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THEORETICAL STUDY OF VARIOUS AIRPLANE MOTIONS AFTER INITIAL DISTURBANCE

By Fr. Haus

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I. INTRODUCTION

The study of the dynamic stability of airplanes by the method of small motions involves the determination of oscillatory and aperiodic motions. There are a great number of well-established methods that permit us to calculate the periods and the damping factors of these motions but which do not, in general, give any indication of the amplitudes of the motions set up. We shall in the present paper, with the aid of a number of numerical examples, attempt to clarify the phenomena arising after a series of typical initial disturbances. It will be found that the results of these calculations contribute to an understanding of the mechanism of the motions which are set up, and it is our belief that the publication of these results, although limited to particular cases, will facilitate the study of the motions and flight paths. In what follows, we shall strictly separate the study of the longitudinal from that of the lateral motions and shall not consider the complete theory.

NOTATION

The OX, OY, and OZ axes will be assumed fixed on the airplane (fig. 1). The projections of the velocity on the axes will be denoted by u, v, and w, respectively. They define the angles of attack i, and of yaw j, these two angles fixing the position of the airplane on its path. When the components v and w are small with respect to u, we may write:

$$i = -\frac{w}{u} \qquad j = +\frac{v}{u}$$

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*"Etude théorique de quelques trajectoires d'avions apres perturbation initiale." Bulletin du Service Technique de L'Aéronautique, No. 17, June 1937.

and have, moreover, the relation

$$V = \sqrt{u^2 + v^2 + w^2} = u$$

The projections of the instantaneous rotational velocity Ω on the three axes are p, q, r, constituting the rolling, pitching, and yawing angular velocities. The rolling is positive if it tends to lower the right wing and raise the left; the pitching is positive if it tends to nose the airplane down, while the yawing is positive if it tends to turn the airplane toward the left. The controls corresponding to these rotations are, respectively, the ailerons, elevator, and rudder, their deflections α, β, γ being positive when they tend to turn the airplane in the positive sense of the rotations.

The position of the system of axes OXYZ fixed to the airplane, will be determined as a function of a system of spatially fixed axes $OX_0Y_0Z_0$ by means of the three angular displacements defined in figure 2. The derivatives of these angles

$$\frac{d\varphi}{dt} = \varphi'$$

$$\frac{d\theta}{dt} = \theta'$$

$$\frac{d\psi}{dt} = \psi'$$

are connected with the angular velocities p, q, r by the geometric relations

$$p = \varphi' \cos\theta - \psi' \cos\varphi \sin\theta$$

$$q = \theta' + \psi' \sin\varphi$$

$$r = \varphi' \sin\theta + \psi' \cos\varphi \cos\theta$$

II. LONGITUDINAL MOTION

Method Used

The equations of motion referred to axes fixed to the airplane are, for nonuniform flight (fig. 3)*

$$\left.\begin{aligned} T + X + P \sin\theta &= \frac{P}{g}\left(\frac{du}{dt} + qw\right) \\ Z - P\cos\theta &= \frac{P}{g}\left(\frac{dw}{dt} - qu\right) \\ M + T_s &= \frac{Pr^2}{g}\,\frac{dq}{dt} \end{aligned}\right\} \qquad (I)$$

to which should be added the relation

$$q = \frac{d\theta}{dt}$$

The equations may be written:

$$\left.\begin{aligned} \frac{du}{dt} &= f_1\ (u,\ w,\ q,\ \theta) \\ \frac{dw}{dt} &= f_2\ (u,\ w,\ q,\ \theta) \\ \frac{dq}{dt} &= f_3\ (u,\ w,\ q,\ \theta) \\ \frac{d\theta}{dt} &= f_4\ (u,\ w,\ q,\ \theta) \end{aligned}\right\} \qquad (II)$$

At the instant t_0, let the values satisfying the system be u_0, w_0, q_0, θ_0, and let us give the airplane an initial disturbance, so that at the instant $t_0 + \delta t$ the variables have the values $u_0 + \delta u$, $w_0 + \delta w$, etc. We may replace the four variables by their increments δu, δw, δq, $\delta\theta$ with respect to their initial values. This

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* In the section on longitudinal motion, r denotes the radius of gyration with respect to OY.

substitution, if the increments are assumed to be small, makes it possible to linearize the system of differential equations. We thus obtain:

$$\left.\begin{aligned}
\frac{d(\delta u)}{dt} &= \frac{\partial f_1}{\partial u}\,\delta u + \frac{\partial f_1}{\partial w}\,\delta w + \frac{\partial f_1}{\partial q}\,\delta q + \frac{\partial f_1}{\partial \theta}\,\delta\theta \\
\frac{d(\delta w)}{dt} &= \frac{\partial f_2}{\partial u}\,\delta u + \frac{\partial f_2}{\partial w}\,\delta w + \frac{\partial f_2}{\partial q}\,\delta q + \frac{\partial f_2}{\partial \theta}\,\delta\theta \\
\frac{d(\delta q)}{dt} &= \frac{\partial f_3}{\partial u}\,\delta u + \frac{\partial f_3}{\partial w}\,\delta w + \frac{\partial f_3}{\partial q}\,\delta q + \frac{\partial f_3}{\partial \theta}\,\delta\theta \\
\frac{d(\delta \theta)}{dt} &= \frac{\partial f_4}{\partial u}\,\delta u + \frac{\partial f_4}{\partial w}\,\delta w + \frac{\partial f_4}{\partial q}\,\delta q + \frac{\partial f_4}{\partial \theta}\,\delta\theta
\end{aligned}\right\} \quad \text{(III)}$$

In this new system the disturbances δu, δw, δq, $\delta\theta$ are the variables and the partial derivatives are constants whose values are determined by the initial conditions. This system is linear and readily integrated, yielding the variables δu, δw, δq, $\delta\theta$ as functions of the time after an initial disturbance. The integrated system is of the form:

$$\left.\begin{aligned}
\delta u &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t} \\
\delta w &= l_1 C_1 e^{\lambda_1 t} + l_2 C_2 e^{\lambda_2 t} + l_3 C_3 e^{\lambda_3 t} + l_4 C_4 e^{\lambda_4 t} \\
\delta q &= m_1 C_1 e^{\lambda_1 t} + m_2 C_2 e^{\lambda_2 t} + m_3 C_3 e^{\lambda_3 t} + m_4 C_4 e^{\lambda_4 t} \\
\delta \theta &= n_1 C_1 e^{\lambda_1 t} + n_2 C_2 e^{\lambda_2 t} + n_3 C_3 e^{\lambda_3 t} + n_4 C_4 e^{\lambda_4 t}
\end{aligned}\right\} \quad \text{(IV)}$$

In the above system the four values of λ are the solutions of the characteristic equation of the system:

$$\begin{vmatrix}
\frac{\partial f_1}{\partial u} - \lambda & \frac{\partial f_1}{\partial w} & \frac{\partial f_1}{\partial q} & \frac{\partial f_1}{\partial \theta} \\
\frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial w} - \lambda & \frac{\partial f_2}{\partial q} & \frac{\partial f_2}{\partial \theta} \\
\frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial w} & \frac{\partial f_3}{\partial q} - \lambda & \frac{\partial f_3}{\partial \theta} \\
\frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial w} & \frac{\partial f_4}{\partial q} & \frac{\partial f_4}{\partial \theta} - \lambda
\end{vmatrix} = 0 \quad \text{(V)}$$

The groups of coefficients $l_1 m_1 n_1$, $l_2 m_2 n_2$, etc., will be obtained in accordance with the method of Lagrange, by means of any three of the four equations of system (VI), into which are successively introduced the four values of λ:

$$\begin{aligned}
&\frac{\partial f_1}{\partial u} - \lambda + l\,\frac{\partial f_1}{\partial w} + m\,\frac{\partial f_1}{\partial q} + n\,\frac{\partial f_1}{\partial \theta} = 0\\
&\frac{\partial f_2}{\partial u} + l\left(\frac{\partial f_2}{\partial w} - \lambda\right) + m\,\frac{\partial f_2}{\partial q} + n\,\frac{\partial f_2}{\partial \theta} = 0\\
&\frac{\partial f_3}{\partial u} + l\,\frac{\partial f_3}{\partial w} + m\left(\frac{\partial f_3}{\partial q} - \lambda\right) + n\,\frac{\partial f_3}{\partial \theta} = 0\\
&\frac{\partial f_4}{\partial u} + l\,\frac{\partial f_4}{\partial w} + m\,\frac{\partial f_4}{\partial q} + n\left(\frac{\partial f_4}{\partial \theta} - \lambda\right) = 0
\end{aligned} \qquad \text{(VI)}$$

and the constants of integration C_1, C_2, C_3, and C_4, the latter depending on the initial conditions chosen.

The characteristic equation (V) is generally written:

$$\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0 \qquad \text{(VII)}$$

This equation, in general, possesses imaginary roots. If the four roots λ are imaginary

$$\lambda = a \pm bi$$

$$\lambda' = a' \pm b'i$$

equations (IV) may be written:

$$\begin{aligned}
\delta u &= e^{at}\rho_1 \sin(bt + \varphi_1) + e^{a't}\rho_1' \sin(b't + \varphi_1')\\
\delta w &= e^{at}\rho_2 \sin(bt + \varphi_2) + e^{a't}\rho_2' \sin(b't + \varphi_2')\\
\delta q &= e^{at}\rho_3 \sin(bt + \varphi_3) + e^{a't}\rho_3' \sin(b't + \varphi_3')\\
\delta \theta &= e^{at}\rho_4 \sin(bt + \varphi_4) + e^{a't}\rho_4' \sin(b't + \varphi_4')
\end{aligned} \qquad \text{(VIII)}$$

In the above form they permit us easily to follow the different disturbances which the airplane undergoes.

When the four roots of the characteristic equation are imaginary, the motion of the airplane consists of the superposition of two oscillatory motions. In each of these ρ represents the maximum possible amplitude, $\sin (bt + \varphi)$ determines the sinusoidal form, φ being the phase shift, and b a value such that the period $T = 2\pi/b$, e^{at} (equals 1 for $t = 0$) represents the damping.

We then proceed as follows:

1. Knowing the characteristics of the airplane studied, we determine its characteristic equation (V) and obtain the roots λ. From these roots we can find the period T and the damping factor e^{at}. These characteristics depend on the airplane and not on the initial disturbance considered.

2. We next determine by equations (VI) the groups of factors l, m, n, these factors being similarly independent of the initial disturbance considered. When the roots λ are conjugate imaginary the factors l, m, n will also be conjugate imaginary and we may write:

$$l_1 = \alpha + \beta i \qquad m_1 = \gamma + \delta i \qquad n_1 = \epsilon + \varphi i$$

$$l_2 = \alpha - \beta i \qquad m_2 = \gamma - \delta i \qquad n_2 = \epsilon - \varphi i$$

3. We now introduce the initial conditions of the motion, that is, the disturbance considered. The latter is expressed by a real value of one or several of the variables δu, δw, δq, $\delta \theta$, and from it we may determine the constants of integration, which are conjugate imaginaries:

$$C_1 = A + Bi \qquad C_3 = A' + B'i$$

$$C_2 = A - Bi \qquad C_4 = A' - B'i$$

The quantity $C_1 e^{\lambda_1 t} = C_2 e^{\lambda_2 t}$ will assume the following forms:

$$\begin{aligned}
C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} &= C_1 e^{(a+bi)t} + C_2 e^{(a-bi)t} \\
&= e^{at} \left[C_1 (\cos bt + i \sin bt) + C_2 (\cos bt - i \sin bt)\right] \\
&= e^{at} \left[(C_1 + C_2) \cos bt + i(C_1 - C_2) \sin bt\right] \\
&= e^{at} \left[2 \cos A\, bt - 2B \sin bt\right] \\
&= e^{at} \rho_1 \left[\sin \varphi_1 \cos bt + \cos \varphi_1 \sin bt\right] \\
&= e^{at} \rho_1 \sin (bt + \varphi_1)
\end{aligned}$$

with

$$\rho_1 = 2\sqrt{A^2 + B^2}$$

$$\sin\varphi_1 = \frac{2A}{\rho_1}$$

$$\cos\varphi_1 = \frac{-2B}{\rho_1}$$

Similarly, the part of the root $l_1 C_1 e^{\lambda_1 t} + l_2 C_2 e^{\lambda_2 t}$ will assume the following forms:

$$e^{at}\left[(l_1 C_1 + l_2 C_2)\cos bt + i\,(l_1 C_1 - l_2 C_2)\sin bt\right]$$

$$e^{at}\left[(2A\,\alpha - 2B\,\beta)\cos bt - (2A\,\beta + 2B\,\alpha)\sin bt\right]$$

$$e^{at}\,\rho_2\,\sin(bt + \varphi_2)$$

with

$$\rho_2 = 2\sqrt{(A\alpha - B\beta)^2 + (A\beta + B\alpha)^2}$$

$$\sin\varphi_2 = \frac{2\,(A\alpha - B\beta)}{\rho_2}$$

$$\cos\varphi_2 = \frac{-\,2\,(A\beta + B\alpha)}{\rho_2}$$

We may note that ρ_2 may be written:

$$\rho_2 = 2\sqrt{(A^2 + B^2)\,(\alpha^2 + \beta^2)}$$

We see that the values of the amplitudes ρ and of the phase displacements φ depend on the initial conditions. For the same oscillation the ratios ρ_2/ρ_1, ρ_3/ρ_1, ρ_4/ρ_1 are, however, independent of the initial disturbance and depend only on the characteristics of the airplane.

AIRPLANES STUDIED

We shall study a series of airplanes differing only in the coefficient of static stability $\partial C_M/\partial i$. If i may be taken equal to $-\frac{w}{V}$, the derivatives with respect to w figuring in the calculations are merely the derivatives with respect to the angle of attack i multiplied by $-\frac{1}{V}$. We have shown elsewhere that the longitudinal motion of an airplane is sufficiently determined by: five geometric characteristics; seven aerodynamic characteristics; a characteristic depending on the engine propeller unit; and a characteristic depending on the weight. In our example, we have assumed the following values for these elements:

1. Geometric characteristics:

$$l = 2.3 \text{ meters}$$

$$r = 1.3 \text{ "}$$

$$s = 0$$

$$l' = 2.6\, l$$

$$S' = \frac{1}{7} S$$

l' and S' being the lever arm and the area of the horizon-tail surface, respectively.

2. Aerodynamic characteristics:

The quantities C_x and C_z are the coefficients of the aerodynamic forces in the directions of the axes fixed to the wing.

The angle of attack i of the state considered is 0: $C_z = 0.40$.

Minimum drag-lift ratio at this angle of attack: $\beta = 0.125$.

$$\frac{dC_x}{di} = 0.006; \quad \frac{dC_z}{di} = 0.065.$$

The damping of the pitching oscillations will be characterized by a coefficient which we shall write in the form $\frac{dC'_z}{di'}$ as if the entire damping were produced by the action of the horizontal tail surface. In the above expression C'_z would be the lift coefficient of the tail, i' the real angle of attack of the tail.

As a matter of fact, we shall take a numerical value somewhat larger than that corresponding to the derivative of the lift of the tail in order to take account of the damping introduced by the parts of the airplane other than the horizontal tail surface. We shall take:

$$\frac{dC'_z}{di'} = 0.065$$

Finally, we shall take

$$\frac{dC_M}{di} = \mu$$

the coefficient of static stability, as the variable element in the example.

As the variations of the static stability may be obtained by a simple displacement of the center of gravity without varying the tail surface, we may assume it to be possible to vary μ without varying the other characteristics.

3. Element characterizing the engine-propeller unit.

We write:

$$\frac{dT}{dV} = - h \frac{T}{V}$$

giving h the value 0.5.

4. Element characterizing the weight of the airplane.

The weight per square meter will be taken as equal to 40 kilograms. From these data the result follows immediately that the speed of the airplane under the conditions considered is 40 meters per second (90 miles per hour).

The above elements permit us to determine the characteristic biquadratic equation in λ^4.

CALCULATED OSCILLATIONS

The equation was solved for eight values of μ. The roots are given in the following table, the coefficient of static stability being evaluated by taking degrees for the unit angle.

Values of μ	Values of λ			
	Short-period oscillation		Long-period oscillation	
$\mu = 0.008$	$\lambda = -3.75 \pm 3.02i$		$\lambda = -0.0408 \pm 0.238i$	
$\mu = 0.006$	$\lambda = -3.75 \pm 2.5i$		$\lambda = -0.0412 \pm 0.213i$	
$\mu = 0.004$	$\lambda = -3.74 \pm 1.86i$		$\lambda = -0.0437 \pm 0.19i$	
$\mu = 0.002$	$\lambda = -3.73 \pm 0.928i$		$\lambda = -0.0508 \pm 0.141i$	
$\mu = 0.001$	$\lambda = -4.58$	$\lambda = -2.86$	$\lambda = -0.056 \pm 0.092i$	
$\mu = 0.0$	$\lambda = -5.18$	$\lambda = -2.255$	$\lambda = -0.1275$	$\lambda = 0$
$\mu = -0.001$	$\lambda = -5.6$	$\lambda = -1.8$	$\lambda = -0.2307$	$\lambda = +0.075$
$\mu = -0.002$	$\lambda = -5.97$	$\lambda = -1.455$	$\lambda = -0.2895$	$\lambda = +0.127$

The motion studied is stable when the roots are negative or when, if imaginary, their real parts are negative. The factor e^{at} or $e^{\lambda t}$ approaches, in this case, zero as t increases.

The time after which the factor e^{at} attains the value $\frac{1}{2}$ is given by:

$$T_{\frac{1}{2}} = \frac{ln_2}{|a|} = \frac{0.692}{|a|}$$

When the coefficient of static stability has a sufficiently large value the characteristic equation defines in general two very different types of oscillation. One oscillation is of short period and is very damped; the second has a long period and is less damped. As the static stability decreases the motion ceases to be oscillatory. In the example given, the rapid oscillation loses its character somewhat earlier than the slow oscillation. The periods and damping are given in the table below.

	Short-period oscillation (seconds)		Long-period oscillation (seconds)	
μ	T	$T_{\frac{1}{2}}$	T	$T_{\frac{1}{2}}$
0.008	2.08	0.185	26.4	17
.006	2.51	.185	29.5	16.8
.004	3.38	.185	33	15.9
.002	6.7	.186	44.5	13.6
.001	aperiodic		68	12.4
.00	aperiodic		aperiodic	

When $\mu = 0$, the two kinds of oscillations vanish, the roots becoming real and one of them zero.

