TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 966
(Supplement)

SUPPLEMENT TO

COMPARISON OF AUTOMATIC CONTROL SYSTEMS

By W. Oppelt

Luftfahrtforschung
Vol. 17, No. 2, February 20, 1940
Verlag von R. Oldenbourg, München und Berlin

Washington
August 1941

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 966 (SUPPLEMENT)

SUPPLEMENT TO

COMPARISON OF AUTOMATIC CONTROL SYSTEMS*

By W. Oppelt

In a previous report (NACA T.M. No. 966) dealing with a reciprocal comparison of different control systems, the regulator was classified into "faultproof" regulator and actual regulator which is subject to faults. The errors of the actual regulator were divided into characteristic groups of "sluggish," "susceptible to oscillation," and "afflicted with friction." These three groups of errors have a certain general validity to the extent that each control system is subject to mass, friction, and elasticity; hence these errors always will be more or less prominent. In addition to that, every regulator has faults due to its particular design, therefore peculiar to it alone. To enable the use of convenient instrumental aids, the strict realization of the regulating principle is abandoned for an approximation. This, of course, involves commulative errors now associated with the instrumental design of the regulator and a further restriction of regulator capacity. These aspects form the subject of the present article, which is restricted to the most essential relationships.

The analysis deals with the indirect regulator, wherefrom the behavior of the direct regulator is deduced as a limiting case. The prime mover is looked upon as "independent of the load": a change in the adjusting power (to be applied) for the control link (as, for example, in relation to the adjusting path η with pressure valves or the rudder of vessels) does not modify the action of the

---

*"Vergleichende Betrachtung verschiedener Regelaufgaben unter Berücksichtigung der apparativen Verwirklichung des Regelgesetzes." Luftfahrtforschung, vol. 17, no. 2, Feb. 20, 1940, pp. 59-64.

prime mover.* Mass forces and friction are discounted; "clearance" also is discounted in the transmission links of the regulator.

The action of the particular types depends essentially upon the method by which the auxiliary force is introduced; this is dependent upon the characteristic diagram of the energy connector.

1. THE CHARACTERISTIC DIAGRAM OF THE ENERGY CONNECTOR

The diagram shows how the running speed $\dot{\eta}$ of the control link is affected by the commands from the detector elements. This relationship may be independent of time or it may be related to it in any one form.

The energy connector "independent of time" is classified as "steady" and "in stages"; the latter can be further subdivided into single stage or multistage arrangements. The resultant characteristic diagrams together with some typical versions of electric energy connectors (switches) are shown in figure 1. The method of execution is, of course, not the decisive one for the curve of the regulator process; the characteristic diagrams can also be obtained in the same way by hydraulic, pneumatic, optical, and other energy connectors. (See reference 1.)

On the "time related" energy connector, the command transmitted to the prime mover depends upon the setting of the energy connector and upon the time; the command is executed in steps, the "step connector." The three types of time-independent energy connector also can be designed

---

*For example, on a valve-controlled hydraulic prime mover with incompressible transmission medium, this load dependence is generally disregarded. Even a slight valve opening brings the entire static prepressive into play; when the valve is closed, the arrangement is "self-locking." In this case therefore the mass forces in the prime mover and the control link are usually disregarded. In a hydraulic prime mover with compressible transmission medium or a jet nozzle regulator (where the pressure exerted rises first with the jet nozzle setting); this, in general, is no longer the case with great masses.

with respect to time. The steady energy connector becomes the "steady step connector," and the stagewise connector becomes the "stagewise step connector." The characteristic curves are illustrated in figure 2 in typical versions of an electric connector (switch).

2. EFFECT OF THE CHARACTERISTIC CURVES OF ENERGY CONNECTOR ON THE REGULATOR PROCESS

The effect of these curves differs on running-speed regulators and setting regulators; hence this division is maintained in the following: (The speed regulator is named here first because a setting regulator is evolved from it by incorporating a return.) For the classification of discontinuous regulating processes, it is expedient to consider, besides the function itself, its temporal derivation in respect to time.

The general result for discontinuous regulator processes is such that they do not become quiescent in their theoretical position but are in continuous "pendulum motion" which disappears only when the regulating system (for example, by friction or clearance) has a sufficiently wide dead zone. The chosen classification - pressure, rpm, temperature, and automatic directional control - is affirmed here. Tables I to III show that the phase curve φ relative to the control-link curve η lags least on the pressure regulator, and most with respect to time on the automatic directional control.

a) Regulator With Conjugate Running Speed

Methodological reasons militate against the application of the running-speed regulator on the rpm control and the automatic directional control, as pointed out in another article; hence these cases are not enumerated. The effect of the single energy connector characteristic curves with a given curve $\varphi(t)$ is illustrated in figure 3. According to figure 3 the results previously achieved on the "actual regulator" are now strictly applicable to the case of the "steady energy connector independent of the time." They obviously hold true also in close approximation for the multistage and for the steady (or multistage) step connector.

Only in the case of the small or the single-stage connector do the changes become appreciable enough to warrant special consideration. For its behavior within the zone immediately adjacent to the theoretical position every unsteady regulator should be looked upon as small stage; only by a division which becomes infinitely fine stage at this point would an approximately continuous transition to the theoretical state be guaranteed. Therefore let us glance at the motion forms of the regulating process on the single-stage energy connector so as to bring out this limit of motion behavior of the regulator.

