r_1$. The mass of the plate constituting the wall of the resonator is denoted by $m_2$ and the mass of the equivalent piston is consequently $\gamma m_2$. The elasticity and friction of the equivalent piston are designated $e_2$ and $r_2$.

In setting up expressions for the kinetic energy connected with the associated mass and for the dissipating function, it was assumed that these magnitudes depend on the relative velocity of the wall and of the air plug. In computing the potential energy of the volume of the resonators, the fact that this computation depends on the volume displacement $X = (\gamma\Sigma - \sigma)\xi_2 + \sigma\xi_1$ and will be equal to $1/2\ EX^2$ where $E = \rho c^2/v$ must be considered. For the kinetic energy $T$, the dissipating function $F$, and the potential energy $V$ of the entire system, the following expressions are obtained:

$$T = \frac{1}{2} m_1'\dot{\xi}_1^{\,2} + \frac{1}{2} m_1''(\dot{\xi}_1 - \dot{\xi}_2)^2 + \frac{1}{2} m_2\dot{\xi}_2^{\,2} \tag{1a}$$

$$F = \frac{1}{2} r_1(\dot{\xi}_1 - \dot{\xi}_2)^2 + \frac{1}{2} r_2\dot{\xi}_2^{\,2} \tag{1b}$$

$$V = \frac{1}{2} EX^2 + \frac{1}{2} e_2\xi_2^{\,2} \tag{1c}$$

The equations of motion by the Lagrange method are obtained from equations (1) and have the form

$$\begin{gathered}
m_1''(\ddot{\xi}_1 - \ddot{\xi}_2) + m_1'\ddot{\xi}_1 + r_1(\dot{\xi}_1 - \dot{\xi}_2) + E\sigma\left[\sigma\xi_1 + (\gamma\Sigma - \sigma)\xi_2\right] = \sigma p_0 e^{j\omega t} \\
-m_1''(\ddot{\xi}_1 - \ddot{\xi}_2) - r_1(\dot{\xi}_1 - \dot{\xi}_2) + E\left[\sigma\xi_1 + (\gamma\Sigma - \sigma)\xi_2\right](\gamma\Sigma - \sigma) + \\
m_2\ddot{\xi}_2 + r_2\dot{\xi}_2 + e_2\xi_2 = (\gamma\Sigma - \sigma)\ p_0 e^{j\omega t}
\end{gathered} \tag{2}$$

For the forces acting on the wall piston, the value of the magnitude $(\gamma\Sigma - \sigma)p_0e^{j\omega t}$ and not the entire pressure $(\Sigma - \sigma)p_0e^{j\omega t}$ must likewise be assumed because the work done by the pressure force over the entire surface of the plate must be replaced by the work of a certain equivalent force applied at the center, as shown by Schuster (reference 11).

By solving the problem for the steady state of vibrations and by setting $\dot{\xi}_1 = \dot{\xi}_{10}e^{j\omega t}$ and $\dot{\xi}_2 = \dot{\xi}_{20}e^{j\omega t}$, the following expressions for the amplitudes of the velocities may be obtained from equation (2):

$$\dot{\xi}_{10} = \frac{\sigma z_2 + \gamma\Sigma z_2'}{z_1z_2 + z_1'z_2'}p_0 \qquad \dot{\xi}_{20} = \frac{-\sigma z_1' + \gamma\Sigma z_1}{z_1z_2 + z_1'z_2'}p_0 \tag{3}$$

where

$$\left.\begin{aligned} z_1 &= r_1 + j\left[(m_1'+m_1'')\omega - \frac{E\sigma^2}{\omega}\right] \qquad z_2 = r_2 + j\left[m_2\omega - \frac{e_2 + (\gamma\Sigma-\sigma)\gamma\Sigma E}{\omega}\right] \\ z_1' &= j\left[m_1'\omega - \gamma\frac{\Sigma\sigma E}{\omega}\right] \qquad z_2' = r_1 + \left[m_1''\omega + \frac{(\gamma\Sigma-\sigma)\sigma E}{\omega}\right] \end{aligned}\right\} \tag{4}$$

The acoustic impedance of the entire system will be equal to the ratio of the amplitude of the pressure $p_0$ to the volume velocity $\dot{X}_0$ or

$$Z = \frac{p_0}{\dot{X}_0} = \frac{p_0}{\sigma\dot{\xi}_1 + (\gamma\Sigma-\sigma)\dot{\xi}_2} = \frac{z_1z_2 + z_1'z_2'}{(\gamma\Sigma-\sigma)(\gamma\Sigma z_1-\sigma z_1') + \sigma(\sigma z_2+\gamma\Sigma z_2')} \tag{5}$$

After transformation of preceding equation

$$Z = \frac{a - b\omega^2 - \frac{c}{\omega^2}+j\left(d\omega - \frac{e}{\omega}\right)}{f + j\left(g\omega - \frac{h}{\omega}\right)} \tag{6}$$

where

$$
\left.
\begin{aligned}
a &= r_1r_2 + e_2(m_1'+m_1'') + E\left[m_1''\gamma^2\Sigma^2 + m_1'(\gamma\Sigma-\sigma)^2 + m_2\sigma_2{}^2\right] \\
&\cong r_1r_2{}^2 + e_2m_1 + E(m_1\gamma^2\Sigma^2 + m_2\sigma^2) \\
b &= m_1'm_1'' + m_2m_1' + m_2m_1'' \cong m_1m_2 \qquad c = E\sigma^2e_2 \\
d &= r_1(m_1'' + m_2) + r_2(m_1'' + m_1') \cong r_1m_2 + r_2m_1 \\
e &= r_1e_2 + E(r_1\gamma^2\Sigma^2+r_2\sigma^2) \qquad f = r_1\gamma^2\Sigma^2 + r_2\sigma^2 \\
g &= m_1''\gamma^2\Sigma^2 + m_1'(\gamma\Sigma-\sigma)^2 + m_2{}^2\sigma^2 \cong m_1\gamma^2\Sigma^2 + m_2\sigma^2 \\
h &= e_2\sigma^2
\end{aligned}
\right\} \tag{7}
$$

In these expressions, simplifications were made in the second parts of the equations on the assumption that $\sigma \ll \Sigma$, as is usually satisfied in practical cases. Thus, $(\gamma\Sigma-\sigma)^2 \cong \gamma^2\Sigma^2$. In the case of very thin sheets where $m_1' \ll m_1''$, the same result is obtained for any ratio of $\sigma$ to $\Sigma$. Moreover, $m_1'$, $m_1'' \ll m_2$ may always be assumed. The aforementioned magnitude $m_1 = m_1' + m_1''$ represents the entire mass that moves with velocity $\xi_1$ and is the equivalent mass in the throat of the resonator. Satisfying the condition $\sigma \ll \Sigma$ is also required in order that the associated mass may be computed by the Rayleigh formula $m_1'' = \rho\sigma^2/K$, where $K$ is the conductivity equal to the diameter of the opening $D$. If $\Sigma/\sigma < 10$, a correction on the closeness of the openings must be introduced in computing $K$ and therefore $K > D$. The theory of the problem and the value of the correction are given by Fok in reference 12.