In passing to the condition of static instability the total motion is the sum of four periodic motions, three of them corresponding to disturbances decreasing with time, the fourth corresponding to a disturbance increasing with time and indicating dynamic instability. The table of roots shows clearly that the unstable motion corresponds to one of the components of the slow oscillation. The aperiodic motions which replace the rapid oscillation do not cease to be stable within the limits of static instability considered.

When the motions are aperiodic the equations defining the motion cannot be put in the sinusoidal form VIII and must remain in the exponential form IV.

CHARACTER OF THE OSCILLATIONS

The character of the two types of oscillations is well known. The rapid oscillation is primarily that about the center of gravity of the airplane, while the slow oscillation, on the contrary, is one of the path described by the center of gravity in the vertical plane. The latter type of oscillation originates in the irregularities produced by any disturbance of the equilibrium of the forces applied to the airplane. It is possible by approximate methods to study each of these types of oscillations independently.

a) Rapid oscillation.- In studying the rapid oscillation by the method utilized by Munk - taking account of the loss in altitude of the airplane - it will be found that the restoring moment is not proportional to dC_M/di, but to

$$\frac{dC_M}{di} + \frac{dC_z}{di}\,\frac{dC'_z}{di'}\,\frac{S'l'}{Sl}\,\frac{gl'}{V^2}\,\frac{1}{C_z}$$

and the damping is not proportional to

$$\frac{dC'_z}{di'}\,\frac{S'l'^2}{Sr^2}$$

but to

$$\frac{dC'_z}{di'}\,\frac{S'l'^2}{Sr^2} + \frac{dC_z}{di}$$

This explains why, so long as the maximum lift is not attained, the rapid oscillation could be stable even if the coefficient of static stability is negative.

The rapid oscillation, at small angles of attack, is strongly damped and the function $e^{at}\rho \sin(bt + \varphi)$, as a result of the values of a and b, respectively, practically ceases to have an oscillatory character. If, for example:

$$a = -3.75 \qquad b = 1.56$$

then

$$T = \frac{2\pi}{b} = 4$$

$$T_{\frac{1}{2}} = \frac{0.692}{|a|} = 0.185$$

which means that after about 1/20 of a period the amplitude is again affected by a factor 0.5. In the case where $\varphi = \frac{T}{4}$ the amplitudes start out from ρ, but decrease more rapidly than according to the sinusoidal curve, the damped motion being that shown in figure 4. The airplane is, in fact, very energetically brought back to the angle of attack which corresponds to a zero moment about the center of gravity.

The same does not apply at the large angles of attack. At the maximum lift $\frac{dC_z}{di} = 0$, and tests on numerous models show that dC'_z/di' is also very small at this instant, whereas dC_M/di, on the contrary, maintains its former value. The theory thus predicts that the rapid oscillation may not be damped at the large angles of attack.

b) Slow oscillation.- The slow oscillation may be studied separately by examining the modifications undergone by the path when the velocity is subject to a disturbance. It will readily be found that the flight path will be of an oscillatory character. These oscillations are necessarily accompanied by disturbances of the other variables. Taking account of the fact that the airplane strongly responds to the applied moments M and of the tendency, through the short-period oscillations, to assume rapidly the angle of attack of equilibrium, it might be supposed that these long-period oscillations are effected at a strictly constant angle of attack. This conclusion would, however, be premature. The moment M is, in fact, a function of two variables w and q, and we may write:

$$dM = \frac{\partial M}{\partial w}\, dw + \frac{\partial M}{\partial q}\, dq$$

The rapid oscillation does not permit the continued existence of moments M and therefore, $dM = 0$. The irregularity of the flight path, however, corresponds to the existence of angular pitching velocities dq, so that the condition $dM = 0$ implies the existence of disturbances of angle of attack δw or δi connected with δq by

$$\frac{\partial C_M}{\partial w}\,\delta w + \frac{\partial C_M}{\partial q}\,\delta q = 0$$

We thus have:

$$\delta q \left(- \frac{l}{V}\,\frac{S'l'^2}{Sl^2}\,\frac{dC'_z}{di'}\right) = -\,\delta w\,\frac{\partial C_M}{\partial w}$$

$$= \delta w\,\frac{\partial C_M}{\partial i}\,\frac{1}{V}$$

whence

$$\frac{\delta q}{\delta w} = -\,\frac{1}{l}\,\frac{Sl^2}{S'l'^2}\,\frac{\frac{\partial C_M}{\partial i}}{\frac{\partial C'_z}{\partial i'}}$$

This condition will only be true provided that:

1) ρ_3 and ρ_2 are in the previously given ratio, and

2) δq and δw corresponding to the short-period oscillation are out of phase by π.

In the case of the numerical examples, we should have:

$\rho_3 = 0.01385\ \rho_2$ for the airplane for which $\mu = 0.002$

$\rho_3 = 0.0352\ \rho_2$ $\mu = 0.008$

It may be seen from the numerical tables computed, that this relation is practically verified and that the phase displacements are by $\pi \pm 3^\circ$.

DETERMINATION OF THE MOTION AFTER INITIAL DISTURBANCE

In the general motion each of the elementary motions is involved to a degree that depends on the nature of the initial disturbance. Preliminary studies on automatic stabilizers have led us to investigate how a given initial disturbance is distributed between the two types of oscillations and we here give the results of these computations. The general method indicated above has been applied to a sufficiently large number of particular cases. We have, however, limited our computations, as far as longitudinal motion is concerned, to six airplanes instead of eight, eliminating the intermediate case $\mu = 0.001$ as well as that for the unstable airplane $\mu = -0.002$. The method used becomes in fact essentially inapplicable when the disturbances are no longer small. Since certain disturbances increase when there is instability the method is at fault in cases of instability, and we have therefore limited ourselves to the study of the unstable case $\mu = -0.001$, which leads to a less rapid increase in the disturbances.

We shall examine the motion of the airplanes for the following initial disturbances:

1. Increase in the angle of attack caused by a sudden pitching motion.

2. Increase in the angle of attack caused by a vertical gust.

3. Increase in the relative velocity due to a gust acting in the direction of the flight path.

4. Sudden engine failure.

In a general way we assume the initial disturbance to be sufficiently large. It may be objected that we are here departing from the assumption of small motions. The size of the disturbance has been chosen, however, with a view toward obtaining variables that may readily be handled numerically. Since all the equations are linear the initial disturbance may be divided by an arbitrary number and all the amplitudes will be divided by the same number.

DISPLACEMENT OF THE AIRPLANE IN UNDISTURBED AIR

Suppose the airplane has undergone a sudden disturbance displacing the airplane by a certain angle with respect to the horizontal and the flight path. We shall assume a displacement of 0.2 radian in a direction to nose the airplane up. The airplane will then undergo a disturbance in altitude $\delta\theta = -0.2$ radian or $-11.46°$, and a disturbance in the angle of attack $\delta i = +0.2$ radian, corresponding to $\delta w = -0.2\ V = -8$ m/s (18 m.p.h.).

The numerical values found for ρ, φ, etc. are given in table I, and the diagram, giving the disturbances δu of the velocity u, (which may be assumed to be the same as V), $\delta\theta$ of the inclination θ, and δi of the angle of attack i, is shown on figure 5. These curves, as well as the figures of the numerical table, show how the initial disturbance affecting the angle of attack and the attitude, is distributed between the two oscillations.* The airplane is suddenly raised as a result of the excess of lift and tends to nose down if stable - these two phenomena decreasing the angle of attack.

The most stable airplane noses down energetically by the action of the rapid oscillation. After a fraction of a second the disturbance of the angle of attack is practically annulled by the joint effect of this diving action and the vertical acceleration which turns the flight path upward.

The strong curvature of the $\delta\theta$ curve corresponds to the vanishing of the rapid oscillations. At the instant when these vanish the airplane is at an angle of attack differing little from the normal angle of attack but stalled by 6° and on a rising flight path. A lack of equilibrium will be felt in the forces and the long-period oscillation will arise from this fact.

The airplane only slightly stable likewise noses down through the effect of the short-period oscillation but the action is less accentuated. The flight path meanwhile curves upward. At the instant when the rapid oscillation ceases to be felt, the airplane has nosed down only 2°, but its flight path has been raised and the airplane will be found on a rising flight path of about 9°, the disturbance in the angle of attack always remaining small. The nonequilibrium of the forces is greater than in the preceding case, and the resulting long-period oscillation will be of greater amplitude.

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*On this figure, as well as on those following, the negative values of $\delta\theta$ are plotted above the x-axis so as to facilitate reading the diagrams. Upwardly inclined motions will then correspond to rising curves and vice versa.

The long-period oscillation is very clearly brought out by the disturbances in the velocity V, to which it corresponds. The disturbances in the angle of attack which accompany the long-period oscillation are greater on a less stable than on a very stable machine. The case of an airplane statically neutral requires no remarks. In the case of the statically unstable airplane the disturbance of angle of attack decreases at first as a result of the upward curvature of the path; the latter is not stable, however. The airplane enters a condition of constantly increasing angle of attack, leading inevitably to such disturbances that the method of small motions, after a certain time, ceases to apply.

DISTURBANCE OF ANGULAR VELOCITY

The disturbance above considered, suddenly increasing the angle of attack and the orientation in space of an important angle, is not actually realized in practice. It may, of course, be imagined that a localized gust strikes the rear of the airplane, but the latter cannot instantly attain the final angle of attack assumed. The airplane will pass through all intermediate angles of attack and the effects of excess lift and of the static stability will make themselves felt during these states.

In order to analyze the case, let us imagine that the disturbance applied is an impulsive angular velocity tending to turn the nose up $\delta q < 0$. From the diagrams of V, θ, i, we have found that for $\delta q = - 0.96$ radian per second (which is a considerable value), the ultimate effects of the disturbance were the same as those of the initial disturbances of attitude and angle of attack studied above. The immediate effects of the disturbance evidently differ. The angle of attack starts from zero and tends to increase but the increase is rapidly stopped and the long-period phenomena are the only ones that subsist after about two seconds. It does not appear necessary to comment any further on the diagrams of figure 6, constructed for the initial disturbance $\delta q = - 0.96$ radian.

DISTURBANCES OF THE SURROUNDING MEDIUM

The effect of the disturbances in the surrounding medium may be studied by the following device. Let us assume that the ambient air, instead of being undisturbed, is subject to a velocity U', having the components u' and w'. Let U of components u and w be the absolute velocity of the airplane. The relative velocity may be written:

$$V = \sqrt{(u - u')^2 + (w - w')^2}$$

and the angle of attack as

$$i = - \frac{w - w'}{u - u'}$$

It is always possible, in the case of a given airplane, to investigate the motion corresponding to an equilibrium of forces in an atmosphere having any absolute velocity whatever, since the relative velocity and the angle of attack are independent of the disturbance velocity. It is therefore simple to investigate the motions which the airplane would assume during disturbances of the surrounding medium.

Let R be the condition of the airplane for which $u'_1 = 0$ $w'_1 = 0$. This condition corresponds to the values u_1 and w_1 of the absolute velocity of the airplane. In a disturbed atmosphere A_2 for which $u'_2 \neq 0$ $w'_2 \neq 0$ the airplane, flying at the same angle of attack and same relative velocity, will be in a state R_2 characterized by different absolute velocities u_2 and w_2 determined by $u_2 = u_1 + u'_2$ $w_2 = w_1 + w'_2$. If the airplane flying in state R_1 suddenly passes from the atmosphere A_1 to the atmosphere A_2, it will cease to be in equilibrium. It will not be able to maintain its state R_1 and will tend to assume state R_2. The motions of the airplane could be calculated by considering the state R_2 as that of equilibrium, and the airplane as deviating from it by

$$\delta u = - u'_2$$

$$\delta w = - w'_2$$

This artifice enables us to study the effect of sudden gusts on the airplane.

ASCENDING GUST

Let an ascending gust, characterized by $u'_2 = 0$ $w'_2 = 8$ m/s act suddenly on an airplane in horizontal flight. The state R_1 in the atmosphere A_1 assumed to be undisturbed, is characterized by a system of given values u_1, w_1, θ_1 (fig. 7). The state R_1 in the atmosphere A_2 is a transitory state and not one of equilibrium. The angle of attack is very large (fig. 8) and the flight path of the airplane will be modified. The airplane will try to adjust itself to a new state of equilibrium R_2 in the atmosphere A_2. It is easy to see that this state will be characterized by the same values of the relative velocity, angle of attack, and inclination to the horizontal as state R_1, but the absolute velocity U_2 will be different. The latter becomes ascending, the airplane being subject to the motion of the surrounding medium (fig. 9).

We may study this transitory period by seeking the effect of an initial disturbance $\delta w = -8$ m/s (fig. 10). The airplane will be lifted suddenly and if it is statically stable, it will start a diving motion. These two motions have the effect of tending to close the angle included between the directions OX and V of the figure, each of these axes approaching the other.

If the airplane has great static stability, it will oscillate more rapidly and the angle of attack of the state will be regained by means of a displacement $\delta\theta$ larger than would be the case if the airplane had been less stable. The static stability increases the disturbance in the attitude $\delta\theta$ due to the rapid oscillation produced by a vertical gust. The axis of the airplane should, however, in its final state, regain its initial attitude θ. A large static stability therefore results in momentarily removing the airplane from its final attitude and increasing the nonequilibrium of the forces acting on the moving center of gravity. This explains why the amplitude of the long-period oscillations will be greater on a very stable airplane than on a less stable one. A neutral airplane will be lifted without its axis OX undergoing any change in attitude $\delta\theta$. It is the U_1 axis which will be effective in destroying the disturbance of the angle of attack δi. The case of an unstable airplane is sufficiently explained by the figure and requires no remarks.

HORIZONTAL GUST

By the same procedure we have calculated the effect of a horizontal gust of 10 m/s (22 m.p.h.) acting in the opposite direction to the motion of the airplane. The initial disturbance is an increase δu. After the effect of this disturbance has disappeared the airplane regains its horizontal flight at the original angle of attack but at a different altitude. The airplane undergoes at first a vertical acceleration. Its angle of attack therefore decreases, and if it is stable it noses up and then starts the motions characteristic of the long-period oscillation. Figure 11 shows the action on the four stable airplanes considered, on a neutral airplane, and on an unstable airplane. The disturbance of the horizontal velocity gives rise to disturbances in the attitude associated with the slow oscillation and their effects in certain cases become far from negligible.

DISTURBANCE OF THE FLIGHT PATH

We shall consider the case of a disturbance $\delta\theta$ acting alone - the velocity V, angle of attack i, and angular velocity q not being disturbed. Such a disturbance occurs under special circumstances. It would result, for example, from the same angular displacement of the airplane and its flight path. The case where an initial state and final state are characterized by sensibly the same angle of attack and same velocity V, but where the paths and the orientation of the OX axis differ by the same quantity, is one that may occur in practice, namely, that of the sudden failure of the engine. Denote the propeller advance diameter ratio V/nD by γ. On some airplanes, when

$$s \frac{dT}{d\gamma} + \frac{\partial M}{\partial \gamma} = 0$$

the sudden failure of the engine does not change the moment about the center of gravity. When this is true the airplane, for the same deflection of the elevator, will be in equilibrium with its engine cut off and on a descending flight path over which it will travel with the same velocity and at the same angle of attack as on the horizontal path with the engine running.

The horizontally flying airplane, subject to sudden engine failure, is then at an angle of attack and velocity of equilibrium corresponding to the new conditions, but on a path not sufficiently descending, and departs from its state of equilibrium by an angle which we consider as the initial disturbance $\delta\theta < 0$. It is therefore sufficient if we make the preceding calculation taking $\delta\theta$ as the disturbance.

Our figures have been drawn for $\delta\theta = -0.2$ radian in order to maintain the same magnitude of initial disturbance and facilitate comparison with the preceding figures. This number does not correspond to any actual value of the minimum drag-lift ratio because the latter would be tan 11.4° or 0.225, that is, a very poor drag-lift ratio. The amplitudes read off on the diagram should be diminished in this ratio if the effect of the engine failure is truly to be taken into account. The diagram is nevertheless of some use. It shows that with a nonequilibrium of the power as the initial cause the phenomenon is exclusively one of slow oscillation.

ANGULAR VELOCITIES AND ACCELERATIONS

The preceding diagrams are concerned with the values of the variables V or u, θ, and i or w. We have not indicated the angular velocities, the latter being obtained by determining the slopes of the θ curves. It is equally possible to determine the accelerations or the intensity of the apparent weight. As regards the sensations of the passengers, the two most important variables are: the inclination to the horizontal θ and the magnitude of the apparent weight J_z.

The quantity J_z measured by the accelerometer, is the function

$$J_z = \frac{d\delta w}{dt} - qu + g \cos \theta$$

Under normal conditions J_z has the value g and may equally well we called the acceleration. It can be calculated as a function of the values a, b, ρ, and φ given in the tables, observing that

$$\frac{d\delta w}{dt} = \rho_2 e^{at} \left[a \sin (bt + \varphi_2) + b \cos (bt + \varphi_2)\right] + \rho'_2 e^{a't} \left[a' \sin (b't + \varphi'_2) + b' \cos (b't + \varphi'_2)\right] \quad (X)$$

It may also be determined from the i and V curves. We have, in fact,

$$m \left(\frac{d\delta w}{dt} - qu\right) = F_z - mg \cos \theta$$

whence

$$\left(\frac{d\delta w}{dt} - qu + g \cos \theta\right) = \frac{1}{m} F_z$$

and from the angle of attack i and the velocity, we may obtain F_z as

$$F_z = S\, C_z \frac{aV^2}{2g}$$

whence

$$J_z = g \frac{S}{P} \frac{C_z\, a\, V^2}{2g}$$

The second procedure for determining J_z is the more rapid if we are given the curves of i and V.

