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1. INTRODUCTION

In order that a lifting propeller of diameter $d = 2r_o$, i.e., of disk area $F = d^2 \frac{\pi}{4} = r_o^2 \pi$ may develop a thrust S, it is necessary that the air be given a downward acceleration by the propeller. This results in an axial velocity w' through the propeller disk. The minimum power required of the propeller is therefore

$$L_o = S\ w' \qquad (1)$$

where S is the propeller thrust.

In the presence of the ground beneath the propeller the downward current of air, that is, the propeller slipstream, will be deflected and the air flowing up to the propeller will be obstructed. Assuming a constant thrust, the axial velocity w' will then be changed with a corresponding change in the power $L_o = S\ w'$. In particular, the propeller performance is better near the ground as compared with that high above it. In what follows an estimate will be made of the magnitude of this ground effect. For the two cases where the distance a of the propeller from the ground is very small, and very large, respectively, in comparison with the propeller radius r_o the relations may be simply expressed.

We consider first the effect of the ground, assuming that the thrust is held constant, as may be done by a suitable change in the propeller speed. If this is not done the thrust will change with decreasing distance from the ground as a result of the change in the axial velocity. This change of thrust at constant propeller speed will be separately considered in section 4.

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*"Die Hubschraube in Bodennähe." Zeitschrift für angewandte Mathematik und Mechanik, vol. 17, no. 2, April 1937, pp. 68-72.

We shall assume for our discussion purpose that the propeller thrust S is uniformly distributed over the entire propeller disk area F and that the rotation of the slipstream may be neglected. Each air particle in passing through the propeller will then experience a pressure increase

$$p_o = \frac{S}{F} \tag{2}$$

When the particle at some distance from the propeller again arrives at a region of undisturbed pressure, this pressure increase is converted into kinetic energy

$$\frac{\rho}{2} w_o^2$$

The fluid which has passed through the propeller therefore flows off with a velocity w_o for which

$$\frac{\rho}{2} w_o^2 = p_o = \frac{S}{F}, \qquad w_o = \sqrt{\frac{2S}{\rho F}} \tag{3}$$

At the edge of the slipstream the pressure prevailing everywhere is that of the surrounding air and since we may to a sufficient degree of accuracy set the latter pressure equal to the constant pressure at a large distance from the propeller, the velocity over the entire surface of the slipstream is w_o, and only in the interior of the slipstream is there an excess pressure in the neighborhood of the propeller and hence also a velocity less than w_o.

2. DISTANCE OF PROPELLER FROM GROUND SMALL IN COMPARISON WITH PROPELLER RADIUS

For a propeller near the ground this "down flow" velocity w_o is horizontal and directed outward radially (fig. 1). If the distance from the ground is very small compared with the propeller radius, the conversion of pressure into velocity occurs in the immediate neighborhood of the propeller-disk area in a region whose radial extension is small compared to the propeller radius. We may therefore, without too great an error, consider the propeller rim as straight (instead of circular). To the fluid passing through the propeller (an appreciable flow occurs only in the neighborhood of the rim), we now apply the momentum theorem. The excess pressure acts in a horizontal direction between the propeller and the ground. If the distance of the propeller from the ground is a and we consider a layer of thickness δ at right angles to the propeller rim, the pressure will act on an area $a\delta$ and the force exerted will therefore be

$$P = p_o\ a\ \delta = \frac{\rho}{2}\ w_o^2\ a\ \delta \tag{4}$$

The flow to the propeller is uniform on all sides so that there is no horizontal impulse. The horizontal force due to the excess pressure must, therefore, be equal to the reaction of the fluid flowing through. If the volume flowing through the propeller is Q then the amount flowing through a portion of the rim of length δ is

$$Q' = \frac{Q}{d\pi}\ \delta \tag{5}$$

The momentum of this volume of fluid is

$$J = \rho\ Q'\ w_o = \rho\ Q\ \frac{\delta}{d\pi}\ w_o \tag{6}$$

By setting J and P equal to each other and making use of equation (3), we obtain the volume flowing through per second:

$$Q = \frac{1}{2}\ w_o\ a\ d\pi = \frac{a}{d}\ 2F\sqrt{\frac{2S}{\rho F}} \tag{7}$$