α) Single-Stage Time Independent Energy Connector

On the pressure system to be regulated, the course of the regulating process following an impulse results in a decaying periodic motion with increasing frequency during the damping-out operation. (See table I.)

The rate of adjustment $\dot{\eta}$ of the control link is constant; from the earlier equation (1) of the system to be regulated, an e-function is afforded for the rate of change $\dot{\varphi}$ of the stage. The free oscillation curve of the state is built up from pieces of curves having their maximum at the intersection with the control-link curve; the free oscillation curve of the phase change $\dot{\varphi}$ consists of pieces of e-functions. The rate of displacement of the control link has the opposite prefix if the stage passes through the neutral position. In that instance the two pieces f_{10} and f_{20} below the e-function of the phase derivation $\dot{\varphi}$ must be equal to each other because individually they represent an indication for the maximum amplitude of state φ. Obviously the two succeeding pieces f_{11} and f_{21} must be smaller, which is symptomatic of continuous damping out and increasing frequency of the process. In reality, starting from a certain point of the damping-out oscillation process, the defects of the control device become so great that by reason of mass effect, friction, play, etc., the time lag in the reversing process reaches the order of magnitude of the regulating process. Then the undamping effect of this time lag makes possible a permanent oscillation. With properly chosen friction or clearance, the process can reach the stopping stage in the dead zone which is thus produced.

The temperature system to be regulated already embodies a time lag because of its construction; hence even the connecting mechanism operating without play, friction, and connection lag manifests permanent oscillations. They show a definite amplitude; at great deflections, as on the pressure regulator, where lower frequencies occur, the undamping effect of this lag is still too small.

The diagram in table I portrays the conditions by permanent pendulation. Besides (η) and (φ) the behavior of the energy input $\bar{\eta}$ (lag with respect to time) (equation 3c) also is shown. The areas f_{10}, f_{20}; f_{11}, f_{21};... can be consecutively equal to one another and hence affirm a permanent pendulation.

β) Single-Stage Step Connector

Theoretically, the "step control free from clearance and friction" cannot operate without pendulum motion; following any minor departure from the theoretical state the control link makes a "step," which effects a deflection of the state toward the other side, and which, after the subsequent connection, is cancelled again, etc. The result is a permanent pendulum process building up around the neutral position. The magnitude of these pendulations is obviously so much greater as the control link is shifted farther by each step, that is, as the step duration S and the rate of displacement $\eta_0^{\bullet}$ of the control link is greater; likewise, at long pauses P between the individual steps the amplitude of this pendulation is greater than for shorter pauses, because the system has more time to follow the accomplished control-link shift. At very short pauses, however, the system to be regulated has not the time between steps to change the prefix of the departure so that repeated connection toward one side and then the other results. (See fig. 4.) With this alternating connection the amplitude of the pendulation itself increases again in consequence of the then greater adjustment of the control link. The limit of duration P_{min} of the connection pause, which must be maintained* if repeated connections are to be avoided, can be taken for the pressure control from the pendulum curves. (See fig. 4.)

---

*In practice it is not always possible to maintain this limit, because the connection pause may not be made too long, in order to avoid excessive departure from the theoretical state within it.

This limit is definitely reached by the pressure control when, during a connection pause, the state φ has reached its highest possible deflection $\frac{S\dot{\eta}_o}{2z}$ and following one connection, the next one occurs after such a pause that state φ has exactly passed through the theoretical state. The amount of this pause follows from

$$\frac{S\dot{\eta}_o}{2z} = \frac{S\dot{\eta}_o}{z}\left(1 - e^{-\frac{t}{T_a}}\right)$$

at

$$P_{min} \approx 0.7\ T_a - S$$

The conditions on the temperature control are similar; the pendulum process can be seen in table I. In consequence of the lag in the system to be regulated, the pause must last longer here than on the pressure control if repetitious connections are to be avoided. The same holds true for the energy input as for the pressure control. Because the shifting of state φ relative to energy input η cannot exceed T_a even in the most adverse case, it affords

$$P_{min} \approx 0.7\ T_v + T_a - S$$

b) Regulator With Conjugate Setting

A classification according to the energy connector characteristic is not sufficient for subdivision of the zone for this regulator. It must also include the manner by which the conjugate setting in the amplifier comes into being. This can be accomplished by incorporating a so-called lead back or return, "control with return"; or the amplifier itself coordinates a certain control-link setting to each energy-connector setting, "regulator with setting-yielding amplifier."

The effect of the different energy connector characteristic curves for the control with conjugate setting within a given time rate of change $\varphi(t)$ is illustrated in figures 5 and 6. The control with return manifests a control-link lag in respect to its theoretical setting; the control with setting-yielding amplifier is fault-proof as a continuous regulator (because in this representation inertia, susceptibility to oscillation, and friction are disregarded), but manifests a jumpy behavior as stagewise and stepwise regulator.

