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No. 810

IMPACT OF A VEE-TYPE SEAPLANE ON WATER WITH
REFERENCE TO ELASTICITY

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IMPACT OF A VEE-TYPE SEAPLANE ON WATER WITH
REFERENCE TO ELASTICITY*

By F. Weinig

The theory developed by H. Wagner for the computation of the landing impact on water for a rigid float is extended to include elastic floats by introducing the concept of an equivalent rigid bottom to substitute for the actual elastic bottom.

OBJECT OF INVESTIGATION

The theory developed by H. Wagner for the computation of the landing of float bottoms on the water is extended to include bottoms having elasticity in order to take account of the elasticity factor on the landing impact.

WAGNER'S THEORY OF THE IMPACT OF VEE-TYPE FLOATS
WITH RIGID BOTTOMS

Let the downward velocity of the seaplane V just before impact be denoted by V_0 . During the immersion process which is assumed to start at $t = 0$, $V = V(t)$.

In investigating the impact we assume for convenience that the float is at rest while the water is in motion relative to it. Essentially this flow is similar to that of an infinite fluid about a flat plate at rest, the width $2c$ of the plate corresponding to the instantaneous width of the impact area of the float (fig. 1). The velocity of

*"Berücksichtigung der Elastizität beim Aufschlag eines gekielten Flugzeugschwimmers auf das Wasser."
Luftfahrtforschung, vol. 13, no. 5, May 20, 1936, pp. 155-159.

the water particles at the free surface ($X > c$) is

$$v = v_n = \frac{V}{\sqrt{1 - \frac{c^2}{X^2}}}$$

The rise Y of the water measured from the instant of immersion is

$$Y = \int_0^t v_n dt = \int_0^t \frac{V dt}{\sqrt{1 - \frac{c^2}{X^2}}}$$

The width of the impact area increases with time; i.e., $c = c(t)$, so that since $t = t(c)$, $V = V(c)$ and with

$$u(c) = \frac{V}{dc/dt}, \quad y = \int_{c=0}^{c=X} \frac{u(c) dc}{\sqrt{1 - \frac{c^2}{X^2}}}$$

As soon as the water particle, at position X reaches the edge of the impact area, $c = X$ and $Y = Y_b$, so that

$$Y_b = \int_0^X \frac{u(c) dc}{\sqrt{1 - \frac{c^2}{X^2}}}$$

We set

$$y = \frac{Y_b}{B/2}; \quad x = \frac{X}{B/2}; \quad \xi = \frac{c}{B/2}; \quad \eta = \int_0^\xi \frac{u(\xi) d\xi}{\sqrt{1 - \left(\frac{\xi}{x}\right)^2}} \quad (1)$$

Let the bottom shape be expressed by:

$$\eta(x) = \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \dots \quad (2)$$

Then $u(\xi) = \gamma_1 + \gamma_2 \xi + \gamma_3 \xi^3 + \dots$

Since $c < X$, if we set $\frac{c}{X} = \frac{\xi}{x} = \sin \alpha$, then expression

(2) becomes $\eta(x) = \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \dots$

$$= x \int_0^{\pi/2} (\gamma_1 + \gamma_2 x \sin \alpha + \gamma_3 x^2 \sin^2 \alpha + \dots) d\alpha$$

$$u(\xi) = k_1 \beta_1 + k_2 \beta_2 \xi + k_3 \beta_3 \xi^2 + \dots \quad (2a)$$

where

$$\left. \begin{aligned} k_1 &= \frac{2}{\pi} &= 0.636; & k_2 &= 1 &= 1.000 \\ k_3 &= \frac{2}{\pi} \times \frac{2}{1} &= 1.272; & k_4 &= \frac{3}{2} &= 1.500 \\ k_5 &= \frac{2}{\pi} \times \frac{2}{1} \times \frac{4}{3} &= 1.696; & k_6 &= \frac{3}{2} \times \frac{5}{4} &= 1.875 \\ k_7 &= \frac{2}{\pi} \times \frac{2}{1} \times \frac{4}{3} \times \frac{6}{5} &= 2.040; & k_8 &= \frac{3}{2} \times \frac{5}{4} \times \frac{7}{6} &= 2.18 \end{aligned} \right\} (3)$$

