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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 677

STRESSES DEVELOPED IN SEAPLANES WHILE TAKING OFF AND LANDING

By Rudolfo Verduzio

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DEFINITION OF THE PROBLEM

The many recent improvements in aircraft whose weight, however, cannot be increased beyond certain limits, have led to a more intensive study of aircraft structures. This has resulted in a more rigorous application of modern principles of construction, based on a better knowledge of the effects of oscillations and vibrations, thus greatly reducing the frequency of failures. Really dangerous vibrations are due to the coexistence of minimum dimensions, elasticity of materials and faulty design. Moreover, failures may result from deformations causing variations in the aerodynamic forces of the same frequency as the vibrations themselves. There is need, therefore, of a thorough analysis of the forces producing the greatest stresses both in air at rest and when more or less disturbed.

Much progress has been made in the knowledge of aerodynamic reactions, to which engineers in every country have contributed, so that all the principal countries have published instructions for aircraft designers. Progress has also been made in the determination of the possible reactions on every part of an aircraft under all the various conditions to which it may be exposed, especially in taking off and in landing.

On an airplane an elastic system, generally consisting of pneumatic tires and shock absorbers, absorbs the vertical component of the kinetic energy at the moment of landing. This phenomenon (fig. 1) has a double vibration

*"Sollecitazioni alla partenza ed all'ammaramento negli idrovolanti," a paper presented at the twentieth annual meeting of the Societa Italiana per il Progresso delle Scienze, Milan, Sept. 12-19, 1931. L'Aerotecnica, Nov., 1931, pp. 1343-1405.

with one frequency twice the other, but rather slow. The rapid oscillation occurs in the middle with a frequency of about an oscillation and a half in the second minute, which is not excessive considering the elasticity of the system. It is always possible, therefore, to arrange things so that, in normal landings, the maximum impact force does not exceed a certain fixed value of five to six times the weight of the airplane.

In the case of a seaplane, however, the lack of elastic shock absorbers, the presence of which might be quite dangerous, especially in taking off, makes it necessary to give some consideration to the phenomenon of landing. Special consideration must be given the process of taking off, since even moderately rough water may develop rather large stresses. The accident of June 5, in the waters of Capri, to our Minister of Aeronautics, whose skill in piloting is well known, serves to demonstrate the importance of investigating the stresses developed on the bottom of floats. The purpose of this communication is to show what has been accomplished in Italy and other countries and to draw a few useful conclusions.

LANDING OF A FLAT-BOTTOMED FLOAT

In May, 1931, Dr. Friedrich Seewald published a report, in which he gives a general view of the impact phenomena during the take-off and landing of a seaplane. (Reference 12.) After a thorough examination of the head resistance of floats in taking off and of its determination by means of models, establishing its dependence on the angle of impact or on the moment about a transverse axis (already established by the British in 1920), he discusses the impacts of a float while taking off and landing. The phenomenon of alighting on smooth water is much simpler than on rough water, where high momentary stresses are developed.

Qualitatively the impact phenomenon under discussion is defined by elementary mechanics and may be compared with that of a body falling from a certain height on smooth water, coming first in contact with the surface and then gradually immersing at a finite velocity until brought to rest. Assuming the water and the body to be nonelastic, the impact force and, consequently, the pressure would be infinitely great, while the time required for the float to lose its finite velocity would be infinitely small, and the quantity of water instantaneously displaced by the body would be finite. The fact that the body is not destroyed is due to the elasticity of the latter and of the water but, in the case of a hull with a flexible bottom, it is easily demonstrated that the elasticity of the water is so small in proportion to that of the hull that the water may be considered nonelastic.

When the falling body touches the surface of the water, the bottom layer of the body is retarded, while the remaining parts continue their motion, thereby producing an elastic force due to the consequent deformation. The lowest stratum of the body and the upper stratum of the water in contact with it, will be accelerated downward by the elastic force, while the rest of the body is being retarded. So long as the velocity of the upper part of the body is greater than that of the accelerated water, the body will continue to be compressed, and consequently the acceleration communicated to the water underneath will increase up to a maximum corresponding to the maximum compression of the body. From this instant the body gradually resumes its original form, and the impact force gradually returns to zero. In the majority of cases the body is then deformed in the opposite direction and the impact force acts as a tensile force. The complete phenomenon of compression and tension may be repeated several times with gradually damped successive oscillations. Only the part of the phenomenon corresponding to the first compression is of static interest, as this produces the greatest stresses.

In the case of a flat-bottomed hull or float, the lowest stratum of the body, which first comes in contact with the surface of the water, is retarded, while the other parts of the float or seaplane continue, in the first instant, to move at the original velocity and are then gradually retarded as the elastic members are compressed, while a force begins to act on the bottom of the float, and the underlying mass of water is accelerated. The greater the elasticity, the more gradually the force generated on the water increases and, consequently (the total acceleration of the water being distributed over a longer time), the smaller the impact force. This principle applies to any form of float but, when the bottom is flat and parallel to the surface of the water, it depends only on the elasticity, provided the impact force is finite.

The magnitude of the impact force depends not only on the elasticity, but also on the shape of the surface coming in contact with the water, since this determines the mass of water to be accelerated. Hence, in the case of a V-bottomed float, at the beginning of the impact on smooth water, only a small mass of water is accelerated, but, as the immersion increases, the mass of accelerated water also increases until the float has reached its full depth. Thus the V bottom acts as a shock absorber, distributing the force of the impact over a longer period of time. Only the elasticity affects the value of the acceleration, while the V bottom affects the mass of water accelerated. This would call for a pronounced V bottom which, however, would impair the hydrodynamic qualities of the hull or float. A moderate V bottom is the most practical solution.

In landing on rough water, the phenomenon is the same qualitatively, but the impact force cannot be determined by any theory that does not take into account the character of the waves, which vary in height and direction and in the relative position of their surface and of the bottom of the float. A V bottom may even happen to be in the same condition as a flat bottom (tilted), in which case only the elasticity can prevent rupture. Hence it is particularly dangerous to land on rough water with the waves parallel to the direction of landing. Experience has shown, however, that floats with V bottoms behave better, even in this case.

When the waves meet the bow of the float, which is usually the case, the phenomena now under consideration (taking off and landing) are almost the same as on smooth water, but the velocity of the wind is greater, because the component of the air velocity due to the inclination of the surface of the waves is greater. Hence the forces are greater and only a suitable bottom angle can keep them within tolerable limits. Since the structural elasticity of the bottom and of the other parts is already sufficient for the brief period of impact, any greater elasticity would produce appreciable deformations of the bottom of the hull, which might unfavorably affect the take-off of the seaplane. It is fortunate that the oscillations due to the wave motion are considerably slower than those due to the impacts, and it is therefore possible in most cases to construct bottoms with such a degree of elasticity as to be considered rigid with respect to the effect on the hydrodynamic lift, but sufficiently yielding as regards the momentary impacts.

LANDING OF A V-BOTTOMED FLOAT

A thorough investigation of the landing of seaplanes has been made by Dr. Herbert Wagner. (References 16 and 17.) At the instant a V bottom first touches the surface of still water, a certain quantity of water is accelerated downward, so that the float is subjected to an upward force J. The accelerated liquid is thrust out laterally and the level of the water is raised on the sides of the float. Since the atmospheric pressure on the surface of the water is uniform, the pressure gradient due to the displacement of the water is perpendicular to the surface at the point considered, so that the resulting acceleration and velocity of the particles of water near the surface are normal to the surface itself. From this it follows that, at a distance from the surface of contact, the elevation of the water surface being very small, the velocity is very small and nearly vertical. The velocity increases toward the edge of the contact surface, and at the edge of contact, if the velocity is high enough, the water is sprayed laterally. Here the pressure is the greatest, and the energy contained in the spray corresponds closely to that of the impact.

Along a section of the surface of contact, the pressure results from the reaction of the water against its downward motion. In the middle of the V bottom, the water already has a downward motion, while in flowing toward the edge of the contact surface, the water, which even had an upward motion at first, is given a downward motion, especially at the edge of the contact surface. Hence the bottom pressure gradually decreases from a maximum at the edge of contact to the center and then gradually increases. If a float has a relatively small mass, it may happen that, toward the end of the impact phenomenon, at the center of the float, the pressure resulting from the immersion is smaller than the negative pressure due to the existing velocity of the water previously accelerated downward. There is then a negative pressure at the center of the float, while the high pressure at the edges has the final effect of retarding the impact.

Lastly, when the edge of the contact area is at the edge of the bottom, the impact phenomenon ends with the gradual immersion of the float, supported by the static lift of buoyancy, which was negligible at first.

A detailed analysis of the flow leads to the assumption that, in the vicinity of the spray and therefore for large inclinations of the water surface, the velocity of the superficial particles of water may be represented as in Figure 2, i.e., as the geometric sum of a constant velocity tangent to the surface and of a like horizontal velocity, each of which is the velocity increment of the impact zone. Hence the spray has nearly twice this velocity.

As shown in Figure 2, the magnitude of the impact force depends on the initial velocity V_0 at which the float strikes the water, as well as on the ratio of the size of the craft to that of the wave. Thus, if the waves are small (fig. 3), the forward and after steps of the seaplane strike the crests of the waves more or less simultaneously and there is no appreciable pitching. If, on the contrary, the waves are large as compared with the seaplane, the latter may strike the wave on the forward part of the hull and may bounce off, especially if the bottom is broad. The speed of the seaplane is reduced by the impact and, the controls having largely lost their efficacy, the seaplane is in danger of injury from the subsequent impact. If the after part of the hull first strikes the wave, the retardation is slight and the seaplane, after pitching forward, strikes the water at increased speed on the whole bottom area of its forward part and is in danger of being staved in.

POSSIBILITY OF MODEL TESTS

The act of taking off does not differ substantially from that of landing, excepting that the succession of events occurs in the reverse order. The act of taxying on smooth water and especially on rough water, where the greatest stresses are produced, is like that of taking off.

Having thus qualitatively defined the phenomenon of the impact of the hull or floats of a seaplane in the above-mentioned maneuvers, let us see whether it is possible to determine the magnitude of the impact force and its distribution or, which amounts to the same thing, the distribution of the water pressure and the accelerations undergone by the c.g. of the seaplane. Since the greater stresses are produced on rough water, in which case no rigid theory is possible, and since, on the other hand, the theoretical investigation is only approximate, even on smooth water for which there are experimental correction coefficients and factors, it is necessary to resort to experimental results for establishing the maximum limits of the impact forces and accelerations, while the more or less approximate theories give us an idea of the magnitudes involved in the impact phenomenon under consideration.

When a float or hull strikes the water or moves along its surface, a force is produced which is chiefly the resultant of three components, namely, the frictional resistance of the surface under water, the inertia of the accelerated particles of water and a force due to the water pressure, which varies with the depth of immersion. These three principal components do not follow the same laws when the measurements are varied. The forces due to the inertia of the water obey the law of the squares for the resistance, or the force increases proportionally to the product of the density of the water, the area of the immersed portion of the body and the square of the velocity (laws of Newton). The forces due to the water pressure vary according to the laws of Froude, geometric similarity being assumed, on condition that the ratio between the impact and static pressures remains constant in both cases. The need of respecting the above-mentioned laws defines the relations between the velocities and dimensions of similar bodies. Hence, if there were no friction, it would be possible to transfer the results obtained with any float to another geometrically and structurally similar one. However, since there are frictional forces, the transfer can be made with respect to the magnitude of Reynolds Number. This new condition reduces the ratio of similarity to unity for the same velocity. It is concluded, therefore, that exact results must be obtained through experimentation with full-sized craft.

The impact forces in which we are now interested are, however, of a higher order of magnitude than the frictional forces developed in taxying, so that we may disregard the frictional forces and those which form waves, in comparison with the forces of inertia, and therefore consider only the laws of Newton in the transition from models to full-sized craft. It should be noted, however, that the elasticity of the whole greatly affects the magnitude of the impact forces and, since elastic similarity is impossible between a small model and a full-sized craft, only the experiments made with the latter can be considered.

Experiments with models can serve only for floats with sharp V bottoms, in which the elastic effect is almost negligible. Experimentation with models may also serve for the general investigation of impact phenomena, provided, however, the factors present have been defined; or for special investigations. In this connection we will mention the experiments of S. Watenabe (reference 18) made with cylinders terminating in cones of 160° at the vertex and dropped into still water,, which showed (fig. 4) that the theoretical curve is closely approximated to the experimental one by means of an empirical correction, namely, by multiplying the relative ordinates by the constant coefficient 1.16; or as if the angle of aperture of the cone were somewhat greater, i.e., the aperture formed by the water raised around the cone (fig. 5). This also explains some other discrepancies between the theory and Watenabe's experiments.

The preceding restrictions do not apply, however, to researches in connection with the motions of rolling and pitching on the water, since the impacts, being of very short duration, produce accelerations considerably smaller than those due to the hydrodynamic and aerodynamic reactions on the hull.

BRITISH FULL-SCALE EXPERIMENTS

The oldest important experiments with full-sized seaplanes, regarding the stresses developed on the bottom of the floats while taking off and alighting, were made in 1919 and 1920 by the British National Physical Laboratory in collaboration with the Marine Aircraft Experimental Establishment, Isle of Grain.

Experiments with the very similar flying boats F 3 and H 16, gave take-off speeds of 104 to 113 km (64.6 to 70.2 mi.) per hour for the F 3, and 115 to 123 km (71.5 to 76.4 mi.) per hour for the H 16, and respective weights of 4,500 and 5,000 kg (9,921 and 11,023 lb.). In these experiments it was found that, when taxying on rough water at a speed between 50 and 70 km (31.1 and 43.5 mi.) per hour, the pressures were greater than in still water, and that the maximum pressure of about 0.456 kg/cm² (6.49 lb./sq.in.), encountered on the H 16, was near the axis of the hull about halfway between the bow and the step.* An exceptional pressure of 0.576 kg/cm² (8.19 lb./sq.in.) was found near the step, behind which there was a pressure of only 0.03 kg/cm² (.427 lb./sq.in.). At the extreme front end, the pressure did not exceed 0.183 kg/cm² (2.60 lb./sq.in.). The maximum pressures were recorded while taxying rapidly on the crests of the waves. While taking off, the maximum pressure of 0.443 kg/cm² (6.30 lb./sq.in.) was encountered halfway between the bow and the step, when the waves were 70 cm (27.56 in.) high and the wind was light. Near the step the pressure was about 0.281 kg/cm² (4.00 lb./sq.in.). In a normal landing on smooth water the maximum pressure was about 0.281 kg/cm². In rough water it reached 0.443 kg/cm² (6.30 lb./sq.in.) near the keel in the middle of the front part of the float, but this pressure was local, the pressure in the vicinity being only 0.352 kg/cm² (5.01 lb./sq.in.). In a fast landing, a pressure of 0.611 kg/cm² (8.69 lb./sq.in.) was registered at the same point and pressures of 0.352 and 0.443 kg/cm² (5.01 and 6.30 lb./sq.in.) in the vicinity.

Similar experiments were undertaken in 1924 on a P 5 flying boat weighing 5,210 kg (11,486 lb.), but the shocks produced in the various maneuvers were no less severe than in 1920, though the pressures were lower. No measurement was made aft of the step. The results of these important experiments were published in Reports and Memoranda Nos. 683 and 926, of the British Aeronautical Research Committee. (References 1 and 2.)

Important investigations were recently made in the United States. We will first mention, however, a few experiments made in England by the Short Brothers in 1929. The object of these experiments was to determine the water pressures on the bottom of seaplane floats moving at a high uniform speed. We represent in Figure 6 the distribution of the pressures measured on a flat plate towed at 16.5 km (10.25 mi.) per hour with an incidence of 10°, and in Figure 7 the same distribution under the bottom of a float model, corresponding to full-scale tests at a

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*It appears that the step was invented by Ramus in 1872, when there was no such use for it as now. Ramus wished to use it on ships, but Froude readily demonstrated that it would be only a disadvantage.

speed of about 55 km (34 mi.) per hour. A. Gouge, General Manager and Chief Designer of the Short Brothers, observed that, since the results may differ as much as 10 per cent from those of full-scale tests, only the lines of equal pressure are approximately correct. The maximum pressure was found to be on a part of the keel in front of the main step. This point of maximum pressure shifts as the speed increases, its change in location being a function of the speed. For a given point X, suitably selected, the maximum measured pressure is 0.422 kg/cm² (6.00 lb./sq.in.), corresponding to a speed of 70 km (43.5 mi.) per hour. Above this speed, the location of X is outside the water, and the pressure at that point is zero. (Fig. 8.) The distribution and intensity of the water pressures on the bottom vary with the sharpness of the bottom.

Many British treatises have recently been published on the problem of the directional stability of seaplanes while taking off and landing and on the total water resistance to the floats.

AMERICAN FULL-SCALE EXPERIMENTS

The most important and conclusive investigations, both as regards the quantity and the quality of the results obtained, were made by the National Advisory Committee for Aeronautics at the request of the Bureau of Aeronautics of the Navy Department. (References 13, 14, and 15.)

The results were published in Technical Reports Nos. 290, 328, and 346 by F. L. Thompson, in the years 1928, 1929, and 1930, respectively, and refer to tests made on floats and hull, as follows:

1. On a seaplane with central float;
2. On a twin-float seaplane;
3. On a flying boat.

The seaplane used for the first series of tests was a Vought UO-1, a two-place, single-engine biplane with wooden float. Its specified stalling speed was 55.5 miles per hour, and its gross weight was 2,764 pounds. The single step was 2.5 inches high, and the angle of the after keel was 5°.