By dividing the numerator and denominator of equation (6) by $\sigma^2\gamma^2\Sigma^2$ and by making the aforementioned approximations in equations (7), the following value for $Z$ is obtained:

$$
Z = \frac{R_1R_2+M_1E_2+(M_1+M_2)E-M_1M_2\omega^2 - \dfrac{EE_2}{\omega^2} + j\left[(R_1M_2+R_2M_1)\omega - \dfrac{R_1E_2+(R_1+R_2)E}{\omega}\right]}{R_1+R_2+j\left[(M_1+M_2)\omega - \dfrac{E_2}{\omega}\right]} \tag{8}
$$

where

$$R_1 = \frac{r1}{\sigma^2}; \quad M_1 = \frac{m1}{\sigma^2}; \; R_2 = \frac{r2}{\gamma^2\Sigma^2}; \quad M_2 = \frac{m2}{\gamma^2\Sigma^2}; \; E_2 = \frac{e2}{\gamma^2\Sigma^2}; \quad E = \frac{\rho c^2}{v} \tag{9}$$

These magnitudes are the parameters of the system expressed in the acoustic system of units.

The expression (8) may also be written in the form

$$Z = R + jY \tag{10}$$

where

$$R = \frac{R_1R_2(R_1+R_2) + (R_1M_2+R_2M_1)^2\omega^2 - 2R_1M_2E_2 + \dfrac{R_1E_2{}^2}{\omega^2}}{(R_1+R_2)^2 + \left[(M_1+M_2)\ \omega - \dfrac{E_2}{\omega}\right]^2} \tag{11}$$

$$Y = \frac{\left[(R_1{}^2M_2+R_2{}^2M_1) - M_1(M_1+2M_2)\ E_2 - (M_1+M_2)E\right]\ \omega + M_1M_2(M_1+M_2)\omega^3}{(R_1+R_2)^2 + \left[(M_1+M_2)\ \omega - \dfrac{E_2}{\omega}\right]^2} + \frac{\dfrac{2(M_1+M_2)EE_2 + M_1E_2{}^2 - (R_1+R_2)^2\ E - R_1{}^2E_2}{\omega} - \dfrac{EE_2{}^2}{\omega^3}}{(R_1+R_2)^2 + \left[(M_1+M_2)\ \omega - \dfrac{E_2}{\omega}\right]^2} \tag{12}$$

It is not difficult to show[1] that the impedance of this acoustic system in the approximation corresponding to equations (7) has its electrical analogy in the system shown in figure 4.

If the frequency of the resonator $\omega_1$ and of the resonator front wall $\omega_2$ are introduced

[1]The proof is given in the work of V. A. Tokar.

$$\omega_1 = \sqrt{\frac{E}{M_1}} \qquad \omega_2 = \sqrt{\frac{E_2}{M_2}} \tag{13}$$

the expressions for R and Y assume the form

$$R = R_1 \frac{R_1 R_2\left(1 + \frac{R_2}{R_1}\right) + M_2{}^2\omega^2\left[\left(1 + \frac{\omega_2{}^2}{\omega^2}\right)^2 + \frac{R_2}{R_1}\frac{M_1{}^2}{M_2{}^2}\right]}{R_1{}^2\left(1 + \frac{R_2}{R_1}\right)^2 + M_2{}^2\omega^2\left[1 - \frac{\omega_2{}^2}{\omega^2} + \frac{M_1}{M_2}\right]^2} \tag{14}$$

$$Y = \frac{R_1{}^2 M_2\omega\left[1 + \frac{R_2{}^2}{R_1{}^2}\frac{M_1}{M_2} - \left(1 + \frac{R_2}{R_1}\right)^2\frac{M_1}{M_2}\frac{\omega_1{}^2}{\omega^2} - \frac{\omega_2{}^2}{\omega^2}\right]}{R_1{}^2\left(1 + \frac{R_2}{R_1}\right)^2 + M_2{}^2\omega^2\left[1 - \frac{\omega_2{}^2}{\omega^2} + \frac{M_1}{M_2}\right]^2} +$$

$$\frac{M_1 M_2{}^2\omega^3\left(1 - \frac{\omega_2{}^2}{\omega^2} + \frac{M_1}{M_2}\right)\left[\left(1 - \frac{\omega_1{}^2}{\omega^2}\right)\left(1 - \frac{\omega_2{}^2}{\omega^2}\right) - \frac{M_1}{M_2}\frac{\omega_1{}^2}{\omega^2}\right]}{R_1{}^2\left(1 + \frac{R_2}{R_1}\right)^2 + M_2{}^2\omega^2\left[1 - \frac{\omega_2{}^2}{\omega^2} + \frac{M_1}{M_2}\right]^2} \tag{15}$$

## 2. RESONANCE FREQUENCIES OF SYSTEM

In order to find the resonance frequencies of the system, the condition $Y = 0$ must be satisfied and results, as is easily seen from equation (12), in the solution of a cubic equation in $\omega^2$. The roots of the equation for the resonance frequencies can be found by assuming the absence of dissipative terms, that is, for $R_1 = R_2 = 0$. The equation for determining the resonance frequencies can then be obtained from equation (15) in the form

$$\left(1 - \frac{\omega_2{}^2}{\omega^2} + \frac{M_1}{M_2}\right)\left[\left(1 - \frac{\omega_1{}^2}{\omega^2}\right)\left(1 - \frac{\omega_2{}^2}{\omega^2}\right) - \frac{M_1}{M_2}\frac{\omega_1{}^2}{\omega^2}\right] = 0 \tag{16}$$

This equation breaks down into two others. If in expression (8) for the impedance $(M_1 + M_2)\omega - E_2/\omega$ is set equal to 0 that corresponds to $1 - \omega_2{}^2/\omega^2 + M_1/M_2 = 0$ gives a root of equation (16) equal to

$$\omega' = \sqrt{\frac{E_2}{M_1 + M_2}} \tag{17}$$

then, according to equation (8), the impedance will have a maximum but not a minimum value; that is, for the frequency $\omega'$ there is no resonance but "antiresonance" (resonance of the currents in the circuit of fig. 4). In this case, the two masses $M_1$ and $M_2$ in the circuit (fig. 4) are connected in series, which means that the velocities $\dot{\xi}_1$ and $\dot{\xi}_2$ are of opposite sign.

