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# AIRPLANE STABILITY IN TAXYING*

By E. Anderlik

One of the vital operating attitudes of an airplane is the time of take-off, and the question arises as to how the dimensions of the landing gear affect the stability of an airplane while rolling in horizontal position.

The problems of the stability theory, presumedly known, refer to the free, level and three-dimensional motion with three and six degrees of freedom. Ordinarily, they are too complicated to afford immediate practical results for the designer; for example, a dynamical stability investigation rarely exerts any essential effect on the design of an airplane.

The stability analysis of an airplane while rolling is much more simplified to the extent that it can be obtained for numerical data which can be put to practical use in the design of landing gear dimensions. Every landing gear type attains to a critical ground friction coefficient that decides the beginning of instability, i.e., nosing over. This study has, in addition, a certain interest for the use of wheel brakes.

Several premises are called for. The airplane rolls over flat, level ground and at steady speed u. The stability of this motion is to be examined by adducing disturbances to this originally uniform motion and follow its course in time. The speed maximum u is assumed such that the gross weight is partly carried by wing lift, and partly supported by the ground. A second premise is that only the wheels touch the ground and the longitudinal airplane axis is parallel with the ground surface.

While rolling the airplane executes a mechanical motion with two degrees of freedom, which can be forced by a certain elevator and throttle setting. It is clear that a

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*"Über die Stabilität des Flugzeugs beim Rollen." Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 28, 1932, pp. 280-283.

comprehensive analysis of the stability would have to include the actually accelerated and decelerated motion at take-off and landing; but mathematical difficulties prevented this general investigation from being completed. The shock absorption was disregarded for similar reasons.

Another important problem is the stability at landing when landing gear and tail skid already touch the ground. This study, significant for brake design, can be reduced to the relative balance of the forces, inertia included, acting on the airplane; but this is beyond the scope of the present report.

Rolling of the airplane with constant speed $u$ is expressed by the following equations of equilibrium:

$$S - R - \frac{\rho}{2} F u^2 c_w = 0 \quad . \; . \; . \tag{1}$$

$$G - N - \frac{\rho}{2} F u^2 c_a = 0 \quad . \; . \; . \tag{2}$$

$$-\frac{\rho}{2} F u^2 l c_m + N a' - R b' = 0 \tag{3}$$

Herein $S$ = thrust, $G$ = gross weight, $N$ = wheel pressure, $R$ = friction, $\rho$ = air density, $c_a$, $c_w$ and $c_m$ the well-known air coefficients, $F$ = wing area, and $l$ = distance of center of pressure $O$ from the axis of the tail surfaces; $a'$ and $b'$ denote rear and height position of point of contact $B$ of the wheel with reference to a system of coordinates, the origin of which is coincident with the c.g. of the airplane.

The friction is

$$R = f N,$$

$f$ being the friction coefficient.

As to the prefixes used in equations 1 - 3, be it noted that the $x$ direction in the direction of motion, the $y$ direction is positive downward. Clockwise moments are positive. $c_m$ positive signifies a nose-heavy moment and carries - (minus) sign.

The disturbed motion of the rolling airplane is represented as plane motion by statement of the speed of an airplane point and by the angular velocity around this point, i.e., about the axis which passes through this point and is at right angle to the plane of motion. Admittedly, the speed changes as the reference point changes, whereas the angular velocity remains constant.

Disregarding the rolling of the wheels, the velocity component of contact point B is horizontal. The motion of the airplane can be expressed by the horizontal velocity of point B and by the angular velocity about this point.

As the airplane changes its originally horizontal position through an angle $\vartheta$ ($\vartheta = 0$ in smooth rolling), the angular velocity is $\omega = \frac{d\vartheta}{dt}$, ($t$ = time). The velocity of the c.g., has the horizontal and vertical components $u_s$ and $v_s$, interdependent with $u$ and $\vartheta$.

$$\left.\begin{aligned} u_s &= u - b' \frac{d\vartheta}{dt} \\ v_s &= a' \frac{d\vartheta}{dt} \end{aligned}\right\} \tag{4}$$

($\vartheta$ = positive for positive rotation of airplane, that is, fore part of airplane rises.)