The most important accelerations are those associated with the short-period oscillation. The latter may be found either by examination of the expression (X) or of the curves of velocity and angle of attack. In the expression (X) at the start of the phenomenon when t is very small, the first term is the preponderant one since the quantities ρ, a, and b entering as factors, are larger when they are relative to the rapid oscillation than when they characterize the slow oscillation. The diagrams also show us that the large variation of i leading to large values of C_z and J_z are due to the rapid oscillations. In the long-period oscillation the angles of attack and velocities V vary simultaneously. Since the velocity V increases when the angle of attack i decreases, the variations in the acceleration are not very large. It is important to note that the acceleration determined by the preceding method, in the case of a vertical gust, corresponds to the roughest sort of calculation. It is assumed that the gust is suddenly set up, that the airplane from the first instant lies entirely within this gust, and that any phenomenon of elasticity of the wings will not diminish or complicate the action. Numerous works based on more probable assumptions have been published. In a remarkable paper published many years ago, E. B. Wilson assumed as the law for the set-up of the gust, the following expression:

$$w = J\,(1 - e^{-rt})$$

Other authors have assumed a sunusoidal law which is generally arbitrary. The latter assumption leads immediately to resonance phenomena if the period of the airplane is the same as that of the gust. Fisher, Bryan, and Jones have studied the effect of sudden and gradual gusts by methods which have become more and more accurate. Kussner, in a very well-known work of his, takes into effect the elasticity of the wings. Since our object is to compare the effect of different disturbances, we have not gone into the subject in more detail.

EFFECT OF THE ACTION OF THE CONTROLS

The method outlined above enables us in certain particular cases to determine how an airplane responds to a manipulation of the controls. Let us examine figure 13, which gives C_M as a function of i and β. Assume the airplane to be flying level at the angle of attack i_1 with the control deflected at angle β_1, this condition corresponding to a velocity V_1. If the pilot gives the control a deflection β_2, the point A passes to B. At B the moment M is no longer zero and the point will be displaced from B to C along the curve β_2. The airplane will be in rotational equilibrium about the center of gravity only at the angle of attack corresponding to point C. Let i_2 be this angle of attack, to which will correspond a speed V_2 different from V_1. We shall assume that the maneuver is made without altering the throttle and that the velocity V_1 is greater than the velocity corresponding to minimum power. Under these conditions the useful power sufficient for making the airplane fly at the velocity V_1 will not be sufficient to make the airplane fly at velocity V_2. The path of the airplane will not be able to remain horizontal but will incline by the angle ζ. The axis of the airplane will be lowered by $\zeta + (i_1 - i_2)$. At the instant when the pilot in deflecting the control to β_2, passes from point A to point B he may be considered to have departed from his final state C by amounts: $\delta i = i_1 - i_2$, $\delta V = V_1 - V_2$, $\delta \theta = (i_1 - i_2) + \zeta$, where ζ is computed from the polar of the airplane and the variation of the thrust T with velocity.

We have applied this computation to a numerical example, choosing the same airplane as above. At about the velocity of 40 m/s (90 m.p.h.), it was found that states differing from each other by a difference in velocity $\delta V = \pm 1$ m/s are characterized by a difference in angle of attack of $\mp 0.308^\circ$.

Similarly, basing our computations on an assumed polar, we found that a variation in velocity of ± 1 m/s requires that the slope of the flight path vary by ∓ 0.0070, corresponding to $d\zeta = -0.400^\circ$. These data were applied by us to the numerical example by multiplying them by the factor necessary to obtain amplitudes that may easily be studied.

Consider, for example, the passage from the level flight condition at velocity $V = 27$ m/s to a state where $V = 40$ m/s by means of a deflection $\Delta\beta$ required to produce a $\Delta i = -4^\circ$. If this deflection $\Delta\beta$ is suddenly applied, the airplane is deviated from its final condition by:

$$\delta V = -13; \quad \delta i = +4^\circ; \quad \delta\zeta = -5.2^\circ$$

Now $\delta\theta = \delta\zeta - \delta i$, so that $\delta\theta = -9.2^\circ$, one portion of which represents the change in angle of attack, and the other the change in the flight path. It suffices to reduce the amplitudes of the disturbances of figures 10, 11, and 12 in the desired ratio and add them. Figure 14 (continuous line) shows the result of this computation for the airplane of coefficient $\mu = 0.004$. The diagram of the angles of attack shows that the final angle of attack is not established rapidly. We may trace separately the effect of each of the elements of the initial disturbance on the variable δi (fig. 15). The effect of the initial disturbance in the angle of attack decreases very rapidly. The initial disturbances of velocity and attitude, however, also have an effect on the variable i - an insufficient velocity tending to increase the angle of attack, and similarly for a not sufficiently descending path. These two initial disturbances act on the long-period oscillation. It may be seen, in the example chosen, that in spite of the rapidity with which the short-period oscillation tends to bring the airplane to its final angle of attack, the airplane may in certain cases remain considerably deviated from its final position by the effect of the components of δi that depend on the long-period oscillation.

Remarks

1. If we assume that the deflection $\Delta\beta$ has not been instantaneously applied, but in a progressive manner (for example, one-third at time zero, the second third at time 0.5 second, and the remainder at time $t = 1$ second), the curve of variation of angle of attack loses the abrupt character which it has during the first second and becomes regular with its appearance not appreciably modified thereafter. This condition corresponds to the dash-dot curve of figure 14.

2. The airplane will more quickly attain its final angle of attack if the pilot, to start the maneuver, gives the control a greater deflection than the amount $\Delta\beta$, which he must maintain at the end of the phenomenon.

3. The computation is made on the assumption that the pilot does not vary the engine throttle. If the pilot increases the power the final path will no longer be descending. The effect of such a maneuver would have to be separately studied.

MORE COMPLICATED MANIPULATIONS OF THE CONTROLS

The same procedure may, of course, be applied to analyze the effect of more complicated maneuvers. Let an initial state be characterized by a given velocity attitude, angle of attack, and deflection β. For example, in the case of the airplane considered above, $V = 27$ m/s; $i = +4^\circ$; $\theta = -4^\circ$.

The pilot deflects his control by $\Delta\beta$, and the airplane starts the motions which tend to bring it to a final state B. He applies this deflection $\beta + \Delta\beta$, however, only within a limited time interval, and re-establishes the deflection β after n seconds. At this instant the airplane has not yet attained its state B but is in an intermediate state. We see on the figure that the momentary characteristics after 7 seconds are: $V = 34.7$ m/s; $i = 1^\circ$; $\theta = +13.4^\circ$; $q = +1^\circ$ per second.

The final state of equilibrium, however, defined by the position of the control, has become state A, and the airplane deviates from it by $\delta V = + 7.7$; $\delta i = - 3^\circ$; $\delta\theta = 17.4^\circ$; $\delta q = + 1^\circ$ per second. Taking these values as an initial disturbance with respect to state A, the changes in the variables may be easily determined. The results of the computation are indicated as discontinuous lines on figure 14. The computations have been made neglecting the effect of the disturbance δq, which is less important than the three preceding ones.

Remarks

1. The curve of the angle of attack drawn as a thin dot-dash line corresponds to the assumption of a more gradual maneuvering of the elevator.

2. We have calculated the curve of J_z, the apparent weight or acceleration, for the maneuver described - consisting of nosing down the airplane for 7 seconds, then nosing up. The result of the computation is given on figure 16.

Tests

It was possible for us to obtain records of the variables defining the longitudinal motion of an airplane. The test was carried out on a Fairey "Fox" with the Bouny equipment, which will be described in detail in a succeeding Bulletin. Records of "phugoid" oscillations are sufficiently numerous. Nevertheless, we believe it useful to publish here the results of the measurements we have made.

Although this test has been conducted on an airplane differing from the one employed in our calculations, the curves obtained show a qualitative correspondence with the calculated curves that appears to justify comparison. The polar and the curve of moments were known from a tunnel test conducted under the usual conditions on a model not provided with a propeller. The airplane itself had a loading of 58.5 kg/m². At the speed of 45.2 m/s, it flew at a lift coefficient of 0.458, corresponding to an angle of attack of 3.6°. The coefficient of mean static stability measured in the interval $i = + 6$ to $i = - 4$ may be determined from figure 17,

$$\frac{dC_M}{di} = 0.00417$$

The coefficient of effectiveness of the elevator for negative deflections was:

$$\frac{dC_M}{d\beta} = 0.01308$$

The conditions of equilibrium, according to the tunnel tests, show that the flight at $C_z = 0.458$ and $i = 3.6^\circ$, should be made at an angle of elevator deflection of $\beta = -2.55^\circ$. Actually, equilibrium was obtained at $\beta = -3.6^\circ$, which is a sufficiently good agreement.

The test consisted in carrying out the following maneuver: Having attained the steady state at velocity $V = 45.2$ m/s, which here is state A, the pilot pushes on the stick and lowers the elevator, the mean deflection becoming $\beta = -1.5^\circ$, the pilot not touching the throttle. The airplane dives and tends toward a new position of equilibrium. The pilot, however, does not wait until the final state B is attained. After having maintained the deflection $\beta = -1.5^\circ$ for 7 seconds, he pulls back on the stick, fixes it in its initial position and allows the airplane, after some oscillations, to return to its initial state A. The test is carried out while recording:

V is the velocity on the flight path.

V_v, vertical component of the velocity measured with a variometer.

β, setting of the elevator.

i, angle of attack.

J_z, component of the apparent weight in the direction of the OZ axis.

J_x, component of the apparent weight in the direction of the OX axis.

q, angular velocity about the OY axis.

We thus have 7 test curves.

The variometer is an apparatus whose readings show considerable lag, and can only be used when corrected. The curve of corrected vertical velocities is shown on one of the diagrams of figure 18. A knowledge of V_v and V gives the angle of slope of the path and the angle θ may be calculated from ζ and i. Finally, having determined θ, it is possible to calculate q and check the readings of the indicator of the angular velocity of pitch.

We shall now consider the curves distinguishing three portions:

1. The flight is assumed rectilinear and uniform at state A, the deflection being -3.6°, the part of the curves to the left of point 1.

2. A period of 7 seconds duration, during which the airplane undergoes manouvering which would lead to state B if the deflection $\beta = -1.5^{\circ}$ were maintained. This period corresponds to the portion 1-2 of the curves.

3. The period following the return to the setting $\beta = -3.6^{\circ}$, during which the airplane oscillates and tends to regain its initial state A - that is, the part of the curves to the right of point 2. The relation:

$$\frac{J_z}{g} \frac{P}{S} = C_z \frac{aV^2}{rg}$$

permits us to determine C_z and hence, i.

For the angle-of-attack curve the computation gave a curve which differed slightly from the one recorded, the latter lagging behind the computed curve and at the minimum angle of attack not coming down so low. It was possible for us to show that there was a systematic error in the reading of the vane due to a play of about 1.5°, a fact which explains in part the lag of the record. The Bouny accelerometer is an instrument with which we are very familiar and whose readings are very accurate. Under these conditions we think that the computed curve corresponds to that of actual angles of attack.

The curve of angular velocities has been obtained with the aid of an apparatus designed to measure the angular velocities which are produced in spinning and which are of the order of 1 to 2 radians per second. This apparatus did not have the desired sensitivity for measuring angular velocities which are of the order of 0.05 radian per second. We were able, however, to calculate the angular velocities q from the angles θ. The angular velocities thus calculated are, however, obtained after a rather larger number of operations, among which is the correction for the indications of the variometer. Under these conditions, the agreement of the two curves is not to be considered as very poor.

We shall now consider the curves in detail. After the elevator has been deflected from $\beta = -3.2^\circ$ to $\beta = -1.5^\circ$ the airplane tends toward a final state B. The latter, corresponding to a displacement $\Delta\beta = 1.7^\circ$, is characterized by a final angle of attack:

$$i_f = 3.6^\circ + \Delta i$$

where

$$\Delta i = -\frac{0.0138}{0.00477}\Delta\beta = -5^\circ$$

Therefore, $$i_f = 3.6^\circ - 5^\circ = 1.4^\circ$$

corresponding to a lift coefficient:

$$C_z = 0.458 - 0.0695\,\Delta i = 0.111$$

and a velocity $$V = 92 \text{ m/s}$$

The airplane used is thus extremely sensitive to the elevator controls. The deflection was applied during 7 seconds, within which time the airplane began a series of motions which would have led to state B if the deflection had been applied long enough. This period is characterized by the immediate appearance of a component of the velocity directed downward by the decrease in the angles of attack and acceleration J_z, and by the increase in the velocity along the flight path. We may note that the increase in the velocity is made at first rather slowly, thus confirming the computations given in figure 14.

Let us now examine the effect of the return to the initial elevator deflection. At the instant when the stick is thrown back to its original position (point 2), the curve of accelerations is suddenly modified, this being the curve which indicates most exactly the instant when the maneuver has been executed. The angle of attack is instantly changed, the velocity V continues to increase for several seconds, and the airplane continues with a vertical velocity of descent - the latter, however, attaining its maximum a little after the maneuver. The different variables vary in such a manner as to regain the values corresponding to the initial state. The diagram shows up the long-period oscillations with particular clearness, a large amplitude being obtained, due to the size of the initial disturbance imparted to the airplane.

We do not consider it necessary to comment further on the experimental curve obtained. The curve has the same appearance as that computed and drawn on figure 14 for a similar maneuver but corresponding to a different airplane, and thus proves the value of the computations for the theoretical determination of the flight paths.

It is part of our program to carry out complete computations, using the characteristics of the airplanes available for our tests.

III. LATERAL OR TRANSVERSE MOTION

Method and Equations

The equations of translation along the OY axis, and rotation about the OX and OZ axes are those that determine the lateral stability. These equations are:

$$\frac{P}{g}\left(\frac{dv}{dt} + ru - pw\right) = Y - P \sin \varphi$$

$$A \frac{dp}{dt} - E \frac{dr}{dt} + qr (C - B) - Epq = \Sigma L$$

$$C \frac{dr}{dt} - E \frac{dp}{dt} + pq (B - A) + Eqr = \Sigma N$$

where A, B, and C are the moments of inertia. If the axes of coordinates coincide with the principal axes of inertia the product of inertia E is zero, and this we shall assume in what follows.

To the above equations must be added two geometric relations connecting the angular velocities p and r with the derivatives $\frac{d\varphi}{dt}$ and $\frac{d\psi}{dt}$, and resulting from the definition of the rotations:

$$p = \frac{d\varphi}{dt} \cos \theta - \frac{d\psi}{dt} \sin \theta \cos \varphi$$

$$r = \frac{d\varphi}{dt} \sin \theta + \frac{d\psi}{dt} \cos \theta \cos \varphi$$

These relations may be written:

$$\frac{d\varphi}{dt} = p \cos \theta + r \sin \theta$$

$$\frac{d\psi}{dt} = \frac{1}{\cos \varphi} (r \cos \theta - p \sin \theta)$$

The system of equations is of the form:

$$\frac{dv}{dt} = f_1 \ (v, p, r, \varphi, \psi)$$

$$\frac{dp}{dt} = f_2 \ (v, p, r, \varphi, \psi)$$

$$\frac{dr}{dt} = f_3 \ (v, p, r, \varphi, \psi)$$

$$\frac{d\varphi}{dt} = f_4 \ (v, p, r, \varphi, \psi)$$

$$\frac{d\psi}{dt} = f_5 \ (v, p, r, \varphi, \psi)$$

and may be linearized as in the study of the longitudinal stability, the disturbances δv, δp, δr, $\delta\varphi$, $\delta\psi$ becoming the variables.

The integrated system, after the above equations have been reduced to a linear system, depends on an algebraic equation of the fifth degree in λ in place of one of the fourth degree. This equation will be:

$$\begin{vmatrix} \frac{\partial f_1}{\partial v} - \lambda & \frac{\partial f_1}{\partial p} & \frac{\partial f_1}{\partial r} & \frac{\partial f_1}{\partial \varphi} & \frac{\partial f_1}{\partial \psi} \\ \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial p} - \lambda & \frac{\partial f_2}{\partial r} & \frac{\partial f_2}{\partial \varphi} & \frac{\partial f_2}{\partial \psi} \\ \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial p} & \frac{\partial f_3}{\partial r} - \lambda & \frac{\partial f_3}{\partial \varphi} & \frac{\partial f_3}{\partial \psi} \\ \frac{\partial f_4}{\partial v} & \frac{\partial f_4}{\partial p} & \frac{\partial f_4}{\partial r} & \frac{\partial f_4}{\partial \varphi} - \lambda & \frac{\partial f_4}{\partial \psi} \\ \frac{\partial f_5}{\partial v} & \frac{\partial f_5}{\partial p} & \frac{\partial f_5}{\partial r} & \frac{\partial f_5}{\partial \varphi} & \frac{\partial f_5}{\partial \psi} - \lambda \end{vmatrix} = 0$$

It will be found, however, that the derivatives of the functions with respect to ψ are zero. The fifth degree equation admits of a zero root $\lambda = 0$. This facilitates the analytic study since the characteristic equation becomes one of the fourth degree when this particular solution is eliminated, and the mathematical study is made by methods similar to those employed in the study of the longitudinal stability.

The existence of this particular root corresponds to a certain physical fact. In the study of the longitudinal motion it is found that certain projections of the external forces depend on the angle θ. When θ is not zero, the axis of the airplane is inclined upward or downward with respect to the horizon, and the weight has along the OX axis a component which adds to or subtracts from the propeller thrust. When an airplane is dynamically stable, it returns to its initial state after a series of oscillations. The forces acting on the airplane should therefore regain their initial values, and this result can be obtained only if the airplane regains its initial longitudinal attitude.

In the study of the lateral motion we meet with the angular magnitudes φ and ψ. The component of the forces along the OY axis depends on the angle φ, since the weight has a lateral component when the airplane is inclined. If the airplane is dynamically stable it returns, after any disturbance, to its initial state and should therefore regain its initial inclination φ. The same does not hold for the angle ψ. Whatever the final position of the airplane the projections of the weight on the axes are independent of the rotation ψ. There exists no force or moment which is a function of the azimuth. Hence, when an airplane, dynamically stable, returns after a disturbance to its initial state, it should regain a motion characterized by the same velocities and by the same angles φ and θ as the initial motion but not necessarily by the same angle ψ. The airplane, after a disturbance, does not possess any weathercock stability - all the derivatives with respect to ψ of the moments and forces being zero. The existence of a solution $\lambda = 0$ is the mathematical consequence of this fact.

