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THE HYDRODYNAMIC THEORY OF DETONATION*

By Heinz Langweiler

SUMMARY

The author derives equations containing only directly
measurable constants for the quantities involved in the
hydrodynamic theory of detonation. The stable detonation
speed, D, is revealed as having the lowest possible value
in the case of positive material velocity, by finding the
minimum of the Du curve (u denotes the speed of the gases
of combustion). A study of the conditions of energy and
impulse in freely suspended detonating systems leads to
the disclosure of a rarefaction front traveling at a lower
speed behind the detonation front; its velocity is com-
puted. The latent energy of the explosive passes into the
steadily growing detonation zone – the region between the
detonation front and the rarefaction front. The conclu-
sions lead to a new definition of the concept of shatter-
ing power. The calculations are based on the behavior of
trinitrotoluene.

INTRODUCTION

The chemical change in an explosive substance, such
as the propelling charge in a firearm, generally takes
place at a linear combustion speed in fractions of meters
per second, whereby this rate at which the conversion zone
penetrates the explosive substance is a function of the
gas pressure.

Berthelot (reference 1), in 1881, discovered the pe-
culiar fact that under suitable conditions, substantially
higher conversion speeds are feasible. In his experiments
with explosible gas mixtures in long tubes, he ascertained
propagation speeds in the conversion zone of several kilo-
meters per second, whereby this speed remained constant
over the entire conversion distance and even remained un-
affected by the initial gas pressure.

Berthelot and Dixon essayed a molecular kinetic explanation of the detonation processes and, in fact, achieved with many gases, practical values for the detonation speed.

The purely thermodynamic and aerodynamic treatment of the detonation process which disregards special molecular concepts, is attributable to Schuster (reference 2), and Chapman (reference 3), who utilized the findings obtained by Riemann (reference 4), and Hugoniot (reference 5) in the treatment of the linear compression shock in gases.

The extension of these macroscopic theories to include solid explosive substances, is due to Jouguet (reference 6) and R. Becker (references 7 and 8), the latter dealing largely with the cause of detonation stability. He proves, with the help of the thermodynamic entropy laws, the validity of Chapman's term for detonation speed. Schmidt's more recent studies of the detonation process of solid explosive substances (reference 9), containing a wealth of experimental material, are particularly well-suited to prove the validity of the hydrodynamic theory of detonation.

However, there is a gap in the orthodox theory. One obtains for the combustion gases behind the detonation front, a speed in the direction of the advancing front. The consequences of the existence of such a speed on the total system, are not considered for the reason that only the measurable quantity of this speed of the combustion gases is of influence for the detonation speed (reference 10). (Schweikert refers to this difficulty, but is of the opinion that it would void the whole hydrodynamic theory of detonation.)

To illustrate  the defect of the present theory, the derivation of Chapman and Becker is briefly reviewed.


CHAPMAN AND BECKER EQUATIONS


A tube of 1 square centimeter section contains an explosible mixture under pressure $p_0$,  density $\rho_0$, specific volume $v_0$,  and temperature $T_0$  (fig. 1).

The front of the conversion moves toward the right into the explosive at the empirically constant detonation speed $D$.  In zone  $F$  the chemical change takes place,

the product of which, the gases of combustion, also moves toward the right at speed $u$.

For an observer, at rest relative to the combustion gases, hence also moving rightward at speed $u$, the pressure $p_1$, density $\rho_1$, specific volume $v_1$, and temperature $T_1$, prevail. Certain relations must exist between these macroscopic quantities which result from the general physical laws governing the preservation of energy, impulse, and mass.

Visualize an observer in the conversion zone $F$ moving at detonation speed toward the right. To him the process of conversion must appear stationary if $D$ is constant. The explosive moves at speed $D$ into zone $F$ from the right, while the combustion gases leave the zone at speed $D - u$ toward the left.

The preservation of the mass gives:

$$D \rho_0 = (D - u) \rho_1 \tag{1}$$

or

$$D v_1 = (D - u) v_0$$

The momentum of the explosive substance moving into the zone per second from the right is

$$B_0 = \rho_0 \, D \, D$$

The momentum of the combustion gases leaving the zone per second is

$$B_1 = \rho_1 (D - u)(D - u)$$

The difference $B_0 - B_1$ of both momentums must be equal to the effective pressure difference. Hence,

$$B_0 - B_1 = p_1 - p_0$$

and, in connection with equation (1), the momentum equation

$$\left. \begin{array}{l} p_1 - p_0 = \rho_0 \, D \, u \\[2mm] (p_1 - p_0) \, v_0 = D \, u \end{array} \right\} \tag{2}$$

From the right, four different forms of energy enter the zone per second:

1.  The latent chemical energy.  If this amounts to $U$ per unit mass, the leftward moving energy per second is:

$$\rho_0 \, D \, U$$

2.  The kinetic energy

$$\frac{1}{2} \rho_0 \, D \, D^2$$

3.  The heat contents of the undecomposed explosive gas

$$\rho_0 \, D \, c_v \, T_0$$

if $c_v$ denotes the specific heat at constant volume per unit mass.

4.  The external energy

$$p_0 \, D$$

Leaving the conversion zone per second, toward the left, are:

1.  The kinetic energy of the combustion gases:

$$\frac{1}{2} \rho_1 \, (D - u) \, (D - u)^2$$

2.  The heat content of the combustion gases

$$\rho_1 \, (D - u) \, c_v \, T_1$$

3.  The external energy

$$p_1 \, (D - u)$$

The law of the preservation of energy therefore gives the next equation

$$\left.
\begin{aligned}
\rho_0 \, D \, U + \frac{1}{2} \rho_0 \, D \, D^2 + \rho_0 \, D \, c_v \, T_0 + p_0 \, D \\
= \frac{1}{2} \rho_1 \, (D - u) \, (D - u)^2 \\
+ \rho_1 \, (D - u) \, c_v \, T_1 + p_1 \, (D - u)
\end{aligned}
\right\} \quad (3)$$

or, in conjunction with equations (1) and (2)

$$\frac{1}{2}(p_1 + p_0)(v_0 - v_1) = c_v(T_1 - T_0) - U \qquad (4)$$

whereby

$$u = (v_0 - v_1)\sqrt{\frac{p_1 - p_0}{v_0 - v_1}} \qquad (5)$$

and

$$D = v_0 \sqrt{\frac{p_1 - p_0}{v_0 - v_1}} \qquad (6)$$

as is directly obtainable from equations (1) and (2).

Now the substitution of $p_1$ and $v_1$ or $p_0$ and $v_0$ for $T_1$ and $T_0$ in equation (4) through the phase equations, gives the relation between $p_1$ and $v_1$, with the constants of the original gas as parameters.