The horizontal rolling motion is disturbed by the change in horizontal speed $\Delta u$ of point B and by the angle of slope $\vartheta$. The angle of attack changes as $\Delta\alpha$, where

$$\Delta\alpha = \vartheta + \frac{a'}{u} \frac{d\vartheta}{dt} \tag{5}$$

The change in angle of attack, induced by the turning motion of the airplane, is approximately put at $\frac{a'}{u} \frac{d\vartheta}{dt}$.

The equations of motion of the airplane can be expressed in the previously introduced system of coordinates as follows:

$$m \frac{du_s}{dt} = S - R - \Delta R - W + A \frac{a'}{u} \frac{d\vartheta}{dt} \tag{6}$$

$$m \frac{dv_s}{dt} = G - N - \Delta N - A - W \frac{a'}{u} \frac{d\vartheta}{dt} \tag{7}$$

$$J \frac{d^2\vartheta}{dt^2} = M - n'V \frac{d\vartheta}{dt} + (N+\Delta N)(a'+b'\vartheta) - (R+\Delta R)(b'-a'\vartheta) \tag{8}$$

Herein

$$\left.\begin{aligned} m &= \text{mass} \\ J &= \text{moment of inertia} \end{aligned}\right\} \text{ of airplane}$$

$$\left.\begin{aligned} \Delta N \\ \Delta R \end{aligned}\right\} = \text{changes in wheel pressure and friction during disturbed motion}$$

$A$ = lift

$W$ = drag

$M$ = moment of air loads referred to c.g.

$V$ = resultant airplane speed

$n'$ = damping coefficient

In connection with these equations, lift and drag form angle $\frac{\pi}{2} - \varphi$, and $\varphi$ with the horizontal, whereby

$$\varphi = \text{arc tan } \frac{v_s}{u_s}$$

in addition, we assumed $\cos \varphi = 1$ and $\sin \varphi = \frac{a'}{u} \frac{d\vartheta}{dt}$. (The relative speed is shown in Figure 2.) Due allowance was made in the moment equation, that airplane rotation $\vartheta$ modifies lengths $a'$ and $b'$ to $a' + b'\vartheta$ and $b' - a'\vartheta$.

Furthermore, $V$, $A$, $W$, and $M$ are expressed as:

$$\left.\begin{aligned} V^2 &= u_s^2 + v_s^2 = u^2 - 2\,u\,b' \frac{d\vartheta}{dt} + (a'^2 + b'^2)\left(\frac{d\vartheta}{dt}\right)^2 \\ A &= \frac{\rho}{2} F\, V^2 c_a\, (\alpha + \Delta\alpha) \\ W &= \frac{\rho}{2} F\, V^2 c_w\, (\alpha + \Delta\alpha) \\ M &= \frac{\rho}{2} F\, V^2 c_m\, (\alpha + \Delta\alpha)\, l \end{aligned}\right\} \quad (9)$$

where $\alpha$ = angle of attack in equilibrium attitude.

Coefficients $c_a\,(\alpha + \Delta\alpha)$ etc., are written $c_a$ +

$\frac{dc_a}{d\alpha}\Delta\alpha$, etc., and $\frac{d\dot{c}_a}{d\alpha}$ etc., later designated by $c_a$ etc. Note, that $\Delta u$, $\vartheta$, $\frac{d\vartheta}{dt}$ are small quantities whose powers and products can be disregarded. With a thought to (1), (2), and (3) equations (6), (7), and (8) now become

$$m\frac{d\Delta u}{dt} + \Delta u\,\rho F u c_w + f\,\Delta N - b'm\frac{d^2\vartheta}{dt^2} + \frac{d\vartheta}{dt}\rho F u\left[-\frac{a' c_a}{2} + \frac{a'\dot{c}_w}{2} - b\,c_w\right] + \vartheta\frac{\rho}{2}F u^2\dot{c}_w = 0 \tag{10}$$