The equation for finding the resonance frequencies is thus obtained by equating to zero only the second part of equation (16).

$$\left(1 - \frac{\omega_1{}^2}{\omega^2}\right)\left(1 - \frac{\omega_2{}^2}{\omega^2}\right) - \frac{M_1}{M_2}\frac{\omega_1{}^2}{\omega^2} = 0 \tag{18}$$

By solving this equation, the following expression is obtained (by setting $\mu = M_1/M_2$)

$$\omega^2{}_{res} = \frac{\omega_1{}^2 + \omega_2{}^2 + \mu\omega_1{}^2}{2} \pm \sqrt{\frac{\omega_1{}^2 + \omega_2{}^2 + \mu\omega_1{}^2}{2} - \omega_1{}^2\,\omega_2{}^2} \tag{19}$$

It is assumed that the magnitude

$$k^2 = \frac{\mu\omega_1{}^2}{\omega_1{}^2 + \omega_2{}^2} = \frac{\frac{M_1}{M_2}}{1 + \frac{\omega_2{}^2}{\omega_1{}^2}} = \frac{1}{\frac{M_2}{M_1} + \frac{E_2}{E}} \tag{20}$$

which is termed the coupling coefficient of the two systems, is small relative to unity. Thus

$$\omega^2{}_{res} \cong \frac{\omega_1{}^2 + \omega_2{}^2}{2} \pm \frac{\omega_1{}^2 + \omega_2{}^2}{2}$$

and for the resonance frequencies

$$\omega_{01} \cong \omega_1 \qquad\qquad \omega_{02} \cong \omega_2 \tag{21}$$

The resonance frequencies will be approximately equal to the frequencies $\omega_1$ and $\omega_2$ for the condition $k^2 \ll 1$. This condition will hold either for $\omega_2 \gg \omega_1$ (rigid wall with high natural frequency) or for the condition $M_2 \gg M_1$. The latter condition is generally realized in practical resonance systems with the exception of the cases where light sheets with very small perforation coefficients are used. For $\omega_{01}$ and $\omega_{02}$, the following approximate expressions may be obtained from equation (19):

$$\left.\begin{aligned}
\omega_{01}^2 &\cong \omega_1^2 \left[1 - \frac{1}{1 + \frac{1}{\mu}\left(\frac{\omega_2^2}{\omega_1^2} - 1\right)}\right] \\
\omega_{02}^2 &\cong \omega_2^2 \left[1 + \frac{1 - (2 - \mu)\frac{\omega_1^2}{\omega_2^2}}{1 + \frac{1}{\mu}\left(\frac{\omega_2^2}{\omega_1^2} - 1\right)}\right]
\end{aligned}\right\} \tag{22}$$

For the values of $\mu$ usually encountered $(\mu \leq 1)$, these expressions are suitable for $\omega_2 \gg \omega_1$. In this case, small correction terms in the brackets are included and consequently $\omega_{01} < \omega_1$ and $\omega_{02} > \omega_2$. If $\omega_1 = \omega_2$, equations (22) are not suitable. In the case $\omega_2 < \omega_1$, equations (22) is applicable for $\mu \ll 1$ and gives

$$\omega_{01}^2 = \omega_1^2(1 + \mu)$$

$$\omega_{02} \cong \omega_2^2 \left[1 + \frac{-2\frac{\omega_1^2}{\omega^2}}{-\frac{1}{\mu}}\right] = \omega_2^2\left(1 + 2\frac{E}{E_2}\right)$$

Thus, for $\omega_2 < \omega_1$ there is obtained $\omega_{01} > \omega_1$ and $\omega_{02} > \omega_2$. The shift of the resonance frequencies relative to the frequencies $\omega_1$ and $\omega_2$ is shown schematically in figure 5.

In the case $\omega_2 \gg \omega_1$, the antiresonance $\omega'$ will lie between $\omega_{01}$ and $\omega_{02}$ and

$$\omega_{02}^2 = (\omega')^2 = \frac{E_2}{M_2} - \frac{E_2}{M_1 + M_2} = \frac{E_2 \; M_1}{M_2(M_1 + M_2)} = \omega_2^2 \; \frac{\mu}{1 + \mu}$$

that is, $\omega_{02} > \omega'$. Conversely

$$(\omega')^2 - \omega_{01}^2 = \frac{E_2}{M_1 + M_2} - \frac{E}{M_1} = \frac{\omega_2^2}{1 + \mu} - \omega_1^2 \cong \frac{\omega_2^2}{1 + \mu} > 0$$

Thus

$$\omega_{01} < \omega' < \omega_{02} \tag{23}$$

In those cases where the partial frequencies $\omega_1$ and $\omega_2$ are near each other and $\mu \ll 1$, all three roots of equation (16) lie near each other. Hence, in the region near the root, the second term in the numerator Y in equation (15) will be small in comparison with the first term containing the large factor $R_1^2$. As a result, neglecting the dissipative term in the resonance equation (Y = 0) is no longer permissible. The function Y in this region is determined by the dissipative term and equating this term to zero will give to a first approximation the equation for determining the resonance frequency

$$1 + \frac{R_2^2}{R_1^2} \mu - \left(1 + \frac{R_1}{R_2}\right)^2 \mu \frac{\omega_1^2}{\omega^2} - \frac{\omega_2^2}{\omega^2} = 0 \tag{24}$$

from which

$$\omega_0^2 \cong \frac{\mu \left(1 + \frac{R_2}{R_1}\right)^2 \omega_1^2 + \omega_2^2}{1 + \frac{R_2^2}{R_1^2} \mu} \tag{25}$$

Thus, instead of two resonance and one antiresonance frequencies, only one resonance frequency is obtained. The character of the function Y in these two cases is shown in figure 6.

## 3. SOME SPECIAL CASES

The probable magnitude of the impedance for various special cases is considered.