The seaplane used for the second series of tests was a TS-1 single-place, one-engine biplane, that could be equipped with either floats or wheels. As a seaplane it had a specified gross weight of 2,123 pounds and a landing speed of 60 to 65 miles per hour. The single step was 2 inches high at the keel, and the angle of the after keel was 5.5°.

In both seaplanes the deck line was parallel to the thrust axis. In the UO-1 the bottom V was 140° (angle of V of 20°), remaining uniform from bow to stern. In the TS-1 the bottom V was 146° (angle of V of 17°), being almost uniform toward the bow, but increasing to 148° (angle of V of 16°) at the stern. The essential characteristics of these two seaplanes are shown in Figures 9 and 10.

The flying boat used in the third series of tests was a Curtiss H-16 twin-engine biplane weighing about 10,000 pounds and landing normally at a speed of about 50 miles per hour. It was constructed of wood. The side sponsons, or fins, extended the bottom lines considerably beyond the true chines. It had two steps, the main step being the forward one. The keel angle between the steps was 4°. The bottom V had an aperture of 137° at the stern and 138° at the bow. The geometrical characteristics are shown in Figure 11.

At various points or stations on the bottoms of the floats, as indicated in Figures 9-11, the water pressures were measured by means of special recording units, each unit being a solenoid which deflected a beam of light by steps when the current was varied by the action of four pistons. Each piston was held in place by a spring of known tension.* When the force acting on the piston exceeded the tension of the spring, it caused a slight displacement of the spring, thereby closing an electric circuit which indicated that the original tension of the spring had been exceeded. Each unit contained four of these pistons or plungers with four distinct progressive adjustments of the springs. In this way the effective pressure on the bottom was comprised between two successive adjustments.

*The pistons were actually restrained by air pressure, rather than by springs as stated in the original text. The general principle of operation was as stated, however.

The object of the tests being to reproduce the most difficult conditions for landing and for other maneuvers of a seaplane on the water, they were conducted with the greatest skill and boldness, and not without danger to the pilot, since the stresses were liable to rupture the planking or other parts exposed to the water. The tests were:

a) Take-offs from smooth and rough water;
b) Taxying on smooth and rough water;
c) Landing with engine stopped;
d) " " " running;
e) " " bouncing or porpoising;
f) " at high speed;
g) Cross-wind landing with wind from the right;
h) " " " " " " left.

The taxying tests were made at low speed, in order not to have the hull on the step, and consequently unresponsive to the controls, and at different speeds, or on the step, with the craft obedient to the controls. These tests yielded a complete series of data from which were derived the following diagrams pertaining to the static properties of the flying boat.

The tests made with the UO-1 led to the observation that the highest pressures occurred immediately forward of the step for the full width of the float bottom. Going forward from the step, the region of high pressures narrowed toward the keel and the magnitudes of the maximum pressures decreased. Figures 12 and 13 give a clear idea of the phenomenon.

The maximum pressures occurred in rough water, especially in taking off and in porpoising, or in fast landings and in taxying, or even in poor handling on perfectly smooth water. The latter may produce high pressures even where (the extreme stern, for example) rough water does not generally produce excessive stresses.

The water pressures on the bottom of the floats occur as successive impulses. It was noted, however, that the highest pressures occurred in the vicinity of and forward of the step, and with greater frequency, but their duration was less than 0.05 second and often much less; but other pressures with a duration of even 0.1 to 0.5 second occurred at other points simultaneously with the above and were repeated several times during a single run. The duration of the pressures always diminished, however, toward the bow. Abaft the step the pressures were of very short duration (0.02 to 0.01 sec.). These results agree very well with those obtained by the British in 1920 and 1924, to which we have already referred.

The second series of experiments, based on the results of the first, was more conclusive and enabled the determination of accelerations whose maximum values (up to 4.3 g) were obtained in fast landings and in porpoising. The pressures during these maneuvers are shown in Figure 14, which may be considered as a sufficiently accurate representation of the distribution of the maximum pressures in hard landings with porpoising, although the maximum pressures do not occur simultaneously. Since these are limited to small zones and short periods of time (0.01 to 0.05 sec.), Figures 15 and 16 give an idea of the duration and travel of the high pressures above certain values already established, corresponding to two steep landings with porpoising. In these figures there are two series of curves, which represent the travel and duration of pressures exceeding 5 to 6 lb./sq.in. along the keel and along the chine. The areas subject to high pressure are shown by the shaded areas in Figures 15 and 16 on one-half the projected float area. The intervals were chosen to show the areas when the high pressure is under the c.g., in the middle of the forebody and near the bow. In the middle of the forebody the high-pressure area is a maximum, although the pressure is lower than at the other two points. (Fig. 14.)

In addition to the above pressure, concentrated in a small area, it is necessary to consider a smaller pressure acting over a larger region and of a duration of at least 0.25 second following the high pressure. This was found to have a mean value of about 3 pounds per square inch. It may therefore be assumed that this pressure existed from the main step to the position of the high-pressure area at any instant. Figure 17 is a rectangular representation of the pressures on the bottom of the forward part of the float taken from Figures 14-16. The existing loads are obtained by multiplying these pressures by the corresponding areas on which they act. Figure 17 shows the area of the diagram, which evidently results in kilograms per meter. This, multiplied by the aperture of the floats, gives the effective load. There is also shown, in correspondence with each diagram, the ratio between its area and the weight of the aircraft divided by the corresponding aperture of the float. Hence this ratio represents the dynamic load factor or, which amounts to the same thing, the contingency factor for the given evolution.

The maximum contingency factor in the preceding tests was found to be 6.8, corresponding to an acceleration of 4.3g at the c.g. The discrepancy in these figures is due chiefly to inaccuracy in the assumed load distribution, particularly in the sustained pressure extending from the step to the high-pressure area. A small difference due to flexibility of the structure is also to be considered, and there is a possibility that the load is unequally distributed between the two floats. Lastly, the point of application of the resultant force can be determined from the load distribution.

This investigation ended with the distribution of the pressures abaft the step and in its immediate vicinity, where negative pressures occurred in taking off. These negative pressures, which fluctuated greatly, were quite low and are of but little importance for the static structure of the float bottom. At high speeds, however, they may cause considerable vibrations of the bottom and must be taken into account.

The third series of tests, made with a large flying boat, completed the program. The results in landing and in taxying are plotted in Figures 18 and 19, which confirm what we have already said and show that, in landing, the maximum load is near the step, while, in taxying, there is a more extensive region where the pressure is greatest at the step and keel and decrease rapidly toward the bow and chine, forming a triangular distribution. Between the two steps there is still considerable pressure. Abaft the second step the pressure is very small.

The pressure distribution and its relation to the reaction of the water is shown in Figures 20 and 21, which represent the pressures and accelerations in two exceptionally hard normal landings. Figure 20 corresponds to a glide without leveling off, so that the heaviest shock (acceleration 4.7g) is experienced at the first contact with the water. Figure 21, on the other hand, represents a landing with light successive shocks with the heaviest one last (2.5g), after about 3 seconds. Both figures show a very small duration of the high pressures. On examining these figures and the location of the pressure stations (fig. 11), the high water pressures are found to act on a small portion of the whole bottom, with triangular distribution.

The maximum accelerations were 4.7g vertical, 0.9g longitudinal, and 0.7g lateral. The maximum vertical acceleration occurred in the above-mentioned steep landing, while the maximum longitudinal acceleration was experienced in taxying on large waves, and the maximum lateral acceleration was produced while landing in a cross wind or maneuvering on large waves. In these tests it was found that the vertical acceleration of the c.g. was about 2g less than that of the bottom.

The following may be considered as the critical load conditions:

a) Vertical, applied under the c.g. in landing;
b) Vertical, applied in the middle of the forebody of the hull, due to the effect of the waves in taking off;
c) Longitudinal, while taking off or landing on rough water;
d) Lateral, while landing in a cross wind.

Figure 22 represents the load corresponding to case a), and the inclination of the boundary of the pressure region can be held at 30° on the plane of the keel. The distribution of the load between the high-pressure region (which can be kept uniform) and the low-pressure region (also to be kept uniform) is in the ratio of 3.6 : 1.1. Figure 23 represents the load corresponding to case b), the limiting angle again being assumed to be 30°, but the high pressure cannot be kept uniform and varies from keel to chine in the ratio of 1 : 0.4, while the load distribution between the two zones is in the ratio of 1 : 1. Lastly, it is well to observe also that the ratio of the loads in cases a) and b) is about 1 to 0.64.

This triple series of tests leads to the following conclusions:

A) Pressures on the Bottom

For small seaplanes with lateral or central floats:

1. The maximum vertical pressures are about 0.7 kg/cm² (9.96 lb./sq.in.) at the step and both for special landings on the stern and near the bow;

2. Only the region near and abaft the step has light and even negative pressures;

3. The maximum pressures act on a small area and only for 0.01 to 0.05 second;

4. The secondary pressures are always small in comparison with the maximum, and are greater near the step;

5. The conditions for the widest distribution of high pressures occur in landings accompanied by porpoising and tend to develop high pressures near the bow. The greatest accelerations of the c.g. of the seaplane are also produced in this case.

For large seaplanes with central hull:

6. The maximum pressures occurred while landing and were about 1 kg/cm² (14.22 lb./sq.in.) near the step and dropped to 0.8 kg/cm² (11.38 lb./sq.in.) at the keel on the middle of the bottom of the forebody of the hull.

7. Between the steps there was a nearly uniform pressure of 0.55 kg/cm² (7.82 lb./sq.in.).

The distribution of the maximum pressures near the step is very similar in all three cases, but their magnitude is greater for the flying boat. This is due to the inclination of the keel toward the bow (figs. 9-11), which localizes the pressures, as also to the angle of the bottom V and to the landing speed. The fact that the floats showed high pressures at the stern, which the hull did not, is attributed to greater pitching moments in the float seaplanes than in the boat seaplane due to the greater height of the c.g. above the step. This also explains the greater bow pressures in the first two series of tests.

B) Accelerations

8. the vertical acceleration component of 4.7g for the c.g. is not exceeded in a dangerous landing, but the bottom of the float probably undergoes an additional acceleration of 2g in the hardest landings. In taking off, however, it may be assumed that the vertical acceleration of 3g is not exceeded, even when the water is rough.

9. In taking off from rough water, there were horizontal accelerations of 0.9g and in landing with a cross wind there was an acceleration of 0.7g, but, under these conditions, there was danger of submerging the wing.

C) Contingent Load

As regards the magnitude of the contingent load acting on the bottom of the float, it may be assumed to be the mass of the seaplane multiplied by the acceleration of its c.g., a fact which is verified in practice, the part of the water reaction absorbed by the flexibility of the bottom being offset by the load supported by the wings.

FULL-SCALE GERMAN TESTS

Another method of measurement was used by the Germans in their 1929 tests. This consisted in scratching with a diamond point on glass, without any transmission lever, the elastic deformations of the struts connecting the float with the fuselage and sometimes those of the resisting portion of the bottom of the float. The records thus traced were measured with a microscope. Vibrations and the effects of inertia were thus avoided. The bottom pressure was recorded by a similar instrument. A thin circular plate, fitted over the portion of the bottom where it was desired to measure the pressure, transmitted the deflections produced by the pressure without the use of a lever or other similar recording device. There was no danger of secondary effects from the oscillation of the plate itself, because of the much greater frequency than that to be measured. The seaplane used for the tests was a Heinkel HE9, D1617 single-engine, two-seat monoplane with two lateral floats, attached by steel-tubing struts of 48 kg/mm² (68,273 lb./sq.in.) tensile strength. It had a weight loaded of 3,000 kg (6,614 lb.) moment of inertia about the lateral axis of 1,000 kg m s² (7,233 ft.-lb.sec.²) wing loading of 63.2 kg/m² (12.94 lb./sq.ft.), power loading of 4.25 kg/hp (9.24 lb./hp), and a landing speed of 80 to 90 km/h (49.7 to 55.9 mi./hr.). The floats had either flat bottoms, with a weight of 143 kg (315.3 lb.) each and a transverse moment of inertia of 49.9 kg m s² (360.9 ft.-lb.sec.²) or V bottoms with $\beta = 161^\circ$, a weight of 147.3 kg (324.74 lb.), and a transverse inertia moment of 47.6 kg m s² (344.3 ft.-lb.sec.²). Each float had a volume of 3 m³ (105.94 cu.ft.). The other dimensions are shown on Figure 24. The results of the tests, as published by Wilhelm Pabst (reference 10) can be grouped in two categories: one corresponding to the stresses in the connecting members between the floats and the fuselage; the other corresponding to the pressure measurements on the bottom.

The stresses in a member can be divided into principal stresses corresponding to the structural arrangement and into secondary stresses due mainly to the moments, to the joints, to the welding, and to the vibrations. Many tests were therefore necessary for obtaining a clear idea of the internal stresses but, given the form of the tubes, the tension in the extreme forward fibers could be assumed to be proportional to the stressing of the members, and the total tension of the axis is therefore determined. The impact forces are obtained by finding the resultants of the vertical and horizontal components of the forces in the members for each point of attachment.

The tests consisted of landing with idling engine and of taking off from relatively calm and rough water in winds up to 8 m/s (26 ft./sec.) and gusts up to 13 m/s (43 ft./sec.).

Figures 25 and 26 show the directions and points of application of the impact forces in taking off and landing, while Figures 27 and 28 show the time intervals between the impacts during the same maneuvers. Figures 27 and 28 also give the load factors corresponding to the various impacts, which closely approximate the factors obtained in America.

The fact that some forces are not normal to the bottom of the float, as they should be, since the water can act only perpendicularly to the bottom of the float itself, the frictional force being negligible, may be due to errors of measurement or interpretation, or to the tubes not being of exactly the calculated cross sections, with possibly neglected fixation moments and possible irregularities of the float bottom. Lastly, Figure 29 represents the curves of the maximum vertical-impact forces encountered in the structures of the floats in the various take-offs and landings, and for various seaways. (Figures 38 and 39.) These curves terminate in straight lines passing through the step. In fact, since the resultant must be at a distance of half the length of the contact surface, which is rectangular, the impact force must diminish in proportion as the length of contact diminishes, other conditions remaining the same. The inclination of the straight lines depends on the landing speed, the angle of impact and the shape of the float bottom. Since, as we shall see, the length of the contact surface in taking off or landing depends on the form of the float bottom, other conditions being the same, the inclination of these lines can give the measurement of the effect of the bottom V. The theory really indicates a somewhat greater effect, but this is attributable to the deformation of the bottom and to the effect, recorded in the Watenabe tests, of raising the water on the sides of the float.

Since, for a given float, the magnitude of the shock depends on the length of the contact surface, which depends on the condition of the water, this length may be taken as a criterion of the landing possibilities of said float. The ratio between this length and that of the forebody of the float may be taken as the criterion of the landing safety, independently of the seaway and of the pilot's skill. Lastly, knowing the maximum length of the impact surface for the normal landing speed and considering the maximum impact angle (taking account of the weakest part of the float bottom), one can determine the water conditions under which the given seaplane can take off and alight.

The pressure measurements on the bottom are not very important, because of their small number. The single instrument was used for a wooden float of the HE5 type and installed 35 cm (13.78 in.) forward of the step and 20 cm (7.87 in.) from the keel.

a) On smooth water.- While taking off, pressures were measured of 1 to 1.35 kg/cm² (14.22 to 19.20 lb./sq.in.) with one case of 1.6 kg/cm² (22.76 lb./sq.in.); while landing, 1.1 to 1.6 kg/cm² (15.64 to 22.76 lb./sq.in.), with two cases of 1.95 and 2.1 kg/cm² (27.73 and 29.87 lb./sq.in.).

b) On rough water.- While taking off, 1.25 to 1.9 kg/cm² (17.78 to 27.02 lb./sq.in.); while landing, 1.45 to 1.75 kg/cm² (20.62 to 24.89 lb./sq.in.); frequency of 70 to 100 per second.

Measurements were also made of the deflections of parts of the bottom, such as the planking, under the action of pressure, and the mean load was calculated by means of previously determined constants. The results, though less accurate than those obtained by direct measurement of the pressure, agree with the results of the American experiments already recorded.

KARMAN'S THEORY (Reference 6)

Professor Theodor von Karman, while in the United States in 1929, conceived a simple theory for determining a formula for the pressure on the bottom of a float while landing, and developed an equation which gives the approximate magnitudes of the forces involved.