α) Control With Return

The problem of maintaining the control-link setting η proportional to the command i from the detector element is itself a regulating problem. Here can be found a system to be regulated, namely, the amplifier system; also a state of the system to be regulated, namely, the setting η of the control link; and lastly a control link, that is, the energy connector with its setting k. The amplifier responds to a given setting of the energy connector with a certain rate of displacement $\dot{\eta}$ of the control link, whence the equation of the system to be regulated follows at

$$R\dot{\eta} = k$$

It corresponds to the equation for the rpm system to be regulated. It is obtained by disregarding all mass forces in the amplifier. If they are allowed for, the equation reads:

$$J\ddot{\eta} + R\dot{\eta} = k$$

The problem is similar to that of the automatic directional control.

The principle by which the problem is solved consists in making the setting of the control link proportional to the departure from the theoretical state, or in other words, in making the setting of the energy connector dependent upon the departure $(i - \eta)$:*

$$k = a_1 (i - \eta)$$

---

*In this calculation the coefficients of the return ratio are put down equal to 1, since $\eta = i$ is to result at neutral energy connector setting. The change in regulator effect due to change in return is not analyzed here; it results in a change in proportionality factors a, b, and c.

This relationship is simply accomplished by giving the energy connector, besides the command i of the detector, an effect corresponding to the control-link setting η, that is, a "return effect." The equation of the regulated system, which in this case is the regulator, reads:

$$R\dot{\eta} + a_1\eta = a_1 i \qquad \text{mass forces disregarded} \qquad (1a)$$

$$J\ddot{\eta} + R\dot{\eta} + a_1\eta = a_1 i \qquad \text{mass forces included} \qquad (1b)$$

Equations (1a) and (1b) follow as a final result from the above line of reasoning where the setting η of the control link represented the quantity to be regulated. Moreover, this η is a part of the control link actually to be considered in the regulating process (for example, pressure regulator). Thus a system regulated by a return control is termed a "boxed regulator system." Both parts of equation (1) show the behavior of the return regulator discounting its mass forces; equation (1a) shows it as a sluggish regulating system. During an ordered change in control-link setting η, the control link can follow only when it departs from its momentary theoretical state, for only then is the necessary deflection k of the energy connector available. The effects on the regulating process were discussed earlier under "sluggish controls." With allowance for the mass forces (equation (1b)), the return control is susceptible to oscillation, as explained elsewhere. If, in addition, the return mechanisms were accompanied also by perceptible friction, then the arguments advanced under "regulator with friction" would apply.*

---

*The fact that an rpm control is transformed in a setting control by the simple expedient of incorporating a lead back or return, and the further fact that a system to be regulated under the effect of a setting control acts different, more damped, than under the effect of rpm controls is frequently termed the "damping effect of the return." This term seems misleading; the "damping" effect is not due to the return but to the then existing conjugate setting. It would, of course, exist as well if this conjugate setting had been brought about by any other means. The return should be looked upon simply as a means to reach the conjugate setting. Even the expression "volume return" should be avoided if used to denote the fact that a vapor pressure control can be effected when vapor pressure and vapor volume enter the basic principle.

It is readily apparent that on the "amplifier to be regulated" the "resetting effect a_i" (which indicates how far the "control link" - energy connector - is adjusted by a certain deflection $i - \eta$) may not exceed a certain measure if disturbing natural motions of the amplifier are to be avoided; in other words, the rate of adjustment of the control link for a given deflection of the energy connector should not be excessive. The appearance of disturbing natural motions constitutes a limit of amplifier serviceability.

In figure 7, which illustrates the customary devices for transmitting the return effect, the command i initiated by the detector element appears in the form of "steering," the return mechanism as "regulator."

In order to neutralize permanently active outside disturbances, the term $c \int \varphi \, dt$ (equation 6) is introduced. Instrumentally this can be accomplished by an integrating mechanism (for instance, a small motor loaded by the deflection φ) which forms this term $c \int \varphi \, dt$, but in common practice the "flexible" return is usually preferred. (See fig. 8.) The return effect arriving at the energy connector is made to remain neutral by some expedient, such as a spring F. In order that a given return command actually dies down to zero within a specified time, the return mechanism is made flexible by means of a brake cylinder B, a bypass opening O, or an equalizing tube R. (See reference 2.) The device operates on the following principle: A certain length of the return connection (in the mechanical or hydraulic case) defines a certain neutral position of the control link; changes of this length also change the neutral position. A permanent outside disturbance of the equilibrium in the regulator process stipulates, on the other hand, a permanent equalizing deflection in control-link deflection. Spring F, however, acts upon the flexibility of the return until this equalizing deflection is reached without a departure φ from the theoretical state.

Mathematically the behavior of the flexible return is expressed as follows:

Force on the spring = force on the brake cylinder

$$f_1 r = r_1(\eta - r)^{\cdot},$$

where

r return command, path of point P_1,

f_1 elastic constant of spring F,

r_1 brake resistance of cylinder B.

For the rate of adjustment $\dot{\eta}$, the omission of the mass forces had given

$$\dot{\eta} = \frac{k}{R}$$

and

$$k = a_1 (i - \eta)$$

whence follows

$$R\dot{\eta} + \left(1 + R\,\frac{f_1}{r_1}\right) a_1 \eta = a_1 i + a_1\,\frac{f_1}{r_1} \int i\,dt \qquad (2)$$

According to equation (2) the desired principle is rigorously adhered to by vanishing inertia, since, in general, the effect i will be a pure resetting effect $a\varphi$.* The inertia of the regulator cannot, of course, be eliminated by the flexible return. If the flexibility $\frac{f_1}{r_1}$ of the return is increased continuously, the setting control ultimately reverts to an rpm control; the effect i disappears more and more relative to displacement effect

$\frac{f_1}{r_1} \int i\,dt$.**

---

*The amplifier with flexible return integrates all commands reaching the energy connector, a fact which must be borne in mind when other quantities (such as damping effects b, etc.) besides deflection φ intervene.