$$\Delta u\,\rho F u c_a + \Delta N + a'm\frac{d^2\vartheta}{dt^2} + \frac{d\vartheta}{dt}\rho F u\left[\frac{a'\dot{c}_a}{2} + \frac{a' c_w}{2} - b\,c_a\right] + \vartheta\frac{\rho}{2}F u^2\dot{c}_a = 0 \tag{11}$$

$$\Delta u\,\rho F u\,l\,c_m + \Delta N\,(f\,b' - a') + J\frac{d^2\vartheta}{dt^2} + \frac{d\vartheta}{dt}\rho F u\,l\left[\frac{a'\dot{c}_m}{2} - b'c_m + n'\right] + \vartheta\left[\frac{\rho}{2}F u^2 l\,\dot{c}_m - N\,(a'f + b')\right] = 0 \tag{12}$$

Here it was already taken into account that $\Delta R = f\,\Delta N$. There are two degrees of freedom for this guided airplane motion, whereas it requires three equations because of the appearance of the unknown force of reaction $\Delta N$.

As known, the solution for $\Delta u$, $\Delta N$ and $\vartheta$ is written in form $K\,e^{\lambda t}$, wherein $t$ = time and $K$ and $\lambda$ are constants. The definition of $\lambda$ yields the algebraic equation

$$\begin{vmatrix} m\lambda+\rho F u\,c_w & f & -\lambda^2 b'm+\lambda\rho F u\left[-\frac{a'c_a}{2}+\frac{a'\dot{c}_w}{2}-b'c_w\right]+\frac{\rho}{2}Fu^2\dot{c}_w \\ \rho F u\,\dot{c}_a & 1 & +\lambda^2 a'm+\lambda\rho F u\left[\frac{a'\dot{c}_a}{2}+\frac{a'c_w}{2}-b'c_a\right]+\frac{\rho}{2}Fu^2\dot{c}_a \\ \rho F u\,l\,c_m & (fb'-a') & \lambda^2 J+\lambda\rho F u l\left[\frac{a'\dot{c}_m}{2}-b'c_m+n'\right]+\frac{\rho}{2}Fu^2 l\,\dot{c}_m-N\,(a'f+b) \end{vmatrix} = 0 \tag{13}$$

Substituting $\eta$ for $\lambda$, where

$$\eta = \frac{m}{\frac{\rho}{2} F u} \lambda$$

equation (13) assumes, after several rearrangements, the form of

$$\begin{vmatrix} \eta + 2c_w & f & \eta a (\dot{c}_w - c_a) + \dot{c}_w C \\ 2c_a & 1 & \eta^2 a + \eta a (\dot{c}_a + c_w) + \dot{c}_a C \\ 2c_m & f & b - a \quad \eta^2 D + \eta[a\dot{c}_m + 2n] + \left[\dot{c}_m - \frac{N}{\rho F u^2}[af + b]\right] C \end{vmatrix} = 0 \qquad (14)$$

Here we put

$$a = \frac{a'}{l}, \quad b = \frac{b'}{l}, \quad n = \frac{n'}{l}, \quad C = \frac{2m}{\rho F \, l}, \quad D = \frac{J}{m \, l^2}$$

Quantities $c_m$ and $\frac{N}{\rho F u^2}$ can be defined from (2) and (3). With the introduction of the nondimensional quantity $g_o = \frac{G}{\rho F u^2}$,

$$\left.\begin{aligned} c_m &= (a - f\, b)\,(g_o - c_a) \\ \frac{N}{\rho F u^2} &= g_o - c_a \end{aligned}\right\} \qquad (15)$$

is obtained.