Starting with the assumption that, at any instant, the inertia of the seaplane and of the masses of water and air involved is constant, and neglecting the mass of the air, which is small with respect to the mass of the water and which corresponds to a half-cylinder of radius c, one may write

$$V = \frac{V_o}{1 + \frac{\rho \pi c^2}{2 m}} = \frac{V_o}{1 + \mu} \qquad (1)$$

where (fig. 30), in the immersion time t, with velocity V in the liquid of density ρ, of the float with a bottom angle of $\beta^\circ = (180 - 2\alpha)$, c is the half-width of the immersed part of the float, of which we will consider a length equal to 1 and assume that the landing speed is V_o and that the corresponding mass per unit length is m. For simplicity we may write the ratio of the masses

$$\mu = \frac{\rho \pi c^2}{2 m} \tag{2}$$

Since y represents the depth of immersion in the time t, we may write

$$V = \frac{dy}{dt} = \tan \alpha \frac{dc}{dt} \tag{3}$$

The impact force per unit of length will be

$$\frac{J}{l} = m \frac{d^2 y}{dt^2} = \frac{\pi \rho V_o^2}{(1 + \mu)^3} \frac{1}{2 \tan \alpha} \tag{4}$$

and the mean pressure will be

$$p_m = \frac{J}{2c} = \frac{\pi \rho V_o^2}{(1 + \mu)^3} \frac{1}{2 \tan \alpha} \tag{5}$$

Evidently the maximum value of this mean pressure is at the first contact, when

$$\max p_m = \pi \rho V_o^2 \frac{1}{2 \tan \alpha} \tag{6}$$

which corresponds quite well with the American tests.

The possible objection to this formula is that, when the bottom angle is 180°, $p_m = \infty$. This is due to the fact that Karman began with the assumption that water is incompressible, and that the float bottom is perfectly rigid. By assuming, instead, that the momentary pressure increase in a fluid occurs with the velocity v of the propagation of sound in the fluid, we obtain

$$p_m = \rho v V_o \tag{7}$$

which gives exaggerated values, because no allowance is made for the elasticity of the bottom and for that of the whole seaplane structure.

THEORY OF THE FLAT ELASTIC BOTTOM

A year after Karman, Dr. Wilhelm Pabst published a more complete theory of the landing impact of seaplanes. (Reference 9.)

After calling attention to the fact that, considering the velocity of seaplanes, the water resistance is largely due to the mass of water accelerated, the wave-producing and frictional resistances being negligible, he shows that the force of the impact is greatly affected by the elasticity of the seaplane, since the compressibility of water is negligible in comparison. For the determination of the total impact force, he shows that the portion of the float bottom subjected to the impact depends on the mutual positions of the float bottom and wave surface coming in contact and that it is necessary to make simplifying assumptions, such as are made by German naval architects, for certain water conditions and lastly to define certain forms of landing. He then assumes that the maximum shock supportable by a seaplane is produced in landing as shown in Figure 31, or in taking off at insufficient speed.

As Karman shows the accelerated mass of water to be that contained in a half-cylinder of diameter equal to the width of the immersed part of the bottom, thus, adopting the hypothesis of Lamb and assuming the density ρ of the water to be constant, Pabst considers, for an absolutely rigid plate of width $2c$ and infinite length, a value of the accelerated water mass given by the formula

$$\Delta M_a = \rho \frac{\pi}{2} c^2 \Delta l. \qquad (8)$$

But, differing from Karman, he assumes that this formula, valid for infinitely long bottoms, is not applicable in practice, due especially to the fact that the width $2c$ of the zone of contact is greater than the length l, and therefore the value of the accelerated mass is modified. Pabst therefore made an accurate series of tests for determining the mass of water accelerated by a plate of finite length and came to the conclusion that (fig. 32), for large values of $l/2c$, the curve is parallel to the line $\rho \frac{\pi}{2} c^2 l$ (formula 8), while the curve approaches $\rho \frac{\pi}{4} c\, l^2$ for very small values. For values of $\frac{l}{2c} = 1$ to

2, which he assumed to be the values corresponding to float bottoms, it is necessary to subtract the constant $\rho \frac{\pi}{2} c^3$ (which must be considered as the edge effect) from $\rho \frac{\pi}{2} c^2 l$. Hence the mass of water accelerated by the seaplane is

$$M_a = \rho \frac{\pi}{2} c^2 l \left(1 - \frac{c}{l}\right) \tag{9}$$

Figure 33 is a schematic representation of a seaplane-float system. The mass M of the seaplane is concentrated in its c.g. A spring, assumed to have no mass and exerting a force of $J = Kf$ is connected with the float bottom, also assumed to be without mass. The latter accelerates a mass of water M_a in an immersion time t initiated with a vertical velocity V_o and being (during the time t) the weight of the seaplane (minus the lift of the wings) νP. The equations of motion (considering the elasticity and disregarding the damping) are then

$$\begin{aligned} M \frac{d^2 x_1}{dt^2} &= K f - \nu P \\ M_a \frac{d^2 x_2}{dt^2} &= - K f \end{aligned} \tag{10}$$

where $x_1 - x_2 = L - f$, L being the initial length of the spring. For flat bottoms M_a = constant, while for V bottoms with straight sides $M_a = \varphi\, x_2{}^2$. In general, for bottoms of any shape, $M_a = f\,(x_2)$. The integrations can be made either graphically or analytically. For flat bottoms the maximum impact force is

$$J_{max} = \sqrt{(\nu\, g\, M)^2 + V_o{}^2\, K\, M} + \nu\, g\, M \tag{11}$$

in which

$$M = \frac{M\, M_a}{M + M_a}$$

For a first approximation, observing that the mass M_a is about 20 per cent of the mass of the seaplane, that ν is less than unity, and that K is very large in seaplanes, we can write

$$J_{max} = V_o \sqrt{K \frac{M\, M_a}{M + M_a}} = V_o \sqrt{K\, M\, \varphi} \tag{12}$$

The formula, obtained by integration from equations (10),

$$J = Kf = \sqrt{(\nu g M)^2 + V_0^2 \ KM} \sin\left(\sqrt{\frac{K}{M}} t - \delta \right) + \nu g M \quad (13)$$

where

$$\text{arc tan } \delta = \frac{\nu g}{V_0} \sqrt{\frac{M}{K}}$$

gives the value of the elastic impact J and shows that the latter represents a vibration of the system whose mass is the sum of the various masses of the seaplane. The initial conditions of vibration are given by the relative motion of the masses at the beginning of the impact.

The eccentric impact is obtained approximately by substituting, for the actual mass of the seaplane, the reduced mass

$$M' = M \frac{i^2}{i^2 + r^2} \quad (14)$$

where i is the radius of inertia and r is the distance of the impact force from the c.g.

This diagraming of the seaplane with a flat-bottomed float yields excessive results, since the elasticity is really distributed throughout all the parts of the seaplane. This fact led Pabst to investigate the problem by assuming a succession of rigid masses elastically connected with one another. The division of the seaplane into two masses leads to serious analytical difficulties, though it finally yields simple results. Figure 34 represents the abovementioned graphic method. M_1 and θ_1 represent the mass and inertia moment of the fuselage, wings, tail and engine, rigidly connected with the line AB. M_2 and θ_2 are the mass and inertia moment of the rigid float, connected with the fuselage by two elastic members assumed to have no mass but having elastic constants K_ν and K_h. The float bottom, without mass, is elastic with constant K_2 and accelerates the water mass M_a. The eccentricity of the impact is represented by r. The other quantities are clearly indicated in Figure 34.

By applying the equations of Lagrange, we obtain

$$\frac{d \dfrac{\partial E}{\partial q'_i}}{dt} - \frac{\partial E}{\partial q'_i} = F_i \quad (15)$$

which represents the kinetic energy of the whole system with the reduced external force F_i for the coordinate q_i of the point i of the mass considered, q'_i being the derivative of q_i with respect to t, successively to the quantities M_1, θ_1, M_2, θ_2, M_a, bearing in mind that the external forces are the elastic reactions connected with the very complex system of equations:

$$M_1 \frac{d^2 x_1}{dt^2} + (K_\nu + K_h) f_{12} + (K_\nu a - K_h b) \varphi_{12} = 0$$

$$\theta_1 \frac{d^2 \varphi_1}{dt^2} + (K_\nu a - K_h b) f_{12} + (K_\nu a^2 + K_h b^2) \varphi_{12} = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} - (K_\nu + K_h) f_{12} - (K_\nu a - K_h b) \varphi_{12} + K_2 f_{23} = 0 \qquad (16)$$

$$\theta_2 \frac{d^2 \varphi_2}{dt^2} - (K_\nu a - K_h b) f_{12} - (K_\nu a^2 + K_h b^2) \varphi_{12} + K_2 r f_{23} = 0$$

$$M_a \frac{d^2 x_3}{dt^2} - K_2 f_{23} = 0$$

which, after a double differentiation of the formulas

$$x_1 - x_2 = L_1 - f_{12}\,; \quad x_2 + r\,\varphi_2 - x_3 = L_2 - f_{23}\,; \quad e\,\varphi_1 - \varphi_2 = \varphi_{12}$$

yield the system of equations for the eccentric impact of the two-mass system for any arbitrary distribution of elasticity between the two masses

$$\frac{d^2 f_{12}}{dt^2} = (K_\nu + K_h) \left(\frac{1}{M_1} + \frac{1}{M_2}\right) f_{12} + (K_\nu a - K_h b) \left(\frac{1}{M_1} + \frac{1}{M_2}\right) \varphi_{12} + - \frac{K_2}{M_2} f_{23}$$

$$\frac{d^2 \varphi_{12}}{dt^2} = (K_\nu a - K_h b) \left(\frac{1}{\theta_1} + \frac{1}{\theta_2}\right) f_{12} - (K_\nu a^2 + K_h b^2) \left(\frac{1}{\theta_1} + \frac{1}{\theta_2}\right) \varphi_{12} + - \frac{K_2}{\theta_2} r\, f_{23} \qquad (17)$$

$$\frac{d^2 f_{23}}{dt^2} = K_2 \left(\frac{1}{M_2} + \frac{r^2}{\theta_2} + \frac{1}{M_3}\right) f_{23} - \left(\frac{K_\nu + K_h}{M_2} + \frac{K_\nu a - K_h b}{\theta_2}\right) f_{12} + - \left(\frac{K_\nu a - K_h b}{M_2} + \frac{K_\nu a^2 + K_h b^2}{\theta_2}\right) \varphi_{12}$$

These equations represent compound vibrations similar to the torsional vibrations of shafts.

In order to arrive at a conclusion, Pabst considered the impact centered on the step $(r = 0)$ and made certain simplifications by putting $K_v = K_h = K/2$ and $a = b = l/2$. Under these conditions the system could be integrated and the impact forces

$$J_1 = K_1 f_{12} \quad \text{and} \quad J_2 = K_2 f_{23}$$

corresponding to the masses M_1 and M_2, then become, on the fuselage and accessories (mass M_1)

$$J_1 = V_0 \sqrt{K M} \, (\varphi_1 \sin \lambda_1 t - \varphi_2 \sin \lambda_2 t) \tag{18}$$

where

$$\varphi_1 = \frac{\frac{e\ c}{s}}{2B\sqrt{A+B}} \quad ; \quad \varphi_2 = \frac{\frac{e\ c}{s}}{2B\sqrt{A-B}}$$

$$A = \frac{1}{2}\left[c \frac{r+s}{r\ s} + e \frac{s+w}{s\ w}\right]$$

$$B = \frac{1}{2}\sqrt{\left(c \frac{r+s}{r\ s} - e \frac{s+w}{s\ w}\right)^2 + \frac{4\ e\ c}{s^2}}$$

$$\lambda_1 = \sqrt{\frac{K}{M}(A+B)} \qquad \lambda_2 = \sqrt{\frac{K}{M}(A-B)}$$

and on the float (mass M_2)

$$J_2 = V_0 \sqrt{K M} \, (-\psi_1 \sin \lambda_1 t + \psi_2 \sin \lambda_2 t) \tag{19}$$

where

$$\psi_1 = \frac{e}{2\ B} \times \frac{A+B-C}{\sqrt{A+B}} \quad ; \quad \psi_2 = \frac{e}{2\ B} \times \frac{A-B-C}{\sqrt{A-B}}$$

$$C = c \frac{r+s}{r\ s}$$

and the symbols have the following significations:

$$M = M_1 + M_2 = \text{total mass of seaplane}$$

$$r = \frac{M_1}{M} = \frac{\text{mass of concentrated parts: fuselage, wings, tail, etc.}}{\text{total mass of seaplane}}.$$

$$s = \frac{M_2}{M} = \frac{\text{mass of float}}{\text{total mass of seaplane}}.$$

$$w = \frac{M_a}{M} = \frac{\text{accelerated mass of water}}{\text{total mass of seaplane}}.$$

V_o = vertical speed of descent.

K = total elasticity of seaplane between masses M_1 and M_a.

$$c = \frac{K_1}{K} = \frac{\text{elasticity between the masses } M_1 \text{ and } M_2}{\text{total elasticity of seaplane}}.$$

$$e = \frac{K_2}{K} = \frac{\text{elasticity between the masses } M_2 \text{ and } M_a}{\text{total elasticity of seaplane}}.$$

For using these formulas, it is necessary to know M_1 and M_2, which are easily measured, M_a given by formula (9), and V_o. The elasticities K_1 and K_2 are generally difficult to calculate, K_1 being often more difficult than K_2. These can be determined experimentally, however, and the author shows how this was done in the Ststic Division of the D.V.L., by H. Hertel and Leiss on a twin-float Heinkel HE8. This consists in determining the number of vibrations of the individual parts by means of resonance with the revolution number of a rotating mass eccentrically attached to the fuselage of the seaplane, which is elastically suspended. To determine the elasticity between the float and fuselage, one observes at what revolution number of the rotating mass the whole seaplane vibrates in resonance with it. Knowing this vibration number and the mass and inertia moment of the fuselage and float, the definitive elastic constant is determined by known formulas.

However, the experimental case does not correspond to the simplified assumption $K_v = K_h$, resulting in $K_h = \infty$ by simply applying the preceding formulas to the seaplane in a landing case as represented in Figure 31, the horizontal landing speed being 90 km/h (55.9 mi./hr.), the slope of the flight path about 12°, corresponding to $V_0 = 5.2$ m/s (17.06 ft./sec.), and assuming the length of the bottom striking the water to be about 1.2 m (3.94 ft.) and taking into account that $c = m\ 0.805$, $M = 305$ kg mass, $M_1 = 275$ kg mass, $M_2 = 30$ kg mass, $M_a = 46$ kg mass (equation 9), and that, from the experiment $K_1 = 574{,}000$ kg/m, $K_2 = 3{,}500$ kg/m, $K = 483{,}000$ kg/m, and hence, $r = 0.9$, $s = 0.1$, $w = 0.15$, $c = 1.16$, and $e = 7.25$. He comes to the conclusion that the respective impacts on the fuselage and on the float are

$$J_1 = 17{,}800 \sin 94\,t - 3{,}820 \sin 450\,t$$

$$J_2 = 36{,}800 \sin 450\,t + 4{,}820 \sin 94\,t$$

the frequencies of the impact phenomenon being $n' = 15/s$ and $n'' = 72/s$.*

As was to be expected from the preliminary hypotheses, the impact was purely elastic. In practice the impact is damped by the effect of the internal damping of the material and by friction in the connections, joints, etc. There were no experiments for this damping action. According to Plank, Honda and Konno, the damping due to the material is directly proportional to the velocity of deformation and, by introducing this concept $\left(J = Kf + \beta \frac{df}{dt}\right)$ into the preceding equations, we would have oscillations whose amplitude would decrease more or less rapidly. For plywood, Plank's hypothesis is not even approximately correct. For metals, the damping effect seems to reach values considerably above the proportionality limit. This is why duralumin bottoms can support excessive loads without failure, while merely developing permanent bulges.

It is concluded that the safety against failure, considering the rapidity of the phenomenon, is greater than that shown by the calculation, on the condition of not exceeding the ultimate strength of the material for alternating loads. Within the elastic range, which alone is of interest here, the damping of the oscillations seems to be negligibly small. Especially for seaplanes with du-

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*In a subsequent paper, Pabst modifies somewhat the take-off data to agree better with those obtained experimentally. The few changes are perfectly acceptable.

ralumin floats, we may consider, as the maximum value of the stresses on the seaplane, the value corresponding to the maximum oscillation of the value J_1, since it may be assumed that, owing to the difference between the frequencies n' and n'', at the instant of maximum amplitude of the larger vibration, the smaller and more rapid vibration has already died out. Similar considerations apply to the impact force on a float since, in correspondence with the maximum amplitude of the more rapid vibration, the deflection of the slower vibration is just beginning. Hence the equations for the maximum impact forces may be simplified to

$$\left.\begin{aligned} \max J_1 &= V_0 \sqrt{K\,M}\,\varphi_2 \\ \max J_2 &= \dot{V}_0 \sqrt{K\,M}\,\psi_1 \end{aligned}\right\} \qquad (20)$$

In the publication of January, 1931 (reference 10), Pabst changes the value of M_a (equation 9) to

$$M_a = \rho\,\frac{\pi}{2}\,c^2\,l\,\frac{l}{\sqrt{l^2 + 4\,c^2}}\left(1 - 0.85\,\frac{c}{l}\,\frac{l^2}{l^2 + 4\,c^2}\right) \qquad (21)$$

which yields slightly higher values, though even this formula is empirical. For the preceding arithmetical calculation, we find

$\max J_1 = 17{,}800 \text{ kg}$; $\frac{J_1}{P} = 5.9$ or the bottom pressure

$\max J_2 = 36{,}800 \text{ kg}$; $p = \frac{\max J_2}{\text{area}} = \frac{36800}{2.06} = \text{kg/cm}^2\ 1.8$

Similar calculations are also applicable to flying boats, especially to large ones, the weight of whose engines and fuel is distributed over the wings.