If the height of the stream at a distance r from the propeller axis is h, there is obtained from the relation

$$Q = 2r\ \pi\ w_o\ h \tag{8}$$

by comparison with equation (7)

$$h = \frac{a}{2}\ \frac{r_o}{r} \tag{9}$$

In order to deliver the quantity Q per second through the propeller against the pressure p_o the power required is

$$L_o = p_o\ Q = \frac{a}{d}\ 2S\sqrt{\frac{2S}{\rho F}} \tag{10}$$

For the undisturbed propeller the final velocity in the propeller slipstream is just as large as the downstream velocity of the slipstream disturbed by the ground, namely:

$$w_0 = \sqrt{\frac{2S}{\rho F}}$$

(equation (3)). Furthermore, according to the results of the simple propeller theory (reference 1), the axial velocity in the plane of the propeller disk for the undisturbed propeller is

$$w'_\infty = \frac{w_0}{2} = \sqrt{\frac{S}{2\rho F}} \tag{11}$$

and the volume flowing through per second

$$Q_\infty = F\ w'_\infty = F\sqrt{\frac{S}{2\rho F}} \tag{12}$$

The minimum power required is therefore

$$L_{0\,\infty} = S\ w'_\infty = S\sqrt{\frac{S}{2\rho F}} \tag{13}$$

The effect of the ground for the condition of constant thrust may therefore be simply expressed by the ratio

$$\frac{L_0}{L_{0\,\infty}} = \frac{Q}{Q_\infty} = 4\ \frac{a}{d} \tag{14}$$

3. DISTANCE OF PROPELLER FROM GROUND LARGE COMPARED WITH PROPELLER RADIUS

The essential action of a propeller on the surrounding fluid at a great distance away is that of suction on the fluid. This action may be represented by the field due to a sink into which the air sucked up by the propeller vanished. For a lifting propeller, that is, one without any velocity of its own, the sucked-up air quantity is equal to the volume Q_∞ flowing through the propeller (equation (12)). The additional velocity induced by the propeller at the distance l from the former is accordingly

$$c = \frac{Q_\infty}{4\pi\ l^2} \tag{15}$$

and is directed toward the propeller.

If the propeller is in the neighborhood of an infinite bounding plane but at such a distance away that the disturbance due to this plane is small compared to the axial velocity, the interference action of this plane may be represented analytically by reflection of the propeller and replacing of the reflected propeller by a sink.

If the propeller is at a distance a above the ground, the reflected image, or the substituted sink, is at a distance $2a$ below the propeller (fig. 2). The strength of the sink is Q_∞ (equation 12) and the sink gives rise to a disturbance velocity at the propeller that is directed downward and is equal to

$$w'' = \frac{Q_\infty}{4\pi(2a)^2} = \frac{F}{16\pi\ a^2}\sqrt{\frac{S}{2\rho F}} \tag{16}$$

this component being superposed on the undisturbed axial velocity w'_∞. The resulting axial velocity is therefore,

$$w' = w'_\infty + w'' = \sqrt{\frac{S}{2\rho F}}\left(1 + \frac{1}{16\pi}\ \frac{F}{a^2}\right) \tag{17}$$

Since, with the thrust held constant, the powers are proportional to the axial velocities

$$\frac{L_o}{L_{o\infty}} = \frac{w'}{w'_\infty} = 1 + \frac{1}{16\pi}\ \frac{F}{a^2} = 1 + \frac{1}{64}\left(\frac{d}{a}\right)^2 \tag{18}$$

whereas, with the propeller a small distance away from the ground, the power expended in driving the air through is, according to equation (14), decreased; for a large distance away there is, according to equation (18), an increase, though very slight. Considering the two results, we may represent the effect of the ground on the required power at constant thrust approximately by the heavy broken line on figure 3. The relations (14) and (18) are shown by the lighter lines. The actual function should differ from the broken line only in the rounding of the sharp edge.