The integral system will be analogous to that obtained for the longitudinal stability and will be written:

$$\delta v = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} + C_4 e^{\lambda_4 t}$$

$$\delta p = l_1 C_1 e^{\lambda_1 t} + l_2 C_2 e^{\lambda_2 t} + l_3 C_3 e^{\lambda_3 t} + l_4 C_4 e^{\lambda_4 t}$$

$$\delta r = m_1 C_1 e^{\lambda_1 t} + m_2 C_2 e^{\lambda_2 t} + m_3 C_3 e^{\lambda_3 t} + m_4 C_4 e^{\lambda_4 t}$$

$$\delta \varphi = n_1 C_1 e^{\lambda_1 t} + n_2 C_2 e^{\lambda_2 t} + n_3 C_3 e^{\lambda_3 t} + n_4 C_4 e^{\lambda_4 t}.$$

The disturbance $\delta\psi$ can be obtained only by integration:

$$\delta\psi = \int \frac{d\delta\psi}{dt}\, dt$$

It is known that the characteristic equation in λ always admits in the case of lateral stability a pair of imaginary roots $\lambda_{1,2}$ and two real roots λ_3 and λ_4. The motion will result from the superposition of an oscillation and two aperiodic motions.

STATIC STABILITY IN LATERAL EQUILIBRIUM

The static stability is a phenomenon by which a dynamic restoring moment arises when the airplane has been subject to any displacement with respect to its flight path. The displacement as regards the lateral motion shows up by an angle of yaw j. The yaw takes place whenever the aerodynamic velocity V departs from the plane of symmetry of the airplane and gives rise to two moments L and N. We shall take:

$$L = C_L \, Sb \, \frac{aV^2}{2g}$$

$$N = C_N \, Sb \, \frac{aV^2}{2g}$$

where the reference length b is by convention the span of the airplane.

The derivatives $\frac{\partial C_L}{\partial j}$ and $\frac{\partial C_N}{\partial j}$ determine the static stability of the airplane. They are the coefficients of static stability of roll and of yaw. Since $j = v/V$, we have also:

$$\frac{\partial C_L}{\partial v} = \frac{1}{V} \frac{\partial C_L}{\partial j}$$

$$\frac{\partial C_N}{\partial v} = \frac{1}{V} \frac{\partial C_N}{\partial j}$$

It is readily seen that $\frac{\partial C_L}{\partial j}$ is positive when the effect of the lateral surfaces of the airplane situated above the OX axis is greater than that of the lateral surfaces below the axis and that $\frac{\partial C_N}{\partial j}$ is positive when the effect of the vertical surfaces situated at the rear is predominant. The form of the wings enters as an important factor in the lateral static stability. The dihedral increases $\frac{\partial C_L}{\partial j}$ while the plan form affects $\frac{\partial C_N}{\partial j}$. These coefficients are capable of numerical evaluation. Tests show that the moments L and N depend also on the angle of attack, so that it must always be specified, when a numerical value is given, to which angle of attack it corresponds.

AIRPLANES STUDIED

Two sets of airplanes were studied - differing in their coefficients of static stability of roll and yaw - the same airplanes being used as in the study of longitudinal stability. It is necessary to give the span b and it will be assumed as equal to 10 meters, while the two radii of gyration will be:

$$r_C = 2.5 \text{ m}$$

$$r_A = \quad \text{m}$$

The derivatives of the forces and aerodynamic moments are given by:

$$Y'_v = a_1 S \frac{aV}{2g} \qquad L'_v = a_4 Sb \frac{aV}{2g} \qquad N'_v = a_7 Sb \frac{aV}{2g}$$

$$Y'_p = a_2 Sb \frac{aV}{2g} \qquad L'_p = a_5 Sb_2 \frac{aV}{2g} \qquad N'_p = a_8 Sb_2 \frac{aV}{2g}$$

$$Y'_r = a_3 Sb \frac{aV}{2g} \qquad L'_r = a_6 Sb_2 \frac{aV}{2g} \qquad N'_r = a_9 Sb_2 \frac{aV}{2g}$$

In our computations we shall assume the constant values:

$$a_1 = -0.60 \qquad a_5 = -0.2343$$

$$a_2 = +0.013 \qquad a_6 = -0.0563$$

$$a_3 = +0.065 \qquad a_8 = +0.0168$$

$$a_9 = -0.044$$

It may readily be seen that:

$$a_4 = \frac{\partial C_L}{\partial j} \quad \text{and} \quad a_7 = \frac{\partial C_N}{\partial j}$$

where the angles are expressed in radians.

The stability of roll will be characterized by the following values of a_4:

$$a_4 = 0, \quad 0.02, \quad 0.04, \quad 0.06, \quad \text{and} \quad 0.08$$

In investigating the effect of these variations in stability, we shall combine it with the constant value $a_7 = 0.04$.

The directional stability will be characterized by the following values of a_7:

$$a_7 = -0.02 \quad -0.01 \quad 0 \quad +0.02 \quad +0.04 \quad +0.065$$

In varying a_7, we shall suppose a_4 constant and equal to 0.04.

Remark

We assume, in fact, that the coefficients of stability may vary without having the quantities a_5, a_6, a_8, and a_9, which define the aerodynamic damping, vary. It is important to note this assumption.

SOLUTION OF THE CHARACTERISTIC EQUATION

The movements executed by the ten airplanes studied are not all stable. The criterion of stability of Routh requires, when the characteristic equation is put in the form (VII), that each of the quantities

$$A_1, A_2, A_3, A_4, \quad \text{and} \quad R = A_2 - \frac{A_3}{A_1} - \frac{A_1 A_4}{A_3}$$

be greater than zero.

We shall take the coefficients of static stability as variables

$$a_4 = x \qquad a_7 = y$$

The expressions A_1, A_2, A_3, A_4, and R become functions of x and y. Let us consider the region of the plane determined by the extreme values of x and y from −0.02 to +0.10 (fig. 19).

The quantities A_1 and A_2 are always > 0 for the values of the variables x and y considered. The lines $A_1 = 0$ and $A_2 = 0$ do not cross this region of the plane. The other three relations:

$$A_3 = 0 \qquad A_4 = 0 \qquad R = 0$$

on the contrary, define lines which cross the region of the plane considered. We shall determine a region where stability exists.

Plotting the points characterizing the ten airplanes studied, it may be seen that six are dynamically unstable, and only four are stable. The roots of the equations and the characteristics of the motions are given in the following tables:

VARIABLE STABILITY OF ROLL

Value of a_4	Imaginary roots $\lambda_{1,2}$		Real roots λ_3	λ_4
0.08	-0.3252	±1.442i	-0.02177	-3.97
.06	-.3595	±1.408i	-.007	-3.93
.04	-.385	±1.366i	+.01045	-3.89
.02	-.4126	±1.310i	+.3035	-3.85
.00	-.441	±1.272i	+.492	-3.815

The period T of the oscillation, and the time $T\frac{1}{2}$ required for reducing each elementary motion by half its value, or doubling it, according to whether the movements are stable or unstable, are given in the table below:

Value of a_4	Point	Oscillation T	Oscillation $T\frac{1}{2}$	Aperiodic motion $T\frac{1}{2}$		Aperiodic motion $T\frac{1}{2}$
0.08	A	4.31	2.13	31.8		0.175
.06	B	4.45	1.93	99		.1765
.04	C	4.58	1.8	66.3	unstable	.178
.02	D	4.78	1.68	2.28	unstable	.181
.00	E	4.92	1.57	1.41	unstable	.181

When the directional stability is variable the roots are as follows:

VARIABLE DIRECTIONAL STABILITY

Value of a_7	Imaginary roots		Real roots	
	$\lambda_{1,2}$		λ_3	λ_4
+0.065	-0.3903	±1.688i	+0.0259	-3.895
+.040	-.385	±1.366i	+.01045	-3.89
+.020	-.371	±1.022i	-.0209	-3.89
0	-.264	±.556i	-.2317	-3.89
-.01	+.04265	±.326i	-.846	-3.89
-.02	+.199	±.253i	-1.58	-3.89

and the movements are characterized by

Value of a_7	Point	Oscillation		Aperiodic motion	
			$T \frac{1}{2}$	$T \frac{1}{2}$	$T \frac{1}{2}$
+0.065	1	3.73	1.78	26.8 unstable	0.178
+.04	2	4.6	1.80	66.3	.178
+.02	3	6.15	1.87	32.2	.178
0	4	11.3	2.62	2.91	.178
-.01	5	19.25	16.25 unstable	.795	.178
-.02	6	24	3.48	.437	.178

In what follows we shall denote as the oscillatory motion that which is determined by the imaginary roots; spiral motion as the aperiodic motion which is defined by the root λ_3; and damped motion as the aperiodic motion defined by the root λ_4. We find that the motion corresponding to the fourth root is always stable and strongly damped, and that its characteristics are independent of the coefficients of static stability of the airplane.

When the airplanes A, B, C, D, E are successively examined, it is seen that the oscillatory motion is stable in all cases with practically the same period and effectively damped. The aperiodic motion determined by λ_3 is stable at the beginning but not to a great extent, since the disturbances do not decrease by half after a sufficiently long time. This motion becomes unstable as soon as the limit $A_4 = 0$ is crossed, the disturbances doubling in times which become smaller and smaller. This instability is the well-known spiral instability.

Let us consider the airplanes 1, 2, 6 - differing in their static directional stability. The aperiodic motion λ_3 is unstable at the beginning, the characteristic points lying outside the region of stability. The airplane is subject to spiral instability but the latter disappears as soon as the boundary $A_4 = 0$ is crossed and the motion λ_3 becomes very stable. As the directional stability decreases the period increases, the damping decreases, and the oscillation becomes unstable at the instant the line $R = 0$ is crossed.

CHARACTERISTICS OF THE MOTION

Dynamic considerations thus indicate that the stability coefficients $\frac{\partial C_L}{\partial j}$, $\frac{\partial C_N}{\partial j}$ cannot be given arbitrary values. A simple qualitative explanation of these facts may readily be given. If the two conditions $\frac{\partial C_L}{\partial j} > 0$ $\frac{\partial C_N}{\partial j} > 0$ are merely satisfied without any further investigation of the movements which will be set up, it is possible to have the airplane dynamically unstable.

Let us examine the case of an airplane banked to the left $\delta\varphi > 0$. It will then be under the action of the weight on that side and a disturbance $\delta j > 0$ will take place (fig. 20). The latter disturbance will have two effects:

1. The airplane will have a tendency to right itself under the action of $\frac{\partial C_L}{\partial j}$, the yaw toward the left tending to incline the airplane toward the right.

2. The airplane will have a tendency to yaw to the left since it behaves like a weather vane, $\frac{\partial C_N}{\partial j}$ being positive.

There thus arises a new disturbance, a yaw $\delta r > 0$. In this motion the right wing will be outside of the turn and will be displaced more rapidly than the left wing. Its lift will be greater and it will tend to raise itself further, thus increasing the lateral bank. The moment L is a function of r, the derivative $\frac{\partial C_L}{\partial r}$ is negative, and the yaw δr has a tendency to right the outside wing, which here is the right wing. Two opposite effects are produced and, depending on the proportions of the airplane, one or the other will predominate. If the effect of the static stability of roll is greater, the airplane after yawing will regain its initial state - flying, however, in a direction different from that preceding the initial disturbance $\delta\varphi$. If the effect of the angular velocity r on the rolling moment is greater than the effect of the sideslip j, the second effect will predominate. The bank of the airplane will increase, the airplane will describe a spiral path, and will then be dynamically unstable. In spite of its static stability of direction and roll, it possesses spiral instability since the first stability is too great with respect to the second.

Similarly, we can explain how the oscillation assumes more and more unfavorable characteristics when the stability of roll is too great compared with the yaw stability. Let us imagine that the airplane yaws to the left, the OX axis of the airplane being oriented toward the right with respect to the flight path. If $\partial C_L/\partial j$ is high, the airplane will incline energetically to the right and the roll will be positive. The tendency to yaw to the left will, on the contrary, be small since $\partial C_N/\partial j$ is small. This yaw will not develop, as a secondary effect, any important negative rolling moment. Since nothing opposes the motion of positive roll toward the right, the airplane will bank to this side but it should then begin to yaw in this direction and the same phenomena will be reproduced in the opposite sense. The airplane will have a yawing motion on which is superposed a continuous lateral swinging. For too small values of $\partial C_N/\partial j$, the successive amplitudes increase and the oscillation is unstable.

We shall here discontinue our qualitative explanations and return to the results of our computations. The first point is to determine the relative importance of the elementary motions by calculating their amplitudes for the given initial disturbances.

GRAPHS OF THE MOTIONS

We shall consider three initial disturbances, namely:

1. A disturbance δv of 10 meters per second, corresponding to an angle of yaw of 0.25 radian.

2. A disturbance $\delta\varphi$ of 0.2 radian.

3. A disturbance δr of 0.2 radian per second.

It did not appear necessary to calculate the effect of a disturbance δp. Such a disturbance would cause the airplane to bank and would produce in a short time a disturbance $\delta\varphi$. We prefer to take $\delta\varphi$ directly as the initial motion to be considered rather than study the sequence of successive values of $\delta\varphi$ corresponding to an initial δp. The amplitudes of the components of the motion are given, for each initial disturbance, by the tables which follow. The resulting motions are shown plotted on figures 21 to 23. Several component motions are superposed on these plots. Examination of the curves leads to the following conclusions. In cases where there is stability, for airplanes A, B, 3, and 4, the curves do not present any essential differences. Spiral instability exists for the airplanes 1, 2C, E, and D. It is not in evidence for airplane 2C, since it is very small and the curves are not prolonged a sufficient length of time for the instability to show up. The spiral instability is clearly evident on airplanes E and D. Similarly, the oscillatory instability of airplanes 5 and 6, is clearly indicated. From a direct study of these figures, however, the phenomena cannot be analyzed. To see more clearly, we shall investigate separately:

A. How the initial disturbances considered are distributed among the different partial component motions.

B. In each partial motion which are the variables most affected.

We shall thus determine the relations existing, in each component motion, between the values of the different variables. We know that, for each airplane, these relations are constant and independent of the initial disturbance.

A. DISTRIBUTION OF THE INITIAL DISTURBANCES

Let us examine how the initial disturbances considered are distributed among the partial motions. This distribution will depend on the initial disturbance considered. With the aid of numerical tables, we may easily show how a given disturbance $\delta\varphi$ is distributed among the amplitudes ρ_4, $n_3 C_3$, and $n_4 C_4$. Similarly, we shall investigate how an initial disturbance δr is distributed among the amplitudes ρ_3, $m_3 C_3$, and $n_4 C_4$, and how an initial disturbance δv is distributed among ρ_1, C_3, and C_4. These distributions will occur in proportions which depend on the aerodynamic characteristics of the airplane considered.

We find that in each case the amplitude of the motion corresponding to λ_4 is very small. It is represented by the hatched region on the figures. This explains the fact that the damped motion does not play any important part in the phenomena. The amplitude of the spiral motion corresponding to λ_3 is very large when the initial disturbance is an inclination $\delta\varphi$. When the initial disturbance, on the contrary, is a sideslip v, the oscillatory motion is the greater. In the case of an initial disturbance δr there is some spiral motion, but the oscillatory motion predominates.

B. VARIABLES AFFECTED IN EACH MOTION

For a given airplane, we can characterize each motion by the ratios of the values of the different variables at the same instant. These ratios are independent of the initial conditions and depend only on the stability parameters of the airplane.

The amplitudes of the disturbances of the variables v, p, r, φ associated with the damped motion, are proportional to the quantities C_4 $l_4 C_4$ $m_4 C_4$ and $n_4 C_4$. Those associated with the spiral motion are proportional to C_3 $l_3 C_3$ $m_3 C_3$ $n_3 C_3$. Let us choose as the term of comparison the value of the disturbance of lateral inclination nC (of desired subscript). The disturbance of lateral velocity C gives an angle of yaw when divided by the velocity V. The quantities:

$$\frac{C}{V}\frac{1}{nC} = \frac{1}{Vn}$$

give the constant ratio which connects, in the elementary motion considered, the angle of yaw with the angle of lateral inclination. The quantities l/n give the constant ratio between the angular velocity of roll and the angle of roll, m/l the angular velocity of yaw to the angular velocity of roll, m/n the angular velocity of yaw to the angle of roll. From the numerical tables we may derive the following results.

1. Damped Motion

The ratios $\frac{1}{Vn_4}$, $\frac{l_4}{n_4}$, $\frac{m_4}{n_4}$ do not vary much when the different airplanes studied are examined. It is not necessary to follow the changes of these ratios with the coefficients of static stability, and it will be sufficient to consider mean values for characterizing the phenomena. We find:

$$\frac{1}{Vn_4} = +\ 0.147$$

In the damped motion a positive bank (to the right) is associated with a positive yaw (to the left), the angle of yaw being always 0.147 times the angle of bank. The relation

$$\frac{l_4}{n_4} = \lambda = -\ 3.8$$

shows that a positive bank corresponds to a negative velocity of roll tending to make this disturbance disappear very rapidly (in 1/3.8 seconds if the angular velocity were constant instead of obeying an exponential law). We have, moreover:

$$\frac{m_4}{n_4} = 0.147 \quad \text{and} \quad \frac{m_4}{l_4} = -\ 0.0384$$

These ratios show that a negative rolling motion is associated with a positive but very small yawing motion.

2. Spiral Motion

For the spiral motion the ratios vary from one airplane to another. Let us draw up the tables below:

Airplane	A	B	C	D	E
$\frac{1}{Vn_3}$	-0.0536	-0.0576	-0.0663	-1.985	-1.41
$\frac{l_3}{n_3}$	-0.0217	-0.007	+0.0101	+0.3035	+0.492
$\frac{m_3}{n_3}$	-0.216	-0.216	-0.2315	-4.17	-2.3
$\frac{m_3}{l_3}$	+10	+30.9	-22.9	-13.75	-4.6

Airplane	1	2	3	4	5	6
$\frac{1}{Vn_3}$	-0.0416	-0.663	-0.0115	-0.42	-1.195	-1.66
$\frac{l_3}{n_3}$	+0.0259	+0.0101	-0.0209	-0.23	-0.846	-1.58
$\frac{m_3}{n_3}$	-0.222	-0.2315	-0.229	-0.207	-0.81	-0.91
$\frac{m_3}{l_3}$	-8.6	-22.9	+10.95	+0.893	+0.96	+0.57

We see that the ratio of the angle of yaw to the angle of bank is always negative. When the airplane is inclined to the right (φ positive), it should yaw to the right (j negative). Here again, we have the action of the lateral component of the weight.

When $\frac{l_3}{n_3}$ is positive a lateral inclination is associated with an angular velocity of roll of the same sign tending to increase the inclination, the motion then being unstable. The ratio m_3/n_3 is always negative. The ratio m_3/l_3 reaches rather high absolute values, the angular velocities of yaw becoming in certain cases considerably larger than the angular velocities of roll.