**The definition "flexible return" is not uniformly used. In rpm control design it is called "isodromic" return,

(Continued on p. 11)

On the regulator with return the actual regulating process does not differ much with stepwise and stagewise energy connector from that with steady energy connector, except when the number of steps or stages is small with respect to the time interval during which the regulating process takes place, as in fact is always the case for the behavior of the regulator in neutral position. The regulator process in the zone of the theoretical state is therefore discussed in greater detail. There is, as on the rpm regulator, a pendulum motion about the zero position, in case there is no definite dead zone due to clearance or friction. The (rectangular) curve of the energy connector setting k, the time rate of change of the control-link setting η and the related curve φ of the state are shown in table II. With the chosen prefixes, where a positive η affords a positive φ, the φ curve must be reflected as return curve Rfg, as the regulator is to produce a counteracting negative η to a positive φ. The intersection of the return curve Rfg, which represents the theoretical control link setting, with the curve φ then gives the point in which the energy connector is reversed.

As pressure control, the stagewise regulator with return manifests a pendulum motion which, however, can develop only as a result of a time lag τ in the energy connector. The reason for this is the same as for the rpm regulator. By equal pendulum motion the lag τ can

---

(Continued from p. 10)

in electricity, "elastic." The term "flexible" seems the most appropriate since the intended effect (displacement of neutral position of control link) can be reached only through the flexibility of the return. One also hears occasionally of an additional damping action of the flexibility. But the change from rigid to flexible return implies, unless other instrumental constants are changed at the same time, an undamping, as readily seen from tables II to IV of the preceeding report (NACA T.M. No. 966). That actually under equal operational requirements with flexible return, a regulator process is often regulated with more damping than by rigid return, is due to the fact that then (outside permanent disturbances being continuously equalized without permanent deflection) the size of the regulator effect can be chosen different, whereby the defects of the regulator, such as its inertia, for example, become less effective.

be greater here than on the rpm regulator. On the temperature regulator and the automatic directional control which dispose of their own inertia effects (temperature system to be regulated in the accumulator masses in the energy flow path, the automatic control in the inertia moment of the vehicle), the pendulum motion can develop even without time lag τ.

The stepwise regulator with return always manifests pendulum motions like the corresponding speed regulator. With approach to the theoretical position, every finite step must signify overregulating. Whether the control link is adjusted in one or the other sense depends upon the then existing position of the theoretical state φ to the return curve Rfg of the theoretical control link setting. The shortest pause P_{min} for which simple connection occurs toward either side during the pendulum motion is:

$$\frac{s\dot{\eta}_o}{2z} - \frac{s\dot{\eta}_o}{za} = \frac{s\dot{\eta}_o}{z}\left(1 - e^{-\frac{t}{T_a}}\right)$$

whence

$$P_{min} = - \ln\left(\frac{1}{2} + \frac{1}{a}\right) T_a - S$$

For the temperature regulator with energy input $\bar{\eta}$ similar to the behavior of the pressure regulator, but whose state φ is still farther postponed, it is

$$P_{min} = - \ln\left(\frac{1}{2} + \frac{1}{a}\right) T_v + T_a - S$$

Automatic directional controls and rpm regulators could equally well be fitted with a stepwise return regulator. The pendulum motion of both could take place within a certain range between limits φ_{g_1} and φ_{g_2}. Since neither one of the two systems disposes of its own resetting force, the position of the pendulum curve is directed by the requirement that for every succeeding engaging point the state φ of the system must be alternated to the other side of the curve Rfg, which then is realizable within a certain extent.

β. Regulator With Setting-Yielding Amplifier

Such a case prevails, for instance, on the steady energy connector when the connector simply steers the adjusting moment of the amplifier and this is balanced against a spring as path. Such devices are often employed on simple regulating arrangements and small adjusting powers (for instance, in telephone and telegraph engineering). Since the amplifying mechanism involves masses and the spring always affords a resetting force, the argument involves chiefly "regulators susceptible to oscillation," although the conditions of the "regulator with friction," also must be noted.

Unsteady energy connectors appear here mostly in connection with control-link arrangements which of themselves favor a convenient division of the control-link setting in separate stages. So, for instance, the output of electric furnaces is simply controlled by connecting or disconnecting the heat elements or series resistances. The behavior of the regulator for unsteady cases is illustrated in table III; for the continuous case it affords unrestrictedly the connection to the "actual" regulator, as represented for the pendulum process about the theoretical position; by greater departure φ and multiply divided energy connector, the regulating process proceeds without radical change because of its discontinuity.

As pressure control, the stagewise regulator with setting-yielding amplifier manifests a pendulum motion that can develop only under the effect of a time lag τ in the energy connector. This applies to all the other regulating systems as well; it requires such a lag τ to maintain the pendulum motion. The reason is that the regulator, though unsteady, is otherwise a faultproof setting regulator which (as explained elsewhere) disclosed damped behavior for all regulating systems.*

The stepwise regulator with setting-yielding amplifier manifests, like all step regulators, a pendulum motion, which becomes less with shorter steps and especially smaller stages of control-link setting.