For the eccentric impact, by way of approximation, it is not only possible to consider the same formulas, but also to use the mass M as reduced by formula (14).*

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*For V-bottomed floats, Dr. Pabst proposes a theory identical with Karman's, as already explained by us, and accordingly disregards all elasticity of the seaplane and of the float bottom and develops the following formula, which represents the impact force J of a boat or float seaplane with a bottom V of $\beta^\circ = (180 - 2\alpha)^\circ$.
(Continued at bottom of page 30.)

THEORY OF THE RIGID V BOTTOM

Karman's theory was taken up and completed by Herbert Wagner, as already mentioned. (References 16 and 17.) In order to make it apply to all V bottoms, he expresses the shape of a normal section of the V bottom by

$$y = \alpha_0 x + \alpha_1 x^2 + \alpha_2 x^3 + \alpha_3 x^4 + \ldots\ldots \tag{23}$$

where the constants α are determined by the shape of the bottom, and he assumes (a hypothesis not adopted by Karman) that the values are so (infinitely) small as to make it possible to hold that the final velocities at the surface of the water, exclusive of the spray, are very close to the vertical. Under these conditions at the free surface, the velocity is vertical, while in the contact area the velocity decreases perpendicularly to the bottom, and therefore the motion of the water is approximately the same as that around a flat plate. (Fig. 35.) Hence the velocity v_y is

$$v_y = \frac{V}{\sqrt{1 - \frac{c^2}{x^2}}} \tag{24}$$

at the free surface corresponding to a point x of the bottom, at the instant considered, immersed for the half-width c, V being the velocity of immersion and c being smaller than x.

The velocity at which the width of the contact area increases is dc/dt. Adopted as an independent quantity,

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*(Continuation of footnote, page 29.)

$$\frac{J}{l} = \frac{\pi\ \rho\ V_0^2}{\left(1 + \frac{M}{M_a}\right)^3} \times \frac{c}{\tan\alpha} \times \left(1 - \frac{3}{2}\frac{c}{l}\right) \tag{22}$$

l being greater than $2c$, the usual notation. This formula is identical with Karman's, if we take for the accelerated mass of water that of formula (8) instead of the experimentally correct one (formula 9). According to this theory, the load factor $J/P = u$ is constant for the whole seaplane. The maximum pressure occurs at the beginning of the immersion and has the value indicated by Karman, as given by formula (6).

the ratio of the velocities $\sigma = \frac{V}{\frac{dc}{dt}}$, it follows that the rise η of the superficial particles of water, corresponding to the abscissa x, is

$$\eta = \int_0^t v_y \, dt = \int_0^c \sigma \frac{d\,c}{\sqrt{1 - \frac{c^2}{x^2}}} \tag{25}$$

where $t = t(c)$ and $dt = \frac{1}{\frac{dc}{dt}}\, dc$. Following the phenomenon of immersion, there will come an instant at which the particle of water corresponding to the abscissa x will touch the bottom, when $\eta = y$ and $x = c$ and hence

$$y = \int_{c=0}^{c=x} \sigma \frac{d\,c}{\sqrt{1 - \frac{c^2}{x^2}}} \tag{26}$$

which is equivalent to $y = y(x)$, giving

$$\sigma = \frac{2}{\pi}\alpha_0 + \alpha_1 c + \frac{4}{\pi}\alpha_2 c^2 + \frac{3}{2}\alpha_3 c^3 + \frac{16}{3\pi}\alpha_4 c^4 \ldots\ldots \tag{27}$$

Hence the ratio σ for every half-width c of the contact zone is defined for any given float bottom.

The maximum pressure is in the spray and corresponds to that of the banking up of the water, whose velocity is dc/dt and therefore, having called m the mass per unit length l of the immersed part of the float

$$p_{max} = \frac{\rho}{2} \frac{V_0^2}{(1 + \mu)^2} \frac{1}{\sigma^2} \tag{28}$$

having assigned to $\mu = \frac{\rho \pi c^2}{2 m}$ the same meaning (formula 2) as in Karman's theory and, since, in this assumption, the accelerated water is that of a semicylinder not corrected by Pabst's experimental formula, and also being assumed constant the quantity of motion in the propagation of the phenomenon, for which $V = \frac{V_0}{1 + \mu}$ (formula 1). In the zone of contact $x < c$, the water pressure,

$$p = \frac{\rho\, V_o^2}{(1+\mu)^2}\,\frac{1}{\sigma}\left[\frac{1}{\sqrt{1-\frac{x^2}{c^2}}} - \frac{2\,\mu}{1+\mu}\sqrt{1-\frac{x^2}{c^2}} - \frac{\sigma}{2\left(\frac{c^2}{x^2}-1\right)}\right] \tag{29}$$

is composed of three terms. The first depends on the increase in the size of the impact area; the second, and the negative term, on the retardation of the water and on the elliptical distribution, and the third term, which contains the square of the velocity, is quite small in the central portion of the bottom. Only at the edge, where x approaches c, does it attain any appreciable value. By integrating along the width of the surface of immersion and disregarding the infinitesimal terms, we obtain

$$\frac{J}{l} = \frac{\pi\,\rho\,V_o^2}{(1+\mu)^3}\,\frac{c}{\sigma} \tag{30}$$

The depth of immersion is

$$T = \int_0^t V\,dt = \int_0^c \sigma\,dc \tag{31}$$

EXAMPLES

A) Two floats, in which all the other conditions (m, V_o, shape, etc.) are identical, but the bottom of one is twice as sharp as that of the other, are first compared. For the one with the sharper bottom:

a) In the central part of the float, the impact forces are very nearly half as great as on the one with the flatter bottom. From formula (29), by disregarding the third term, p is obviously proportional to $1/\sigma$;

b) The maximum pressure at the edge of the contact surface is very nearly one-fourth that of the flatter bottom, it being obvious from formula (28) that p max is proportional to $1/\sigma^2$ (equation 30);

c) The total impact force is approximately one-half that of the flatter bottom.

B) By varying the mass loading m on the same float with a concave V bottom, which can be done, V_o being constant, by varying the length l of the impact zone, or by considering two impacts, one on the step and one forward of the step:

d) At the beginning of the impact, the bottom V being rather sharp, the velocity of immersion is great;

e) In the case of small m, the retardation being already sufficient, the impact forces are not great, while, in the case of large m, the float, being but slightly retarded, strikes on the flatter part of the bottom and the pressures are great. If the bottom V becomes zero, the theory gives infinite pressures.

C) By varying the shape of the bottom: Figure 30 represents, for successive contact widths c in meters and for $V_0 = 3$ m/s (9.84 ft./sec.), the time in seconds from the beginning of the impact, the velocity ratio V/V_0 and the impact forces J corresponding to a weight of 4,000 kg (8,818 lb.) per unit length. The impact forces J are also plotted for 8,000 kg (17,637 lb.) and for ∞. The bottom, for which J remains constant during the whole impact, is flat in the central portion, where it has the shape of a parabola. The three examples give a very clear idea of the effect of the shape of the V bottom.

A bottom with straight sides shows a maximum value for J toward the center. In practice the maximum is less, because the bottom is here more yielding than at the keel or chine. The concave bottom V has the maximum pressure at the chine, which may be disadvantageous. For a given bottom V with straight lines, a value of c is attained, at which the impact force is the greatest. It is of no advantage to increase the width of the float bottom beyond this value, unless the bottom V is made a little sharper. It may happen that, for very short impact lengths l, greater impact forces will be produced in floats of longer base, but these are unimportant, due to the smallness of l. For concave V bottoms the impact forces are generally greater than for straight V bottoms, but the forces diminish in proportion as the bottom is widened. A greater width, however, involves greater difficulty of construction and a greater liability of bouncing in a faulty landing.

In the case where the bottom shape varies from section to section, we will consider an element dl of the float for which it is possible to consider the plane phenomenon. The preceding principles apply to all such elements. The immersion depth T with respect to the original undisturbed surface of the water is given by equation (31), when σ refers to the section under consideration, so that, for any immersion depth T, the width c is determined for any section and the total impact force from equation (30) written

$$J = \frac{\pi\ \rho\ V_0^2}{(1+\mu)^3} \int_0^l \frac{c}{\sigma}\, d\,l \tag{32}$$

In this equation, when, in any section, the contact surface reaches the chine, $c = b/2$ and $u = \infty$. Then it follows from equation (2) that

$$\mu = \frac{\pi}{2}\frac{\rho}{M} \int_0^l c^2\, d\,l \tag{33}$$

where M represents the total mass. Having μ, it is possible to calculate p, V, p_{max}, etc.

Thus far a planar flow has been assumed, notwithstanding the escape of the water fore and aft. This phenomenon is negligible, so long as the impact length l is enough larger or smaller than the impact width b, but, in the rather frequent case when l and b are of similar magnitude, the calculated values are subject to considerable variations, due to the variations in the quantities of water accelerated.*

The preceding theory is subject, however, to other corrections (as observed by Wagner), when it is extended to larger values of the bottom V. The ratio between the actual impact force J_ν and the calculated J is given by the formula

$$\frac{J_\nu}{J} = 1 - \frac{\alpha}{\pi} - 0.15\,\frac{\sigma}{\pi} - \frac{\sigma}{\pi}\ln\frac{1}{\sigma} \tag{34}$$

where α is the incidence of the float bottom at the edge of the impact surface.

For the straight-V bottom, Figure 37, in which is also plotted the curve $\frac{1}{\alpha_0}\frac{J_\nu}{J}$, shows that the impact force varies with α_0. Hence the ratio $\frac{J_\nu}{J}$ is approximately equivalent to the ratio of the dynamic lift A_g to

*See what we have already written concerning the theory expounded by Dr. Pabst for taking this fact into account.

$$A_{g_0} = \frac{\rho \pi}{8} b^2 v^2 \kappa \tag{35}$$

corresponding to the bottom V with straight lines α_0 and 0 (b being the width of the step, v the velocity of rolling, and κ the incidence of the float bottom at the step). Figure 37 gives an idea of the behavior of the bottom V in landing or taking off, and it is obvious that the variation in the impact is great, while the dynamic lift in taking off is small.*

We thus have the elements for calculating the reactions under the bottom of a projected seaplane. If M_r represents the reduced mass of the seaplane for a given position of the impact force, then, according to equation (30), the total impact force is

$$J = \frac{\pi \rho V_0^2}{(1 + kl)^3} \frac{c}{\sigma} l \tag{36}$$

where (according to equation (2)),

$$k = \frac{\rho \pi c^2}{2 M_r}.$$

Differentiation according to l yields the length

$$l = \frac{M_r}{\rho \pi c^2} \tag{37}$$

of the contact zone for which we obtain the maximum impact force

$$J_{max} = 0.3 V_0^2 M_r \frac{1}{\sigma c} \tag{38}$$

For each width c there is a maximum impact force. The absolute maximum is obtained with the minimum value of σ c.

---

*In taking off, the hydrodynamic resistance of the float is
$R_a = \kappa A_g + c_f \frac{1}{2} \rho v^2 b l = \frac{1}{2} \rho v^2 b \left(\frac{\pi}{4} \kappa^2 + 0.025 \frac{l}{b} \right)$

The value of the absolute maximum impact force, in the case of a V bottom with straight sides and constant angle, is easily found to be

$$\max J_{max} = 0.835 \frac{V_o^2}{\alpha_o} \sqrt{\rho\ l_{max}\ M_r} \tag{39}$$

where l_{max} is the maximum length of the contact zone, taking into account the seaway which the float must navigate. The maximum J_{max} acts at $\frac{1}{2}\,l_{max}$ ahead of the step, for which point the reduced mass M_r must also be computed. This formula is also approximately correct for a variable bottom V, when α_o has a suitable value.

We must also determine whether, for the given location of the impact force, the bottom has at least the half-length l according to equation (37), or whether, taking into account the seaway which the seaplane must navigate, the calculated length l is still possible. Otherwise we must determine c (from equation 37) for the actually existing l corresponding to this value σc and then calculate J_{max} from equation (38).

For a projected seaplane float, it is then easy to calculate the absolute values J_{max} at various points on the keel and, after determining the possible lengths l of the contact zone, to compute the values of J_{max} corresponding to the lengths l. We thus obtain as many groups of curves as there are assumed widths b of the float at the step, and as many curves as there are assumed shapes of the V bottom. These curves all begin at the step, first follow an almost rectilinear course, then bend and reach zero at the bow of the float. Figures 38 and 39 represent the results of the calculations for the same float weighing 15,000 kg (33,070 lb.) and having a speed of 5 m (16.4 ft.) per second. Figure 38 is for a straight V bottom and Figure 39 for a concave V, the width at the step being respectively 2 and 3 m (6.56 and 9.84 ft.). The impact lengths were 1.5 and 3 m (4.92 and 9.84 ft.). For the float with the straight V bottom, the maximum values were obtained for $c < 1$ m (3.28 ft.) and hence the wide bottom is not subjected to greater impact forces. For the narrow float with concave bottom V, the impact forces are greatest when the contact surface reaches the chine. The wide float is subjected to smaller impact forces. The maximum impact forces are therefore absolute.

Hence it is possible to calculate, for any projected type of float, the reactions on its bottom in landing, when the elasticity is negligible with respect to the effect of the bottom V. The tests made in Germany and summarized by us (fig. 29) practically confirm the curves given by the calculation. (Figs. 38 and 39.)

Dr. Wagner's theory enables us to determine the order of magnitude of the forces and their distribution with sufficient approximation. It is based on the hypothesis of the smallest keel angles for which the elastic effect of the float bottom is important and extends the theory, by means of an empirical formula, to larger angles for which the effect of the elasticity is very much less. This fact must not be overlooked. Dr. Wagner's reasoning on the effect of the elasticity of the bottom for very small keel angles, as a special preexisting bottom shape, must be considered rather bold, since the elastic deformation has two distinct effects, the first one being a variation in the shape, of which Wagner's theory takes account. The second effect is the absorption of a certain kinetic energy, due to internal deformation work of the bottom. This is the greater effect of the two and is not taken into account by Wagner's theory. To this cause in particular are to be attributed any discrepancies between the pressure diagram according to Wagner's theory, as determined across the width of the V bottom, and the diagram obtained by test measurements.

PRESENT ACCEPTANCE REQUIREMENTS

We consider it pertinent to the problem under consideration to mention briefly the present requirements in various countries. These requirements, which are generally applicable in principle, to all cases, are therefore conventional, but give the order of magnitude of the forces to be considered. Italian regulations have always specified that acceptance conditions shall be revised from time to time, as occasion demands. On the other hand, the C.I.N.A. (Commission Internationale de Navigation Aerienne) has not yet established any rules for the acceptance of floats.

For France the Bureau Veritas has fixed the pressure which must be withstood by the bottom of a float and consequently by other parts of the same float. This upward pressure on the bottom is given in kg/cm^2 by the formula

$$p = K \left(\frac{V_{min}}{100}\right)^2$$

V_{min} being the minimum speed of sustentation of the seaplane in km/h, and K, a coefficient dependent on the bottom angle at the step.

$\alpha =$		5°	10°	15°
$K =$	1	0.9	0.7	0.5

This pressure is not uniform, however, along the whole float, but takes the form shown in Figures 40 and 41, according to whether the float has one or two steps. The distribution is uniform according to the width of the float. Moreover, a float rigidly fixed at the height of the step must withstand a bending stress corresponding to the application to its bottom of loads one-half those defined by Figures 40 and 41.

The attachments of heavy parts, capable of developing considerable forces of inertia in taking off and in landing, shall be so calculated as to be able to withstand vertically the weight of the parts supported multiplied by five, or, if it is greater, by the ratio of the load defined by Figures 40 and 41 to the weight P of the seaplane; also to be able to withstand horizontally twice the anticipated force due to the inertia of the masses to be supported. In the case of seaplanes with two floats, each shall be considered as a simple float with the condition that the coefficient K shall be multiplied by 0.8.

The French literature, as well as the Italian, is very poor in comparison with the others mentioned. According to an article by E. Barrillon (reference 3), engineer-in-chief of the naval engineers, in which he dwells on the strength conditions of seaplane floats and investigates the various phases of motion in taking off and in landing, we may conclude, with Boutiron, that one of the principal causes of the delay in solving the problems of taking off and landing will disappear with the establishment of the naval basin at Marignane.

The British, however, on the basis of their own experiments, conclude that:

1. The forebody of a twin-float seaplane must withstand a load of 3.5P uniformly distributed over a zone as wide as the bottom of the float and that extends from the step to the beginning of the V bottom at the bow (fig. 42);

2. All the parts, keel, keelsons, planking, etc. of the forebody must withstand a load five times that acting on these parts when the float is immersed to its water line;

3. The after body must have the same strength as the corresponding part of the fuselage;

4. The ports, windows and gun rings must be reinforced locally at the edges, so as to withstand the stresses. The walkways and floors must be able to support 1,220 kg/m² (249.88 lb./sq.ft.). The United States, as a result of their tests, decided that (always assuming the horizontality of the axis of thrust) it was necessary to consider three dangerous conditions and verify the breaking strength:

1) Normal bow landing; the reaction of the c.g. in the plane of symmetry of the seaplane, but so inclined that, on the axis of thrust, there is a component 1/4 of the other component normal to this (fig. 43) and with a value of 8P', where P' represents the weight of the floats and of their supporting members;

2) Landing on the step with reaction passing through the c.g. normal to the axis of thrust, with the value 8P' (fig. 43);

3) Landing with lateral load. The vertical forces are disregarded but, in the plane normal to the axis of thrust and passing through the center of gravity, a horizontal force of 2P acting on the one float (fig. 44), or equally distributed over both floats and acting at half the depth of the submerged portion of the float, is considered. Moreover, the American rules prescribe for conventional seaplanes the pressure distribution shown in Figure 43.