This relation between $p_1$ and $v_1$ is the so-called "dynamic adiabatic" or Hugoniot curve, named after its inventor (fig. 2).

Accordingly, if in a gas of original state, $p_0$ and $v_0$, a conversion zone travels at constant speed - saying nothing at present about the magnitude of this speed - the state of the combustion gases is represented by some point on the Hugoniot curve. According to equation (6) every point has a certain speed of propagation $D$ and, according to equation (5), a certain speed of combustion gases.

For all points above $G$, there is a speed of combustion gases in the same direction as that of the propagation of the conversion zone. This conversion is associated with a marked pressure rise and passes very quickly. If the conversion of the explosive has the characteristics of this zone, it denotes detonation.

If the explosive mixture burns - i.e., if the speed of transformation is low, whereby the combustion gas pressure $p_1$ decreases relative to $p_0$ and the speed of the combustion gases is contrary to the direction of combustion - then the stage of the combustion gases is represent-

ed by the curve below  E.  The zone  G - E  cannot be prac-
tically realized.

So, even though it is true that at detonation the stage
of the combustion gases, and hence the magnitude of detona-
tion speed, is characterized by a point on the Hugoniot
curve above  G,  this still does not determine which point
defines the actually occurring detonation speed.

The only stable point on the curve, according to
Chapman and Becker, is the contact point  J  of the tangent
from  A.

For angle  φ  of the tangent, it is

$$\tan \varphi = \frac{p_1 - p_0}{v_0 - v_1}$$

or, according to equation (6),

$$D = v_0 \sqrt{\tan \varphi} \tag{7}$$

Since  $\tan \varphi$  is computable from the dynamic adiabatic,
expressions for  D  and  u  containing only macroscopic,
thermodynamic equatioms can be adduced.

These are:

$$D = \frac{k + 1}{k} \sqrt{k \frac{R}{M} T_1} \tag{8}$$

$$u = \sqrt{\frac{1}{k} \frac{R}{M} T_1} \tag{9}$$

The constants

k,  ratio of specific heat

R,  gas constant

M,  molecular weight

apply to the temperature  $T_1$  of the combustion gases.

Forming the difference  a = D - u  gives

$$a = \sqrt{k \frac{R\,T_1}{M}} \qquad (10)$$

But this expression presents the magnitude of the velocity of sound in the combustion gases.

Accordingly, it may be stated:  The detonation speed is given through the sum of the speed of combustion gases and sonic velocity in the combustion gases.

The detonation temperature necessary for computing  D is obtained from the specific heat and the heat content of the explosive whereby the temperature rise due to the density increase on passage through the wave front is still accounted for through the dynamic adiabatic curve.  The detonation speeds computed this way are in good agreement with practical experience, hence affording conclusions regarding the fundamental proof of the concept.

The hydrodynamic derivation of formulas for solid and liquid explosives follows the same method, save that the specific volume of the gas molecules must be allowed for, as is customary in internal ballistics, by the introduction of Abel's equation, a modification of van der Waal's form.

Quantitative calculation of the detonation speed of solid explosives being out of the question, on account of ignorance of the covolume of gases at high pressures and temperatures, the measured detonation speed is used instead for calculating the covolumes.

In any case, the derived expressions indicate an experimentally confirmed relationship with density $\rho_0$, which does not obtain for the gas formulas (8) and (9).


DERIVATION OF NEW EXPRESSIONS


The first task is the derivation of expressions for detonation speed  D,  and material speed  u,  which are free from the concept of detonation temperature and contain solely constants directly amenable to measurement. The derivation is to be carried out direct for solid and liquid explosives.  By disregarding the covolume, the equations become valid for gaseous explosives.

In the following, various concepts of internal ballistics are employed.*

The combustion of a quantity of explosive of weight L in a closed bomb of constant volume V creates a terminal pressure $p_E$, called explosion pressure.

For the covolume of gases per unit weight $\alpha$, Abel's equation gives

$$p_E = f \frac{L}{V - \alpha L}$$

or, for the specific weight of the gases $\frac{L}{V} = \rho_0$,

$$p_E = f \frac{\rho_0}{1 - \alpha \rho_0} \tag{11}$$

f and $\alpha$ can be determined from pressure measurements in the bomb by means of equation (11).

The phase equation

$$p_E (1 - \alpha \rho_0) = \frac{R T_E}{M} \rho_0$$

discloses that the experimentally defined constant f has the value

$$f = \frac{R T_E}{M} \tag{12}$$

with $T_E$, explosion temperature. $T_E$ is the temperature of the explosion gases existing when combustion takes place without external energy input; i.e., for instance, in the closed bomb.

The gas under explosion pressure is to be adiabatically released up to zero pressure by means of a plunger. With k as exponent of the adiabatic release, the energy performed on the piston is

$$A = \frac{p_E \, (V - \alpha \, L)}{k - 1} \qquad (13)$$

which follows directly by integration of the adiabatic
curve, according to Poisson. The energy capacity  A  of
the gas in the form of equation (13) is called the poten-
tial gas energy. But this energy  A  must be equal to
the latent energy contained in the quantity of explosive
L  before combustion. If this amount per unit weight is
U,  we have

$$A = \frac{p_E \, (V - \alpha \, L)}{k - 1} = L \, U$$

or, with observance of equation (11):

$$f = (k - 1) \, U$$

$$k = 1 + \frac{f}{U} \qquad (14)$$

Constants  f  and  U  each have the dimension of an
energy  (m kg) per unit weight (kg), hence the dimension
of a length (m). The exponent  k  is an average value;
but differs very little from that  ($\approx$ 1 percent) computed
from the specific heat, as will be seen from the example
later on.

Other than density  $\rho_0$  of the explosive and covolume
$\alpha_1$  of the detonation gases, the expression for the deto-
nation speed is to contain only the measurable constants
U  and  f.

Figure 3 is another sketch of the detonation process.
For the sake of clearness the two sides of the conversion
zone carry pistons which at pressure  $p_0$  and  $p_1$  act on
the static explosive or the moving combustion gases. The
conception of such pressure forces is necessary as other-
wise no stationary phase would be conceivable.

As the conversion zone moves toward the right for a
distance  D,  the combustion gases move for a distance
u,  in the direction of the advancing zone. Since the
pressure  $p_1$  is to be constant within the combustion gases,
the piston  A  likewise must travel a distance  u,  toward
the right.

The mass, impulse, and energy equations for an observer moving with the detonation front, can be written in known manner. The pressure $p_0$ being very small relative to the detonation pressure $p_1$ ($p_0 = 0$ for solid explosives), its effect can be disregarded.