In general,

$$k_n = \frac{1}{\int_0^{\pi/2} \sin^{n-1} \alpha \, d\alpha}$$

From k_n there is derived the formula

$$k_{n+1} = \frac{2n}{\pi k_n}$$

For large values of n the following approximate formula applies:

$$k_n \sim \sqrt{\frac{2n-1}{\pi}} \quad (n > 4)$$

while for small values of n the approximate formula is

$$k_n \sim \sqrt{\frac{2n-4+\pi}{\pi}} \quad (n < 4)$$

In some cases the bottom shape is better expressed by

$$\eta = \beta_1 x + \beta_n x^n$$

and the value of u is then obtained as

$$u = k_1 \beta_1 + k_n \beta_n x^{n-1}$$

For nonintegral values of n the values of k_n may be read off figure 2.

In order to determine the force P exerted on the float, we again consider the surface of the water to be at rest and the float moving relatively to it with velocity $V = V(t)$. The momentum of the fluid at the lower half plane is known to be

$$J = \frac{\pi}{2} \rho c^2 V = \frac{\pi}{2} \rho \left(\frac{B}{2}\right)^2 V \xi^2 \quad (4)$$

from which, substituting $\frac{dc}{dt} = \frac{V}{u}$, we obtain

$$P = \frac{dJ}{dt} = \pi \rho \left(\frac{B}{2}\right) \frac{V^2}{u} \xi + \frac{\pi}{2} \rho \left(\frac{B}{2}\right)^2 \xi^2 \frac{dV}{dt} \quad (5)$$

The momentum of the seaplane of mass m is equal to $m(V_0 - V)$ and for the case we are here considering where the elasticity, for example, of the landing gear and float bottom is neglected, this is equal to the momentum imparted to the fluid. With

$$\mu = \frac{\pi \rho \left(\frac{B}{2}\right)^2}{2m} \quad (6)$$

$$\frac{\pi}{2} \rho \left(\frac{B}{2}\right)^2 V \xi^2 = m(V_0 - V) \quad (7)$$

and

$$V = \frac{V_0}{1 + \mu \xi^2} \quad (8)$$

From expression (8) it may be seen that $V(\xi)$ does not depend on the bottom shape but only on the width of the wetted surface and on the mass. If the work of deformation is neglected then

$$P = -m \frac{dV}{dt} \quad (9)$$

and we thus obtain

$$P = \pi \rho \frac{B}{2} V_0^2 \frac{\xi}{(1 + \mu \xi^2)^3 u(\xi)} \quad (10)$$

For briefness, we set

$$\pi \rho \frac{B}{2} V_0^2 = P_1; \quad \frac{P}{P_1} = \frac{B}{B_1} \quad (11)$$

so that

$$\frac{B}{B_1} = \frac{\xi}{(1 + \mu \xi^2)^3 u(\xi)} \quad (12)$$

The pressure distribution on the float bottom will now be determined. For $X^2 < c^2$ the velocity potential is:

$$\varphi = -V \sqrt{c^2 - X^2} = -V c \sqrt{1 - \left(\frac{X}{c}\right)^2}$$

The fluid pressure p is, in general,

$$p = -\rho \left[\frac{\partial \varphi}{\partial t} + \frac{1}{2} v_x^2 + F(t) \right]$$

In the case we are considering $F(t)$ is constant since there is no superposition of external variable pressures. On the surface of the water $F(t) = -p_L/\rho$. If p is the pressure above the air pressure p_L then

$$p = -\rho \left[\frac{\partial \varphi}{\partial t} + \frac{1}{2} v_x^2 \right]$$

We have

$$v_x = -\frac{\partial \varphi}{\partial X}$$

so that

$$\frac{p}{\rho} = \frac{V^2}{u} \frac{1}{\sqrt{1 - \left(\frac{X}{c}\right)^2}} + \frac{dV}{dt} c \sqrt{1 - \left(\frac{X}{c}\right)^2} - \frac{1}{2} \frac{V^2}{\left(\frac{c}{X}\right)^2 - 1} \quad (13)$$

The last term of the above equation does not apply to the edge of the impact area and is small compared to the first two terms.