---

*For the temperature regulator it is not immediately apparent, but will be if the $\overline{\eta}$ curve is replaced by (more unfavorable) straight lengths. The picture is then the same as for the stagewise pressure control with conjugate running speed.

As on the stepwise regulator with return, the rpm regulator and the automatic directional control possess, in this instance, potential pendulum zones because neither one of the regulating circuits has a resetting ability of its own.

3. DIRECT AND INDIRECT REGULATORS

The foregoing results are obtained for the most complicated type, the indirect regulator. Quite obviously they then apply also to the direct control without amplifier.

In amount the adjusting ability required of direct controls is such that it can be directly overcome by the energy capacity of the detector. The detector systems which have to give off a comparatively great adjusting moment are therefore usually encumbered with appreciable masses; hence the aspect of "regulator susceptible to oscillation" must be borne in mind. The theoretical considerations of the faultproof regulator naturally apply to both the indirect and the direct regulator.

The direct regulator is usually designed with conjugate setting; the adjusting moment of the detector, which is proportional to the departure φ, is transformed into an adjusting path with respect to a spring; occasionally it is also found as regulator with conjugate running speed. Then the adjusting moment of the detector proportional to departure φ becomes the rate of adjustment of the control link relative to some damping mechanism. As direct control from among the unsteady types, the group "stepwise regulator with setting-yielding amplifier" is particularly used on the temperature control; expansion temperature regulators connect the power to be regulated direct across a contact unit as control link.

Irregular and not clearly definable forces (such as back pressure of valves and throttles, frictional forces, etc.) applying on the regulating system, especially on the control link, must be substantially lower than the forces available for adjustment, in order to give a clear and hence useful behavior of the regulator. Herewith the limit of application of direct regulators is reached.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCES

1. Kniehahn, W.: Die Elemente der Schalt-, Steuer- und Regeltechnik. Masch.-Bau-Betrieb, 1930, pp. 361 and 448.

Kniehahn, W.: Verstärker. Masch.-Bau-Betrieb, 1931, pp. 239 and 272.

2. Himmler, C.: Arch. f. techn. Mess. J060-2.

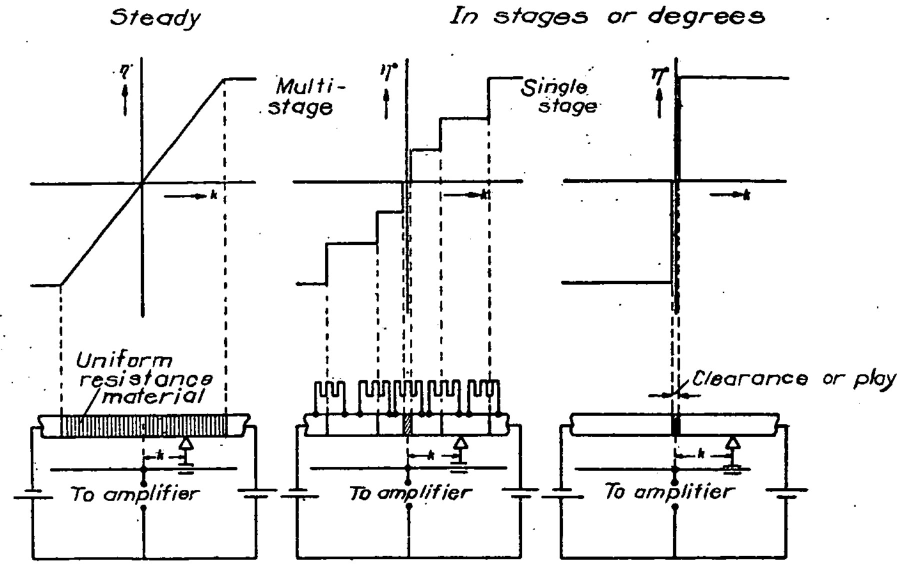


Figure 1.- Characteristic diagrams and typical versions of "energy connector" independent of time.