Germany in its latest regulations, still in the experimental stage, defines the coefficient of impact as the product

$$u = c_0 \; c_1 \; c_2 \; c_3 \; \frac{1 + \sqrt[4]{P}}{1 + \sqrt[4]{P} + \sqrt{P}} \; V^{1.5} \tag{40}$$

where c_0 = 1/125 to 1/95, according to the seaway on which the seaplane must land;

c_1 = 0.9 to 1.1, according to the robustness of the seaplane;

$c_2 = 1 - 0.7 \cos (90^\circ - \alpha^\circ)$, according to the bottom angle at the point of impact;

c_3 = a coefficient to allow for special configuration of the float bottom;

V = horizontal velocity of the seaplane in km/h;

P = weight in metric tons.

The following cases of landing will be considered.

1. Normal: a) on the bow; b) on the step; c) on the stern.

2. On a single float: a) on the bow; b) on the step; c) on the stern.

3. Lateral: a) on the bow; b) on the step; c) on the stern.

The value and point of application of the force in case 1, normal, are indicated on Figures 45 and 46, which refer to the twin-float seaplane and to the flying boat or single-float seaplane, respectively. In case 2, landing with a single float, the same figures apply, but the forces are halved and applied either to the middle of the lateral float or to a quarter of the width of the bottom of the hull. For a lateral landing, case 3, always to be considered in connection with case 2, Figures 47 and 48 apply, on which the magnitude of the horizontal forces are indicated for the three cases a, b, and c with the same points of application as in the normal case.

The safety factor is 1.55 for the twin-float seaplane and 1.8 for the single-float seaplane and for the flying boat. The maximum pressure on the bottom for the calculation of the local strength of the main step will be that corresponding to 1.5 u P, distributed over 20 per cent of the submerged area. The displacement of the principal float or of the hull is at least enough to support the weight of the seaplane multiplied by 1.8 or 2, according to the seaway.

All these regulations are therefore deduced from experiments, but the presence of formula (40) in the German specifications shows that theoretical investigations are also involved.

DEDUCTIONS AND CONCLUSIONS

From the foregoing it may be concluded that a flat bottom undergoes an impact force determined by its own elasticity and by that of the whole seaplane, whose value is given by the second formula (20) or by

$$J_2 = V_0 \sqrt{K\ M}\ \psi_1 \tag{41}$$

where the elastic properties are comprised in K and in ψ_1. A rigid V bottom of a rigid seaplane undergoes an impact force which may be represented by

$$J = V_0^2 \sqrt{\rho\ l\ M}\ \frac{\xi}{\sigma} \tag{42}$$

where the form characteristics of the V bottom are comprised in σ and in ξ. It follows from the foregoing that the impact force is always proportional to the square root of the mass, so that, for seaplanes which are similar geometrically and structurally, the impact force varies as the square root of their weight, and the impact factors

$$u = \frac{J}{P} \tag{43}$$

vary inversely as the square root of the mass or weight.

Thus far the formulas (41) and (42) for J agree. This agreement ceases, however, when we consider the other quantities contained in them. A rigid V bottom of a rigid seaplane comprises, in formula (42) for J, the square of the velocity V_0. The flat bottom resists, however, if it and the whole seaplane are elastic, and comprises the first power of said velocity, formula (41). Neither a V bottom nor a seaplane shall longer be rigid, and therefore even V bottoms must be elastic. The shape of the V bottom determines what element shall have the greater effect in the definition of J, the elasticity or the bottom V. It is doubtless possible to imagine a bottom V so pronounced that the effect of the elasticity is almost negligible, while small values of the bottom V would yield very large values for J, if not favorably affected by the elasticity. For small values of the bottom V, it can be easily included in the case of landing with a flat bottom. Hence, for a given length l of the impact or contact surface, a criterion to be kept in mind is to consider a bottom as V-shaped, if the mean value of its bottom angle is always greater than the maximum heeling possible in landing. In this case, formula (42) is applicable, on condition that it does not yield values greater than those given by formula (41). In any event, there will exist a mean value α_m of the bottom V for which formula (42) yields the same value as the one given by (41) for the same structural and landing conditions. This common value of J, taken as unity, will give, in a certain manner, the measurement of the effect of the bottom V. Greater values will give percentages less than one, while smaller values of the bottom V will give percentages greater than one, which cannot be considered, because the structural elasticity comes into play. For a straight V bottom we have plotted in Figure 49* the curve of the percentile variation of J in terms of the bottom V, having assumed J_o, for $\alpha^o = 8^o\ 30'$, to be equal to unity, being verified for this value and for the length l of the chosen impact surface, the equivalent of J in formulas (41) and (42). From the coordinate points 0 and 100, we have drawn the tangent to the curve, indicating on the curve the point of the coordinates i and α_r. This tangent can represent, with the remainder of the curve, the percentile variation of J for variations in the bottom V when $J_o = 1$. The resulting curvilinear triangle indicates in per cent the part of J due to elasticity, which is disregarded in formula (42). The possible landing speeds can be determined from these curves.

By equating the first members of formulas (41) and (42) for a constant bottom V with straight sides, we obtain

$$\frac{V_o}{\alpha_{m}} = \frac{2}{\pi} \frac{\sqrt{K}}{\sqrt{\rho\, l}} \frac{\psi_1}{\xi}$$

where α is the inclination of the bottom and, since we are dealing with small angles α_m, we can consider the arc as equal to the tangent and the sine. K and ψ_1 are

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*As it is easy to see, the result is a hyperbola referred to the asymptotes, and it is easily shown to be $\alpha_r = 2\alpha_m$, and the ordinate $i = 100/2$.

constant for a given seaplane. If we assume l to be constant and remember that ξ varies by a few per cent, we may consider α_m, and therefore α_r, directly proportional to V_o.

Figure 50 represents, for $J_o = 100$ corresponding to a predetermined velocity V_o, the values of J for various velocities V_o, having chosen these respectively, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3 times V_o, and having designated as abscissas, besides the values of α, also the values $\beta^o = 180^o - 2\alpha^o$. The values of α_m and α_r are all on two straight lines issuing from the origin. Knowing any value of J, Figure 50 gives the impact force in every case. Note that the straight line $y = \frac{1}{4} \frac{V_o}{V_o} \frac{\alpha}{\alpha_m}$ divides the diagram into two parts. The points of this line correspond to $y = \frac{1}{2} \frac{V_o}{V_o}$. For values of $\alpha < 2\,\alpha_m$ the equation is $y = \left(1 - \frac{1}{4} \frac{\alpha}{\alpha_m} \frac{V_o}{V_o}\right)$ while for values of $\alpha > 2\,\alpha_m$ the equation must be $y = \frac{\alpha_m}{\alpha} \frac{V_o}{V_o}$. It is also possible to plot a figure like Figure 50 for V bottoms with curved sides.

In order to utilize the structural elasticity of the seaplane in determining the impact force J_1, let us note that the first of the formulas (20),

$$J_1 = V_o \sqrt{K M}\, \varphi_2 \tag{44}$$

compared with formula (41), gives

$$\frac{J_2}{J_1} = \frac{\psi_1}{\varphi_2} = \sqrt{\frac{A - B}{A + B} \frac{s}{c} (A + B - C)} \tag{45}$$

and, if we disregard the value of C, which is small with respect to $A + B$, and the value of $s^2/r\,w$, which is of a lower order of magnitude with respect to the others, we obtain

$$\frac{J_2}{J_1} = \sqrt{\frac{e}{c} \left(\frac{s}{w} + \frac{s}{r}\right)} \tag{46}$$

which, with the 10 per cent approximation, gives the ratio between the impacts on the bottom and in the center. This formula also shows the manner of introducing the elasticity ratio e/c, the inertia ratio s/w of the float and of the accelerated water, and s/r of the float and all other parts of the seaplane.

We have, therefore, all the data for determining the maximum stresses in landing and in taking off, when the mechanical and structural characteristics of the seaplane are known. What has been said regarding the values of J also applies to the values of u. In the absence of the elastic characteristics, it is necessary to resort to experimental determinations on other seaplanes.

The experimental results lead to the conclusion that the flat bottoms support the greatest load, but the load on straight-sided V bottoms increases proportionally with the increase in the bottom angle. It may be assumed that, for a V bottom with $\alpha_0 = 0.3$, the maximum contingent stresses are about as follows:

A) If the wind velocity is zero or in the direction of motion of the seaplane:

1. In a normal landing on the step, which also corresponds to one made in line of flight, with a horizontal speed of 80 to 120 km (50 to 75 mi.) per hour and the corresponding attitude, the maximum reaction occurs immediately forward of the main step for the duration of 0.01 to 0.05 second with a resultant passing very near the c.g. of the seaplane having a distribution, which may be assumed to be uniform, of about 0.65 kg/cm² (9.24 lb./sq.in.), or triangular in plan and in height, with a maximum value of about 0.8 kg/cm² (11.38 lb./sq.in.) at the apex, decreasing toward the chine to almost 0.4 and toward the bow to almost 0.65. The magnitude of the central resultant may amount to 4.5P, but the bottom of the float supports an acceleration of about 2g more. It is interesting, however, to note that taxying on waves 50 to 80 cm (19.7 to 31.5 in.) high may give similar values of the resultant, because the fall from the wave produces a vertical velocity of 3 to 4 m/s (9.84 to 13.12 ft./sec.), which, combined with that of translation, yields loads nearly equal to the preceding.

2. In landing on rough water, the force is applied toward the middle of the forebody of the float with an intensity slightly less but of the same duration, and with triangular configuration but of constant value to the for-

ward limit, with increasing pressure halfway to the step. The vertical acceleration remains the same, while the horizontal acceleration is somewhat greater and attains a value 0.9g in the direction of the axis.

3. In taking off or taxying in a seaway, the maximum reaction occurs at the bow with an approximate intensity of 2/3 in the first case with resultant of plane of symmetry of the seaplane and inclined aft, so as to form an angle of 10 to 20° with the normal to the line of the keel. As regards the point of application of the resultant, it may be assumed to be at about the center of the forebody of the float.

4. Landing on the after body, which also corresponds to violent pitching on rough water. The resultant is always in the plane of symmetry and applied normal to the keel abaft the step with uniform or trapezoidal distribution, greater pressures toward the stern, intensity of force 0.5, case 1. It is also well to remember the pronounced lift of the wing in this case.

B) In a cross wind:

5. The experiments show an acceleration of about 1/7 of the maximum vertical. Horizontal forces are considered in planes normal to the axis of thrust, passing through the points of application of the forces in cases 1 to 4 and applied at half the immersion depth of the section on which they act and having a magnitude of 1/7 of those corresponding to the preceding cases and with uniform distribution over the side or sides in correspondence with the distribution of the predicted forces.

Translation by Dwight M. Miner,
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for Aeronautics.

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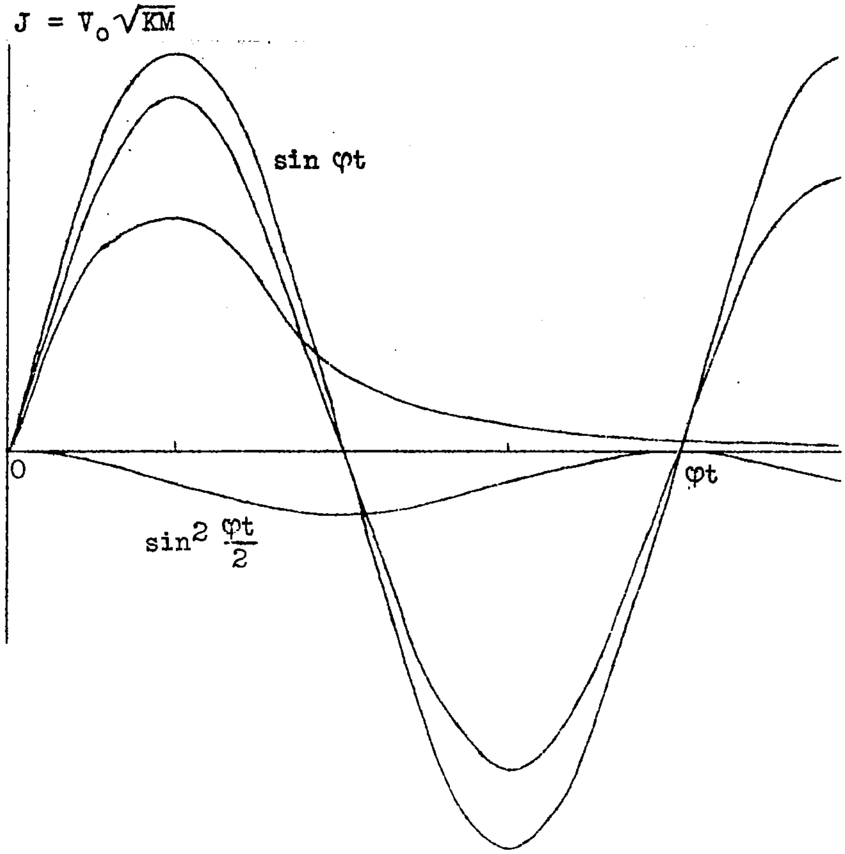
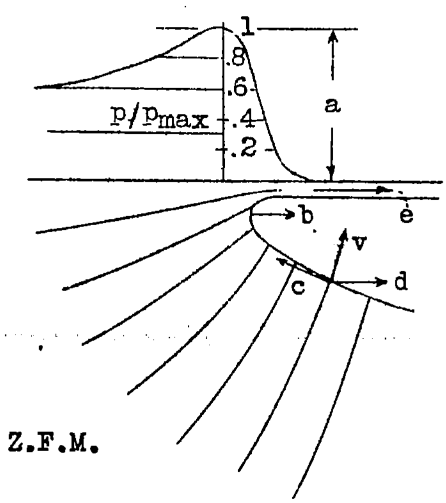


Fig. 1 Coefficient of impact in landing.



a, $p_{max} = \frac{1}{2}\rho\left(\frac{dc}{dt}\right)^2$

b, $\frac{dc}{dt} = \frac{V}{\sigma}$

c, $\frac{dc}{dt}$

d, $\frac{dc}{dv}$

e, $2\,\frac{dc}{dt}$

Fig. 2 Flow and pressure distribution at origin of spray

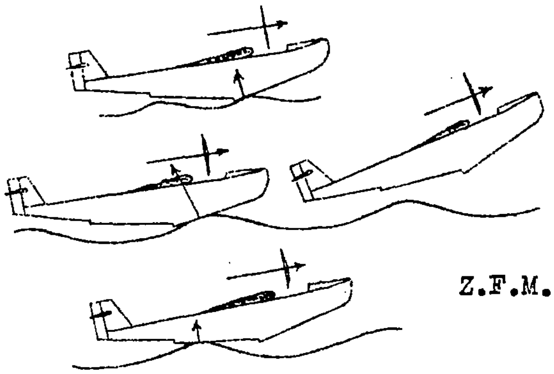


Fig. 3 Effect of size of waves on impact area and on attitude of hull after striking.

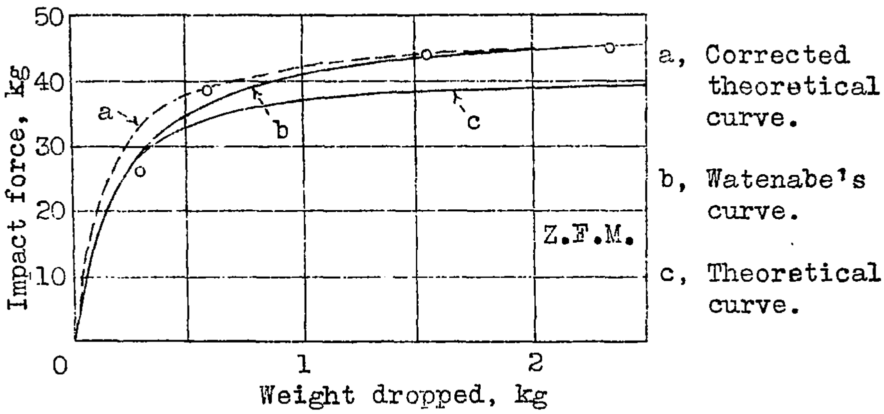


Fig. 4 Watenabe's experiments. Impact force plotted against the weight dropped.