When these quantities are inserted in determinant (14), the latter becomes

$$\varphi_o \eta^3 + \varphi_1 \eta^2 + \varphi_2 \eta + \varphi_3 = 0 \qquad (16)$$

Coefficients $\varphi_o, \varphi_1, \varphi_2, \varphi_3$ can be written as:

$$\begin{aligned} \varphi_o &= a^2 - ab\, f + D \\ \varphi_1 &= a^2[f\, 2(g_o - c_a) + \dot{c}_a + 3c_w] - ab[\, f^2 2(g_o - c_a) \\ &\quad + f\, (\dot{c}_a + 3c_w)] + a\, \dot{c}_m + 2n + D(2c_w - 2c_a f) \end{aligned}$$

$$
\left.
\begin{aligned}
\varphi_2 = {} & a^2[f\ (2c_w\ g_o - 2\dot{c}_w c_a + 2\dot{c}_a g_o - 2\dot{c}_a c_a) \\
& + 2c_w{}^2 + 2\dot{c}_a c_w + 2g_o\ (c_a - \dot{c}_w)] \\
& - a\ b\ [2\ f^2 (g_o - c_a)\ (\dot{c}_a + c_w) \\
& + f\ (2c_a{}^2 + 2\dot{c}_a c_w) + 2\ g_o\ (c_a - \dot{c}_w)] \\
& + a\ [2\dot{c}_m c_w - 2f\ \dot{c}_m c_a] + 4\ c_w\ n - 4\ f\ c_a n \\
& + C(a\,[\dot{c}_a - fg_o + fc_a] + b\ [c_a - g_o - \dot{c}_a f] + \dot{c}_m) \\
\varphi_3 = {} & 2C\left[a\left(-f^2 c_a\ (g_o - c_a) + f(g_o - c_a)\ (\dot{c}_a - c_w)\right.\right. \\
& \left. + \dot{c}_a c_w - \dot{c}_w g_o\right) - b\left(f^2 \dot{c}_a (g_o - c_a)\right. \\
& \left. + f[\dot{c}_a c_w - \dot{c}_w\ g_o + c_a\ (g_o - c_a)\ ] + c_w\ (g_o - c_a)\right) \\
& \left. + \dot{c}_m (c_a f + c_w)\right]
\end{aligned}
\right\} \qquad (17)
$$

The condition for the stability is, that the inequations

$$
\begin{gathered}
\varphi_0 > 0,\ \varphi_1 > 0,\ \ \varphi_2 > 0,\ \varphi_3 > 0 \\
\varphi_1 \varphi_2 - \varphi_0 \varphi_3 > 0
\end{gathered} \qquad (18)
$$

are complied with.

The significance of these stability criteria can also be so expressed that the aperiodic proportion of the disturbed motion changes from damping to intensification, when the condition $\varphi_3 = 0$ is complied with. This condition is called static stability; the root in (16) is real, the transition from damping to intensification is noted as $\eta = 0$.

The condition $\varphi_1 \varphi_2 - \varphi_0 \varphi_3 = 0$ is equivalent in so far as $\eta$ becomes purely imaginary, i.e., the periodic portion of the disturbed motion is neither damped nor intensified.

The primary aim of these stability investigations is to ascertain how the stability of the rolling airplane is influenced by the dimensions of the landing gear. The aerodynamic factors, the constants C and D depend on the general character of the airplane. There remain then the quantities $a$ and $b$ which denote the arrangement of the landing gear, and lastly, the coefficient of ground friction $f$.

The stability conditions can be interpreted as functions of quantities $a$, $b$ and $f$. These functions can be utilized in two directions: one, to examine individual airplanes with definite $a$ and $b$ values and assuming the friction coefficient as variable. Then the latter can be numerically defined from the stability equations. A condition of the ground conformably to this coefficient, produces instability when rolling. This may be called "critical friction coefficient," and with this factor, different airplanes, i.e., types of landing gears can be compared.

A second application of the stability investigation is a more general study of the relationship between $a$, $b$, and $f$ in the manner that the effect of size of the landing gear on the stability of rolling motion is to be principally explained.

Apart from the cited structural quantities $a$, $b$, and $f$, two parameters, $\dot{c}_m$ and $g_0$ are equally important. The first is the criterion of the static stability of the airplane in flight, the second depends on the speed of the rolling airplane.