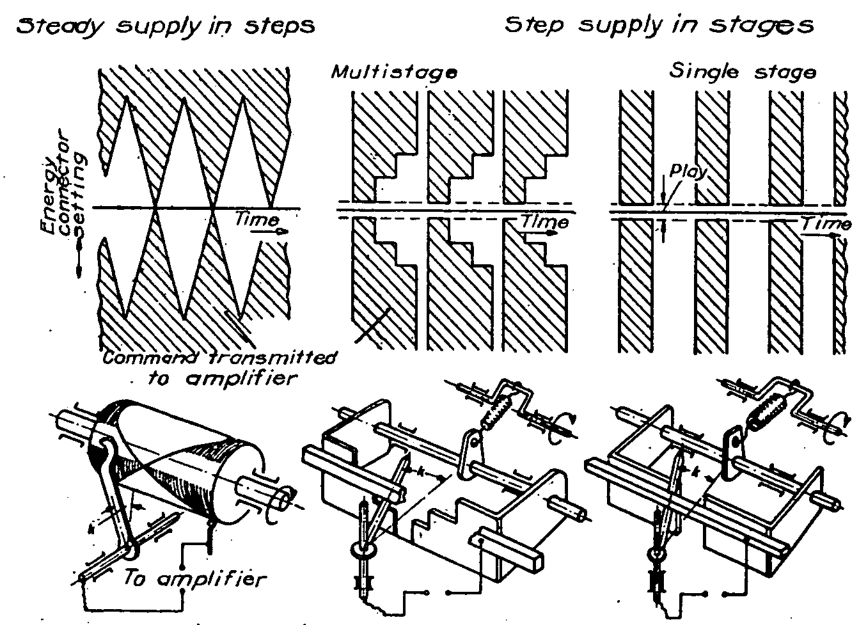


Figure 2.- Characteristic diagrams and typical versions of "energy connector" dependent on time.

Table I.– Unsteady regulator with conjugate running speed.

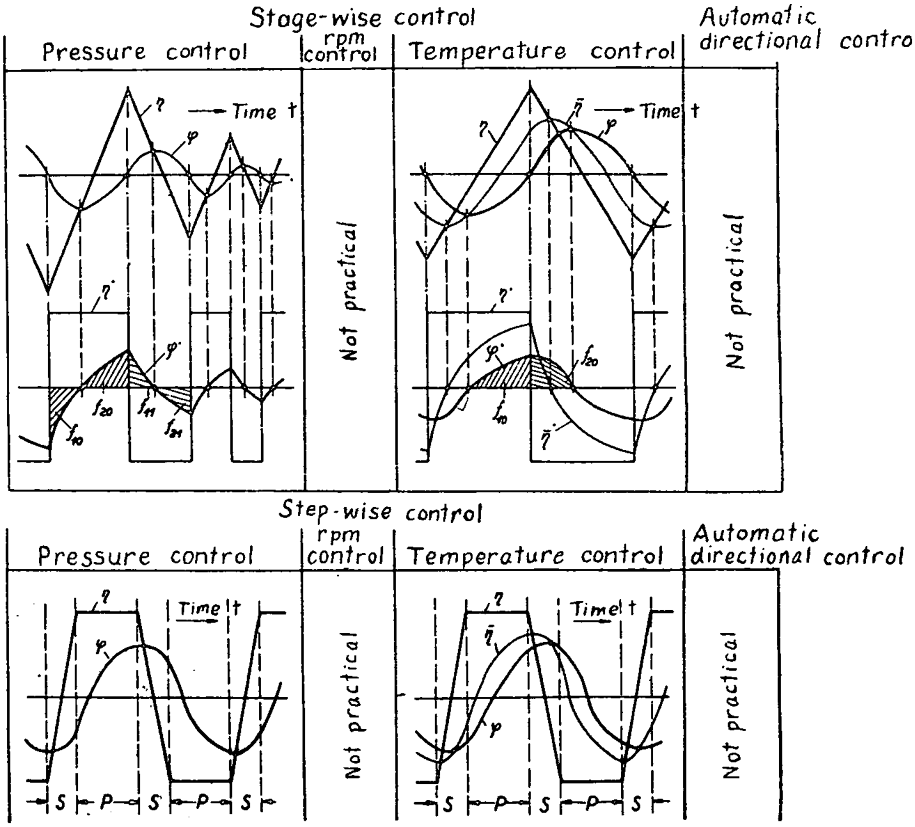
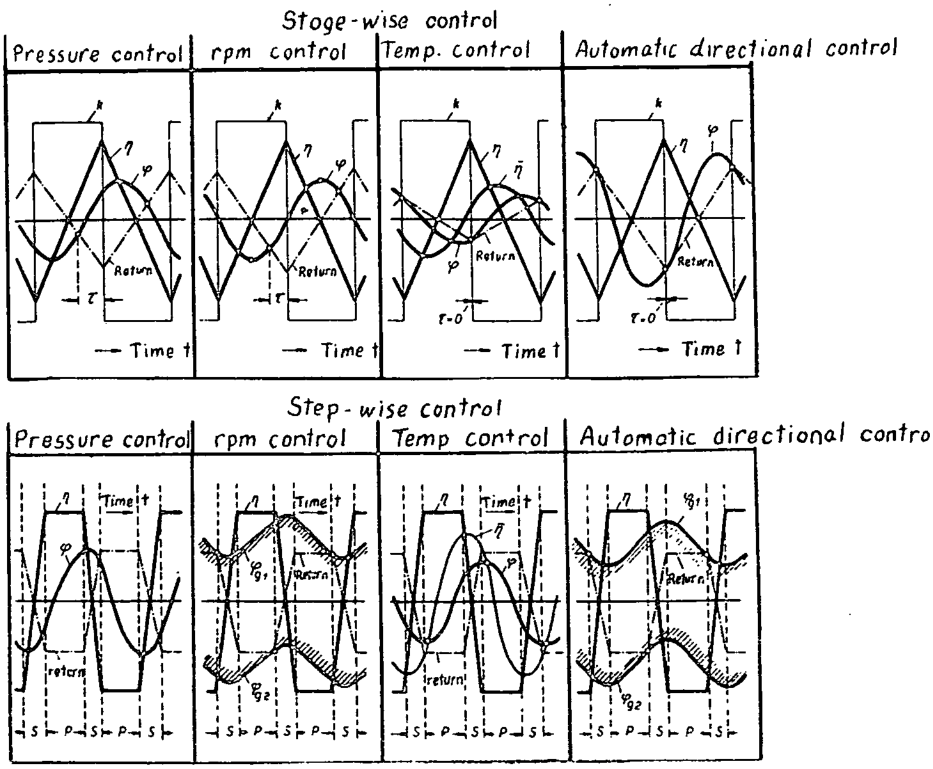


Table II.– Unsteady regulator with conjugate setting through return.