With the aid of (8), (9), and (10), we obtain:

$$p = \frac{P}{\pi \frac{B}{2}} \frac{1}{\xi} \left[\frac{1 + \mu \xi^2}{\sqrt{1 - \left(\frac{x}{\xi}\right)^2}} - 2\mu \xi^2 \sqrt{1 - \left(\frac{x}{\xi}\right)^2} - \frac{u(1 + \mu \xi^2)}{2\left(\frac{\xi}{x}\right)^2 - 1} \right] \quad (14)$$

This formula does not give correct values for the edge of

the impact area. There the flow is similar to that about a planing surface moving sideways with velocity

$$v_1 = \frac{dc}{dt} = \frac{V}{u} \quad (15)$$

The maximum pressure that is set up is therefore,

$$p_{\max} = q_1 = \frac{\rho}{2} v_1^2 \quad (16)$$

or with

$$\left. \begin{aligned} q_0 &= \frac{\rho}{2} V_0^2 \\ p_{\max} &= q_1 = q_0 \frac{1}{(1 + \mu \xi^2)^2 u^2(\xi)} \end{aligned} \right\} \quad (17)$$

THE CONCEPT OF AN EQUIVALENT RIGID BOTTOM TO

REPLACE THE ELASTIC BOTTOM

From equation (12) it may be seen that $B(\xi)$ depends only on the form parameter of the float bottom. Such a parameter may also be found for a float with elastic bottom. Let $y_{\text{rel}}(\xi)$ be the elastic displacement at position ξ due to the load $P(\xi)$, then

$$\eta(\xi) = \eta_0(\xi) + \eta_{\text{rel}}(\xi) \quad (18)$$

where $\eta_0(\xi)$ represents the shape of the nondeformed float bottom and $\eta(\xi)$ the shape of a nonelastic float bottom which replaces that of the elastic float. The results obtained for the rigid float may thus be applied to the case of the elastic float. The equivalent rigid float fully takes into account the rate of expansion of the width dc/dt due to the elasticity; that is, the main part of the deformation; and only a small part of the deformation is neglected, namely, that of the bottom opposite the chord between the keel and edge of the impact area.

We must next consider how y_{rel} is to be determined. It may be seen from equation (14) for the pressure distribution that the relative distribution of the pressure is essentially independent of the shape of the float bottom, i.e., of $u(\xi)$ since the last term in the brackets is very

small compared to the other terms for the greatest part of the range $0 < x < \xi$.

If the elastic deformation for the same relative pressure distribution is proportional to the total load, then the deformation under the load $\underline{B}(\xi)$ at any position ξ may be expressed in the form

$$\eta_{\text{rel}}(\xi) = \underline{B}(\xi) (a_0 + a_1 \xi + a_2 \xi^2 + a_3 \xi^3 + \dots) \quad (19)$$

or with $\xi = x$.

$$\eta_{\text{rel}}(x) = \underline{B}(x) (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots)$$

For briefness we shall denote the expression in parentheses which takes account of the elasticity by $A(\xi)$:

$$(a_0 + a_1 \xi + a_2 \xi^2 + a_3 \xi^3 + \dots) = A(\xi)$$

The coefficients a_0, a_1, a_2, a_3 must be determined, for each type of float, either by experiment or computation. The pressure distributions given by equation (14), which applies to rigid floats, are to be substituted and thus the bending at the edge of the impact area determined.

We have assumed that the elastic deformation was proportional to the load P , but due to the small thickness of the float bottom this assumption will in many cases not prove to be correct. In these cases, however, it may be taken as a first approximation that for a moderate increase in an assumed loading the increase in the deformation is proportional to the load increase.