B. OSCILLATORY MOTION

Let ρ_1, ρ_2, ρ_3, ρ_4 be the maximum amplitudes of the disturbances of the variables v, p, r, φ associated with the lateral oscillation. The ratios of these amplitudes are given in the tables below:

Airplane	A	B	C	D	E
$\frac{\rho_1}{V\rho_4}$	1.06	1.345	1.825	2.94	4.28
$\frac{\rho_2}{\rho_4}$	1.475	1.45	1.42	1.375	1.347
$\frac{\rho_3}{\rho_4}$	1.325	1.67	2.545	3.67	5.3
$\frac{\rho_3}{\rho_2}$	0.9	1.15	1.79	2.67	3.93

Airplane	1	2	3	4	5	6
$\frac{\rho_1}{V\rho_4}$	2.23	1.825	1.45	0.886	0.441	0.307
$\frac{\rho_2}{\rho_4}$	1.715	1.42	1.097	0.622	0.329	0.322
$\frac{\rho_3}{\rho_4}$	3.56	2.545	1.29	0.1785	0.203	0.182
$\frac{\rho_3}{\rho_2}$	2.075	1.79	1.185	0.287	0.618	0.565

The foregoing tables should be completed by the phase differences which determine the sign relations between the disturbances.

Airplane	A	B	C	D	E
$\varphi_1 - \varphi_4$	112°	109°	103°	86°	38°
$\varphi_2 - \varphi_4$	103°	104°	106°	108°	109°
$\varphi_3 - \varphi_4$	16°	22°	15°	-2.5°	-50°
$\varphi_3 - \varphi_2$	-77°	-82°	-91°	-110°	200°

Airplane	1	2	3	4	5	6
$\varphi_1 - \varphi_4$	97°	103°	108°	116°	41°	56°
$\varphi_2 - \varphi_4$	100°	106°	110°	116°	82°	56°
$\varphi_3 - \varphi_4$	9°	15°	21°	41°	240°	316°
$\varphi_3 - \varphi_2$	-81°	-91°	-79°	-75°	+158°	+266°

The value $\varphi_1 - \varphi_4 = x$ indicates that the variable 1 passes through its maximum of positive amplitudes $\frac{x}{2\pi} T$ seconds before the oscillation of variable 4. From an examination of the tables, the following conclusions may be derived. The disturbance in yaw δj is, in general, greater than the disturbance in roll $\delta\varphi$ when the oscillatory motion is stable. When it becomes unstable the ratio $\delta j/\delta\varphi$ decreases. When $\varphi_1 - \varphi_4$ is in the neighborhood of 90°, the oscillation of the variable 1, that is, of the yaw, passes through its maximum positive amplitude about a quarter of a period ahead of the variable 4, the lateral inclination. The yaw to the left (positive) is followed by a bank to the right (positive).

The disturbance δp of the angular velocity of roll is not exactly in quadrature with φ. The phase difference $\varphi_2 - \varphi_4$ varies between 100 and 110° and is not exactly equal to 90°. This may be expected since p is not exactly the derivative of φ but

$$p = \frac{d\varphi}{dt} \cos \theta - \frac{d\psi}{dt} \cos \varphi \cos \theta$$

The ratios ρ_3/ρ_2 result in smaller values than those for the spiral motion. With equal roll the yawing is less in the oscillatory motion than it is in the spiral motion.

Let us now consider the characteristics of the yawing motion. When the airplane is stable, $\varphi_3 - \varphi_4$ is included between 15° and 40°. The yawing motion passes through its positive maximum 1/24 to 1/9 of a period ahead of the positive maximum of φ. The yawing is to the left at the instant when the left wing is rising, the secondary effect of the rotation r thus tending to oppose the roll. The same conclusion may be drawn from $\varphi_3 - \varphi_2$. When the oscillation becomes unstable, for airplanes 5 and 6, the phase difference $\varphi_3 - \varphi_4$ increases; we find 240° and 316° while we have $\varphi_3 - \varphi_2 = 158°$ and 266°. This indicates a reversal in the sense of the phenomena since it is easy to see that for $\varphi_3 - \varphi_2 = 180°$ the two rotations r and p are in opposition. At this instant a positive yaw r (to the left) is associated with a negative roll p (to the right) and the secondary effect of the yaw will be to increase the roll.

The above numerical analysis throws some light on the partial motions, of which the resultant motion is composed. The characteristics of each of the three motions studied above depend on the airplane but are independent of the initial disturbance considered.

CONCLUSIONS

1. The analysis of the lateral motion is more complicated than that of the longitudinal motion. It is thought that the results obtained from the complete numerical computation of examples may be useful in estimating the effect of the different factors. The variable factors here con-

sidered were the coefficients of static stability. The motions computed were determined for a zero angle of attack. It is of course understood that it is not enough that an airplane shall be stable at a low angle of attack. The lateral stability should be investigated, particularly at large angles of attack, and this is a problem which at the present moment is the most difficult of solution. A logical continuation of the above study would be the computation of the motions for flight at large angles of attack.

2. The theoretical determination of the variables v, p, r, φ, and ψ has already been considered in several works. Of these, we may note:

a) The computations carried out by Halliday on a certain airplane (namely, the Bristol "Fighter") for different angles of attack (R. & M. No. 1306). At low angles of attack, the curves found for this airplane have the same general appearance as those computed by us for the stable airplanes.

b) An example computed by Mellvill Jones, in volume V, of W. F. Durand's "Aerodynamic Theory."

REMARKS CONCERNING AUTOMATIC STABILITY

The present investigation may be considered as preliminary to the study of automatic stabilizers. We have sought to determine first how an airplane of average characteristics reacts against the principal disturbances it may encounter. It is only after such a study has been undertaken that one may inquire as to the effect produced by a stabilizer on the natural reactions. We have already published several ideas on the subject in articles devoted for the most part to descriptions of the apparatus proposed.

Without entering here into the general study of automatic stabilizers, we may point out that the present work suggests to us immediately the idea of a stabilizer whose sensitive member would be a wind vane or pressure plate. The elements here considered as variable were the coefficients of static stability - that is, the derivatives of the coefficients of the moments with respect to the angles of attack and of yaw; these angles may be determined by the vanes. If the vanes are utilized so as to operate the controls, an automatic stabilizer is created whose action on an airplane will be analogous to an increase in the static stability.

Let us consider, for example, the longitudinal stability. The moment M is a function of i and of β. The derivative characterizing the moment about the center of gravity is:

$$\frac{dM}{di} = \frac{\partial M}{\partial i} + \frac{\partial M}{\partial \beta} \frac{d\beta}{di}$$

where $d\beta/di$ is determined by the mechanical apparatus connecting the deflection β of the elevator with position of the vane. Our computations are, in fact, referred to the total derivative dM/di. It seems, from the theoretical point of view, all other conditions remaining the same - particularly, the total area of the tail surfaces - that the production of a moment M by means of natural static stability or by means of an automatic stabilizer would be equivalent. The comparison of the curves obtained for different values of μ shows the effect that may be expected from vane stabilizers. The same remark applies to the lateral stability, where a vane could be imagined as acting on the ailerons or the rudder:

$$\frac{dC_L}{dj} = \frac{\partial C_L}{\partial j} + \frac{\partial C_L}{\partial \alpha} \frac{d\alpha}{dj}$$

$$\frac{dC_N}{dj} = \frac{\partial C_N}{\partial j} + \frac{\partial C_N}{\partial \gamma} \frac{d\gamma}{dj}$$

It therefore seems that it should be possible, if an airplane possesses certain unfavorable flight characteristics due to insufficient static stability about one of the axes, to correct this lack of stability by the use of a vane stabilizer, thus avoiding a re-design of part of the airplane.

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20. Mathias, Gotthold: The Calculation of Lateral Stability with Free Controls. T.M. No. 741, N.A.C.A. 1934.

21. Mathias, Gotthold: Supplemental Data and Calculations of the Lateral Stability of Airplanes. T.M. No. 742, N.A.C.A., 1934.

22. Alston, R. P.: Lateral and Directional Control. Aircraft Engineering, February 1936.

LONGITUDINAL MOTIONS

Initial Condition: $\delta w = -8$ and $\delta\theta = -0.2$

μ	0.008	0.006	0.004	0.002		0	-0.001
	Short Period Oscillation					Aperiodic Motion	
ρ_1	0.1775	0.20	0.2558	0.453	C_1	0	-0.015
ρ_2	8.93	9.13	10.5	14.48	$l_1 C_1$	0	-1.135
ρ_3	.763	.70	.619	.619	$m_1 C_1$	0	+.089
ρ_4	.1587	.1532	.1485	.1612	$n_1 C_1$	0	-.016
φ_1	157°	143°	118°	113°	C_2	-0.78	-1.095
φ_2	243°	241°	231°	214°	$l_2 C_2$	-8.5	-6.85
φ_3	-2°	-1°	0	+1°	$m_2 C_2$	0	-.073
φ_4	221°	213°	206°	194°	$n_2 C_2$	0	+.040
	Long Period Oscillation					Aperiodic Motion	
ρ_1'	4.24	5.44	6.86	11.4	C_3	+16.22	+7.95
ρ_2'	.456	.694	.879	1.9	$l_3 C_3$	+3.73	+1.661
ρ_3'	.025	.026	.027	.027	$m_3 C_3$	0	+.012
ρ_4'	.1055	.121	.139	.177	$n_3 C_3$	0	-.052
φ_1'	179°	179°	178°	178°	C_4	0	-6.84
φ_2'	179°	179°	178°	178°	$l_4 C_4$	0	-1.97
φ_3'	355°	354°	352°	355°	$m_4 C_4$	0	-.013
φ_4'	256°	253°	250°	244°	$n_4 C_4$	0	-.18

Initial Condition: $\delta q = 0.96$ rad./sec.

μ	0.008	0.006	0.004	0.002		0	-0.001
		Short Period Oscillation				Aperiodic Motion	
ρ_1	0.243	0.314	0.496	1.231	C_1	+0.189	-0.14
ρ_2	12.25	14.35	19.5	41.0	$l_1 C_1$	+13.2	-10.59
ρ_3	1.05	1.10	1.20	1.75	$m_1 C_1$	-.96	+.83
ρ_4	.218	.241	.288	.456	$n_1 C_1$	+.185	-.148
φ_1	275°	262°	245°	213°	C_2	-1.256	+1.7
φ_2	180°	180°	180°	180°	$l_2 C_2$	-13.65	+10.65
φ_3	295°	300°	308°	327°	$m_2 C_2$	0	+.114
φ_4	153°	153°	154°	161°	$n_2 C_2$	0	-.063
		Long Period Oscillation				Aperiodic Motion	
ρ_1'	4.18	5.25	6.86	10.78	C_3	+15.31	-7.9
ρ_2'	.45	.67	.96	1.80	$l_3 C_3$	+2.95	-1.65
ρ_3'	.025	.025	.027	.025	$m_3 C_3$	0	-.012
ρ_4'	.104	.117	.138	.168	$n_3 C_3$	0	+.052
φ_1'	177°	177°	177°	177°	C_4	-14.27	+6.34
φ_2'	177°	177°	177°	177°	$l_4 C_4$	-3.05	+1.83
φ_3'	353°	352°	351°	352°	$m_4 C_4$	0	+.012
φ_4'	253°	250°	248°	243°	$n_4 C_4$	-.185	+.166

Initial Condition: $\delta w = -8$ m/sec

μ	0.008	0.006	0.004	0.002		0	-0.001
	Short Period Oscillation					Aperiodic Motion	
ρ_1	0.175	0.202	0.257	0.441	C_1	0	-0.011
ρ_2	8.78	9.21	10.1	14.7	l_1C_1	0	-.84
ρ_3	.75	.71	.62	.62	m_1C_1	0	+.066
ρ_4	.156	.155	.150	.163	n_1C_1	0	-.012
φ_1	340°	322°	298°	244°	C_2	-0.75	-1.168
φ_2	246°	240°	233°	212°	l_2C_2	-8.17	-7.3
φ_3	0	0	0	0	m_2C_2	0	-.078
φ_4	218°	212°	205°	194°	n_2C_2	0	+.043
	Long Period Oscillation					Aperiodic Motion	
ρ_1'	4.01	3.848	3.34	2.396	C_3	+0.75	+1.85
ρ_2'	.431	.490	.468	.400	l_3C_3	+.175	+.386
ρ_3'	.0241	.0187	.0130	.0056	m_3C_3	0	+.002
ρ_4'	.10	.086	.068	.037	n_3C_3	0	-.012
φ_1'	1°	2°	4°	9°	C_4	0	-0.669
φ_2'	1°	2°	4°	9°	l_4C_4	0	-.192
φ_3'	186°	179°	177°	177°	m_4C_4	0	-.001
φ_4'	76°	76°	76°	77°	n_4C_4	0	-.017

Initial Condition: δu = 10 m/sec

μ	0.008	0.006	0.004	0.002		0	-0.001
	Short Period Oscillation					Aperiodic Motion	
ρ_1	0.022	0.033	0.036	0.068	C_1	0	-0.005
ρ_2	1.118	1.508	1.425	2.275	l_1C_1	0	-.378
ρ_3	.0056	.1178	.0878	.0973	m_1C_1	0	+.06
ρ_4	.019	.026	.021	.025	n_1C_1	0	-.005
φ_1	20°	12°	330°	320°	C_2	-0.215	-0.26
φ_2	287°	304°	261°	225°	l_2C_2	-2.34	-1.63
φ_3	22°	26°	27°	13°	m_2C_2	0	-.017
φ_4	176°	174°	176°	177°	n_2C_2	0	+.01
	Long Period Oscillation					Aperiodic Motion	
ρ_1'	10.134	10.12	10.310	10.70	C_3	+10.215	+8.48
ρ_2'	1.09	1.29	1.444	1.782	l_3C_3	+2.34	+1.77
ρ_3'	.061	.049	.040	.025	m_3C_3	0	+.013
ρ_4'	.252	.227	.208	.116	n_3C_3	0	-.056
φ_1'	100°	100°	104°	110°	C_4	0	+1.785
φ_2'	100°	100°	104°	110°	l_4C_4	0	+.514
φ_3'	275°	275°	278°	286°	m_4C_4	0	+.003
φ_4'	176°	174°	176°	177°	n_4C_4	0	+.047

Initial Condition: $\delta\theta = -0.2$ rad.

μ	0.008	0.006	0.004	0.002		0	-0.001
	Short Period Oscillation					Aperiodic Motion	
ρ_1	0.0009	0.0012	0.0019	0.0026	C_1	0	0
ρ_2	.046	.056	.076	.086	l_1C_1	0	0
ρ_3	.0039	.0043	.0047	.0037	m_1C_1	0	0
ρ_4	.0008	.0009	.0011	.0010	n_1C_1	0	0
φ_1	94°	82°	65°	33°	C_2	-0.02	0.06
φ_2	352°	15°	4°	14°	l_2C_2	-.22	.375
φ_3	114°	124°	127°	147°	m_2C_2	0	.004
φ_4	332°	332°	334°	340°	n_2C_2	0	.002
	Long Period Oscillation					Aperiodic Motion	
ρ_1'	8.24	9.26	10.39	13.97	C_3	+15.46	+6.1
ρ_2'	.886	1.184	1.451	2.325	l_3C_3	3.56	+1.275
ρ_3'	.050	.045	.040	.0326	m_3C_3	0	+.009
ρ_4'	.205	.207	.210	.218	n_3C_3	0	-.04
φ_1'	180°	180°	180°	180°	C_4	-15.44	-6.12
φ_2'	180°	180°	180°	180°	l_4C_4	-3.3	-1.775
φ_3'	356°	355°	355°	356°	m_4C_4	0	+.012
φ_4'	256°	254°	251°	246°	n_4C_4	-.2	-.162

LATERAL MOTIONS

Initial Condition: $\delta v = +10$ m/sec

Airplanes	A	B	C	D	E
			Oscillation		
ρ_1	9.62	9.70	9.74	9.05	8.46
ρ_2	.335	.261	.188	.106	.067
ρ_3	.300	.301	.338	.2846	.2648
ρ_4	.226	.180	.132	.077	.050
φ_1	86°	86°	89°	76°	71°
φ_2	76°	80°	90°	98°	143°
φ_3	359°	359°	359°	348°	342°
φ_4	333°	336°	344°	350°	33°
			Spiral Motion		
C_3	-0.043	-0.016	+0.023	+1.044	+1.896
l_3C_3	-.004	-.0005	-.0001	-.0040	.0166
m_3C_3	-.0043	-.0015	+.002	+.055	+.077
n_3C_3	+.020	+.0070	-.009	-.013	-.034
			Damped Motion		
C_4	+0.440	+0.338	+0.254	+0.144	+0.079
l_4C_4	-.325	-.257	-.185	-.101	-.025
m_4C_4	+.0103	+.0084	+.0058	+.0032	+.0005
n_4C_4	+.0818	+.0655	+.047	+.0263	+.0006

Initial Condition: $\delta v = +10$ m/sec

Airplanes	1	2	3	4	5	6
			Oscillation			
ρ_1	9.72	9.74	9.83	11.5	6.54	5.92
ρ_2	.186	.188	.214	.203	.122	.247
ρ_3	.387	.338	.217	.058	.040	.090
ρ_4	.109	.132	.170	.323	.370	.800
φ_1	89°	89°	84°	82°	94°	55°
φ_2	90°	90°	90°	89°	87°	50°
φ_3	359°	359°	359°	14°	185°	295°
φ_4	350°	344°	337°	336°	0°	358°
			Spiral Motion			
C_3	0.046	+0.023	-0.073	-1.73	+3.19	+4.60
l_3C_3	-.007	-.0001	-.0003	-.023	+.056	+.110
m_3C_3	+.006	+.002	-.003	-.021	+.054	+.063
n_3C_3	-.028	-.009	+.016	+.103	-.067	-.070
			Damped Motion			
C_4	+0.252	+0.254	+0.282	+0.275	+0.283	+0.532
l_4C_4	-.186	-.185	-.183	-.175	+.161	-.302
m_4C_4	+.0044	+.0058	+.0070	+.008	+.009	+.017
n_4C_4	+.048	+.047	+.049	+.045	+.041	+.077

Initial Condition: $\delta\varphi = 0.2$ rad.