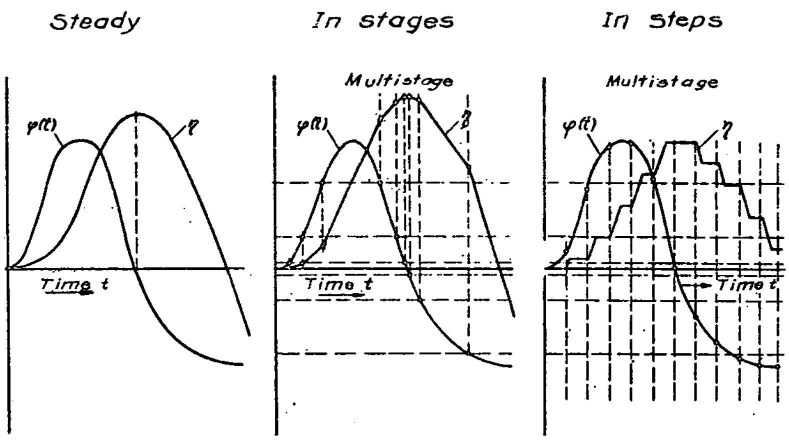


Figure 3.- Behavior of different running speed control systems by given departure $\varphi(t)$.

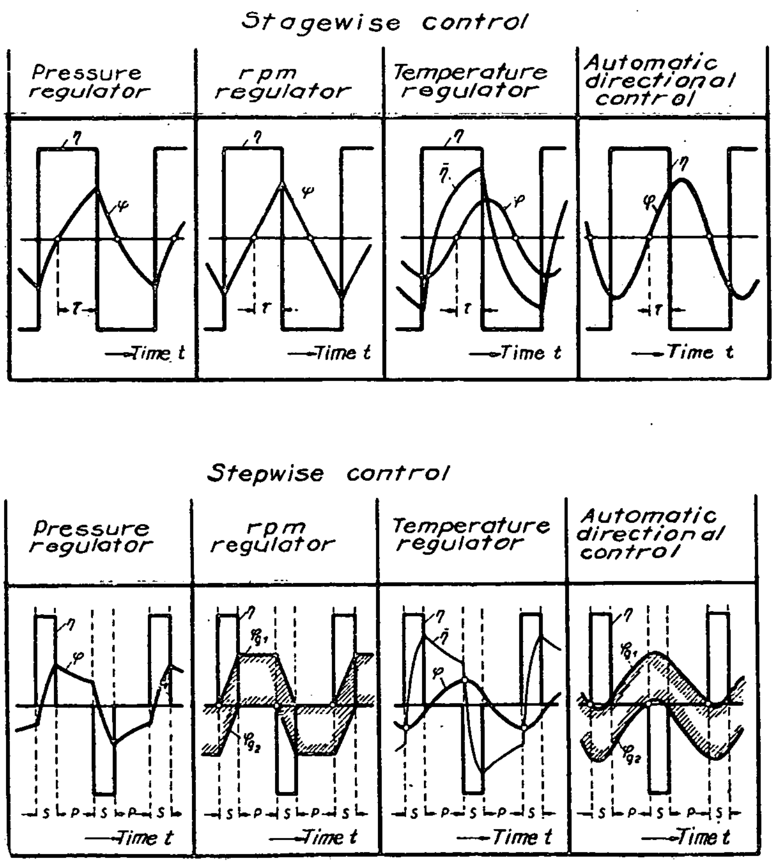


Table III.- Unsteady regulator with conjugate setting through setting yielding amplifier.

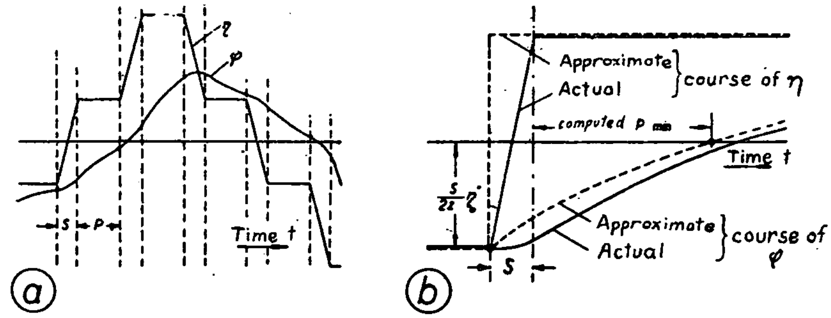


Figure 4.- Explanation of multiple connector on step-regulated pressure control.