In order to determine the deformation it may be assumed first that the load is the same as that exerted on the nondeformed float on impact, the corresponding deformation under this load determined, and then the deformation when the load is slightly increased. In this way an expression for the deformation may be found of the form

$$\eta_{\text{rel}}(x) = \eta_{\text{orel}}(x) + \underline{B}(a_0 + a_1 x + a_2 x^2 + \dots)$$

If we then add η_0 and η_{orel} , we obtain:

$$\eta_0 + \eta_{\text{orel}} = \tilde{\eta}_0 = \tilde{\beta}_1 x + \tilde{\beta}_2 x^2 + \tilde{\beta}_3 x^3 + \dots$$

and the same formulas apply for the determination of the

impact force as for the case where the deformation was taken proportional to the load, provided we replace β_i by β_1 .

This method of computation is admissible provided the equation for y_{rel} gives sufficiently accurate values corresponding to the load P first obtained; otherwise, the computation must be repeated for a new value of P .

PROCESS FOR OBTAINING THE EQUIVALENT FLOAT BOTTOM AND
DETERMINING THE EFFECT OF THE ELASTICITY
ON THE IMPACT FORCE

a) Mathematical formulation of the problem as an integral equation.— We have given $\eta_0(\xi)$, μ , $A(\xi)$ and seek to determine η_{rel} . From equations (12) and (19)

$$\eta_{rel} = \frac{\xi}{(1 + \mu \xi^2)^3} \frac{A(\xi)}{(u_0(\xi) + u_{rel}(\xi))} \quad (20)$$

and therefore,

$$u(\xi) = u_0(\xi) + u_{rel}(\xi) = \frac{\xi A(\xi)}{(1 + \mu \xi^2)^3 \eta_{rel}}$$

and from equations (1) and (18)

$$\eta = \int_0^x \frac{u(\xi) d\xi}{\sqrt{1 - \left(\frac{\xi}{x}\right)^2}} = \eta_0 + \eta_{rel}$$

so that for the determination of η_{rel} , we obtain the following relation:

$$\eta_0(x) = -\eta_{rel}(x) + \int_0^x \frac{\xi A(\xi)}{(1 + \mu \xi^2)^3} \frac{1}{\sqrt{1 - \left(\frac{\xi}{x}\right)^2} \eta_{rel}(\xi)} d\xi \quad (21)$$

This is a nonhomogeneous, nonlinear integral (Volterra) equation of the second class. No direct methods have so far been developed for the solution of equations of this

kind. In the case, however, where there exists a solution $u(\xi) = 0$ the iteration method may probably lead to the solution.

b) Development into series and approximate solutions.-

Instead of carrying out the iteration process a solution was first attempted by development into series and the following series were tried:

$$A. \quad \pi = p_0 + p_1 \xi + p_2 \xi^2 + \dots$$

$$B. \quad \pi (1 + \mu \xi^2)^3 = p_0' + p_1' \xi + p_2' \xi^2 + \dots$$

$$C. \quad \eta_{rel} = \beta_1' x + \beta_2' x^2 + \beta_3' x^3 + \dots$$

These expressions may be simplified, depending on the type of float, by the elimination, for example, of the first term of the series. None of these series, however, converges in general (corresponding to the nonconverging of the series in the development of $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$ for $x > 1$).

It was then attempted to find approximate solutions but it was not possible to obtain good approximations since the system of equations to be solved was no longer linear so that this method, too, was unsatisfactory.

The possibility of using the iteration method was not tried since it could only be applied to individual cases in connection with a graphical process, and the results thus obtained would then have no general significance.

c) Obtaining solutions by applying the inverse process.- To obtain results of more general application, the inverse process may be employed; that is, we start out with the deformed bottom shape and seek to find the original undeformed shape.