Mass:

$$D \rho_0 = (D - u) \rho_1 \tag{15}$$

Impulse:

$$p_1 = \frac{\rho_0}{g} D u \tag{16}$$

($g$ is acceleration of gravity)

Energy:

$$
\left.
\begin{aligned}
& D \rho_0 U + \frac{1}{2g} \rho_0 D D^2 \\
& = \frac{p_1 (D-u)(1-\alpha_1 \rho_1)}{k-1} \\
& + \frac{1}{2g} \rho_1 (D-u)(D-u)^2 + p_1 (D-u)
\end{aligned}
\right\} \tag{17}
$$

The left-hand side of the last equation gives the latent and kinetic energy of the explosive movement per second into the zone from the right.

Behind zone F, leftward, a gas space of magnitude $D - u$ per second is created, wherein the pressure $p_1$ prevails because the piston A, as seen from the detonation front, moves a distance $D - u$ to the left per second. Now the potential gas energy of this newly created space is (similar to equation (13)):

$$\frac{p_1 (D - u)(1 - \alpha_1 \rho_1)}{k - 1}$$

The factor $(1 - \alpha_1 \rho_1)$ accounts for the reduction in gas space due to the covolume of the gases.

The total gas space moves at a speed $(D - u)$ toward

·the left.   Its additional kinetic energy, accordingly, is:

$$\frac{1}{2g} \rho_1 \; (D - u) \; (D - u)^2$$

As seen by the observer, the work $p_1 \; (D - u)$ is performed on the plunger.  The three forms of energy appearing per second on the left side of the detonation front must, together, equal the energy per second entering from the right.  The result is equation (17).

Quantities $p_1$ and $\rho_1$ may be substituted herein through (15) and (16).  With consideration of $U \; (k - 1) = f$, we have:

$$(k - 1) \; D^2 + f \; g = \frac{k - 1}{2} \; (D - u)^2$$

$$+ \; (D - u) \; u \left(1 - \rho_0 \frac{D}{D - u} \; \alpha_1\right) + (k - 1) \; u \; (D - u)$$

and through direct calculation

$$D = \frac{1}{1 - \alpha_1 \; \rho_0} \left(\frac{f \; g}{u} + \frac{k + 1}{2}\right) \tag{18}$$

or abbreviated:

$$\eta = 1 - \alpha_1 \; \rho_0 \tag{19}$$

For negligible covolume $\eta = 1$.  In

$$D = \frac{1}{\eta} \left(\frac{f \; g}{u} + \frac{k + 1}{2} \; u\right) \tag{20}$$

the detonation speed $D$ is shown as function of the sole variable $u$, namely, the material speed.

Since $f$ has the dimension of length and $g$ is the acceleration, the validity of the dimension is proved.

Figure 4 illustrates the function (20).  The existence of a minimum is immediately apparent.  The correlated pair of values is obtained by differentiation of (20):

$$\frac{dD}{du} = 0 = \frac{1}{\eta}\left(-\frac{f\,g}{u^2} + \frac{k+1}{2}\right)$$

Hence

$$u = \sqrt{\frac{2}{k+1}\,f\,g} \tag{21}$$

and, with (20)

$$D = \frac{1}{\eta}\sqrt{2(k+1)\,f\,g} \tag{22}$$

or

$$D = \frac{1}{\eta}(k+1)\,u \tag{23}$$

Equations (21) and (22) are equivalent to Chapman's equations (8) and (9) as closer examination shows. Aside from the effect $\eta$ of the covolume, they merely contain the constant $f$ known from bomb tests and the latent energy $U$, because $k$ itself is $= 1 + f/U$.

From the agreement of the value of the lowest possible detonation speed, according to (20), with that derived as stable from the Hugoniot curve, the following conclusion can be drawn:

The constant detonation speed ultimately attained in a detonation process, is the lowest conversion speed possible, according to the laws of physics, provided that a sufficiently strong initial ignition creates a combustion-gas speed in the direction of the advancing zone of transformation.

The magnitude of the sonic velocity in the combustion gases is determined as follows: If $E$ is the modulus of elasticity of the medium, the sonic velocity $a$ in any medium follows the law:

$$a = \sqrt{\frac{E\,g}{\rho}}$$

The modulus of elasticity for ideal gases is $E = k\,p$ ob-

tained by differentiation of the adiabatic curve

$$p \left( \frac{1}{\rho} \right)^k = \text{constant}$$

But with

$$p_1 \left( \frac{1}{\rho_1} - \alpha_1 \right)^k = \text{constant}$$

as adiabatic curve of the combustion gases, its modulus of elasticity becomes:

$$E = \frac{p_1 \ k}{1 - \alpha_1 \ \rho_1}$$

Accordingly, the velocity of sound in these gases is

$$a = \sqrt{\frac{k \ p_1 \ g}{\rho_1 \ (1 - \alpha_1 \ \rho_1)}} \tag{24}$$

and, after replacing $p_1$ and $\rho_1$ by equations (15) and (16)

$$a = \sqrt{\frac{k \ u \ (D - u)}{1 - \alpha_1 \ \rho_0 \ \dfrac{D}{D - u}}}$$

and, with the abbreviation $\eta = 1 - \alpha_1 \ \rho_0$ :

$$a = \frac{1}{\eta} \ (k + 1) \ u - u \tag{25}$$

On comparison with equation (23), it is seen that the velocity of sound is equal to the difference between detonation speed $D$ and combustion-gas speed $u$, that is:

$$\boxed{a = D - u} \tag{26}$$

The result is in accord with Chapman and Becker's findings.

At sonic velocity itself, the equation is:

$$a = \left(\frac{k + 1}{\eta} - 1\right) u$$

or

$$a = \left(\frac{k + 1}{\eta} - 1\right) \sqrt{\frac{2}{k + 1} f g} \qquad (27)$$

The detonation pressure $p_1$ can be computed according to equation (16):

$$p_1 = \frac{\rho_o}{g} D u$$

The introduction of the values (21) and (22) gives:

$$\boxed{p_1 = \frac{1}{\eta} 2 f \rho_o} \qquad (28)$$

Expressing the gas pressure of the explosives resulting in the original volume under the slow combustion of the explosive, again as explosion pressure $p_E$, and the covolume for this pressure with $\alpha_E$, Abel's equation

$$p_E = \frac{f \rho_o}{1 - \alpha_E \rho_o}$$

is applicable.