Airplanes	A	B	C	D	E
			Oscillation		
ρ_1	1.21	1.25	1.25	13.17	11.82
ρ_2	.042	.034	.024	.154	.094
ρ_3	.038	.039	.039	.413	.370
ρ_4	.028	.023	.017	.112	.070
φ_1	163°	161°	160°	149°	142°
φ_2	153°	156°	163°	170°	213°
φ_3	76°	74°	73°	60°	54°
φ_4	50°	52°	58°	63°	104°
			Spiral Motion		
C_3	-0.373	-0.410	-0.432	-6.89	-7.28
l_3C_3	-.0038	-.0012	+.002	+.0263	+.0635
m_3C_3	-.0373	-.0385	-.037	-.362	-.298
n_3C_3	+.174	+.178	+.183	+.087	+.130
			Damped Motion		
C_4	+0.0207	+0.016	+0.012	+0.074	+0.040
l_4C_4	-.015	-.012	-.009	-.052	-.012
m_4C_4	+.0005	+.0004	+.0003	+.0016	+.0002
n_4C_4	+.0038	+.0031	+.002	+.013	+.003

Initial Condition: $\delta\varphi = 0.2$ rad.

Airplanes	1	2	3	4	5	6
			Oscillation			
ρ_1	1.066	1.25	1.89	4.66	3.85	2.34
ρ_2	.02	.024	.035	.081	.072	.098
ρ_3	.042	.039	.0421	.024	.044	.056
ρ_4	.012	.017	.032	.131	.218	.305
φ_1	164°	160°	155°	156°	192°	193°
φ_2	167°	163°	158°	156°	183°	187°
φ_3	75°	73°	69°	81°	342°	352°
φ_4	67°	58°	48°	40°	101°	137°
			Spiral Motion			
C_3	-0.308	-0.432	-0.798	-1.90	+0.786	+0.522
l_3C_3	+.005	+.002	-.004	-.026	+.014	+.012
m_3C_3	-.041	-.037	-.040	-.023	+.013	+.007
n_3C_3	+.186	+.183	+.173	+.113	-.016	-.008
			Damped Motion			
C_4	+0.012	+0.012	+0.015	+0.010	+0.014	+0.004
l_4C_4	-.009	-.009	-.009	-.006	-.009	-.002
m_4C_4	+.0002	+.0003	+.0003	+.0003	+.0004	+.0001
n_4C_4	+.002	+.002	+.002	+.0016	+.002	+.060

Initial Condition: $\delta r = 0.2$ rad./sec.

Airplanes	A	B	C	D	E
			Oscillation		
ρ_1	5.52	5.74	5.94	4.056	5.88
ρ_2	.192	.155	.113	.048	.047
ρ_3	.172	.178	.185	.128	.184
ρ_4	.130	.106	.080	.035	.035
φ_1	183°	182°	182°	203°	183°
φ_2	173°	177°	185°	224°	253°
φ_3	96°	95°	94°	114°	94°
φ_4	70°	72°	79°	117°	144°
			Spiral Motion		
C_3	+0.278	+0.239	+0.178	+1.63	+0.42
l_3C_3	+.003	+.0007	-.0008	-.0062	-.0037
m_3C_3	+.028	+.022	+.016	+.08	+.0017
n_3C_3	-.129	-.104	-.070	-.020	-.007
			Damped Motion		
C_4	+0.034	+0.011	-0.013	-0.055	-0.157
l_4C_4	-.025	+.008	+.010	+.040	+.048
m_4C_4	+.0008	+.0003	-.0003	-.0012	-.0009
n_4C_4	+.0063	+.0021	-.0025	-.010	-.013

Initial Condition: $\delta r = 0.2$ rad./sec.

Airplanes	1	2	3	4	5	6
			Oscillation			
ρ_1	4.74	5.94	7.79	21.28	8.48	9.22
ρ_2	.091	.113	.136	.369	.158	.386
ρ_3	.188	.185	.173	.107	.098	.219
ρ_4	.053	.080	.134	.601	.481	1.202
φ_1	181°	182°	184°	207°	251°	229°
φ_2	184°	185°	186°	208°	243°	225°
φ_3	92°	94°	98°	133°	40°	30°
φ_4	84°	79°	76°	92°	160°	173°
			Spiral Motion			
C_3	+0.084	+0.178	+0.584	+9.93	+8.02	+7.18
l_3C_3	-.0013	-.0008	+.0027	+.137	+.141	+.181
m_3C_3	+.0011	+.016	+.029	+.122	+.136	+.099
n_3C_3	-.051	-.070	-.127	-.592	-.167	-.108
			Damped Motion			
C_4	-0.010	-0.013	-0.020	-0.054	+0.002	-0.182
l_4C_4	+.0076	+.010	+.013	+.034	-.001	+.104
m_4C_4	-.0002	-.0003	-.0005	-.0016	+.0007	-.0061
n_4C_4	-.0002	-.0025	-.0003	-.0090	+.0004	-.026

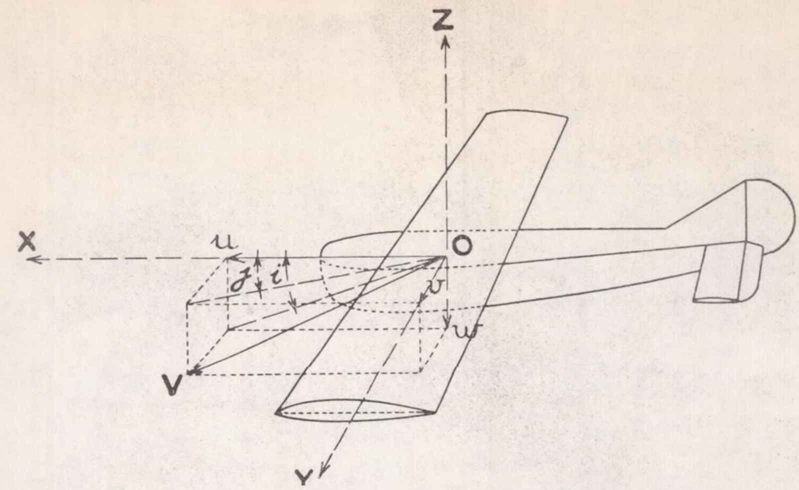
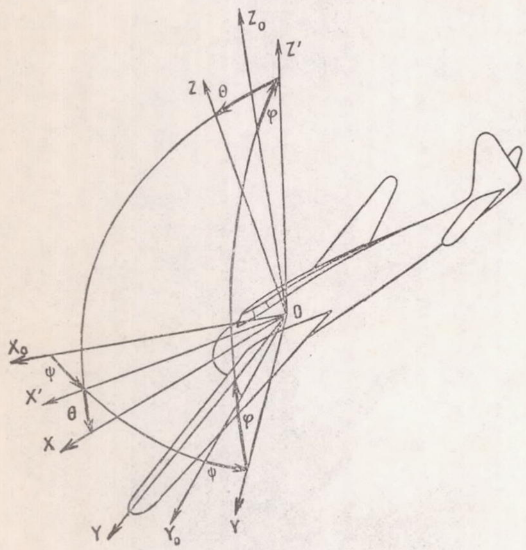


Figure 1.- Scheme of axes.

Figure 2.- Successive rotations.



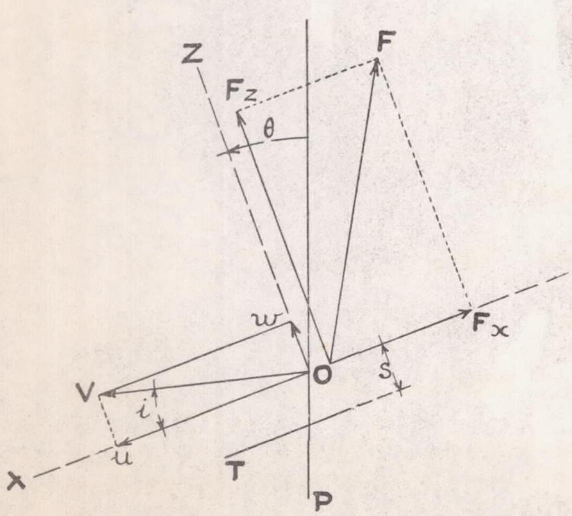


Figure 3.- Longitudinal equilibrium.

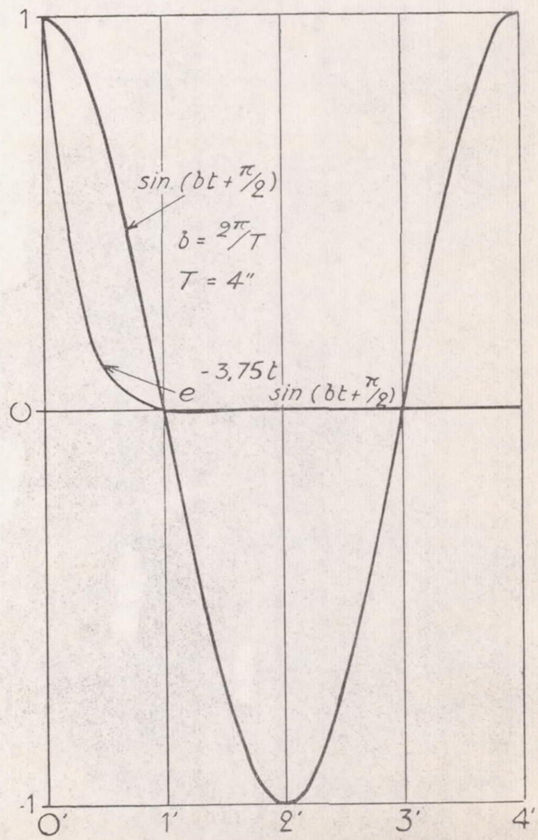


Figure 4.- Dampened oscillation.

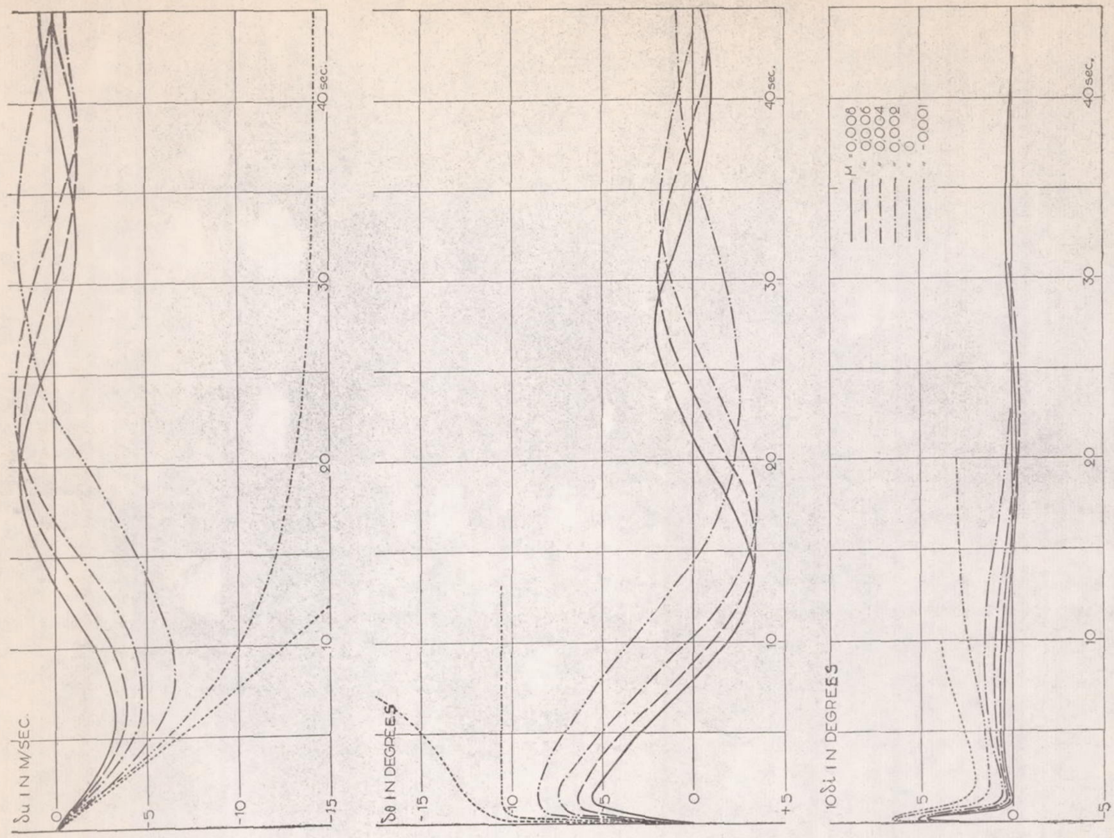


Figure 6.- Effect of a disturbance in angular velocity.

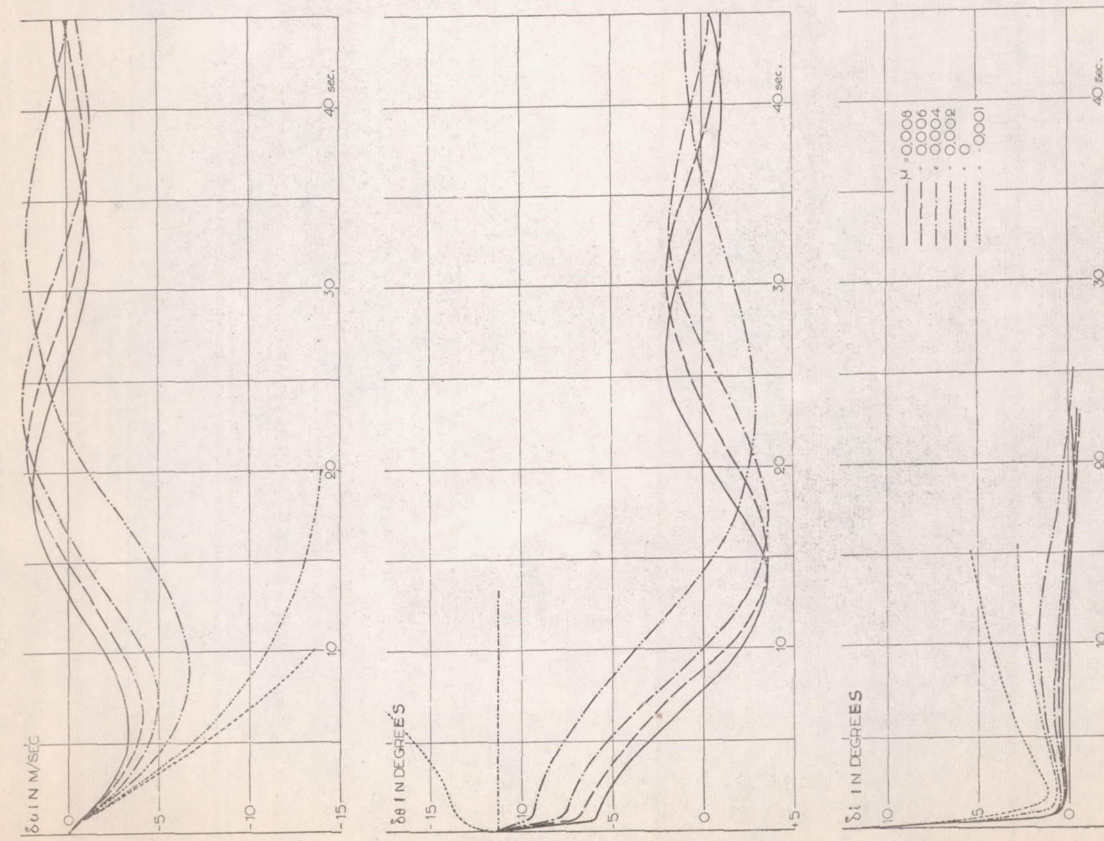


Figure 5.- Effect of an initial disturbance in angle of attack and attitude.

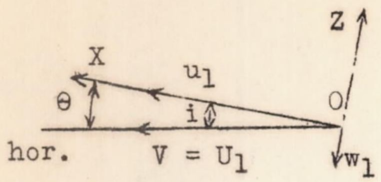


Figure 7

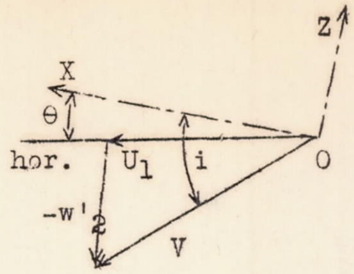


Figure 8

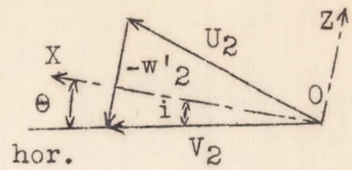


Figure 9

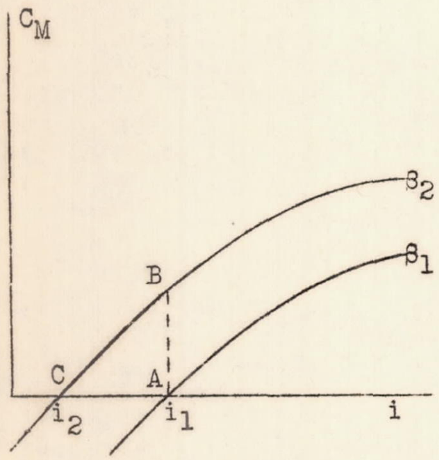


Figure 13.- Angles of attack of equilibrium.

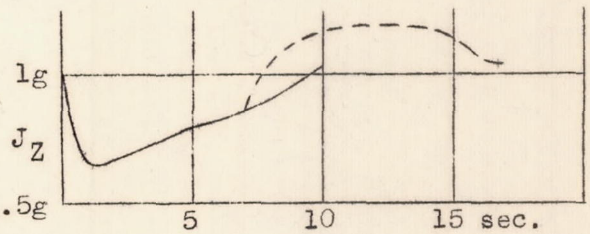


Figure 16.- Computed accelerations.

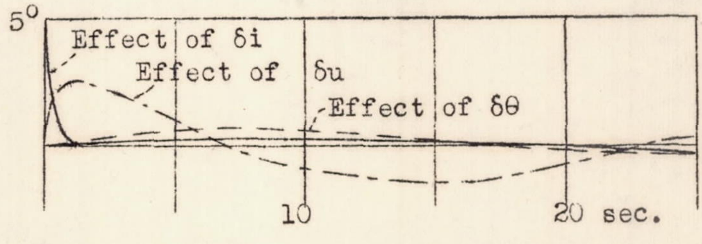


Figure 15.- Components of the disturbance in angle of attack.