(a) Resonator with rigid front wall. In this case, $e_2 \longrightarrow \infty$ and therefore $E_2 \longrightarrow \infty$. Thus, from equation (8),

$$Z = \frac{M_1 - \frac{E}{\omega^2} - j\,\frac{R_1}{\omega}}{-\,j\,\frac{1}{\omega}} = R_1 + j\left(M_1\,\omega - \frac{E}{\omega}\right) \tag{26}$$

The acoustic impedance of a simple resonator with friction $R_1 = r_1/\sigma^2$ in the opening is obtained.

(b) Front wall with no opening. In this case, $\sigma = 0$ and $M_1 = m_1/\sigma^2 = \infty$. By retaining only terms containing $M_1$ in the numerator and denominator of equation (8) and dividing by $M_1$

$$Z = R_2 + j\left(M_2\,\omega - \frac{E + E_2}{\omega}\right) \tag{27}$$

The impedance in the given case is determined by the mass of the diaphragm and the sum of the elasticities of the diaphragm $E_2$ and the air cushion $E$. This case corresponds to the membrane absorber investigated by Meyer (reference 13).

(c) Resonator front wall with only inertia resistance but no elasticity and friction. This case is equivalent to setting $\omega_2 = 0$ and $R_2 = 0$. In this case, which is practically realized in the form of a perforated sheet screen that is freely suspended at a certain distance from the wall and that vibrates without deforming

$$E_2 = 0 \qquad R_2 = 0 \qquad \gamma = 1$$

For the impedance, the following expression is obtained:

$$\omega_0 = \sqrt{\frac{(M_1+M_2)^2 E - R_1{}^2 M_2 \pm \sqrt{\left[(M_1+M_2)^2 E - R_1{}^2 M_2\right]^2 + 4R_1 M_1{}^2 M_2 (M_1+M_2) E}}{2M_1 M_2 (M_1+M_2)}} \tag{29}$$

The minus sign under the root is not applicable because it gives imaginary values for $\omega_0$ and thus the system has only one resonance frequency. As an initial approximation, $R_1$ is set equal to 0 and

$$\omega_0 = \sqrt{\frac{E}{\frac{M_1 M_2}{M_1 + M_2}}} \tag{30}$$

In the given case, $\omega_0$ is determined by the elasticity of the air cushion $E$ and the mass

$$M' = \frac{M_1 M_2}{M_1 + M_2} = \frac{1}{\frac{1}{M_1} + \frac{1}{M_2}} \tag{31}$$

Thus representing a case of the "parallel connection" of the masses $M_1$ and $M_2$. This system is termed an "acoustic mushroom" (Tonpilz) and has a resonance frequency higher than the frequency of the resonator with rigid wall. Such a system may be represented by the scheme of figure 7. The impedance of the system in the region of high frequencies will be

$$Z \cong \frac{R_1 M_2{}^2}{(M_1 + M_2)^2} + j\left[\frac{M_1 M_2}{M_1 + M_2}\omega - \frac{E}{\omega}\right] \tag{32}$$

If the perforated sheet is suspended in a free space far from the wall, $E \cong 0$; for high frequencies, the active resistance will be small in comparison with the reactive and

$$Z \cong j \frac{M_1 M_2}{M_1 + M_2} \omega \tag{33}$$

This expression was obtained in a somewhat different manner by Maliuzhinets (reference 9) and was used by him to compute the sound conductivity of perforated screens.

(d) Layer of porous material. If the dimensions of the openings of the perforations are assumed to be near in value to the pitch of the perforations, then, as follows from the Fok equation (reference 12), the associated mass will be very small; thus, $M_1 \ll M_2$ and equation (28) assumes the form

$$Z \cong R_1 \frac{M_2^{\,2}\,\omega^2}{R_1^{\,2} + M_2^{\,2}\,\omega^2} + j\left(M_2\,\omega - \frac{E}{\omega}\right)\frac{R_1^{\,2}}{R_1^{\,2} + M_2^{\,2}\,\omega^2} \tag{34}$$

This relation is applicable to the case of a material with minute openings (porosity) that is freely suspended at a certain distance from the wall, for example a layer of fabric or a number of superposed meshed layers. Wintergerst (reference 15) gave the theory of the absorption of sound by an infinite layer of a dense porous material with account taken of the sympathetic vibration of the rigid frame of the material. The structure of equation (34), in which in this case $E$ must be set equal to 0, has much in common with the equations of the Wintergerst theory.

As generalizations of cases (a) and (b), the behavior of a resonance system with a yielding wall is considered for frequencies considerably lower and higher than the natural frequency of the resonator wall. In choosing these cases, the magnitude $M_1$ in real systems (as will be evident from additional examples) is assumed to be generally much less than $M_2$ and in rare cases is of the same order as $M_2$ ($\mu \leqslant 1$); the value of $R_2$, however, is less than that of $R_1$. From equation (14), the following relation is obtained for the case $\omega \ll \omega_2$ when considering the fact that the first term in the numerator and denominator will be much less than the second:

$$R\Big|_{\omega\ll\omega_2} \cong R_1 \frac{\left(1 - \frac{\omega_2^{\,2}}{\omega^2}\right)^2 + \frac{R_2}{R_1}\,\mu^2}{\left(1 - \frac{\omega_2^{\,2}}{\omega^2} + \mu\right)^2} \cong R_1 \frac{\left(1 - \frac{\omega^2}{\omega_2^{\,2}}\right)^2}{\left[1 - (1+\mu)\,\frac{\omega^2}{\omega_2^{\,2}}\right]^2} \cong R_1\left(1 + \mu\frac{\omega^2}{\omega_2^{\,2}}\right) \cong R_1 \tag{35}$$

Thus

$$Y\Big|_{\omega\ll\omega_2} \cong R_1\,\frac{R_1}{M_2\omega}\,\frac{\omega^2}{\omega_2^{\,2}} + M_1\,\omega\left[\left(1 - \frac{\omega_1^{\,2}}{\omega^2}\right) + \mu\,\frac{\omega_1^{\,2}}{\omega^2}\cdot\frac{\omega^2}{\omega_2^{\,2}}\right] \cong M_1\,\omega - \frac{E}{\omega} \tag{36}$$

Thus R and Y remain, as in case (a), near those values that hold for the resonance system with an immovable wall.