The equation of static stability of rolling motion of the airplane, according to (17) and (18) is:

$$\begin{aligned}\frac{\varphi_3}{2\ C} = a\ [-f^2\ c_a(g_o - c_a) + f\ (g_o - c_a)\ (\dot{c}_a - c_w) \\ + \dot{c}_a c_w - \dot{c}_w g_o\ ] - b\ [\ f^2 \dot{c}_a(g_o - c_a) \\ + f(\dot{c}_a c_w - \dot{c}_w g_o + c_a(g_o - c_a)) + c_w(g_o - c_a)] \\ + \dot{c}_m(c_a f + c_w) \gtreqqless 0 \qquad (19)\end{aligned}$$

An analysis of this (19) reveals the startling effect of $a$, $b$, $f$ and $\dot{c}_m$. When $\alpha > 0$, that is, when the c.g., is back of the landing gear, the static stability is increased, when $b$ becomes large, i.e., the landing gear too high, the static stability decreases. Static stability is also increased when the airplane is stable in the usual sense of the work, i.e., when $\dot{c}_m > 0$.

It should be remembered that $a$ may be increased only to a certain degree in the positive sense, or else it might happen that the tail of the airplane cannot be raised with the elevator at the right time, when taking off. Quantity $b$ is bounded by the propeller.

The unstabilizing action of the friction becomes readily apparent. An increase in angle of attack increases the lift and decreases the wheel pressure and the friction. The decrease in friction denotes a tail-heavy moment in the same direction as the direction of rotation of the airplane.

These problems were studied by means of concrete examples. Part of the data is given in Figures 3 to 6.

The airplane characteristics were:

$$G = 1{,}500 \text{ kg}, \quad J = 300 \text{ kgm/s}^2, \quad F = 22\text{m}^2, \quad l = 4.9 \text{ m}$$

$$C = \frac{2\,m}{\rho F l} = 22.6, \quad D = \frac{J}{m\,l^2} = 0.08$$

$$c_a = 0.43, \quad \dot{c}_a = 3.9, \quad c_w = 0.06, \quad \dot{c}_w = 0.200,$$

$$\dot{c}_m = 0.128, \; n = 1.09$$

Figure 3 shows function $\varphi_3 = 0$ in the $a$, $b$ plane; $f$ is considered as parameter and $g_o = 0.63$. The influence of the friction is manifest. To examine a concrete landing gear type, use the critical coefficient of friction of Figure 3. At low friction the landing gear could be fitted aft of the c.g., but as the friction increases such an arrangement becomes impractical. The influence of the forward position of the landing gear can be seen in Figure 4, where the increase in critical coefficient of friction by increasing $\alpha$ is distinctly noticeable. To examine the influence of the rate of rolling the stability equation $\frac{\varphi_3}{2C} = 0$ is pictured for $g_o = 0.50$, i.e., at a higher speed. As is seen in Figure 5, the critical friction numbers are higher for $g_o = 0.50$ than for $g_o = 0.63$ (see fig. 3) with fixed $a$, $b$ values. The static stability grows with the speed.

In conclusion the influence of $\dot{c}_m$ of the static stability in flight is to be examined. For comparison consider a neutral airplane that differs from the one treated by the value $\dot{c}_m = 0$. According to Figure 6 the straights for the different friction values meet in point $a = b = 0$, that is, they have shifted parallel in comparison to the stable airplane. The critical friction factors for the fixed $a$, $b$ values have become smaller; the stability in rolling is lower.

By analyzing the periodic solutions of the motion equations, it is found that stability prevails when the periodic parts have a damping factor. The transition from damping to intensification occurs when the root of the typical equation (16) becomes purely imaginary, the condition of which is:

$$\varphi_1\varphi_2 - \varphi_0\varphi_3 = 0 \qquad (20)$$

An approximation is to afford a survey relative to this condition and its importance from the designers' point of view.