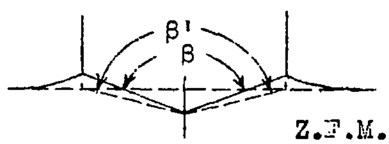
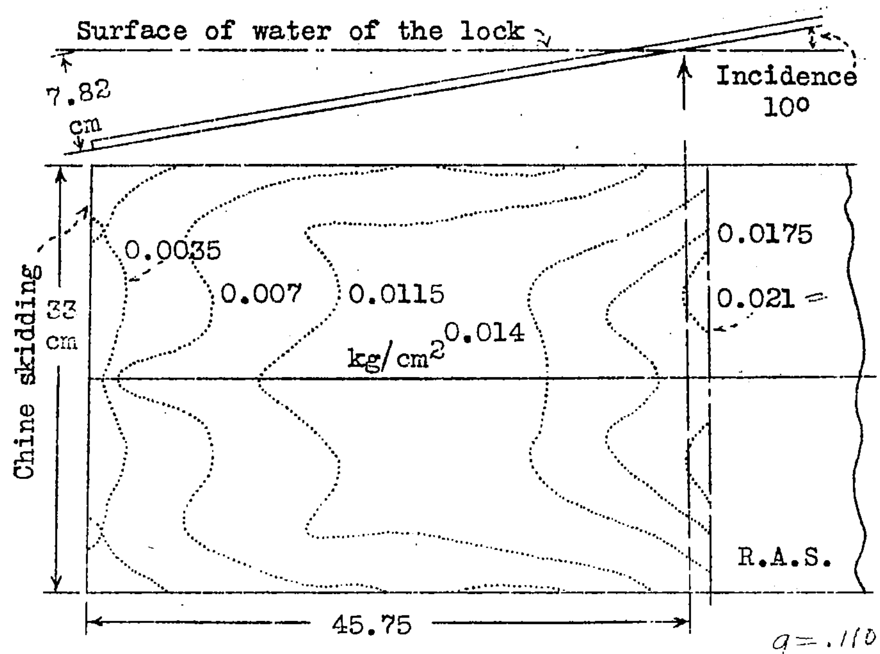


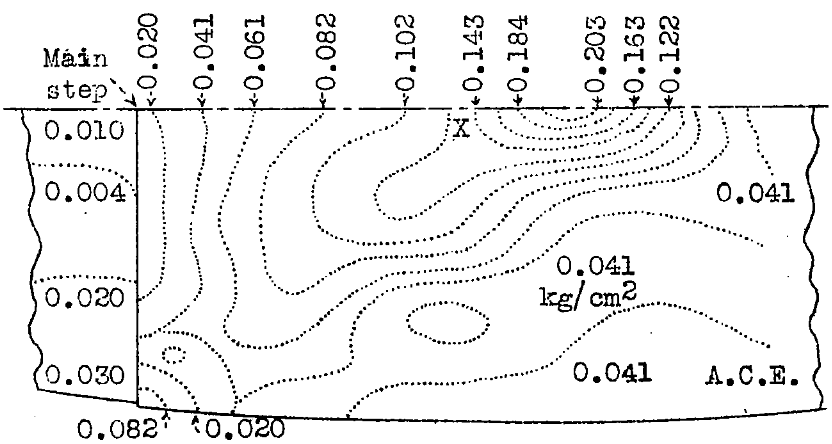
Fig. 5



q = .1108 kg./sq.cm.

.19

Fig. 6 Lines of uniform pressure on a flat plate.
Velocity, 16.5 km/hr



.120

q = 1.199
0.422 max. of X

q = 1.26

$\frac{p}{q}$ = .16

X is point at which pressure was measured in terms of the velocity.

Fig. 7 Lines of uniform pressure. Weight of hull, 8800 kg
Velocity, 55.6 km/hr Automatic trim.

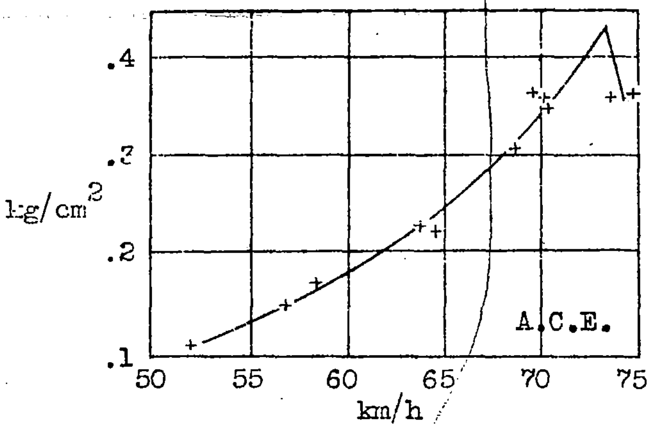


Fig. 8 Pressure at X plotted against velocity.

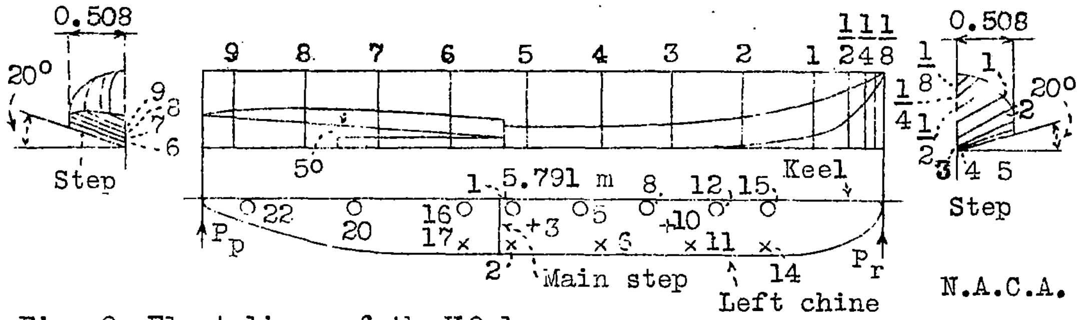


Fig. 9 Float lines of the U O-1

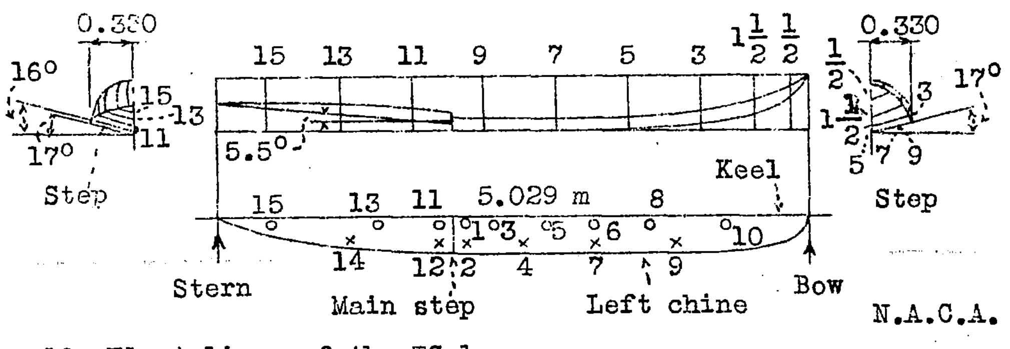


Fig. 10 Float lines of the TS-1

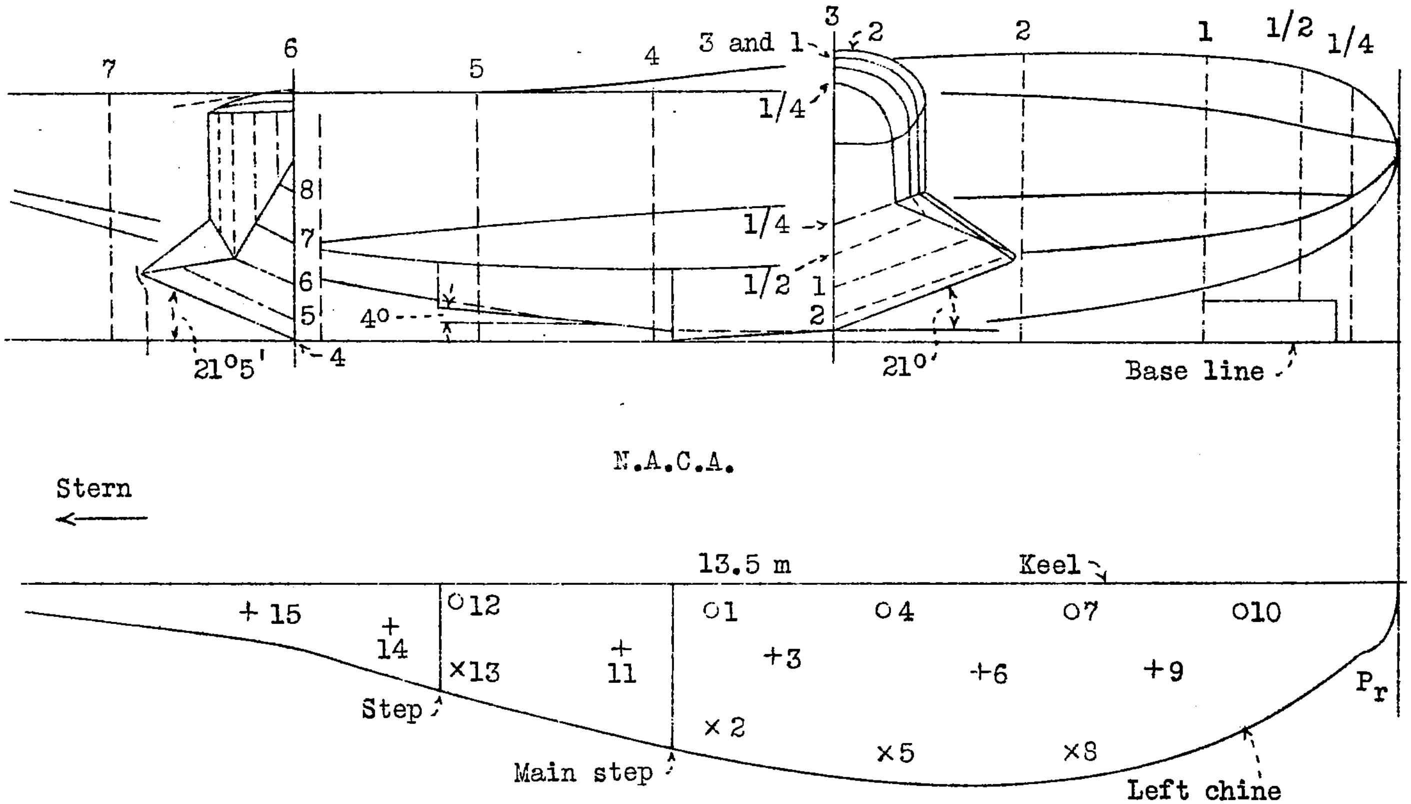


Fig. 11 Hull lines and pressure stations of the H-16 flying boat.

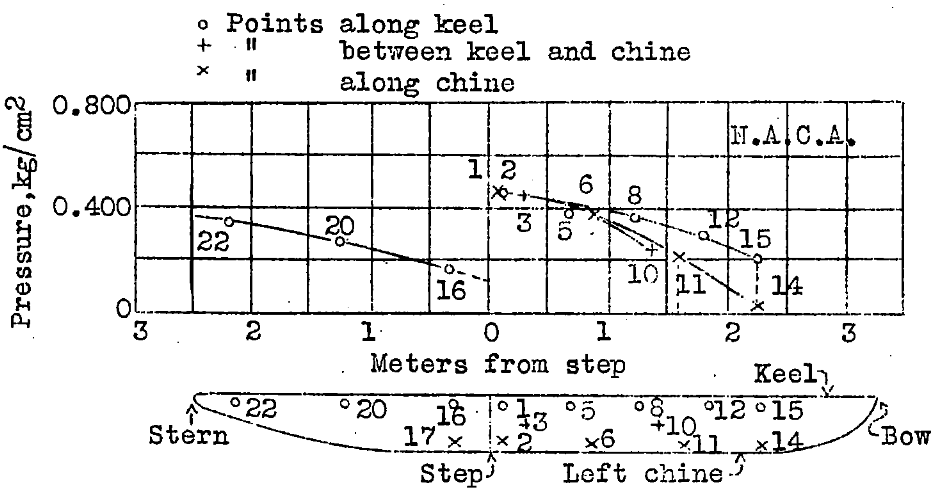


Fig.12 Distribution of maximum water pressure on the UO-1 seaplane float bottom.

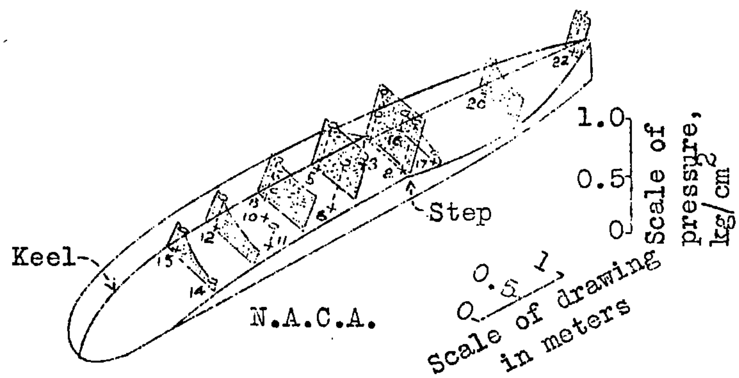


Fig.13 UO-1 seaplane float bottom showing the distribution of maximum water pressures over one side of the bottom.

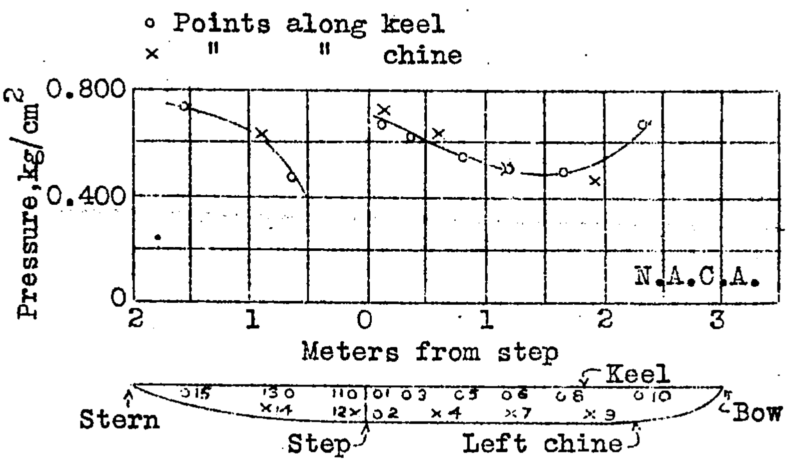


Fig.14 Distribution of maximum water pressures on the float bottom.

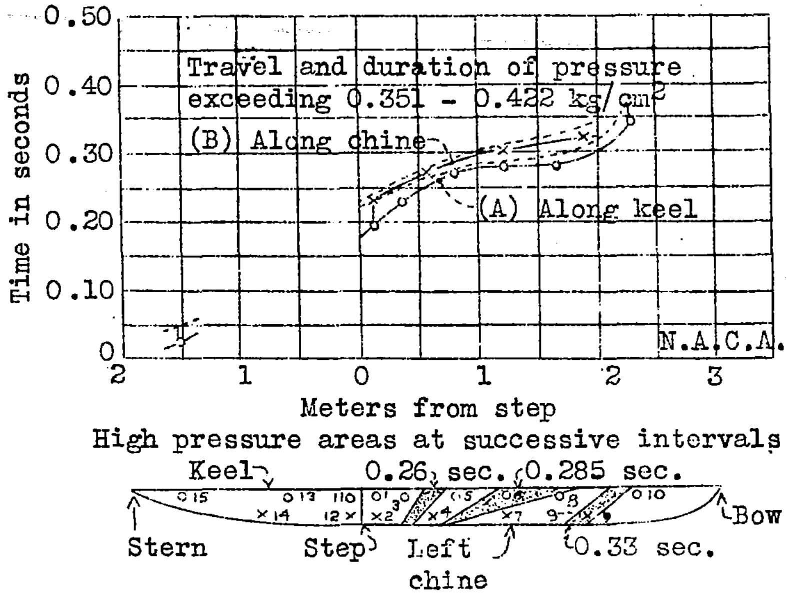


Fig.15 Travel of high local pressure over the float bottom in pancake landing, run 55. Water speed , 77.3 km/h ,smooth water.

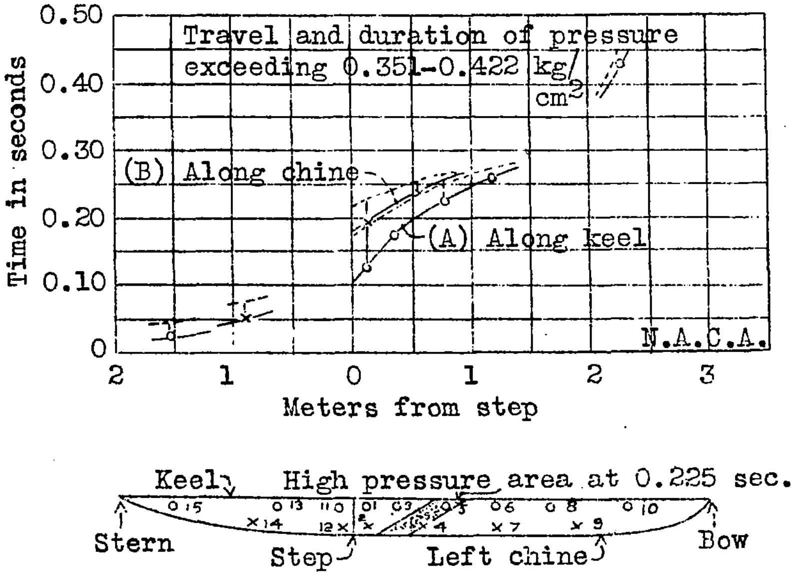


Fig.16 Travel of high local pressure over the float bottom in pancake landing, run 56. Water speed 75.6 to 67.6 km/h , smooth water.