4. EFFECT OF THE GROUND ON THE THRUST

Throughout the previous considerations we had assumed that the thrust was held constant as the propeller approached the ground. Since when this happens the axial velocity through the propeller changes, it is possible to maintain constant thrust only by a simultaneous change in the rotational speed or by a change in the propeller form; for example, by a change in the pitch angle. We shall not, however, here consider any changes in form. If the decrease in thrust for a small rate of advance $\lambda = v/u$ (v = velocity of advance, u = circumferential velocity) is known as well as the thrust for zero rate of advance, then the thrust near the ground may be given for any propeller speed. We shall assume that the thrust coefficient $k_s = \frac{S}{\frac{\rho}{2} F u^2}$ (S = propeller thrust, ρ = air density, $F = r_0^2 \pi$ = propeller disk area) is a linear function

$$k_s = k_{s_0} - \kappa \lambda \tag{19}$$

where k_{s_0} is the thrust coefficient in free air at standstill ($\lambda = 0$) and $\kappa = -\frac{\partial k_s}{\partial \lambda}$ is assumed constant as may be done to some extent for the small pitches of the lifting propellers. If the effective pitch of the propeller is H the thrust is zero for $\lambda = \frac{H}{d\pi}$ so that

$$\kappa = -\frac{\partial k_s}{\partial \lambda} \approx k_{s_0} \frac{d\pi}{H} \tag{20}$$

If the amount flowing through, at constant propeller torque, is decreased by $\Delta Q = Q_\infty - Q$ due to the ground effect, the mean axial velocity is reduced by

$$\Delta w = w'_\infty - w' = \frac{\Delta Q}{F} \tag{21}$$

The propeller therefore behaves as if it had a propulsion velocity

$$v = -\Delta w$$

If, in order to maintain the thrust constant, a reduction in the circumferential velocity from u_0 to u' is necessary, then the rate of advance is

$$\lambda = \frac{v}{u'} = - \frac{\Delta w}{u'} \tag{23}$$

and therefore, according to equation (19),

$$k_s = k_{s_0} + \kappa \frac{\Delta w}{u'} = k_{s_0} \left(1 + \frac{\kappa}{k_{s_0}} \frac{\Delta w}{u'}\right) = k_{s_0} \left(1 + \frac{d\pi}{H} \frac{\Delta w}{u'}\right) \tag{24}$$

The thrust for any number of revolutions per second n or circumferential velocity u is then

$$S = \frac{\rho}{2} F u^2 k_s = \frac{\rho}{2} F u^2 k_{s_0} \left(1 + \frac{d\pi}{H} \frac{\Delta w}{u'}\right) \tag{25}$$

Comparing with the thrust

$$S_0 = \frac{\rho}{2} F {u_0}^2 k_{s_0} \tag{26}$$

there is obtained

$$\frac{S}{S_0} = \left(\frac{u}{u_0}\right)^2 \left(1 + \frac{d\pi}{H} \frac{\Delta w}{u'}\right) \tag{27}$$

If we limit ourselves to effects for which $\Delta w \ll w'$, which is practically always the case, we may replace u' in the small correction member by u_0. Since, furthermore, according to equations (21) and (11)

$$\Delta w = w'_\infty \left(1 - \frac{w'}{w'_\infty}\right) = \sqrt{\frac{S_0}{2\rho F}} \left(1 - \frac{L_0}{L_{0\infty}}\right) \tag{28}$$

where $L_0/L_{0\infty}$ is the function shown on figure 3 for constant thrust, we may express the thrust at any circumferential velocity in terms of this function

$$\frac{S}{S_0} = \left(\frac{u}{u_0}\right)^2 \left[1 + \frac{d\pi}{Hu_0} \sqrt{\frac{S_0}{2\rho F}} \left(1 - \frac{L_0}{L_{0\infty}}\right)\right] \tag{29}$$