We thus have as given $\eta = \eta_0 + \eta_{rel}$ and the function $A(\xi)$, and we obtain in turn P , η_{rel} , and η_0 :

$$\eta_0 = \eta(\xi) - \frac{\xi}{(1 + \mu \xi^2)^3} A(\xi) \quad (22)$$

As an example, we choose the values

$$\left. \begin{aligned} \eta &= \beta_1 x \\ \mu &= 0 \\ A(\xi) &= \varphi \underline{R}(\xi) = \left(\frac{2}{\pi} \beta_1\right)^2 \psi \underline{R}(\xi) \\ \underline{R}(\xi) &= \alpha_0 + \alpha_1 \xi + \alpha_2 \xi^2 + \dots \end{aligned} \right\} \quad (23)$$

which represent a straight V bottom float of a seaplane having a very large mass. We thus obtain:

$$\begin{aligned} \dots \quad \eta_0 &= \beta_1 x - \frac{x}{\frac{2}{\pi} \beta_1} \varphi \underline{R}(x) \\ \eta_0 &= \beta_1 x \left(1 - \frac{2}{\pi} \frac{\varphi}{\left(\frac{2}{\pi} \beta_1\right)^2} \underline{R}(x) \right) \end{aligned}$$

or with

$$\frac{\varphi}{\left(\frac{2}{\pi} \beta_1\right)^2} = \psi = \frac{\varphi}{\gamma_1^2}$$

$$\begin{aligned} \eta_0 &= \eta \left(1 - \frac{2}{\pi} \psi \underline{R}(x) \right) = \beta_1 \left(1 - \frac{2}{\pi} \psi \alpha_0 \right) x \\ &\quad - \beta_1 \frac{2}{\pi} \psi (\alpha_1 x^2 + \alpha_2 x^3 + \alpha_3 x^4 + \dots) \end{aligned}$$

$$u_0 = k_1 \beta_1 \left(1 - \frac{2}{\pi} \psi \alpha_0 \right) - \beta_1 \frac{2}{\pi} \psi (k_2 \alpha_1 x + k_3 \alpha_2 x^2 + \dots)$$

We have

$$\underline{B} = \frac{\xi}{k_1 \beta_1}$$

$$\underline{B}_0 = \frac{\xi}{k_1 \beta_1 \left(1 - \frac{2}{\pi} \psi \alpha_0 \right) - \beta_1 \frac{2}{\pi} \psi (k_2 \alpha_1 \xi + k_3 \alpha_2 \xi^2 + \dots)}$$

so that

$$\frac{\underline{B}}{\underline{B}_0} = 1 - \psi (k_1 \alpha_0 + k_2 \alpha_1 \xi + k_3 \alpha_2 \xi^2)$$

and at $\xi = 1$, where the maximum impact force is to be ex-

pected,

$$\frac{B_{rel}}{B_0} = \frac{B(1)}{B_0(1)} - 1 = -\psi (k_1 \alpha_0 + k_2 \alpha_1 + k_3 \alpha_2 + \dots)$$

The deformation is:

$$\eta_{rel} = \underline{B}(\xi) \underline{A}(\xi) = \frac{\xi}{k_1 \beta_1} \varphi \underline{R}(\xi)$$

and attains its maximum value for

$$\frac{\varphi}{k_1 \beta_1} [\underline{R}(\xi) + \xi \underline{R}'(\xi)] = 0$$

From the above equation

$$\xi = \bar{\xi} = - \frac{\underline{R}(\bar{\xi})}{\underline{R}'(\bar{\xi})} \quad (\text{if } \bar{\xi} < 1)$$

and therefore

$$\bar{\eta}_{rel} = \frac{\varphi}{k_1 \beta_1} \frac{\underline{R}^2(\bar{\xi})}{-\underline{R}'(\bar{\xi})}$$

$$\frac{\bar{\eta}_{rel}}{\eta(1)} = k_1 \psi \frac{\underline{R}^2(\bar{\xi})}{-\underline{R}'(\bar{\xi})}$$

or

$$-\psi = \frac{\bar{\eta}_{rel}}{\eta(1)} \frac{1}{k_1} \frac{\underline{R}'(\bar{\xi})}{\underline{R}^2(\bar{\xi})}$$

and thus (for $\bar{\xi} < 1$)

$$\frac{B_{rel}}{B_0} = \frac{\bar{\eta}_{rel}}{\eta(1)} \frac{\pi}{2} \frac{\underline{R}'(\bar{\xi})}{\underline{R}^2(\bar{\xi})} (k_1 \alpha_0 + k_2 \alpha_1 + k_3 \alpha_2 + \dots)$$