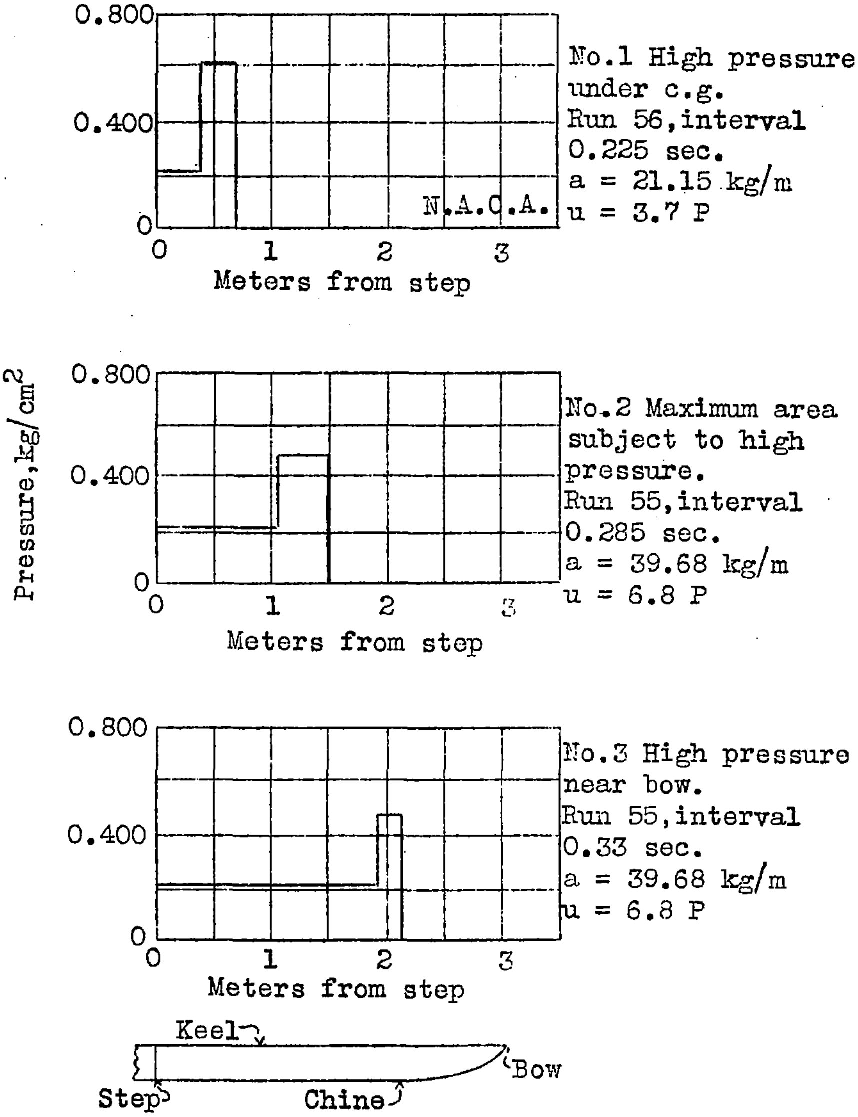


Fig.17 Approximate loads at three intervals for a severe pancake landing.

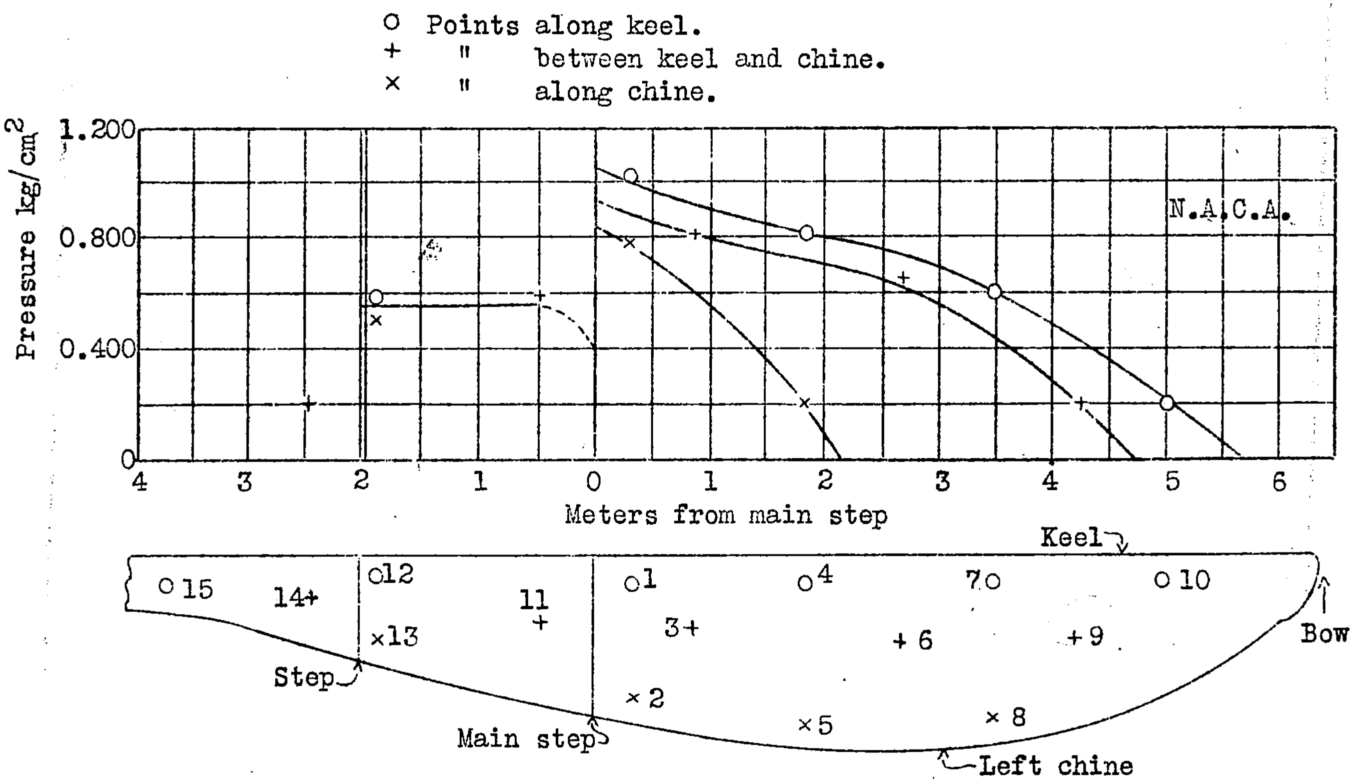


Fig.18 Distribution of maximum water pressures on the H-16 hull in landings.

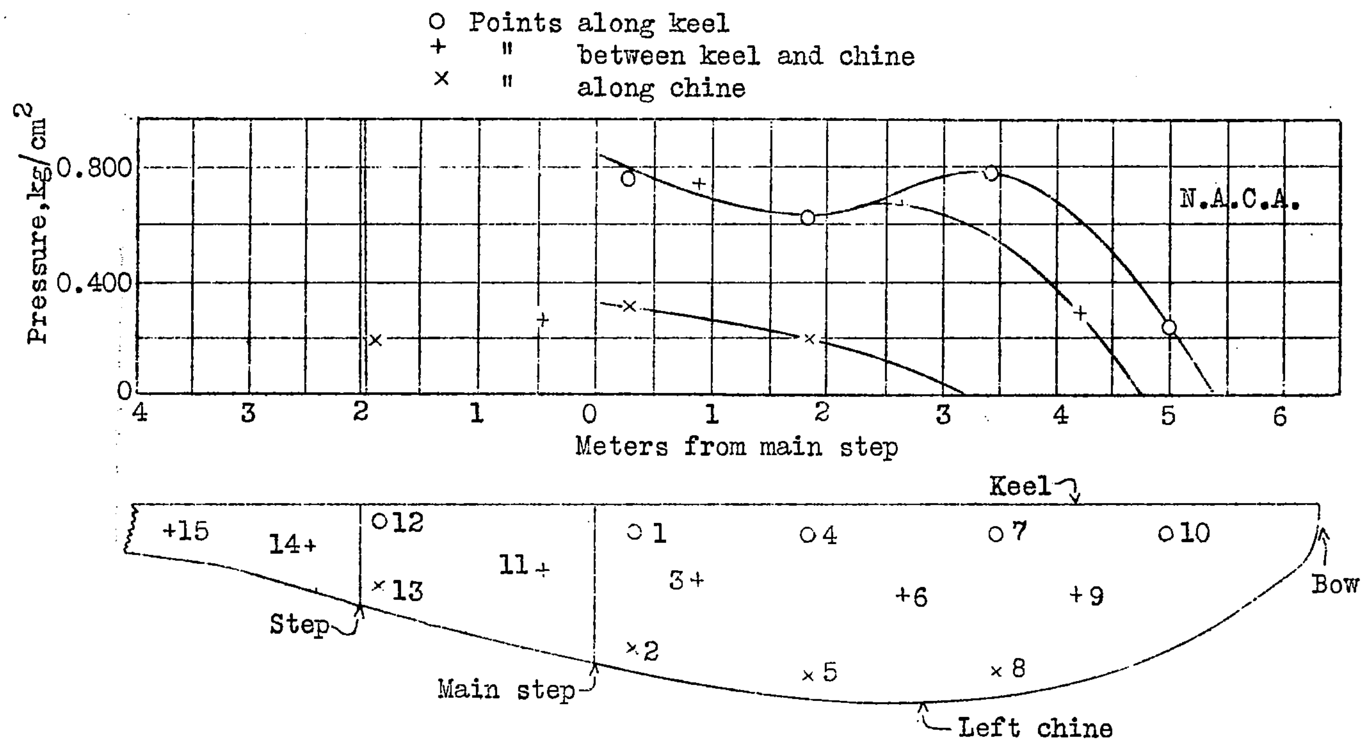


Fig.19 Distribution of maximum water pressures on the H-16 hull in taxying.

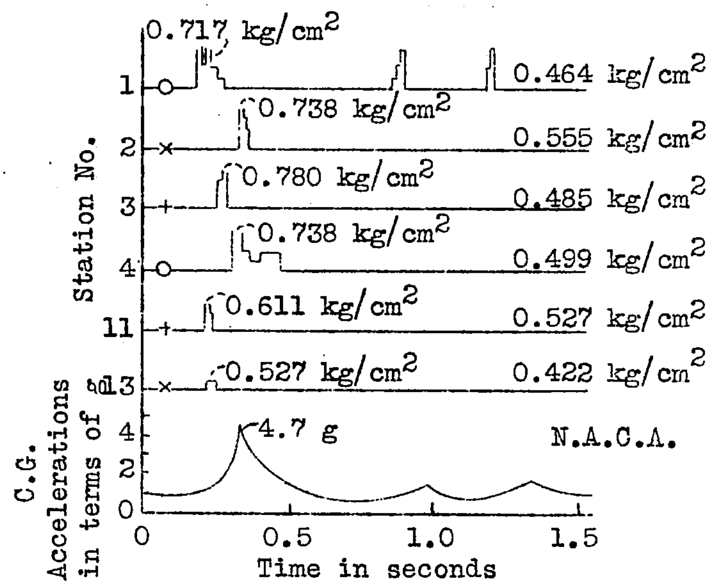


Fig. 20 Water pressure and vertical acceleration records in landing run 53 (pressures of .42 to .56 kg/cm² were not exceeded at other stations).

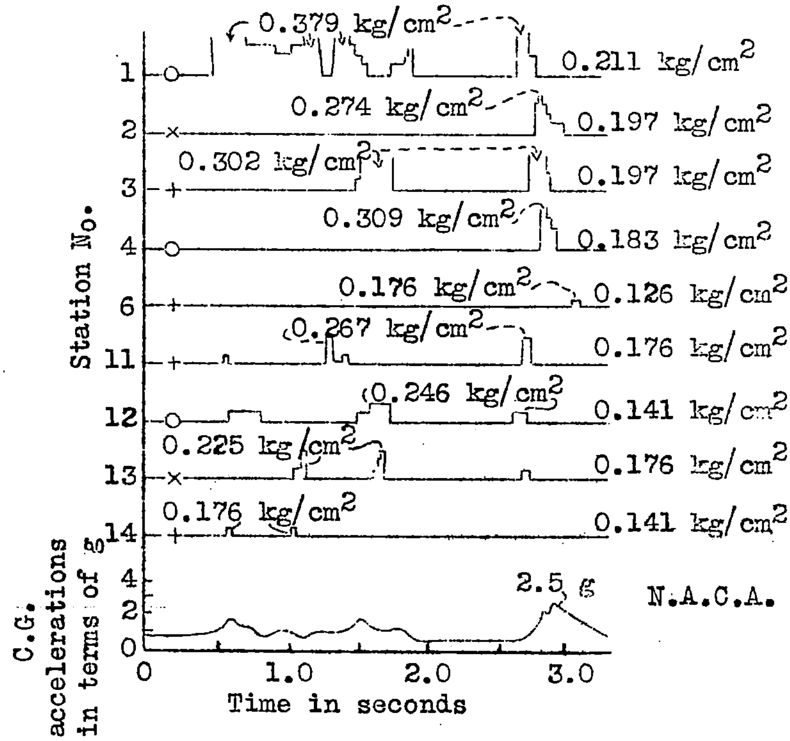


Fig. 21 Water pressure and vertical acceleration records obtained in landing run 90 (pressures of .14 to .21 kg/cm² were not exceeded at other stations.

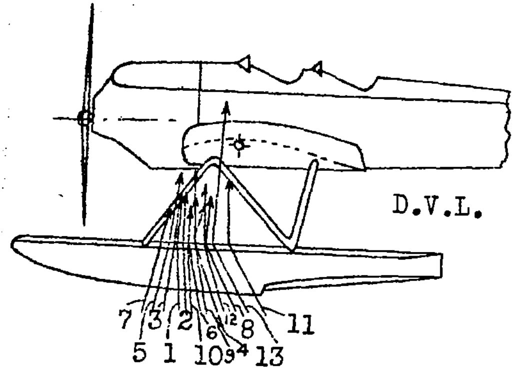


Fig.25 Impact forces in taking off.

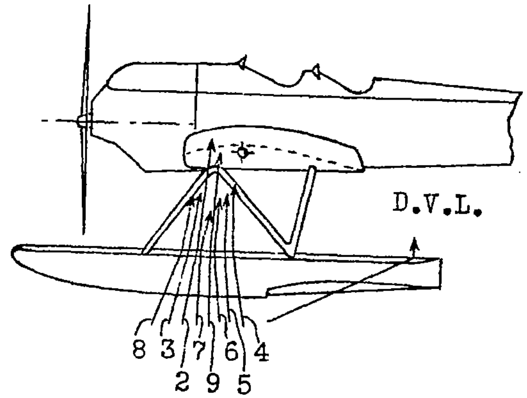


Fig.26 Impact forces in landing.

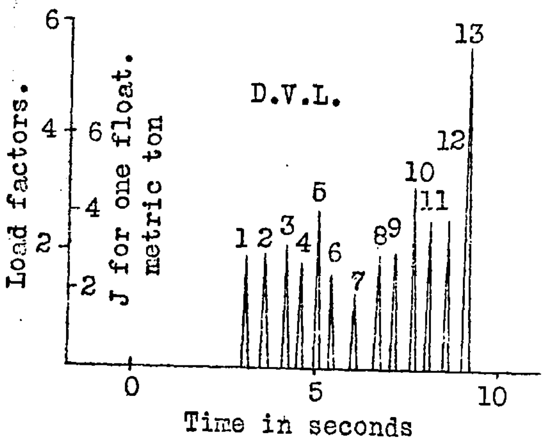


Fig.27 Impact forces in taking off.

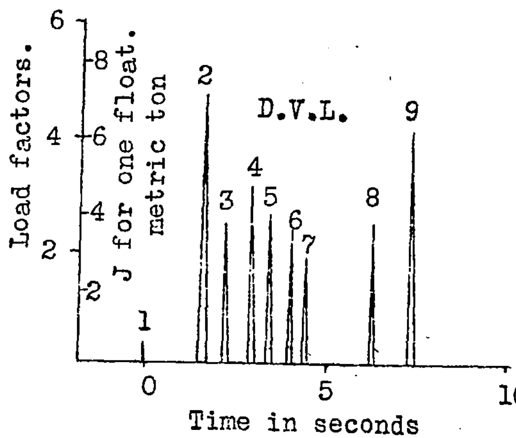


Fig.28 Impact forces in landing.

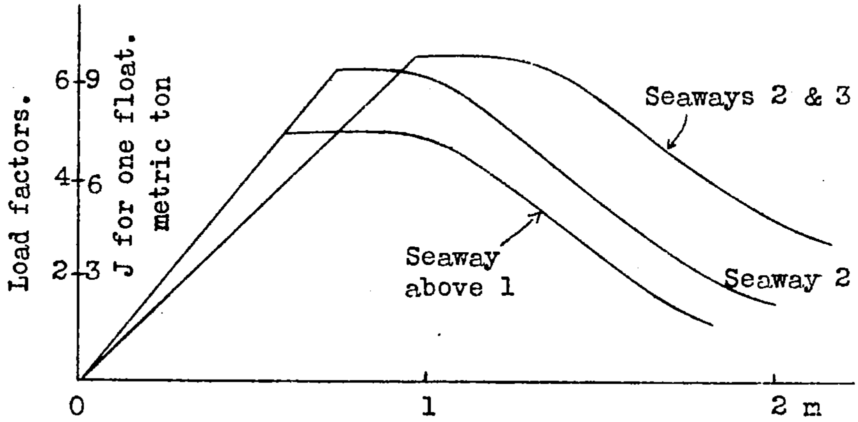


Fig.29 Max. impact force in taking off and in landing.