From equation (28) then follows:

$$p_1 = 2 p_E \frac{1 - \alpha_E \rho_o}{1 - \alpha_1 \rho_o} \qquad (29)$$

and with it, the important result that the detonation pressure is twice as great as the explosion pressure, disregarding for the time being the effect of the difference in covolume. For gases, it is exactly:

$$\boxed{p_1 = 2 p_E} \qquad (30)$$

For the combustion-gas density, it is, according to equation (15), after insertion of the $D$ and $u$ value:

$$\rho_1 = \frac{k + 1}{k + 1 - \eta} \, \rho_0 \qquad (31)$$

The detonation temperature $T_1$ can be computed in the following manner. The combustion gases follow the phase equation:

$$p_1 \left( \frac{1}{\rho_1} - \alpha_1 \right) = \frac{R}{M} \, T_1$$

But, according to equation (12), it is:

$$f = \frac{R}{M} \, T_E$$

and consequently,

$$\frac{R}{M} = \frac{f}{T_E}$$

Hence,

$$T_1 = \frac{p_1 \left( \frac{1}{\rho_1} - \alpha_1 \right)}{f} \, T_E \qquad (32)$$

$p_1$ and $\rho_1$ may be expressed with $D$ and $u$:

$$T_1 = \frac{\rho_0 \, D \, u}{f \, g} \left( \frac{D - u}{\rho_0 \, D} - \alpha_1 \right) T_E$$

Substituting equations (21) and (22) for $D$ and $u$, we have the simple relation:

$$T_1 = \frac{2k}{k + 1} \, T_E \qquad (33)$$

The detonation temperature $T_1$ is, independent of the covolume of the combustion gases, greater by the factor $\frac{2k}{k + 1}$ than the explosion temperature $T_E$ of the explosive.

The aim of the first problem, the derivation of the
quantities incident to the detonation of any explosive,
containing - besides covolume - only the constants  f  and
U,  is herewith reached.  Obviously, equation (22) is ap-
plicable for the determination of covolume  $\alpha_1$  of the com-
bustion gases, in case the detonation speed  $D$  is meas-
ured.

## THEORETICAL PROOF OF THE EXISTENCE OF

## AN INCREASING DETONATION HEAD

The second problem is the study of the consequences
of a stationary detonation process illustrated in figure
3.  The assumption of a plunger  **A**  acting externally at
magnitude of detonation pressure  $p_1$  and moving at com-
bustion-gas speed  u  in direction of the moving detona-
tion front is, as already stated, necessary for the expla-
nation of the stationary processes in the transformation
zone  F.

At the initiation of a detonation process, the plunger
**A**  is represented by the nascent gases of the initiator.
The energy of these gases, however, would only suffice to
drive the detonation front a very short way into the ex-
plosive.  Once the action of the initiator gases has stopped,
the cause for the positive speed of the combustion gases
must naturally cease and the detonation come to an end.

Riemann stated that a shock wave in gases could be
maintained permanently only through external forces.  But
since experiments indicate that a detonation, once initi-
ated, travels with great constancy over any long distance,
the explanation of the detonation process is in need of a
supplement.  This necessity becomes especially evident in a
simple impulse consideration comprising a whole detonating
system.

Visualize a long column of explosive freely suspended
horizontally.  Ignition is to take place at one end.  Af-
ter the detonation wave has reached the other end, the
then-formed gas of pressure  $p_1$  and density  $\rho_1$  would
have to move as a whole at material velocity  u  in the
direction of the detonation front.  Such a motion is, of
course, impossible, because no external forces which could
cause a total impulse, act on the freely floating explosive.

The effect of the initial charge can be disregarded, since the column may be of any length.

An energy balance comprising the whole system also shows the incompleteness of the initially employed presentation; for the total energy of the combustion gases (potential and kinetic) of a freely suspended detonating column of explosive is, according to calculation, greater than the latent energy originally existing in the explosive.

In the calculation this difference is covered by the external energy of the plunger, but in the absence of external energies, as is the case for a column of explosive left to itself, no constant detonation speed would probably exist.

The difficulties introduced with the impulse and energy balance of explosive systems left to themselves, must be voided by assuming a second wave front in the combustion gases. The speed $w$ of this front relative to the quiescent explosive, must be greater than the speed of the gases $u$, but smaller than the detonation speed. On passing through this front the detonation pressure itself must drop to a value below that of the explosion pressure.

Figure 5 illustrates the process of detonation of an explosive substance on the assumption of a rarefaction front advancing at lower speed behind the detonation front. This rarefaction front moves at subsonic velocity relative to the combustion gases; that is, it is debatable, while a rarefaction shock of supersonic velocity would be impossible.

A - B is the explosive column made to detonate. Ignition takes place at A, where a solid abutment rests relative to the original explosive and which is therefore unable, in contrast to a moving plunger, to perform work on the total system.

The significance of this "damming up" is referred to elsewhere. For the present, it may be stated that the omission of A modifies the speed of the rarefaction front but not the detonation speed.

$F_1$ is the detonation front wherein the pressure rises from $p_0$ to the value $p_1$. $F_2$ indicates the rarefaction front, within which $p_1$ drops to $p_2$ and the ma-

terial velocity changes from value  u  to zero.  Nothing
is said about the form of the rarefaction front other than
assuming that its extent is small compared to the distance
$F_1 F_2$.  The region between the two fronts is called deto-
nation head, and the distance  $F_1 F_2$,  the head length.

Figure 5 indicates how, as a result of the difference
in speed of the fronts, the length of the head continues
to grow with time and, hence, with the detonation distance.
Since the abutment  A  is fixed, the gas behind the rare-
faction front  $F_2$  must be at rest; i.e., its material ve-
locity must become zero in case the process in the present-
ed form is to be stationary.  Suppose the values for this
new stage of the combustion gases are:  $p_2$, $\rho_2$, $\alpha_2$.

At present the combustion-gas speed  u  is confined
within the head  $F_1 F_2$.  Before front  $F_1$  and behind
front  $F_2$  the material is quiescent.  As the head becomes
longer in unit time the momentum of the gases within the
fronts increases with respect to time.  The cause for this
growth in momentum lies in the pressure difference  $p_2 -$
$p_0$  existing at either side of the head.  The pressure  $p_2$
must be lower than the explosion pressure  $p_E$,  for the
energy increment of the detonation head is necessarily at
the expense of combustion-gas energy of stage 2.

Assuming the nonexistence of the abutment at  A,  the
inertia resistance of the gases in proximity of  A  is to
be substituted.  Then the gases behind the rarefaction
front are subject to leftward acceleration - i.e., oppo-
site to the gas speed within the head - so that the impulse
law is always complied with.  Such leftward motion of the
combustion gases in stage 2 is followed by a drop in pressure
$p_2$  relative to the pressure with fixed abutment.

But a drop in  $p_2$  calls for a decrease in temporal
rise of gas momentum within the head; i.e., the head length
would increase less rapidly in time or, ultimately, the
speed of the rarefaction front would more closely approach
the speed of the detonation front.  This fact is of impor-
tance for the effect of the explosive, as will be shown
later on.