For $\bar{\xi} > 1$ the greatest deformation is at $\xi = 1$, so that

$$\frac{\eta_{rel}(1)}{\eta(1)} = k_1 \psi \underline{R}(1)$$

or

$$\psi = \frac{\eta_{rel}(1)}{\eta(1)} \frac{1}{k_1} \frac{1}{\underline{R}(1)}$$

and therefore,

$$\frac{B_{rel}}{B_0} = - \frac{\eta_{rel}(1)}{\eta(1)} \frac{\pi}{2} \frac{k_1 \alpha_0 + k_2 \alpha_1 + k_3 \alpha_2 + \dots}{\alpha_0 + \alpha_1 + \alpha_2 + \dots}$$

As a first example (fig. 3a), let the following values be chosen:

$$\eta = \beta_1 x$$

$$\mu = 0$$

and

$$A = \varphi \xi(\xi-1)$$

so that

$$\underline{R} = \xi(1 - \xi) \quad (\alpha_0 = 0, \quad \alpha_1 = 1, \quad \alpha_2 = -1)$$

In this case the bottom (construction A) is deformed between the keel and the chine in such a manner that for a constant impact force η_{rel} would be a parabola.

The maximum deformation is at

$$\bar{\xi} = - \frac{(1 - \bar{\xi})}{1 - 2\bar{\xi}}; \quad \bar{\xi} = \frac{2}{3}$$

so that

$$\frac{B_{rel}}{B_0} = \frac{27}{8} (4 - \pi) \frac{\bar{\eta}_{rel}}{\eta(1)} = 2.89 \frac{\bar{\eta}_{rel}}{\eta(1)}$$

The relative increase in the impact force in this case is therefore 2.89 times the ratio of the maximum deformation $\bar{\eta}_{rel}$ to the dead rise $\eta(1)$.

As a second example (fig. 3b), let the following values be chosen:

$$\eta = \beta_1 x$$

$$\mu = 0$$

and

$$A = \varphi \xi^2$$

so that

$$\underline{R} = \xi^2 \quad (\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = 1)$$

In this case (construction 3b), the bottom is elastic at the sides so that the chine moves up on impact. The form of the A function is then obtained by assuming a rigid bottom with the impact force approximately concentrated near the edge of the impact area.

The maximum deformation is now at $x = 1$, so that

$$\frac{B_{rel}}{B_0} = -2 \frac{\eta_{rel}(1)}{\eta(1)}$$

This construction therefore leads to a decrease in the impact force, namely, by twice the ratio of the deformation at the edge to the final dead rise $\eta(1)$.

For the third example (fig. 3c), let the values chosen be:

$$\eta = \beta_1 x$$

$$\mu = 0$$

and

$$A = \varphi (1 - \xi^2)$$

so that

$$\underline{R} = 1 - \xi^2 \quad (\alpha_0 = 1, \quad \alpha_1 = 0, \quad \alpha_2 = -1)$$

(construction C).

The maximum deformation is at

$$\bar{\xi} = \frac{1 - \bar{\xi}^2}{2 \bar{\xi}}; \quad \bar{\xi} = \sqrt{\frac{1}{3}}$$

so that

$$\frac{B_{rel}}{B_0} = \frac{3}{2} \sqrt{3} \frac{\bar{\eta}_{rel}}{\eta(1)} = 2.6 \frac{\bar{\eta}_{rel}}{\eta(1)}$$

In this case, therefore, the relative increase in the impact force is 2.6 times the ratio of the maximum deformation to the dead rise.