Function $\varphi_2$ can be simplified by a consideration of arrangement of quantities. The terms of the function multiplied by $C$ are large compared to the others; for instance, putting quantities $a$, $b$ and $f$, which practically range between 0 and 0.5, at = 0.3, the terms multiplied by $C$ yield $\sim$ 19.5, the others - 0.202; accruing therefore, when neglected, in a relative error of the order of $10^{-2}$. A satisfactory approximate calculation is

$$\frac{\varphi_2}{C} \sim a[\dot{c}_a - f\ (g_o - c_a)] + b[c_a - g_o - \dot{c}_a f] + \dot{c}_m \qquad (21)$$

Further simplifications may be attained to from an analysis of $\frac{\varphi_1\varphi_2}{C}$ and $\frac{\varphi_0\varphi_3}{C}$. Quantities $a$, $b$ and $f$ practically range between 0. and 0.5; calculation shows $\frac{\varphi_1\varphi_2}{C}$ to be of the order of size of 2 to 3 between the cited practical limits, and $\frac{\varphi_0\varphi_3}{C}$ to be of the order of 0.01 to 0.06. In the practical case, $\varphi_1$ is always positive.

To arrive at a summary regarding the zero points of the stability equation (dynamic stability) $\varphi_1\varphi_2 - \varphi_0\varphi_3 = 0$ the more simple

$$\frac{\varphi_2}{C} \geqq 0 \qquad (22)$$

will have to be examined, which is decisive so that (20) may become = 0 in the practical range. The proof of this statement follows from the consideration of the arrangement of the quantities. Of course, the accuracy of this approximation must be checked when the order of size of the numerical values is at appreciable variance with the erstwhile quoted data. The transition from dynamic stability to instability is thus characterized by the equation:

$$\varphi_2 = 0$$

This equation is illustrated like the criterion of the static stability. The coefficient of friction, considered as parameter, is put at $0 < f < 0.5$. Each parameter value has a corresponding linear function of $a$ and $b$. These straights, plotted in Figure 7, readily show the dependence of the critical coefficient of friction on the dimensions of the landing gear.

As the coefficient of friction increases a consistently higher $\alpha$ value is necessary to assure stability; i.e., the landing gear must be ahead of the c.g. The influence of $\dot{c}_m$, it will be noted, is similar to that established by the examination of the static stability.

Some remarks about $\dot{c}_m$ are to form the conclusion of this report. By $\dot{c}_m$ was merely meant the measure of the static stability, the differential quotient of the moment curve, and in the numerical example this value was taken from the usual stability calculations. The process must be looked upon as an approximation, the accuracy of which becomes questionable, because of the motion of the airplane in proximity of the ground. This factor can be accounted for by reflecting the airplane respectively, the Prandtl vortices of the wings and of the horizontal surfaces about the plane of the ground and then effect the moment and downwash corrections.

These calculations do not obtain to a complete description of the process on the horizontal tail group. Because of the slipstream a twofold action on the horizontal tail group is noticeable. The velocity of flow is increased; the induction effect of the wings on the control surfaces is influenced by an area of discontinuity formed by the jet boundary. An exact recognition of the phenomena is moreover, rendered difficult because of the existence of a radial motion in the slipstream.

The quoted influences should really be amenable to estimation if $c_m$ were to be correctly introduced in the calculations. Apart from these refinements, the application of the stability theory affords a correct numerical study of the interference of the landing gear designs and of the state or conditions of the ground.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