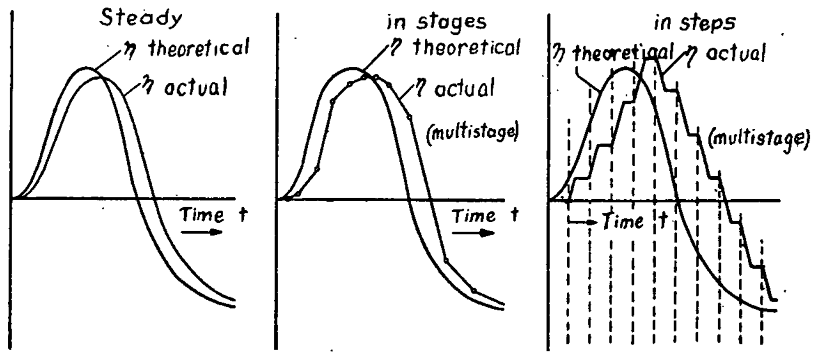


Figure 5.- Regulators with conjugate setting by return at given φ (t).

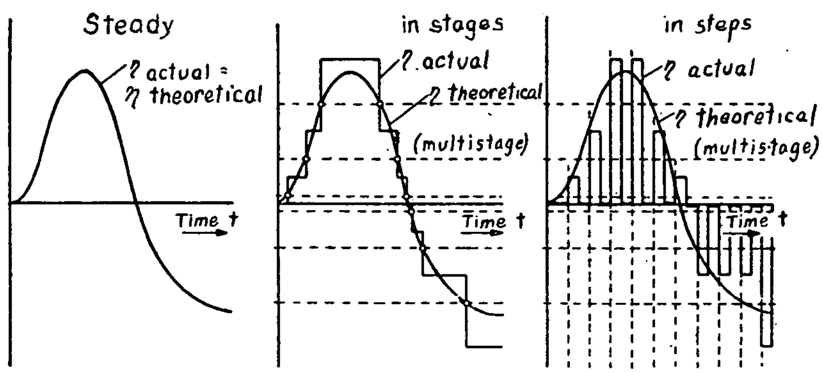


Figure 6.- Regulators with conjugate setting by setting-giving amplifier at given φ (t).

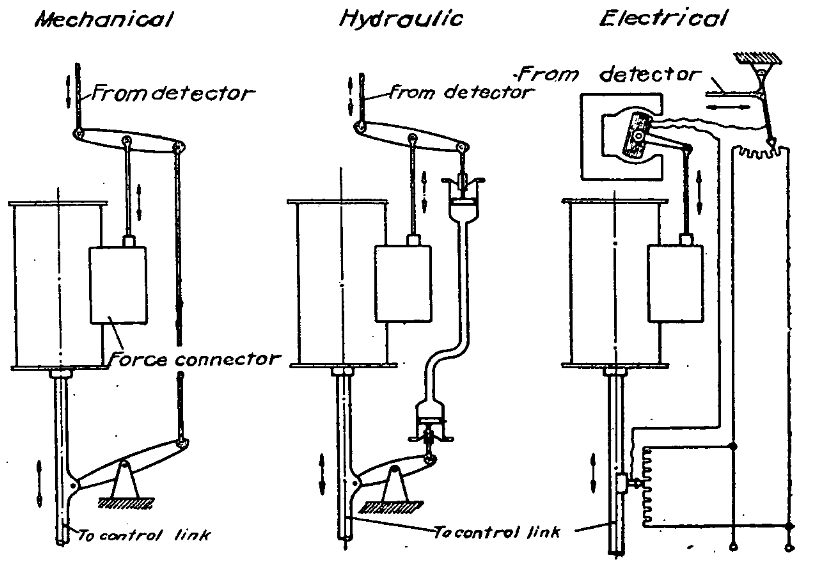


Figure 7.- Diagrammatic sketch of amplifiers with rigid return.

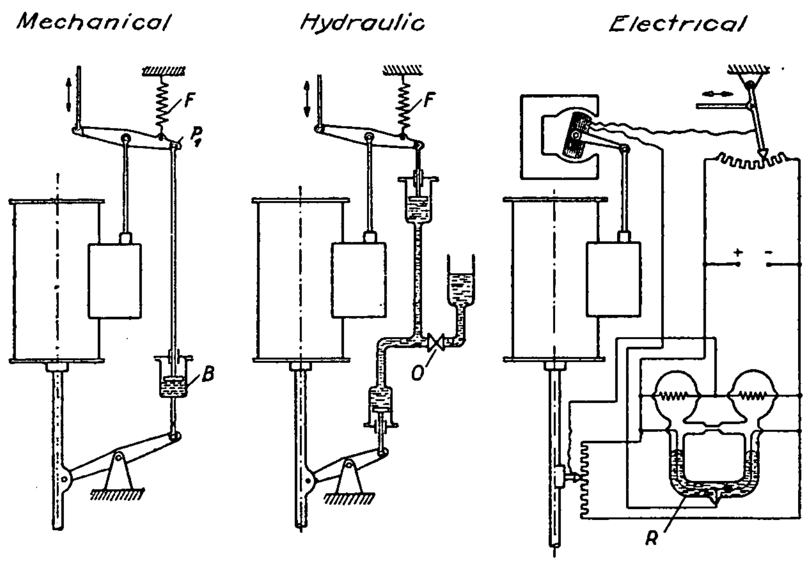


Figure 8.- Diagrammatic sketch of amplifiers with flexible return.