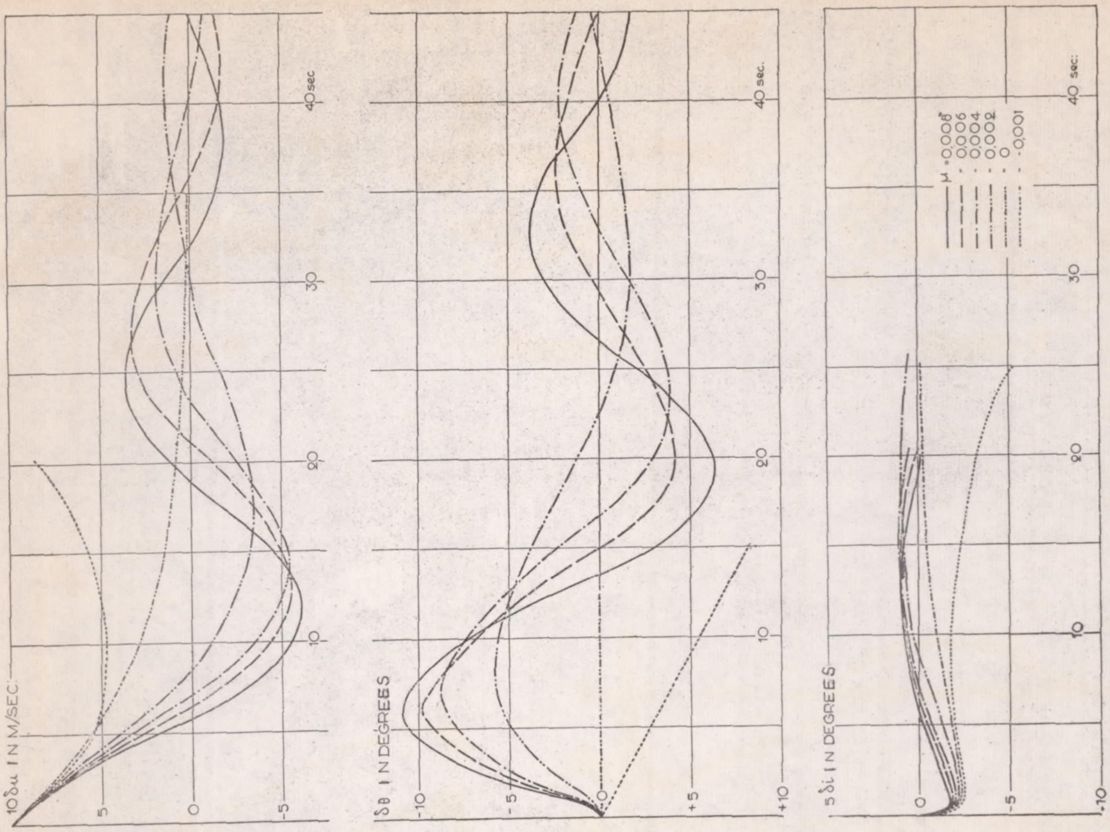


Figure 11.- Effect of a horizontal gust.

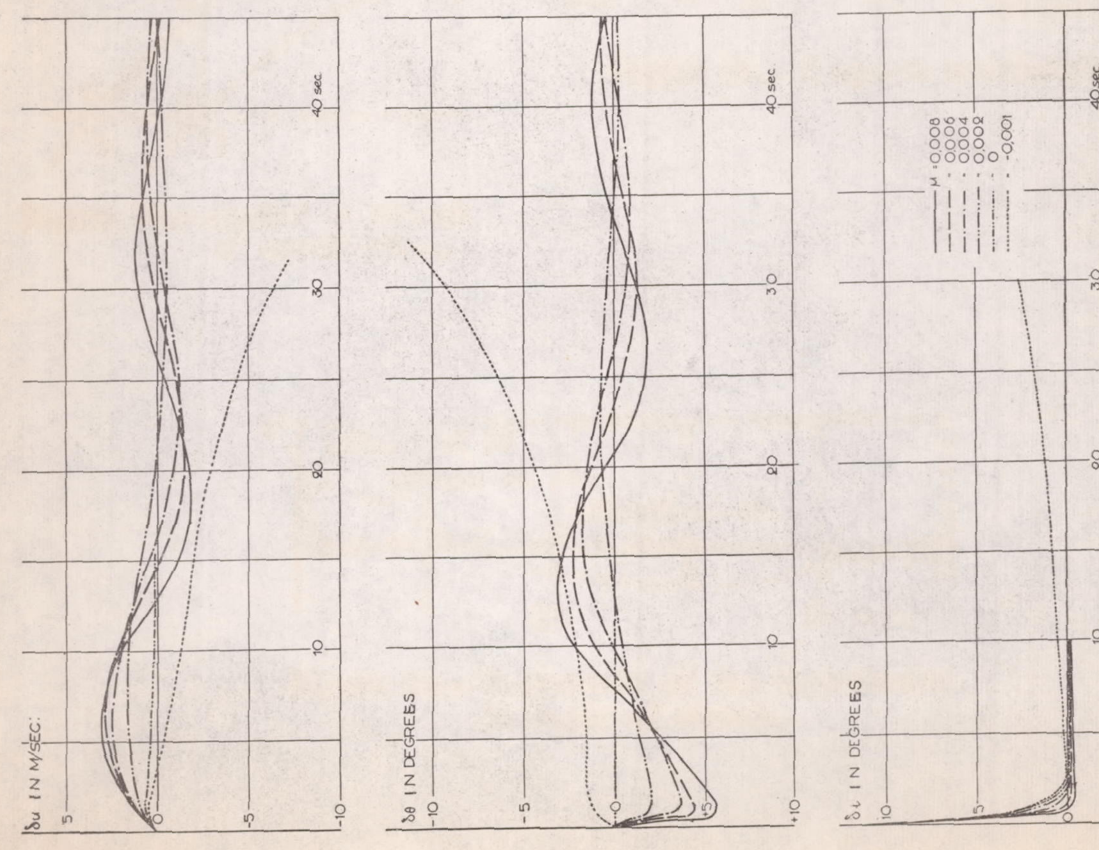


Figure 10.- Effect of a vertical gust.

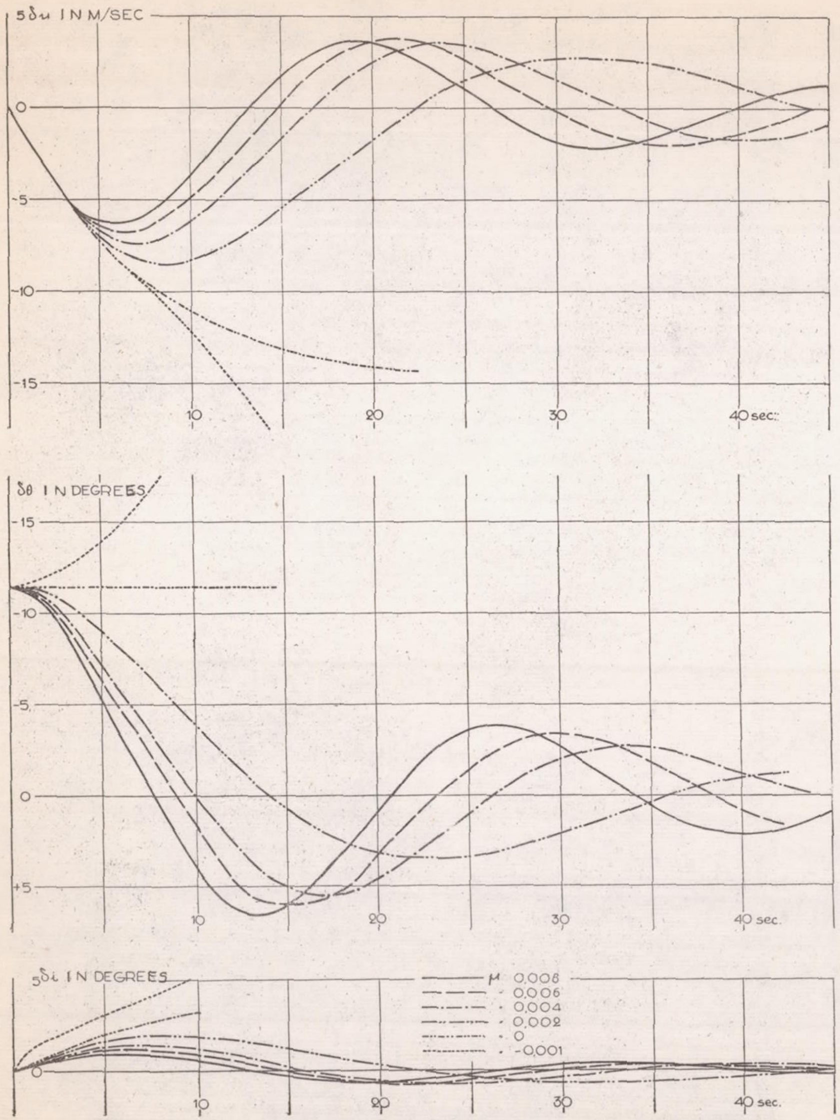


Figure 12.- Effect of a disturbance in attitude.

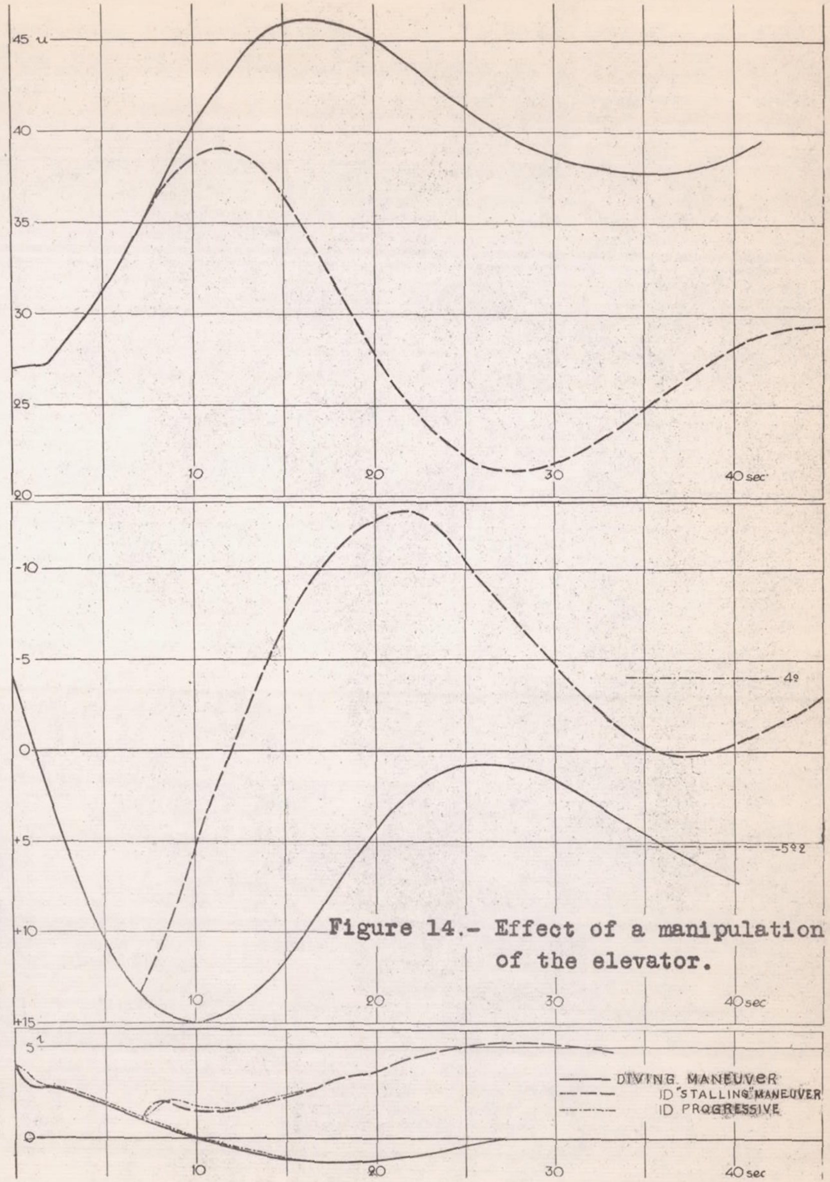


Figure 14.- Effect of a manipulation of the elevator.

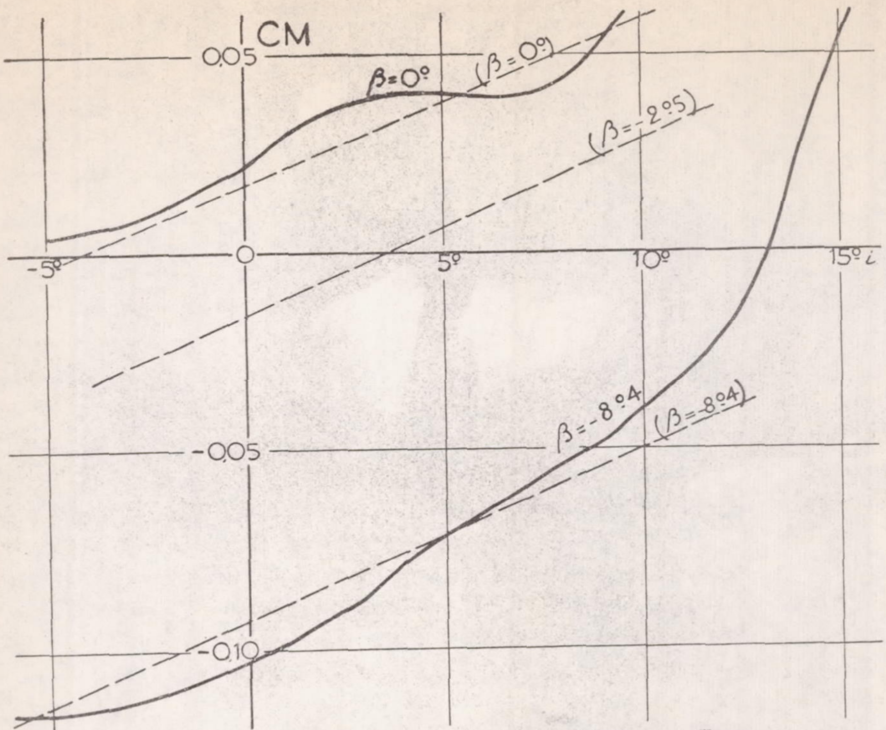


Figure 17.- C_M curve for the Fairey Fox.

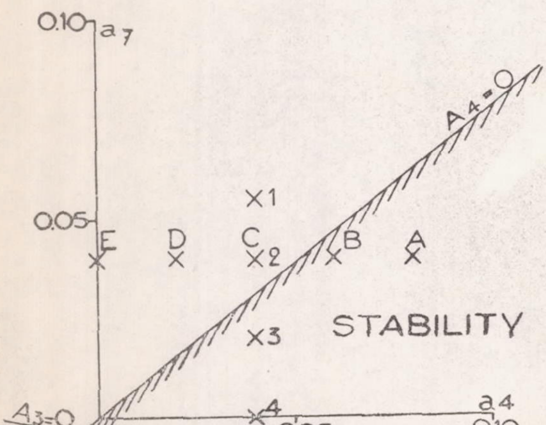


Figure 19.- Regions of stability and instability.

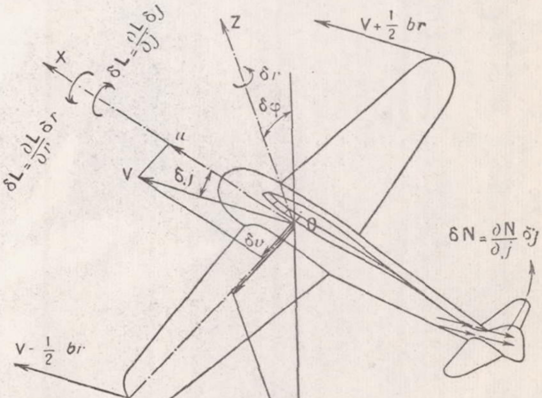


Figure 20.- Secondary effects of yaw and rotations.

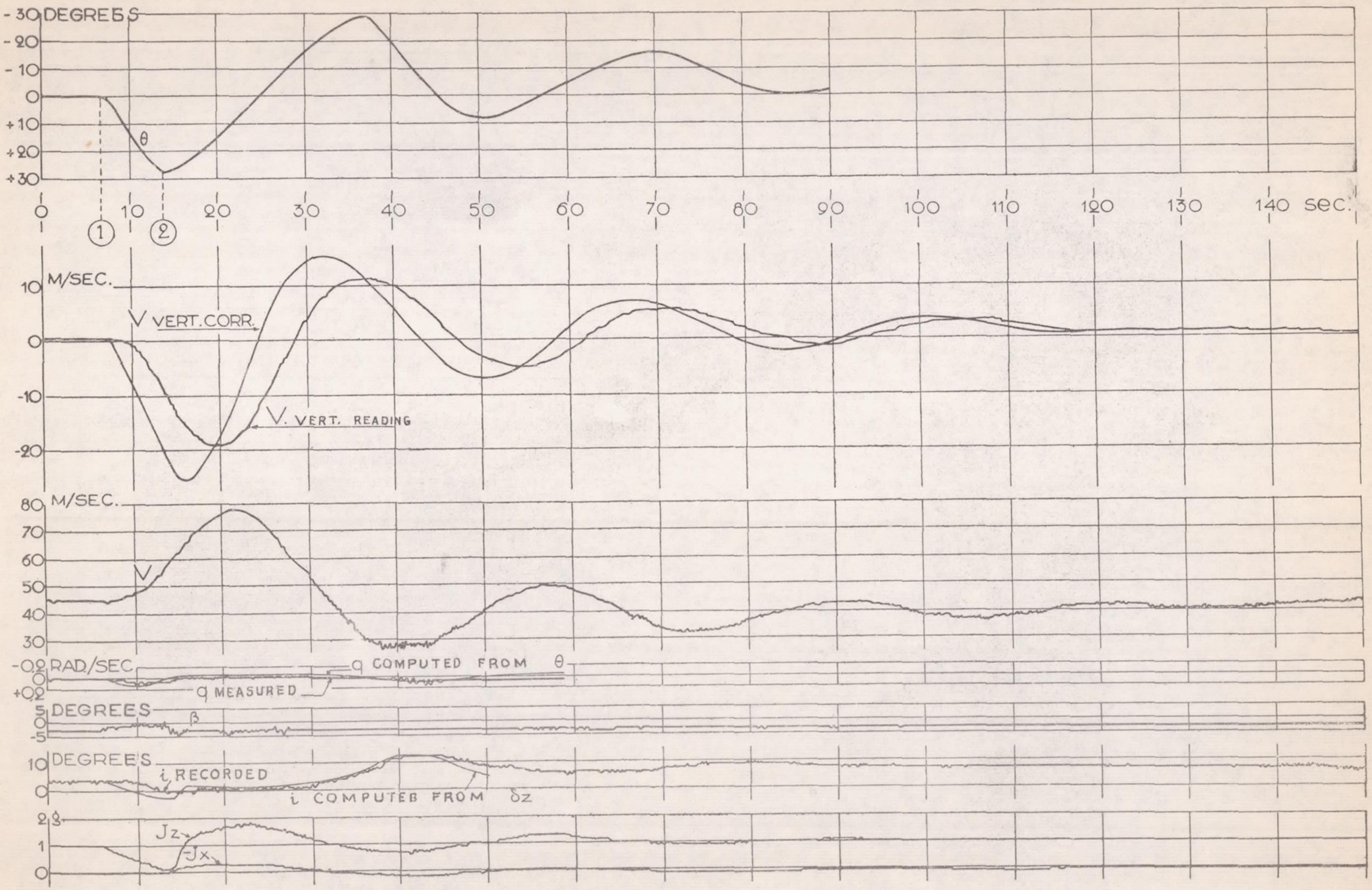


Figure 18.- Experimental curves obtained on the Fairey Fox.