From these examples it may be deduced that under certain conditions the effect of the elasticity of the bottom may be of some importance but in especially unfavorable

cases the effect may even be more considerable, so that no conclusions of general application can be attached to the results obtained from the examples just given.

SUMMARY

For several particular cases of possible float-bottom shapes the impact force due to the elasticity deformation was compared with that obtained for the rigid bottom. The work carried out considers, however, only part of the effect of the elasticity on landing and obviously the less important part. A much more important effect is that of the elastic connection between the principal masses of the seaplane and the floats, and this problem still awaits solution.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

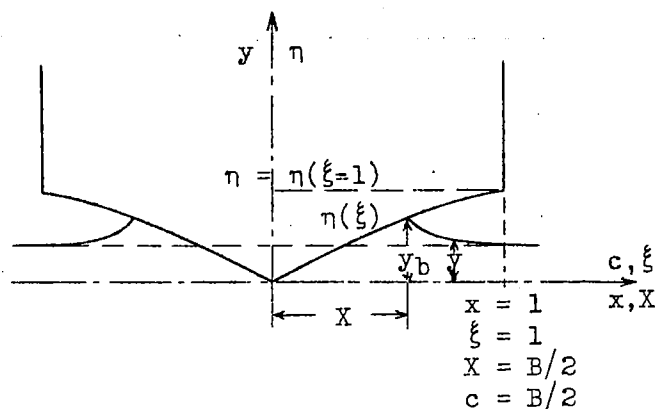


Figure 1.- Notation for rigid V-type float bottom landing on water.

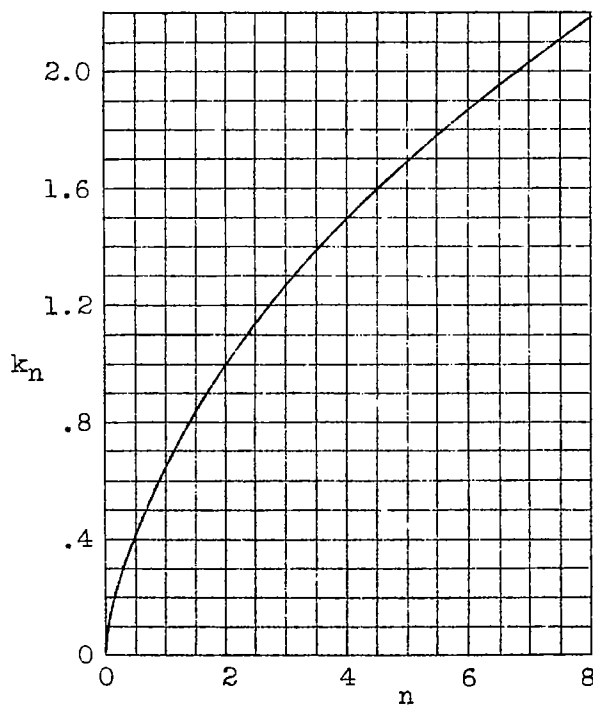


Figure 2.- Values of k_n .

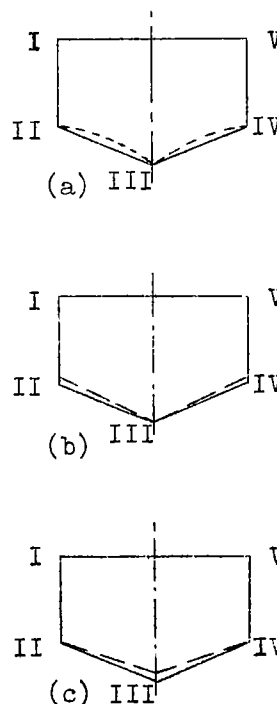


Figure 3.- Several types of possible deformation for float bottoms.

(a) Points I, II, III, IV, and V are mutually fixed and

bottom is deformed between II-III and III-IV.

(b) Points I, III and V are mutually fixed and bottom is deformed at

II and IV. (c) Points I, II, IV and V are mutually fixed and deformation is possible at the keel III.

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