In the following, the quantitative calculation of the
speed of the rarefaction front with fixed abutment  A  is
carried out.  It could also be made for absent abutment,

although not without difficulties because of the nonuniform
processes. Since, as ascertained qualitatively, the ex-
istence of material speed behind the detonation front is
possible by the energy law as well as the impulse law in
case of an extension of the concept regarding the further
destiny of the combustion gases, equations (21) and (22),
established for detonation speed and material speed, will
be considered applicable.

Assume that figures 5c and 5d are instantaneous photo-
graphs of the detonation process with a time interval of 1
second. While within this interval point $H$ of the deto-
nation front has reached by quantity $D$ the point $G$, the
point $C$ of the rarefaction front has traveled the dis-
tance $w$ to point $K$. Consequently, the difference of the
head lengths $CH$ and $KG$ after 1 second, is $D - w$. The
increment of the mass of the head is

$$\frac{\rho_1}{g} (D - w)$$

As this mass moves at material speed $u$, the rise in
momentum of the head per second is

$$\frac{\rho_1}{g} (D - w) u$$

This rise must equal the effective pressure difference at
both ends of the head. This gives the impulse equation
($p_0 \ll p_2$):

$$\boxed{p_2 = \frac{\rho_1}{g} (D - w) u} \tag{34}$$

Each second the latent energy $\rho_0 D U$ is transformed
into potential and kinetic gas energy of the combustion
gases. New energy forms per second, are: potential gas
energy in stage 2, of the order of

$$\frac{p_2 w (1 - \alpha_2 \rho_2)}{k - 1}$$

the potential gas energy at stage 1, of the order of

$$\frac{p_1 (D - w) (1 - \alpha_1 \rho_1)}{k - 1}$$

and the kinetic gas energy at stage 1, of the value of

$$\frac{1}{2g} \rho_1 (D - w) u^2$$

Hence,

$$\rho_0 D U = \frac{p_1 (D - w) (1 - \alpha_1 \rho_1)}{k - 1}$$

$$+ \frac{p_2 w (1 - \alpha_2 \rho_2)}{k - 1} + \frac{1}{2g} \rho_1 (D - w) u^2$$

or (equation 14)

$$\rho_0 D f = p_1 (D - w) (1 - \alpha_1 \rho_1)$$
$$+ p_2 w (1 - \alpha_2 \rho_2) \qquad (35)$$
$$+ \frac{k - 1}{2g} \rho_1 (D - w) u^2$$

The law of preservation of mass for an observer sitting in the rarefaction front gives:

$$\rho_2 w = \rho_1 (w - u) \qquad (36)$$

The solution of the variables $w$, $p_2$, $\alpha_2$, and $\rho_2$ in equations (34), (35), and (36) is contingent upon $\alpha_2$.

In the detonation of gaseous explosives, the covolumes $\alpha_1$ and $\alpha_2$ are negligible, hence $w$ can be computed. For gases, it is:

$$w = K(k) D \qquad (37)$$

whereby $K(k)$ is a function of $k$ only:

$$K(k) = \frac{3 - k + \sqrt{(3k - 1)^2 + 8k (k + 1)}}{4 (k + 1)}$$

Putting $k$ roughly at 1.2, we have:

$$K(k) = 0.8 \quad \text{and} \quad w = 0.8 \, D$$

The speed of the rarefaction front in gases is thus only about 20 percent lower than the detonation speed or, in other words, the head length amounts to about 20 percent of the distance covered by the detonation front.

For solid or liquid explosives whose covolumes $\alpha_1$ and $\alpha_2$ are not negligible, $w$ can be computed in the following manner. As shown by A. Schmidt (reference 9), the covolume $\alpha_1$ of the combustion gases, is a function of the gas density $\rho_1$ and, consequently, of the original density $\rho_0$ of the explosive. He illustrates this relationship as following from detonation-speed measurements for different explosive densities with curves and arrives at a decrease in covolume for increasing gas density, which seems extremely likely.

Now the covolume is not only dependent on the density but, with constant density, on the temperature as well.

Visualize a molecule heap of great density but low temperature. Then the individual molecules penetrate into their common action spheres merely as a result of their need for space (deformation of "molecule envelopes"). If a rise in density is accompanied by constant temperature, the depth of penetration is greater; that is, the covolume becomes less. If, with constant density, the temperature is increased, the molecules - as a result of their then-increased kinetic energy - penetrate still deeper into the envelopes of their neighbors with a further reduction in covolume as a result.

Hence, the factor best suited to describe, as sole independent variable, the behavior of the covolume of a gas, is the pressure because it is, according to the phase equation, proportional to the density as to the temperature.

The relation $\alpha = \alpha(p)$ obtained, say, for a certain gas, may therefore suffice to define the magnitude of $\alpha$ when $p$ alone is known. This functional relationship between the pressure and covolume of a certain explosive

$$\alpha = \alpha(p) \tag{38}$$

can also be established from detonation-speed measurements

for different explosive densities. The covolume $\alpha_1$ is solved according to equation (22), and the correlated pressure $p_1$, according to equation (28). Several pairs then give the desired curve $\alpha = \alpha\,(p)$.

Following this preparatory step, the speed of the rarefaction front is calculable, since the magnitude of $\alpha_2$ is now defined by the existing pressure $p_2$. The four available equations, (34), (35), (36), and (38) contain the four variables $w$, $p_2$, $\alpha_2$, and $\rho_2$. An explicit calculation of $w$ would give an expression too complex to handle. Much quicker results are obtained when computing $\rho_2$ according to (36), and $p_2$ according to (34) for an assumed $w$. The $\alpha = \alpha(p)$ curve gives the correlated $\alpha_2$ value for $p_2$. These four variables must satisfy equation (35). The process is repeated.

The temperature $T_2$ behind the rarefaction front follows from the phase equation at

$$T_2 = \frac{p_2\left(\dfrac{1}{\rho_2} - \alpha_2\right)}{f}\,T_E \tag{39}$$

The quantities in the detonation of trinitrotoluene of density 1.59 kg/liter are now computed by means of the developed equations, on the basis of Friedrich's test data of detonation speeds at different densities quoted by A. Schmidt.

TABLE I

Detonation speed of trinitrotoluene in relation to density according to Friedrich

| Density $\rho_o$ (kg/liter) | Detonation speed $D$ (m/s) |
|---|---|
| 1.0 | 4,700 |
| 1.29 | 5,900 |
| 1.46 | 6,500 |
| 1.59 | 6,900 |

The heat content is:

$$U = 1,085 \quad (cal/kg)$$

$$= 4.63 \times 10^5 \ (m \ kg/kg) \ or \ (m)$$

According to H. Kast (reference 11), the powder constant  f  of trinitrotoluene is

$$f = 8.08 \times 10^4 \ (m)$$

hence the exponent of adiabatic expansion, according to equation (14):

$$k = 1 + \frac{f}{U} = 1.175$$

(For  U  the total heat volume, strictly speaking, should be set up to absolute zero point; but the error caused by referring the recorded heat content to $0°$ C. may be ignored.)