For frequencies considerably higher than the resonance frequency of the wall $\omega \gg \omega_2$, which is a generalization of case (c), the following relation is obtained

$$R\Big|_{\omega \gg \omega_2} \cong R_1 \frac{1 + \frac{R_2}{R_1}\mu^2}{(1+\mu)^2} \cong \frac{R_1}{(1+\mu)^2} \tag{37}$$

Thus

$$Y_{\omega \gg \omega_2} \cong \frac{R_1\left[1 - \frac{R_2^2}{R_1^2}\mu - \left(1 + \frac{R_2}{R_1}\right)\mu\frac{\omega_1^2}{\omega^2}\right]}{M_2\,\omega\,(1+\mu)^2} + M_1\,\omega\,\frac{1 - (1+\mu)\frac{\omega_1^2}{\omega^2}}{1+\mu} \cong \frac{M_1 M_2}{M_1 + M_2}\,\omega - \frac{E}{\omega} \tag{38}$$

For high frequencies, the system thus has a less active resistance and small mass than in the case of an immovable wall. For systems in which $\mu = M_1/M_2 \ll 1$, the effect of the sympathetic vibrations of the wall, even in the case $\omega \gg \omega_2$, is not of great importance.

## 4. VOLUME VELOCITY THROUGH WALL AND RESONATOR THROAT

In order to describe the nature of the phenomenon of the reflection of sound from a resonance system with a yielding wall, the relation between the volume velocities $\dot{X}_{10}$ and $\dot{X}_{20}$ through the area of the wall $\Sigma - \sigma$ and the resonator throat $\sigma$ must be considered. From equations (3) and (4), the following relations are obtained by neglecting the small terms, as was done in deriving expressions (7):

$$\dot{X}_{10} \cong \frac{R_2 + j\left(M_2\,\omega - \frac{E_2}{\omega}\right)}{z'}\,p_0 \tag{39}$$

$$\dot{X}_{20} \cong \frac{R_1 + jM_1\,\omega}{z'}\,p_0 \tag{40}$$

where $z'$ is equal to the numerator of $Z$ in equation (8). For low frequencies $(\omega \ll \sqrt{E_2/M_2})$, the velocity $\dot{X}_{10} \cong (-jE_2/\omega)/(-EE_2/\omega^2) = j\omega p_0/E$ and the volume displacement $X_{10} \cong \dot{X}_{10}/j\omega = p_0/E$, which is the statistical volume displacement of the air in the opening under the action of the pressure $p_0$. The magnitude $\dot{X}_{20}$ will in the given case be very small in comparison with $\dot{X}_{10}$ and the entire process is restricted to the motion of the air in the opening at the immovable wall.

For $\omega \gg \sqrt{E_2/M_2}$

$$\dot{X}_{10} \cong \frac{jM_2\omega}{-M_1M_2\omega^2} p_0 = \frac{p_0}{j\omega M_1}$$

and

$$\dot{X}_{20} \cong \frac{jM_1\omega}{-M_1M_2\omega^2} = \frac{p_0}{j\omega M_2}; \qquad \dot{X}_{10} + \dot{X}_{20} = \frac{p_0}{j\omega \frac{M_1M_2}{M_1 + M_2}}$$

Thus for high frequencies, both the air in the opening and the wall take part in the motion. The ratio of the velocities is determined by the ratio of their acoustical masses. The motions of $M_1$ and $M_2$ occur in the same direction and are in phase.

For frequencies $\omega < \sqrt{E_2/M_2}$, the numerator of $\dot{X}_{10}$ contains a negative reactive resistance and in $\dot{X}_{20}$ a positive resistance. If $R_1$ and $R_2$ are small, the air in the opening and the wall move almost in opposite phases, that is, toward each other. If the condition

$$j\omega\left(M_2\omega - \frac{E_2}{\omega}\right) + jM_2\omega = 0$$

is satisfied, the following equation is obtained:

$$\dot{X}_0 = \dot{X}_{10} + \dot{X}_{20} = \frac{R_1 + R_2}{z'}$$

that is, $X_0$ will be a minimum and, in the case where $R_1 + R_2$ is small, $\dot{X}_0 \cong 0$ is obtained; the impedance $Z = p_0/X_0$ will be large and the system will almost not absorb the sound. This condition will occur for a frequency of antiresonance

$$\omega' = \sqrt{\frac{E_2}{M_1+M_2}} \tag{17}$$

For this frequency, the following relations are obtained (for small $R_1$ and $R_2$):

$$\dot{X}_{10} \cong - j\omega \frac{M_1+M_2}{M_1} \frac{p_0}{E_2}$$

$$\dot{X}_{20} \cong + j\omega \frac{M_1+M_2}{M_1} \frac{p_0}{E_2}$$

The volume velocity of the air in the throat will be equal and opposite in sign to the volume velocity of the resonator wall; that is, all the air compressed by the wall into the concavity is again forced out through the opening and the total volume velocity is near zero.

The behavior of the system for a frequency equal to the natural frequency of the resonator wall must be considered. From equations (3) and (4), as in equations (39) and (40)

$$\dot{\xi}_1 - \dot{\xi}_2 \cong \frac{1}{\sigma z'} \left[R_2 + j\left(M_2\omega - \frac{E_2}{\omega}\right)\right] \tag{41}$$

For $\omega = \omega_2 = \sqrt{E_2/M_2}$, the difference in velocities $\dot{\xi}_1 - \dot{\xi}_2$ will be determined only by the magnitude of the friction $R_2$. If the wall of the resonator is slightly damped $(R_2 \cong 0)$

$$\dot{\xi}_1 - \dot{\xi}_2 \cong 0 \tag{42}$$

Although the friction $R_1$ in the throat is considerable, it will not lead to a dissipation of energy because the first term of the dissipative function (1b) depending on $\dot{\xi}_1 - \dot{\xi}_2$ is near zero. For this case, $R \cong 0$ is obtained from equation (14). The coefficient of sound absorption, as given in reference 6, will be equal to

$$\alpha = \frac{4R \frac{\rho c}{\Sigma}}{\left(R + \frac{\rho c}{\Sigma}\right)^2 + Y^2} \tag{43}$$

where $\rho$ is the density of the air and $c$ is the velocity of sound. For $\omega = \omega_2$ and $R_2 \cong 0$, $\alpha$ will be equal to zero. The range of frequencies in which $\alpha$ is close to zero will be very narrow, as will be subsequently shown for a particular example. If $R_2$ is small but not zero, the following relation is obtained from equation (14) for $\omega = \omega_2$:

$$R \cong \frac{R_1 R_2}{R_1 + R_2} = \frac{R_2}{1 + \frac{R_2}{R_1}} \tag{44}$$

If $R_2 \ll R_1$, then $R \cong R_2$. The values of $R_2$ encountered under actual conditions are already sufficient so that $R$ becomes of the same order as $\rho c/\Sigma$ or even greater than $\rho c/\Sigma$. Accordingly, for $\omega = \omega_2$, a dip is no longer obtained in the curve $\alpha$ but on the contrary a peak may appear.