In particular, for a constant speed $u = u_0$

$$\frac{S}{S_0} = 1 + \frac{d\pi}{Hu_0} \sqrt{\frac{S_0}{2\rho F}} \left(1 - \frac{L_0}{L_{0\infty}}\right) \tag{30}$$

5. ADDITIONAL LOSSES TO BE CONSIDERED

In applying the results given above, it is to be noted that besides the energy required for producing the velocity w' (equation (1)), additional energy is required to overcome the airfoil drag and accompanying disturbing effects. Consequently, the power L_∞ even at an infinite distance from the ground, exceeds the theoretical power $L_{0\infty}$ according to equation (13) by an amount $L_{R\infty}$

$$L_\infty = L_{0\infty} + L_{R\infty} \qquad (31)$$

For constant lift-to-drag ratio of the wing profile this additional power, at constant propeller speed n near the ground, is proportional to the thrust. For a speed n we therefore obtain for this additional power

$$L_R = L_{R\infty} \frac{S}{S_0} \frac{n}{n_0} \qquad (32)$$

where $\frac{S}{S_0}$ is to be taken from equation (29) and $\frac{n}{n_0} = \frac{u}{u_0}$.

Since the power $L_0 = S\,w'$ is proportional to $S^{3/2}$, the total power for any rotational speed n is

$$L = L_0 \left(\frac{S}{S_0}\right)^{3/2} + L_{R\infty} \frac{S}{S_0} \frac{n}{n_0} = L_{0\infty} \left(\frac{L_0}{L_{0\infty}}\right) \frac{S}{S_0} \left(\sqrt{\frac{S}{S_0}} + L_{R\infty} \frac{n}{n_0}\right) \qquad (33)$$

where the values $\frac{S}{S_0}$ and $L_{R\infty}$ are to be computed from equations (29) and (31) and $\frac{L_0}{L_{0\infty}}$ is again the effect at constant thrust.

SUMMARY

On the basis of simple considerations the effect of the ground on the power of a lifting propeller at relatively small and large distances, respectively, from the ground, was determined and represented by a simple broken line. The ground-effect function thus found was used also to determine the variation in thrust near the ground at any num-

ber of revolutions per second. Finally, it is also shown how the effect of the additional propeller losses may be taken into account.

Translation by S. Reiss,
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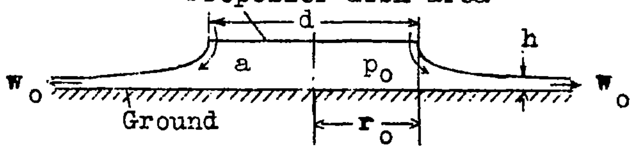

Figure 1

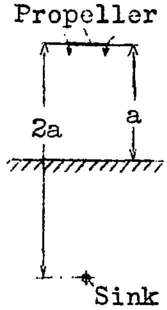

Figure 2

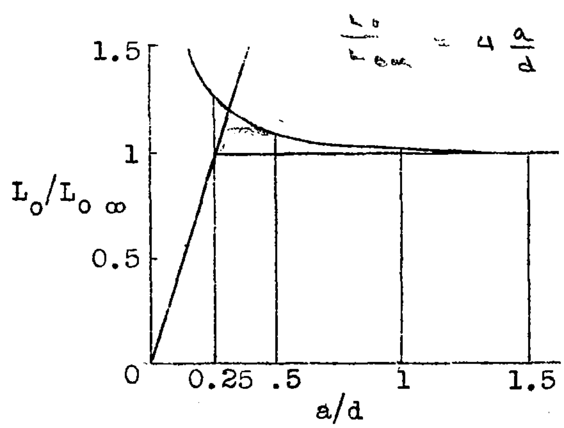

Figure 3