Equation (21) gives the speed of the combustion gases $(g = 9.81 \ (m/s^2))$

$$u = 855 \ (m/s)$$

The factor  $\frac{1}{\eta}$  or the covolume  $\alpha_1$  is obtainable from equation (23) for the different detonation speeds, while the detonation pressure follows from equation (16).

Table II contains the values thus obtained from Friedrichs' measurements.

TABLE II

Covolume  $\alpha_1$  and detonation pressure  $p_1$  of trinitro-toluene according to the developed equations

| Detonation speed D (m/s) | Covolume $\alpha_1$ (liters/kg) | Detonation pressure p (kg/cm²) |
|---|---|---|
| 4,700 | 0.60 | $4.09 \times 10^4$ |
| 5,900 | .53 | $6.62 \times 10^4$ |
| 6,500 | .49 | $8.27 \times 10^4$ |
| 6,900 | .46 | $9.55 \times 10^4$ |

The association between covolume and pressure $\alpha = \alpha(p)$, taken from this table, is graphically shown in figure 6.

The explosion pressure is, according to (11)

$$p_E = 7.18 \times 10^4 \quad (kg/cm^2)$$

Since its solution is contingent upon the knowledge of $\alpha_E$, the calculation must proceed on the basis of the now-known function $\alpha = \alpha(p)$. The correlated covolume is

$$\alpha_E = 0.517 \; (liter/kg)$$

The detonation pressure of trinitrotoluene of density 1.59 kg/liter is thus about 30 percent higher than the explosion pressure which, for gaseous explosives, would amount to 100 percent. The density of the detonation gases behind the transformation zone follows from equation (31) at

$$\rho_1 = 1.82 \; (kg/liter)$$

and the explosion temperature from equation (12) at

$$T_E = \frac{f}{R} M$$

or, since the molecular weight $M = 32.72$,

$$T_E = 3,120^\circ \; K.; \quad T_E = 2,847^\circ \; C.$$

The detonation temperature is, according to (33):

$$T_1 = 1.08 \; T_E; \quad T_1 = 3,380^\circ \; K.; \quad T_1 = 3,107^\circ \; C.$$

These quantities are compared with Schmidt's findings.

Except for $T_1$ the agreement is good. The values for the phase quantities behind the rarefaction front are obtained by approximation.

Table IV gives the quantities involved in the detonation, along with the explosion quantities for comparison.

TABLE III

Comparison of computed detonation quantities of
trinitrotoluene of 1.59 kg/liter density and
6,900 m/s detonation speed

| Detonation quantity | Calculated according to | | |
| --- | --- | --- | --- |
| | Author's equations | Chapman's equations | |
| $k$ | 1.175 | 1.16 | − |
| $u$ | 855 | 891 | (m/s) |
| $\alpha_1$ | 0.46 | 0.45 | (liters/kg) |
| $p_1$ | $9.55 \times 10^4$ | $9.65 \times 10^4$ | (kg/cm$^2$) |
| $\rho_1$ | 1.82 | 1.83 | (kg/liter) |
| $T_1$ | 3,107 | 3,630 | ($^\circ$C.) |

TABLE IV

Quantities involved in the detonation of trinitrotoluene
of 1.59 kg/liter density

| Detonation quantities | Notation | Value |
| --- | --- | --- |
| Density of solid explosive | $\rho_0$ | 1.59 (kg/liter) |
| Detonation speed | $D$ | 6,900 (m/s) |
| Explosion pressure | $p_E$ | $7.18 \times 10^4$ (kg/cm$^2$) |
| Covolume of explosion pressure | $\alpha_E$ | 0.517 (liter/kg) |
| Explosion temperature | $T_E$ | 2,847 ($^\circ$C.) |
| Exponent of adiabatic expansion | $k$ | 1.175 |
| Material speed of combustion gases behind detonation front | $u$ | 855 (m/s) |
| Detonation pressure | $p_1$ | $9.55 \times 10^4$ (kg/cm$^2$) |
| Covolume behind detonation front | $\alpha_1$ | 0.46 (liter/kg) |
| Combustion-gas density behind detonation front | $\rho_1$ | 1.82 (kg/liter) |
| Detonation temperature | $T_1$ | 3,107 ($^\circ$C.) |
| Speed of rarefaction front | $w$ | 3,720 (m/s) |
| Pressure behind rarefaction front | $p_2$ | $5.03 \times 10^4$ (kg/cm$^2$) |

TABLE IV (Cont.)

Quantities involved in the detonation of trinitrotoluene
of 1.59 kg/liter density

| Detonation quantities | Notation | Value |
|---|---|---|
| Covolume behind rarefaction front | $\alpha_2$ | 0.58 (liter/kg) |
| Gas density behind rarefaction front | $\rho_2$ | 1.40 (kg/liter) |
| Temperature of gases behind rarefaction front | $T_2$ | 2,350 ($^\circ$C.) |

The obtained values are illustrated in figure 7. A
striking feature is the great distance of the two fronts,
amounting to almost 50 percent of the travel of the deto-
nation front as compared to about 20 percent for gaseous
explosives.

According to figure 7c, the material speed  u  pre-
vails only between the two fronts, but is zero in the qui-
escent explosive and during the terminal stage of the com-
bustion gases.  Figure 7d gives the energy density along
the detonation path, with allowance for the potential as
well as the kinetic gas energy.  The shifting of the energy
into the detonation head is plainly seen.  The integral of
this energy distribution must naturally equal the total
latent energy of the explosive released by the detonation
front  $F_1$.  Figure 7e illustrates the effect of the mass.
Here also the integral of the mass distribution must agree
with the total mass of explosive engendered by the detona-
tion.

Table V gives the individual energies for a column of
explosive of 1 cm$^2$ section and  D  length at detonation.

The sum of the total energy of the detonation head
and of the potential energy of the residual combustion
gases behind the rarefaction front is numerically equal to
the latent energy.  Consequently, the total energy of the
head is 60 percent of the latent energy of the explosive
involved in the detonation.  Thereby the kinetic energy
within the head is considerably smaller than the potential.