## 5. EXAMPLES OF APPLICATION OF THE THEORY

I. As an example of the application of the obtained relations, a wide-range resonance sound absorber is considered for which the normal incidence of sound $\alpha \geqslant 0.6$ is in the range of 100 to 400 cycles per second; maximum $\alpha = 0.65$ for 214 cycles per second (fig. 8). Such an absorber consists of a sheet of thin iron ($l_0 = 0.05$ cm) placed at 33 centimeters from the wall with openings of diameter $d = 0.4$ centimeter at distances of 2.6 centimeters from each other. The zone of the openings is glued to a wide meshed cotton cloth with friction coefficient $r = 3$ mechanical ohms per square centimeter. A more detailed computation of the sound absorber for a diffusive sound field (reference 14) shows that $\alpha \geqslant 0.5$ in the range from 100 to 1500 cycles, reaching a maximum at about 300 cycles ($\alpha_{max} \cong 0.7$).

The computation of the characteristic for normal incidence with an immovable wall (solid thin curve of fig. 8) was determined from equation (44) in which the elastic reactive resistance of the air layer with a depth of 33 centimeters was computed by assuming a concentrated elasticity. This computation is practically unsatisfactory for frequencies above 400 cycles (a drop to zero is obtained at 500 cycles in the $\alpha$ curve) and is comparable to only similar computations in which the sympathetic vibration of the wall is taken into account and the elasticity $E$ is considered as concentrated.

For the sound absorber considered and from equations (9) (by setting $\beta = \gamma = 1$)

$$M_1 = 3.5 \cdot 10^{-3} \frac{g}{CM^4} \qquad R_1 = 24 \frac{\text{mech.ohm}}{CM^4} \qquad E = 0.63 \cdot 10^4 \frac{\text{dynes}}{CM^3} \text{ sec}$$

$$M_2=5.8\cdot 10^{-2}\ \frac{g}{CM^4} \qquad \omega_1=\sqrt{\frac{E}{M_1}}=1.34\cdot 10^3 \qquad f_1=214\ \frac{\text{cycles}}{\text{sec}} \qquad \mu=\frac{M_1}{M_2}=0.06$$

The value of the elasticity $E_2$ is unknown but depends on the conditions of the attachment of the iron sheet. For example, the frequency of the antiresonance $\omega' = \omega_2/\sqrt{1+\mu}$, approximately equal to the resonance frequency of the wall, is assumed to coincide with the frequency of the resonator $\omega_1$. In order to obtain such a frequency, the iron sheet must be attached to a frame with square cells (for example, $14\times 14\ cm^2$). Under the condition $\omega' = \omega_1$, the distortions introduced by the sympathetic vibrations of the wall will lie in the region of greatest absorption of the resonance system.

The value of $R_2$ is also unknown but probably depends on the nature of the attachment of the iron sheet to the frame and may vary within wide limits. On the basis of test data, it is found that the decrement of the damping of the free vibrations of the iron membranes has the order of magnitude of 0.1. Thus, in the given case, $R_2 \cong 0.1\ R_1$.

In order to describe the changes in the sound absorption for various degrees of damping of the resonator wall, the computation was carried out for the following values (fig. 8): (1) $R_2 = 0$ (dotted curve), (2) $R_2 = 0.1\ R_1$ (solid curve), and (3) $R_2 = R_1$ (dot-dashed curve).

The case for which a drop in $\alpha$ down to zero is obtained can hardly be realized in systems of practical application. Even a very small damping of the iron sheet results in a leveling of the dip in the curve $\alpha$. For very strong damping, a peak in the curve occurs instead of a dip. It can be noted that $\alpha$ changes principally because of the effect of $R_2$ on the active resistance; the reactive resistance Y changes very little. In real systems, the damping of the individual cells of the sheet that form the attachment to the supporting frames will be very different, diverging from the mean in either direction. On the average, for a large area of the absorber, $\alpha$ may be assumed to vary approximately as for the case of an immovable resonator wall.

II. As a second example, a sound absorber with a narrow absorption range $\alpha \geqslant 0.7$ is considered in the region of low frequencies from 50 to 70 cycles per second. By the method of computation given in reference 6, the front wall of the resonators must be placed at a distance of 21 centimeters from the wall to obtain such a sound absorber. If the wall is made of veneer $(l_0 = 0.6 \text{ cm})$, the openings $(d = 3 \text{ cm})$ must be placed at distances of 35 centimeters from each other and a material must be introduced with a friction coefficient of about 0.5 mechanical ohm per square centimeter (gauze on metal mesh of similar thickness). In the given case, the frame cells to which the front wall is attached should be in the form of a square network with a length of 35 centimeters. The corresponding frequency of the square veneer plates of such dimensions is equal to $f_2 = 206$ cycles per second $(\omega_2 = 1.29 \times 103)$. The parameters of the system will have the values

$$M_1 = 0.4 \cdot 10^{-3} \qquad R_1 = 7.1 \cdot 10^{-2} \qquad E = 0.57 \cdot 10^2$$

$$M_2 = 0.62 \cdot 10^{-3} \qquad E_2 = 1.02 \cdot 10^3 \qquad \mu = \frac{M_1}{M_2} = 0.65 \qquad \frac{E_2}{E} = 18$$

where $\beta = 1/5$ and $\gamma = 1/3$.

On the basis of the estimate of the decrement of the damping, it must be assumed that $R_2 < R_1$. The coefficient of coupling in the given case will be small

$$k = \frac{1}{E_2/E + M_2/M_1} \cong 0.05$$

and on the basis of equations (22), the lower resonance frequency of the system $f_{01}$ under the effect of the sympathetic vibrations of the wall may be assumed to change only slightly

$$f_{01} = f_1(1 - 0.027)$$

that is, the resonance frequency is lowered by only 1.6 cycles per second. By solving equation (35), the magnitude $R$ in the region near $f_1$ is found to increase by 11 percent in comparison with $R_1$. The lowering in $\alpha_{max}$ from the value 0.87 (for the case of an immovable wall) to 0.83 results. Thus, the resonance of the wall changes $\alpha$ in the working range only slightly.