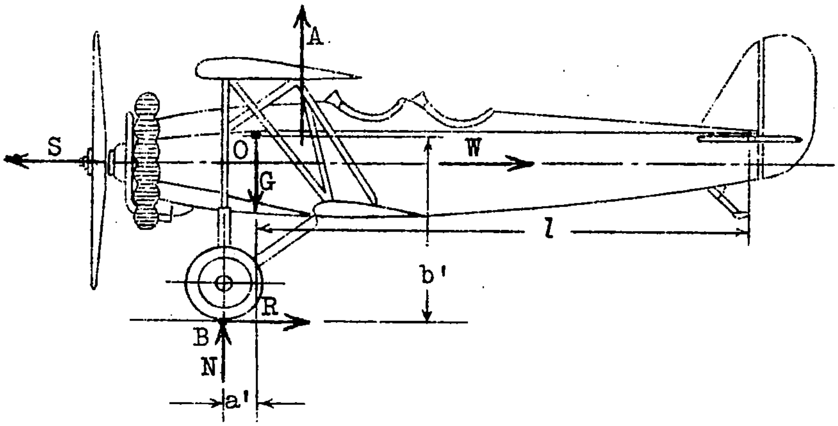



Fig. 1 Forces on the airplane

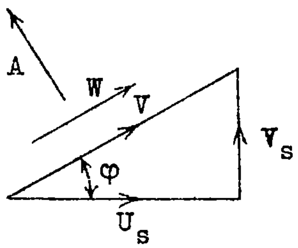



Fig. 2 Velocity component

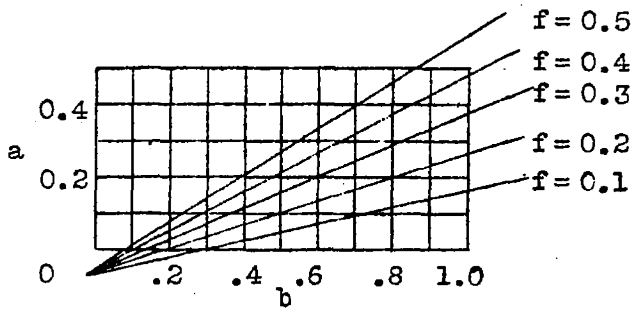



Fig. 3 Function $\varphi_3 = 0$, coefficient of friction f as parameter.

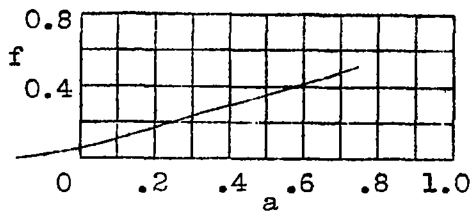



Fig. 4 Effect of forward position of landing gear.

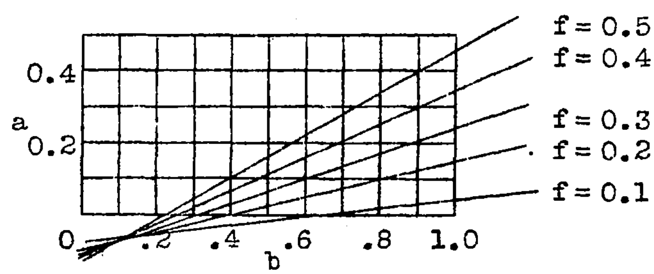
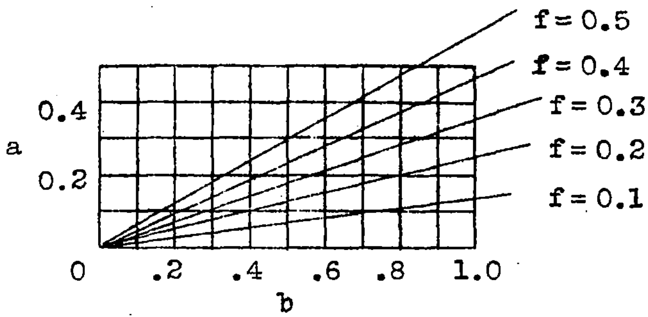
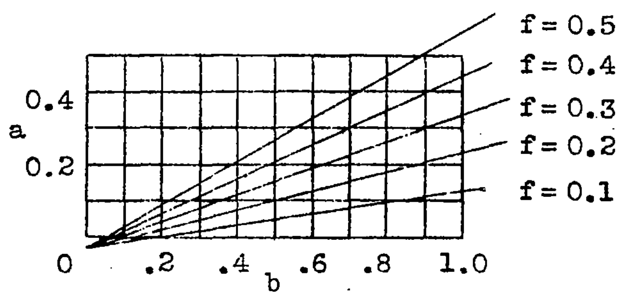



Fig. 5 Function $g_o = 0$, coefficient of friction f as parameter




$\varphi_3\,[a,b,f] = 0$
$g_0 = 0.50$
$c_m = 0$

Fig. 6 Neutral airplane, $c_m = 0$. The straights meet in the origin of the coordinates, that is, are displaced parallel compared to the stable airplane.




$\varphi_2\,[a,b,f] = 0$

Fig. 7 Critical coefficient of friction plotted against landing gear dimensions.