TABLE V

Distribution of energy in a detonating
trinitrotoluene column of density 1.59 kg/liter and length D

| Energy form | Magnitude (m kg) |
|---|---|
| Latent energy | $5.08 \times 10^8$ |
| Potential energy of detonation head | $2.83 \times 10^8$ |
| Kinetic energy of detonation head | $.22 \times 10^8$ |
| Total energy of detonation head | $3.05 \times 10^8$ |
| Potential energy of gases behind rare- faction front | $2.02 \times 10^8$ |

DEDUCTIONS FOR THE CONCEPT OF SHATTERING POWER

Following proof of the existence of a detonation head of increasing length in time as sequence of the physical fundamental laws, the question of shattering power of an explosive is discussed.

Visualize at B (fig. 5) a steel plate hit vertically by the detonation front $F_1$.

R. Becker holds the detonation pressure $p_1$ responsible for the effect on the steel plate. But a pressure alone cannot be decisive for an effect. Assume a closed bomb of very small volume under high pressure $p_1$. Owing to the pressure, the walls expand and the pressure drops immediately to a fraction of the initial value, since the small energy storage is not able to maintain the initial pressure permanently.

While the pressure alone is decisive for the effect on absolutely rigid walls, the magnitude of all effects exerted on deformable bodies (i.e., all actually existing bodies) is contingent upon the available energy.

Take a closed bomb of large content filled with a gas, the pressure of which is lower than the tensile strength of the bomb walls, the potential gas energy being suffi-

cient to withstand the work of deformation. Even then the effect obviously will be zero, on account of the low pressure.

The shattering effect therefore does not increase with the available energy only but also with the height of the pressure.

The basis of our concept of detonation process now suggests consideration of the total energy of the detonation head as energy reservoir for a shattering effect, while the effective pressure is equal to the detonation pressure $p_1$. (The increase in hitting pressure due to the combustion-gas momentum is negligible.)

But the head energy is a function of the amount or volume of detonating explosive. Denoting this energy of a detonating explosive of 1 cm section and 1 m length with $E_K$, we find for the cited trinitrotoluene

$$E_K = 4.22 \times 10^4 \left( \frac{m\ kg}{m\ cm^2} \right)$$

However, this head energy $E_K$ of a certain explosive volume cannot as yet be a fitting term for the shattering power, because the detonation pressure itself must, as qualitatively deduced, be present. The higher the detonation pressure, the higher the detonation speed and the smaller the time interval necessary to course through a 1-meter column. If this time is $t = \frac{1}{D}$ (s), the shattering power $B$ of an explosive appears to be adequately expressed by

$$B = \frac{E_K}{t} = E_K\ D\ \left( \frac{m\ kg}{m\ cm^2\ s} \right)$$

which presents the shattering power by an energy per volume per time; i.e., performance per unit volume. Substituting the kilowatt and liter for m kg/s and cm² m, respectively, we have

$$B = E_K\ D\ (kw/liter) \tag{40}$$

Thereby the head energy $E_K$ for an explosive of 1-meter length and 1-liter volume, i.e., 10 cm² section, is computed in the described manner.

The shattering power could equally be referred to the unit weight of the explosive but since in practice a certain volume is available for an explosive, the unit volume is more appropriate.

For trinitrotoluene of 1.59 kg/liter density, it gives

$$B = 2.98 \times 10^7 \ (kw/liter)$$

It is readily seen that the shattering effect of an explosive is essentially contingent upon the detonating quantity rather than upon the data of detonation pressure only, which is not dependent on the quantity of material.

In the absence of abutment A (fig. 5), the gases in stage 2 move under pressure decrease, opposite to that of the rarefaction front. The rarefaction advances faster as a result of this pressure decrease.

After a certain detonation path, the distance of both fronts therefore is smaller without than with A. Hence, since the head energy, despite the constant detonation pressure, is dependent on A, it is readily apparent why the effects of explosives in the dammed-up stage are greater than those in the free stage.

## CONCLUSION

In conclusion, we shall analyze the detonation mechanism. The accord of the detonation speed derived from the minimum condition with practical experience suggests the assumption that the magnitude of detonation speed is actually dependent upon the macroscopic quantities only, such as heat content, explosion pressure, etc., whereas the physical state of the individual explosive particles is unimportant, as exemplified by the quality of detonation speed of solid and liquid explosives of the same density.

The reaction process, which in explosions and combustions defines the linear speed of transformation, plays no part at all in the phenomenon of detonation.

It may therefore be assumed that the explosive before the detonation front is at a stage independent of its initial physical state. The author, in consequence, assumes

that the radiation of the gases of combustion is sufficient to activate a thin layer of the explosive before the detonation front and here create the explosion pressure $p_E$ and the explosion temperature $T_E$. (Then the magnitude of the detonation speed is given only by the propagation speed of a shock wave in a gas of pressure $p_E$ and temperature $T_E$.) (That this speed of propagation is greater than the normal velocity of sound in this gas, is the result of the high amplitude of the shock.)

A molecular-kinetic analysis of the detonation phenomenon (reference 12) can be made backward from the already available thermodynamic data precisely as the nature of the velocity of sound on the basis of the known thermodynamic formula was recognized as the quantity of the molecular speed of the particular gas multiplied by a certain constant.

Translation by J. Vanier,
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## REFERENCES

1. Bollé, E.:  Handbuch d. physikal. u. techn. Mechanik von F. Auerbach u. W. Hort, vol. 6; ch. on Explosion und Explosionswellen.  Leipzig, 1928.

2. Schuster, A.:  Phil. Trans., vol. 184A, 1893, p. 152.

3. Chapman, D. L.:  Phil. Mag., vol. 47, 1899, p. 90.

4. Riemann, B.:  Göttinger Nachr., vol. 8, 1860, p. 156.

5. Hugoniot, H.:  Jour. d. l'école polytechn.  Paris, 1887 and 1889.

6. Jouguet, E.:  Mécanique des Explosifs.  Paris, 1917.

7. Becker, R.:  Z. f. Phys., vol. 8, 1922, pp. 321-362.

8. Becker, R.:  Physikalisches über feste und gasförmige Sprengstoffe.  Z. f. techn. Phys., vol. 3, 1922, pp. 152 and 249.

9. Schmidt, A.:  Über die Detonation von Sprengstoffen und die Beziehung zwischen Dichte und Detonations- geschwindigkeit.  Z. f. d. ges. Schiess- u. Spreng- stoffw., vol. 30, 1935, pp. 364-369; vol. 31, 1936, pp. 8-13, 37-42, 80-84, 114-118, 149-153, 183-187, 218- 222, 248-252, 284-288, 322-327.