III. As a last example, a resonance sound absorber "without porous material" is considered. For a sound absorber of a relatively low effectiveness $(\alpha \cong 0.3 - 0.4)$, sufficient friction in the openings may be assured without the introduction of special porous material by the internal friction of the air flowing through the small opening alone. Thus, for a sound absorber with $\alpha \geqslant 0.3$ in the range from 100 to 200 cycles per second, a thin sheet of tin plate may be taken $(l_0 = 0.03 \text{ cm})$ with openings $(d = 0.07 \text{ cm})$ placed 3.2 centimeters apart at a distance of 4.85 centimeters from the wall attached to a corresponding cell frame that can be chosen arbitrarily. A cell measuring 6.4 centimeters long has twice the pitch as compared with the network of the perforated sheet. The resonance frequency of the wall will then be of the order of 500 cycles per second. The computation by the preceding method gives $\alpha$ the values shown in figure 9 (thin line); the computation for the immovable wall is shown by the thick line. In the given case, the absorption coefficient in the working range (100 to 200 cycles/sec) is markedly lowered because of the large value $\mu = 1.52$, notwithstanding the fact that $\omega_2 \gg \omega_1$. The dotted curve in the figure gives the result of the computation of $\alpha$ in the case of a nonfixed (freely hanging) sheet. The maximum $\alpha$ is raised to the value 0.98 and shifts upward to 220 cycles; at the lower frequencies in the region less than 100 cycles, $\alpha$ reduces to small values and thus it is evident that for the production of low-frequency sound absorbers of this type, the front wall should be sufficiently rigidly fixed.

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REFERENCES

1. Wintergerst, E., und Klupp, H.: VDI Zeitschr., Bd. 77, 91, 1933.

2. Rzhevkin, S. N.: Jour. Tech. Phys. (USSR), T. VI, 1936, p. 2103.

3. Antsyferov, M. S.: Jour. Tech. Phys. (USSR), T. VI, 1936, p. 2118.

4. Rzhevkin, S. N., Maliuzhinets, N. D., Terosipyants, S. T., and Antsyferov, M. S.: Otchet Otd. akust., str. Dvortsa Sovetov, 1937.

5. Rzhevkin, S. N.: DAN, vol. XVIII, 25, 1938.

6. Rzhevkin, S. N.: DAN, vol. XXII, 568, 1939.

7. Nesterov, V. S.: Jour. Tech. Phys. (USSR), T. IX, 1939, p. 1727; T. X, 1940, p. 617; DAN, vol. XXXI, 879, 1941.

8. Vital, K. A.: Jour. Tech. Phys. (USSR), T. X, 1940, p. 980.

9. Maliuzhinets, G. D.: Otchet Lab. akust., str. Dvortsa Sovetov, 1941.

10. Rzhevkin, S. N.: Jour. Tech. Phys. (USSR), T. V, 1935, p. 1440.

11. Schuster, K.: Handb. d. Exper. Phys., Bd. XVII, Teil 2, 22, 1932.

12. Fok, V. A.: DAN, vol. XXXI, 9, 1941.

13. Meyer, E.: El. Nachr. Techn., 13, 3, 1936.

14. Rzhevkin, S. N., Nesterov, V. S.: Resonance Sound Absorbers (In print).

15. Wintergerst, E.: Schalltechnik, IV, 85, 1931.

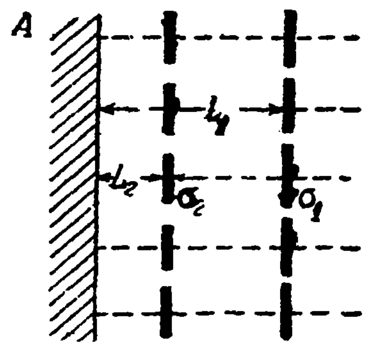



Figure 1.

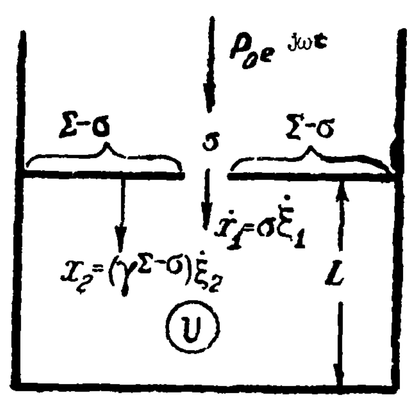



Figure 2.

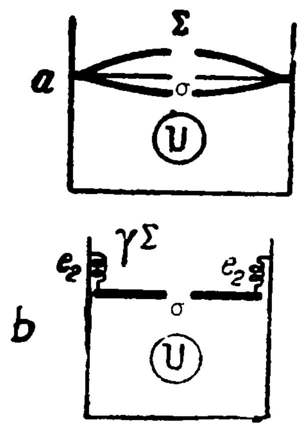



Figure 3.

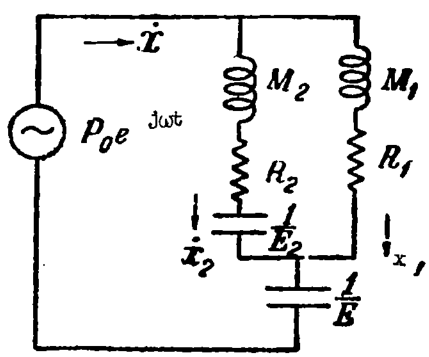



Figure 4.