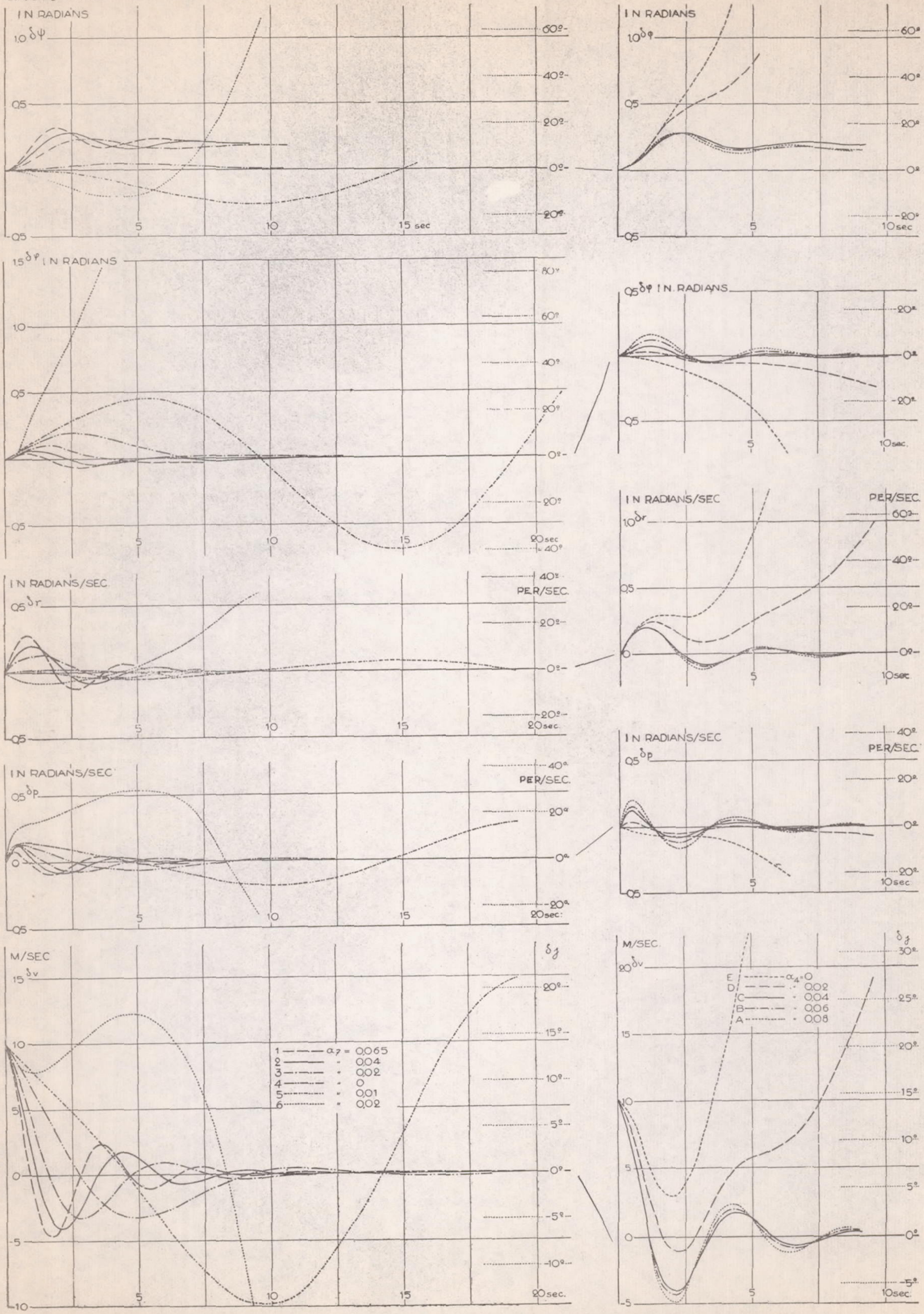


Figure 21.- Effect of an initial disturbance of yaw.

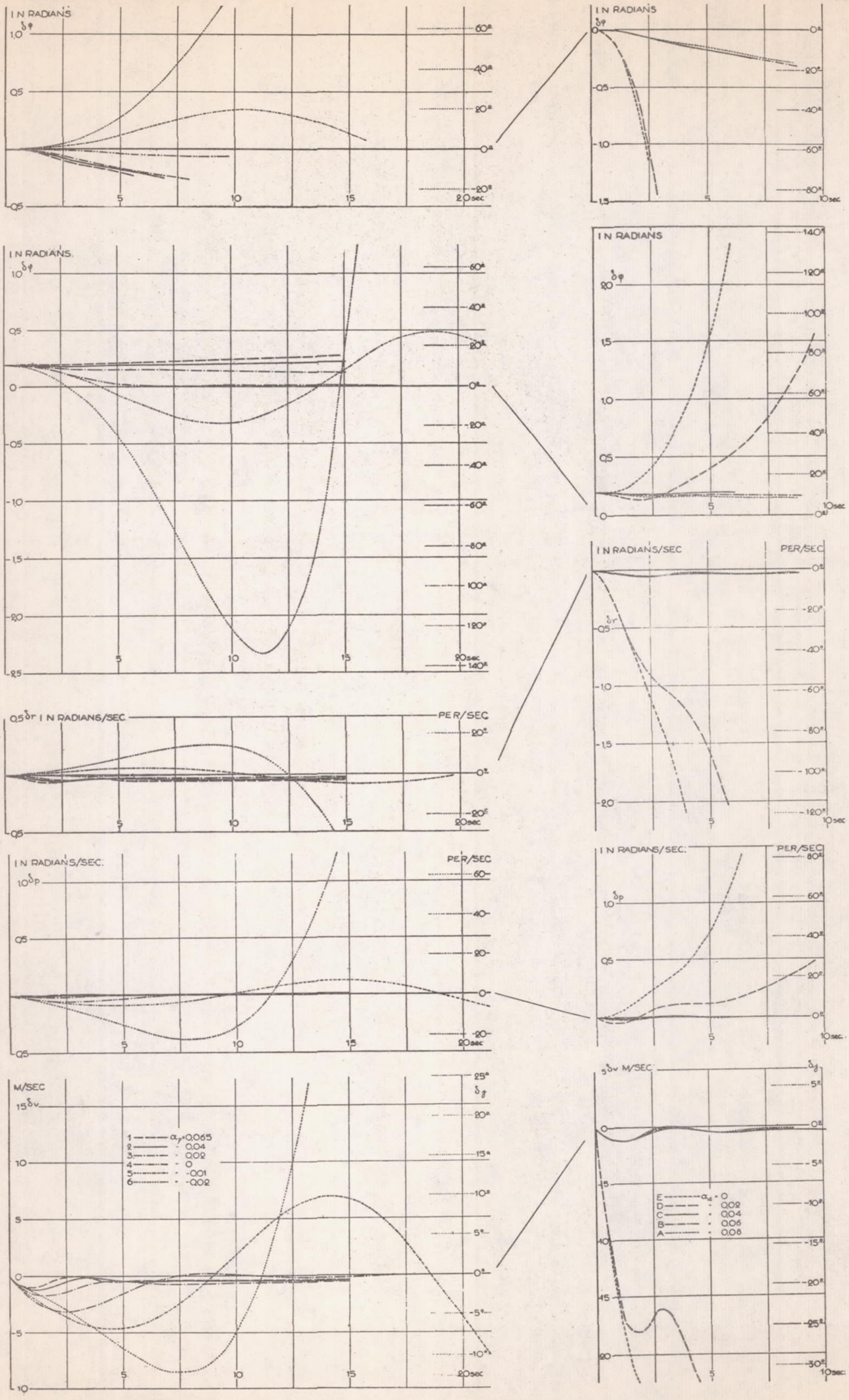


Figure 22.- Effect of a disturbance of lateral inclination.

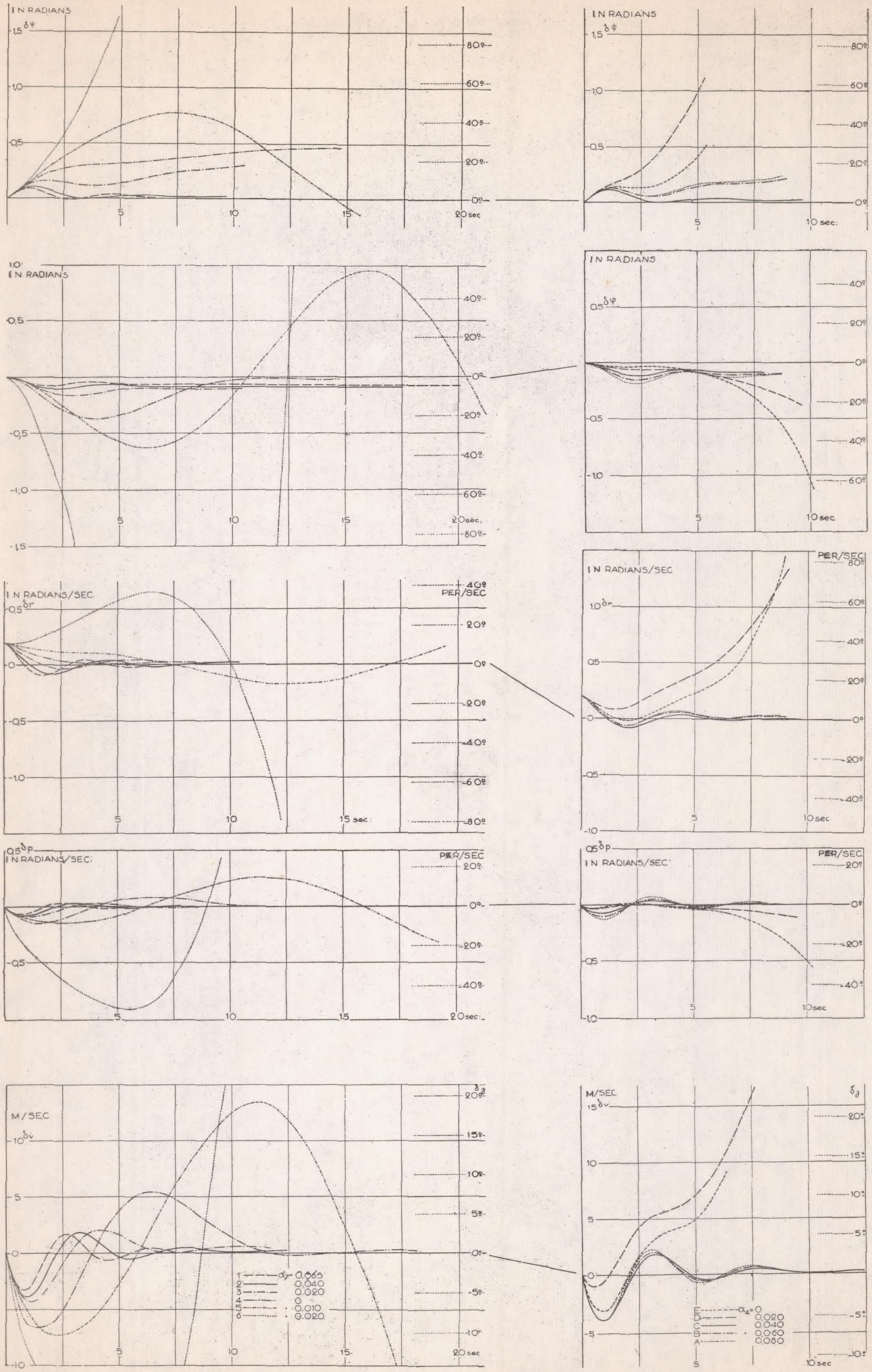


Figure 23.- Effect of an initial disturbance of angular velocity δ_r.

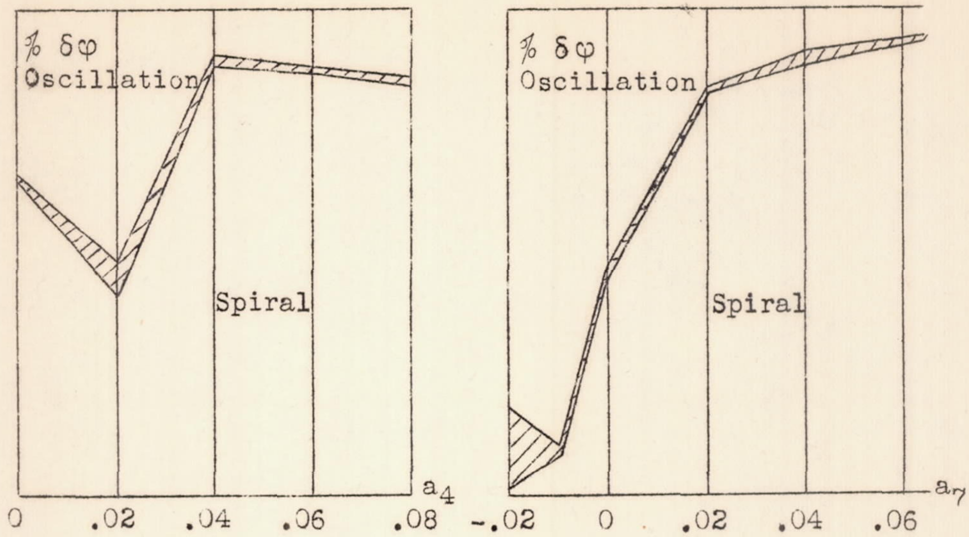


Figure 24.- Distribution of an initial disturbance of lateral inclination.

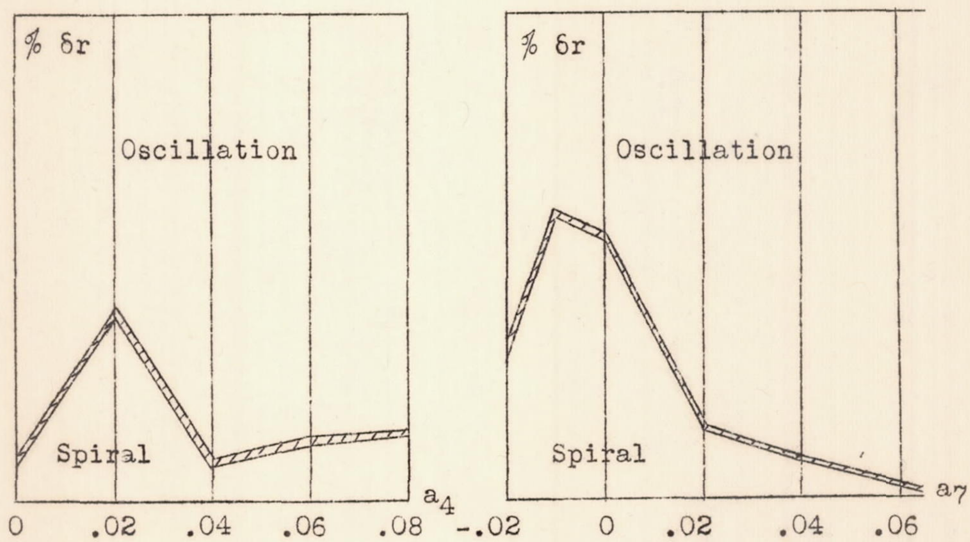


Figure 25.- Distribution of an initial disturbance of angular velocity δr.

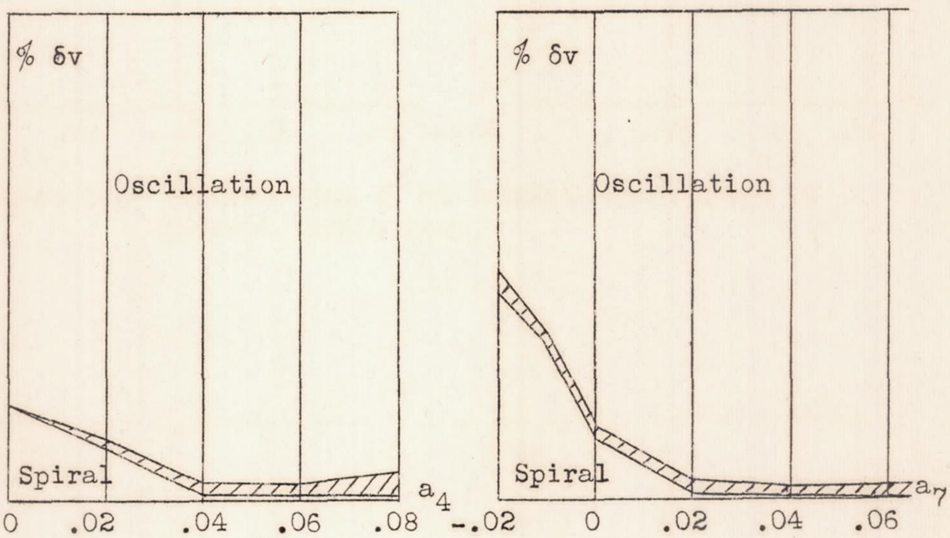


Figure 26.- Distribution of an initial disturbance of yaw.