10. Schweikert, G.:  Z. f. d. ges. Schiess- u. Sprengstoffw., vol. 32, 1937, p. 88.

11. Cranz, C.:  Lehrbuch d. Ballistik, vol. II, p. 107. Berlin, 1926.

12. Finkelnburg, W.:  Zur Theorie der Detonationsvorgänge. Z. f. d. ges. Schiess- u. Sprengstoffw., vol. 31, 1936, pp. 109-113.
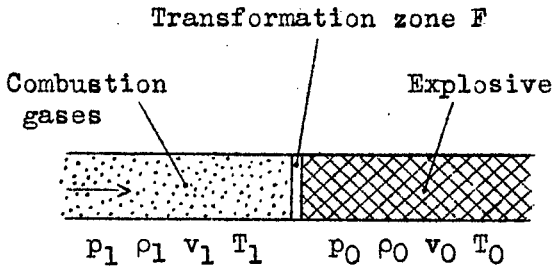
Transformation zone F

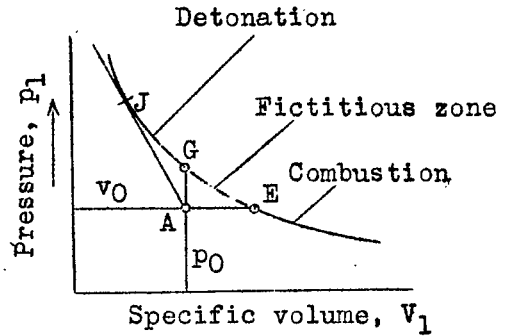Combustion
gases
Explosive

$p_1$ $\rho_1$ $v_1$ $T_1$ $p_0$ $\rho_0$ $v_0$ $T_0$

Figure 1.- Detonation process.

Detonation

Fictitious zone

Combustion

Figure 2.- Hugoniot curve.

Combustion
gases
Transformation zone

Explosive

$\rho_1 p_1 \alpha_1$ $\rho_0$

(a)
$p_1$ $p_0$

u D

(b)
$p_1$ $p_0$

Time interval between a
and b is 1 sec.
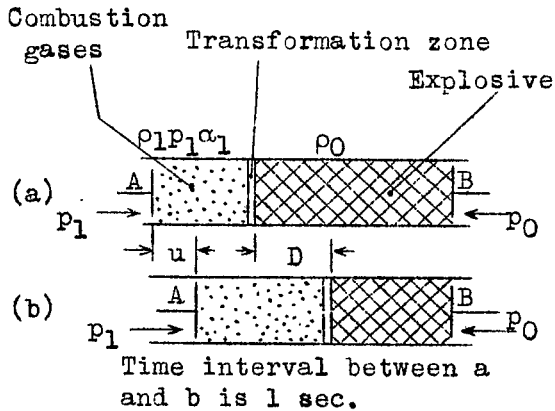
Figure 3.- Improved detonation
process.

Stable value for D

Speed of combustion gases, U
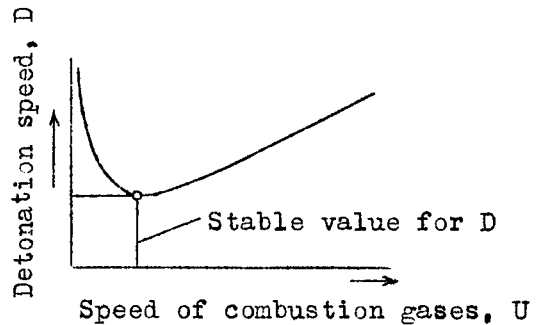
Figure 4.- Detonation speed plotted
against speed of
combustion gases.
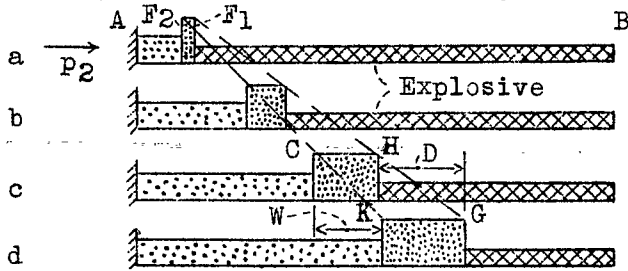
$F_1$ = Detonation front

$F_2$ = Rarefaction front

Figure 5.— Formation of detonation head.

Difference in stages = 1 time unit

$p_2, \rho_2, \alpha_2 \quad p_1, \rho_1 \alpha_1 \quad p_0, \rho_0$
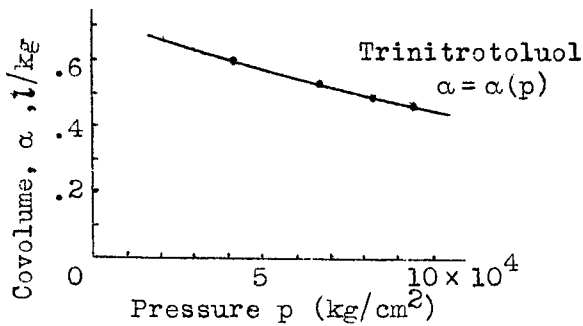
Figure 6.— Covolume of trinitrotoluene plotted against pressure.
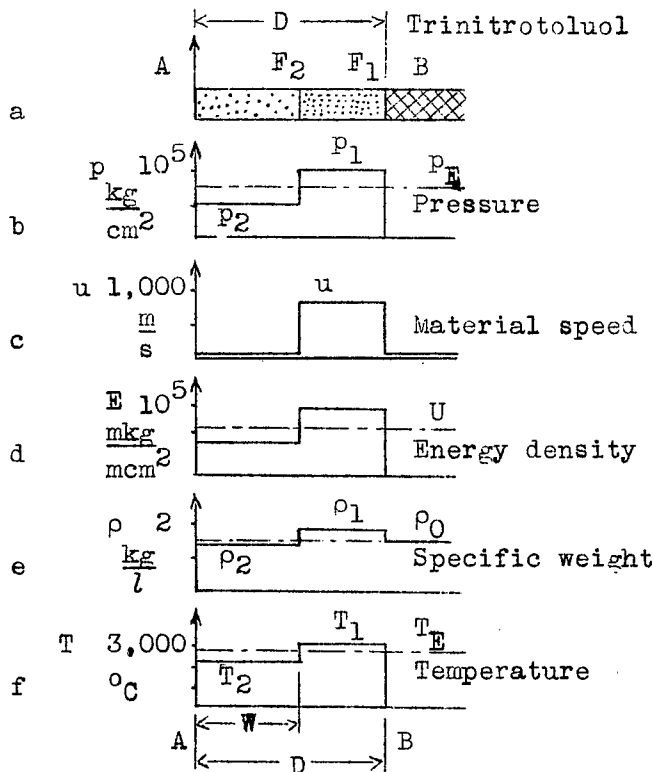
Trinitrotoluol
$\alpha = \alpha(p)$

Figure 7.— Distribution of pressure, material speed, energy density, and temperature in a detonating column of trinitrotoluene of D length.