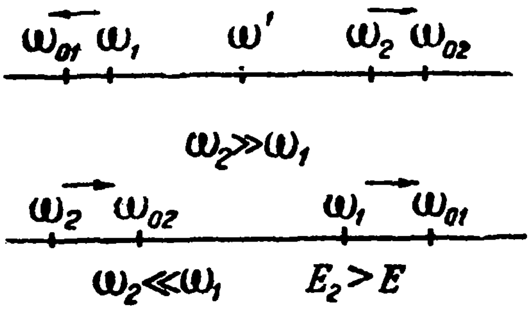
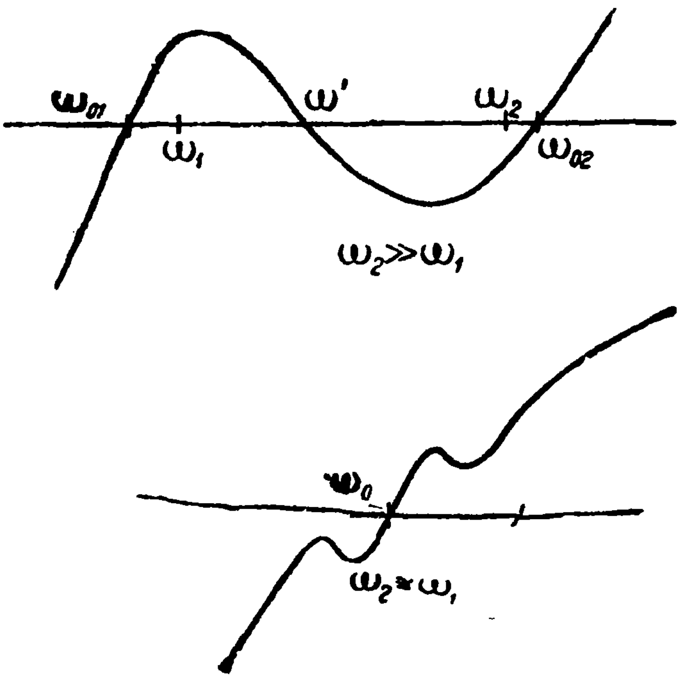



Figure 5.

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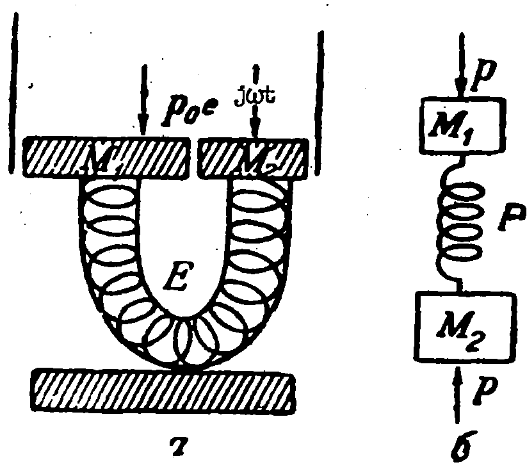


Figure 7.

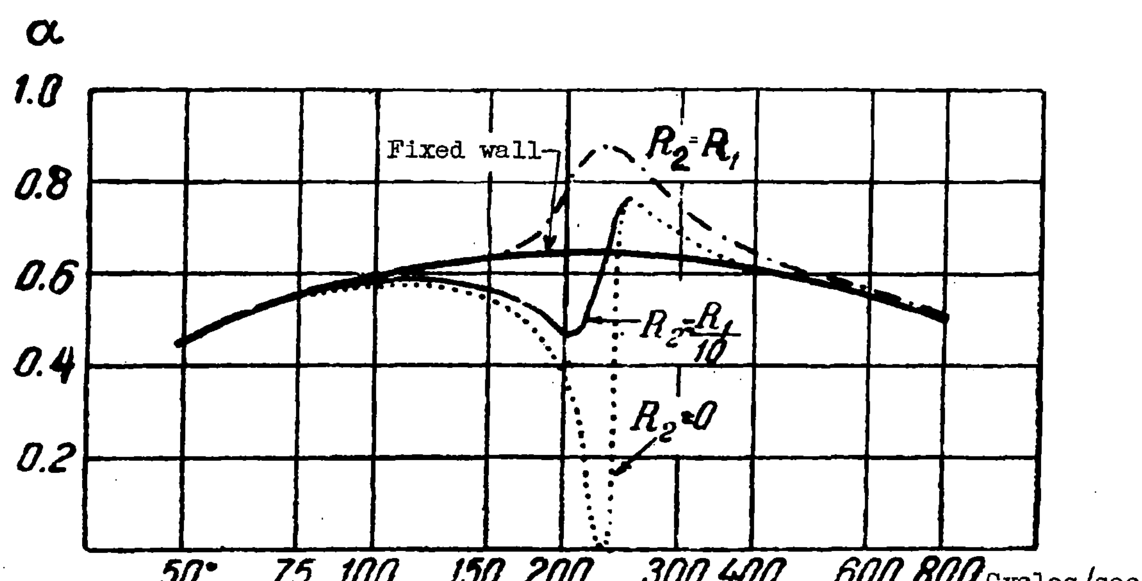


Figure 8.

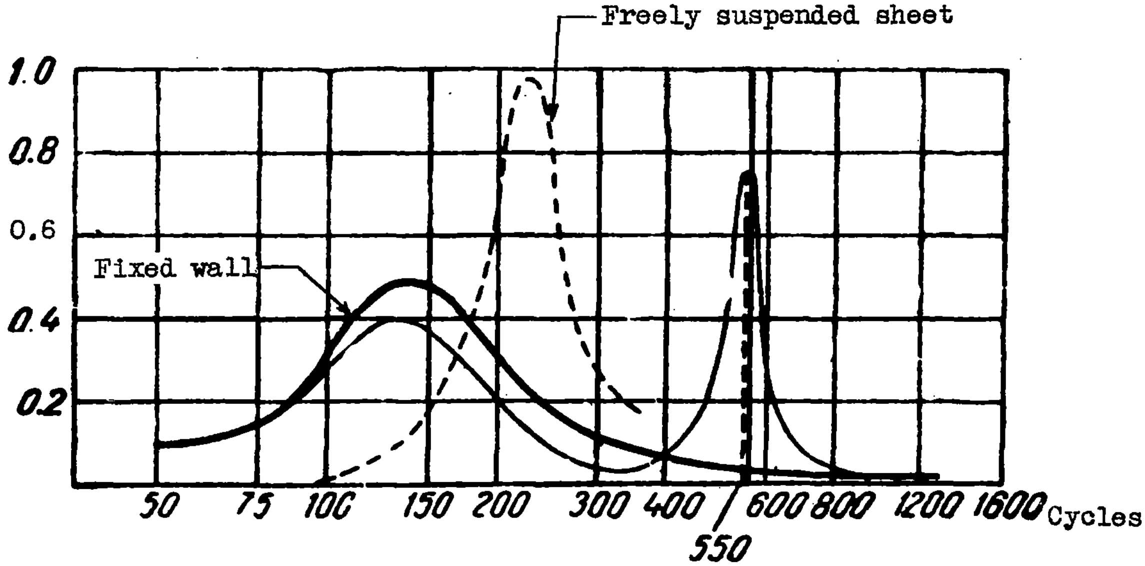
Freely suspended sheet
Fixed wall
1.0
0.8
0.6
0.4
0.2
50
75
100
150
200
300
400
550
600
800
1200
1600
Cycles

Figure 9.