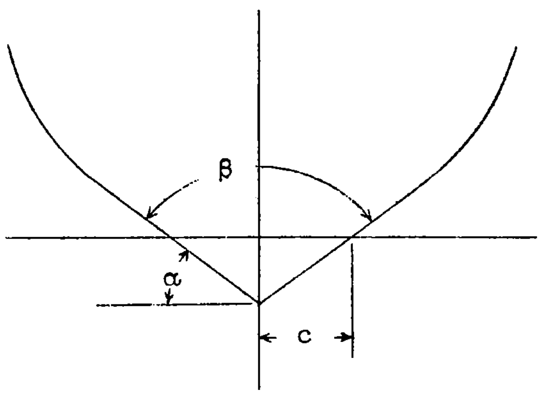


Fig.30

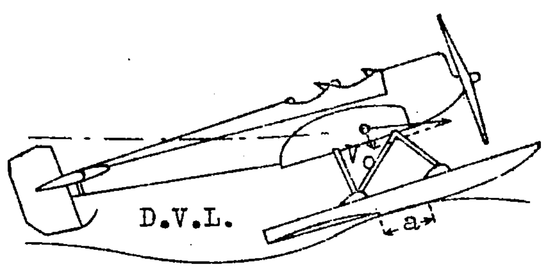


Fig.31 Landing of seaplane on a wave.

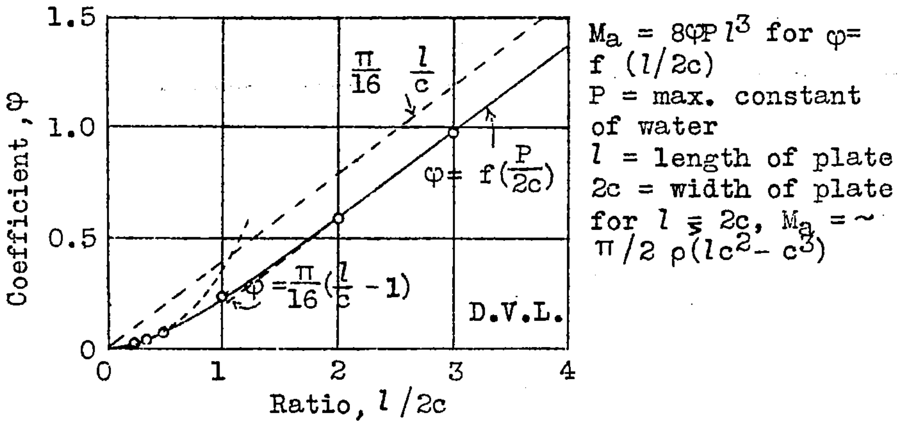


Fig.32 Accelerated water mass with one-sided flow.

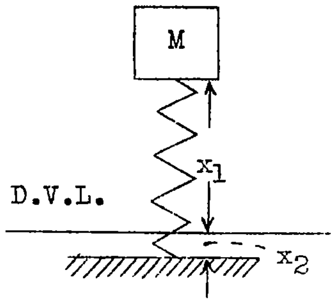


Fig.33 Diagram of seaplane-float system.

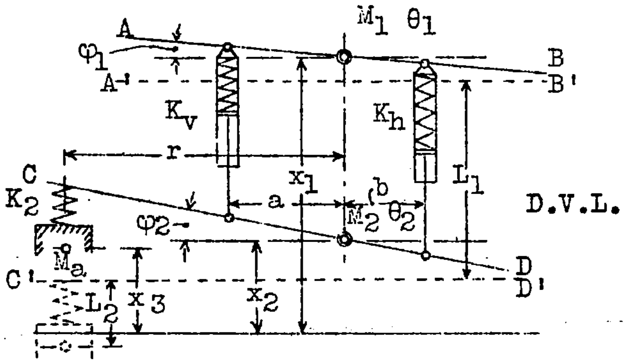


Fig.34 Diagram of eccentric impact of the two-mass system.

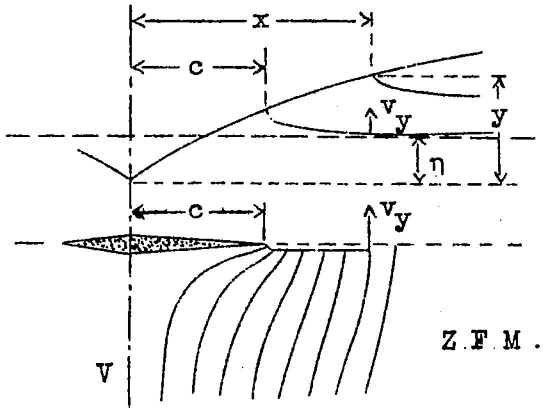


Fig. 35 Field of velocity on impact.

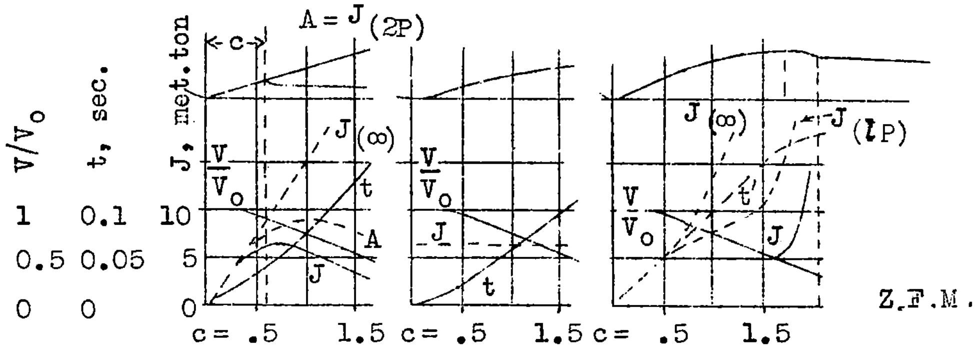


Fig. 36 Variations of J with variations in shape of bottom and in weight P, $V_o = 3$ m/s.

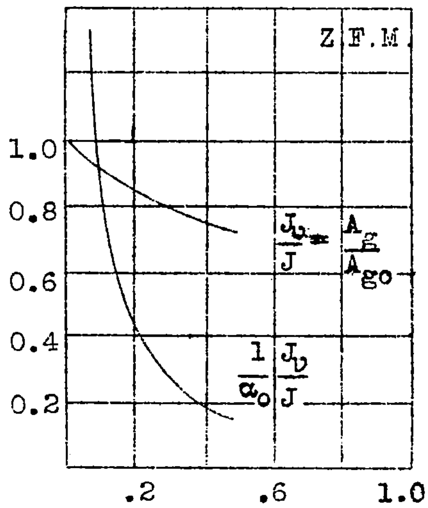


Fig. 37 Variation of impact force.

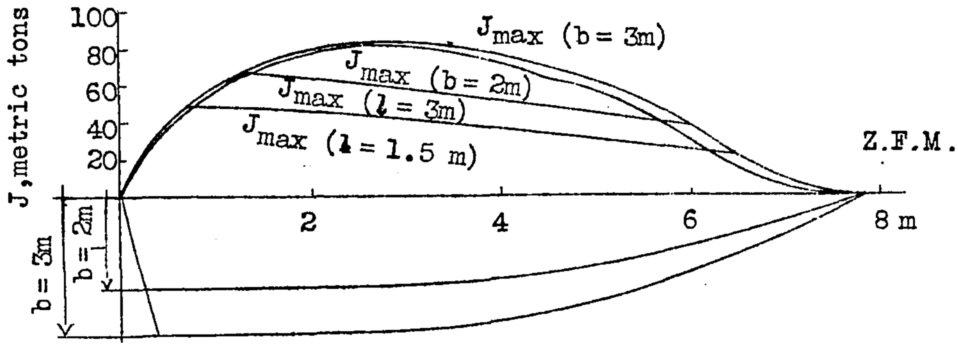


Fig. 38 Impact forces for P = 15 tons and V_0 = 5 m/s for length l = 1.5 and 3 m and maximum impact forces. Straight V. $\alpha_0 = 0.25$

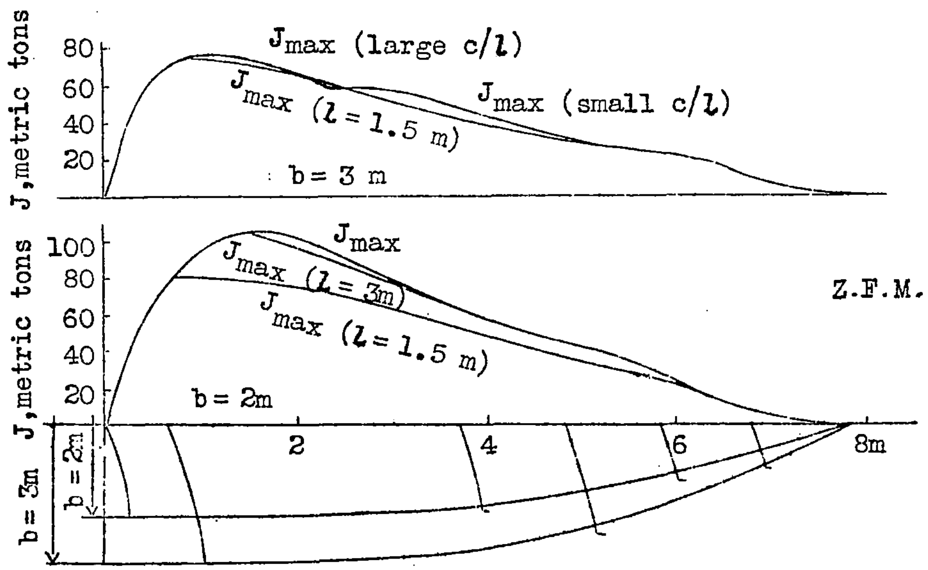


Fig. 39 Impact forces for P = 15 tons and V = 5 m/s for l = 1.5 and 3 m and maximum impact forces. Concave V bottom.

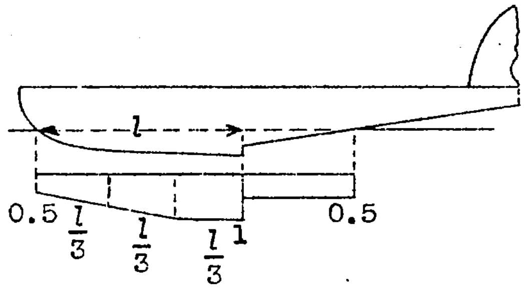


Fig. 40 Pressure distribution according to "Bureau Veritas".

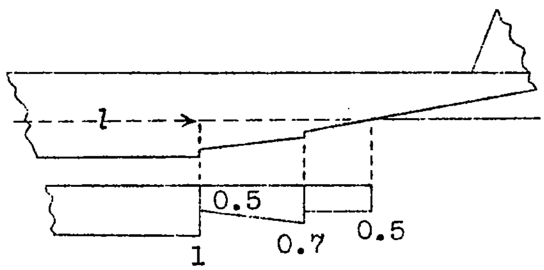


Fig. 41 Pressure distribution according to "Bureau Veritas".

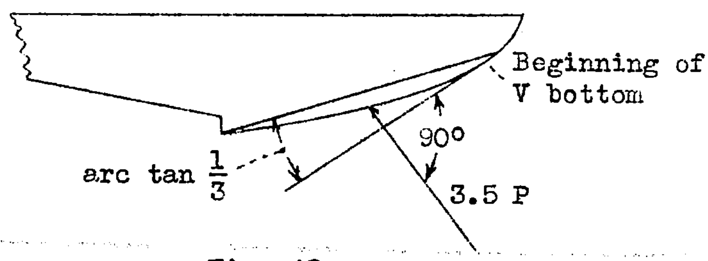


Fig. 42

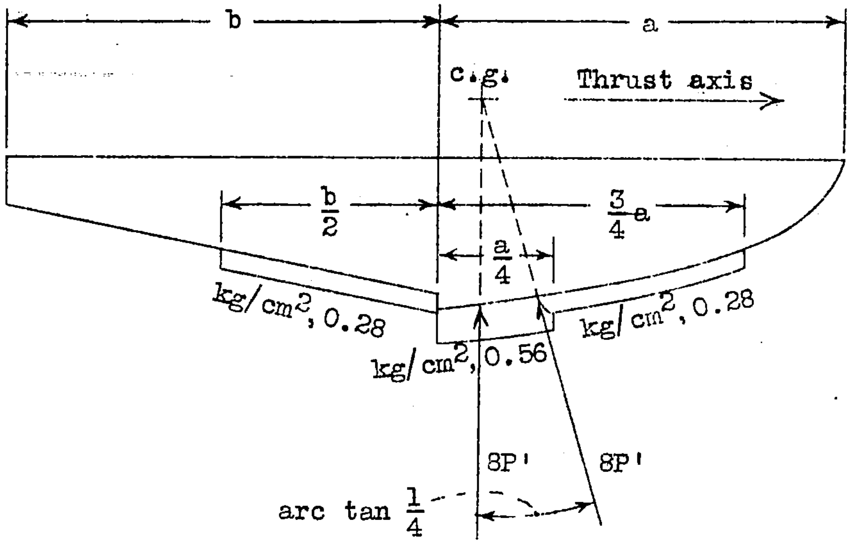


Fig. 43

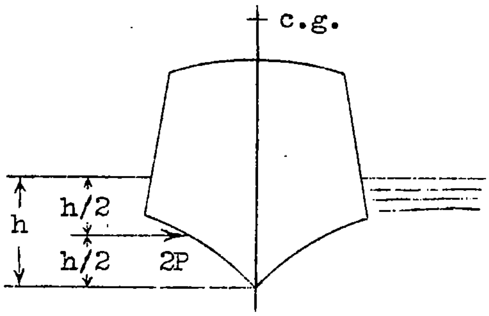


Fig. 44

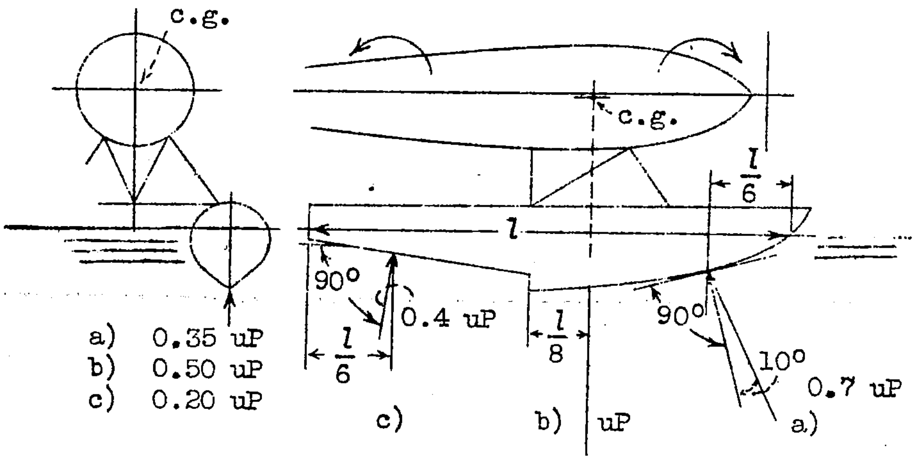


Fig. 45

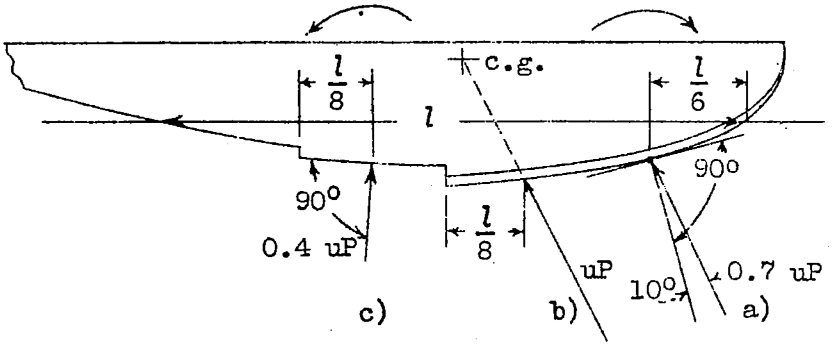


Fig. 46

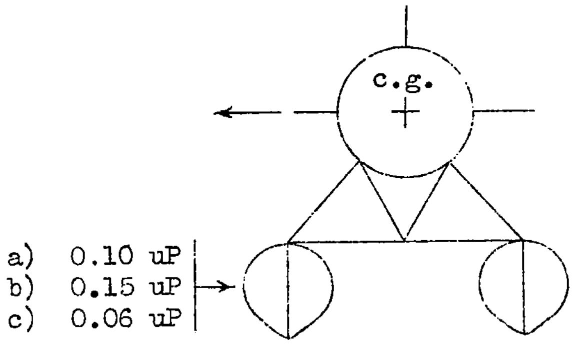


Fig. 47

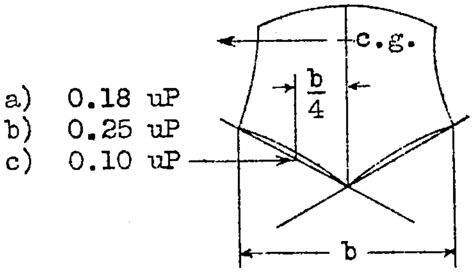


Fig. 48

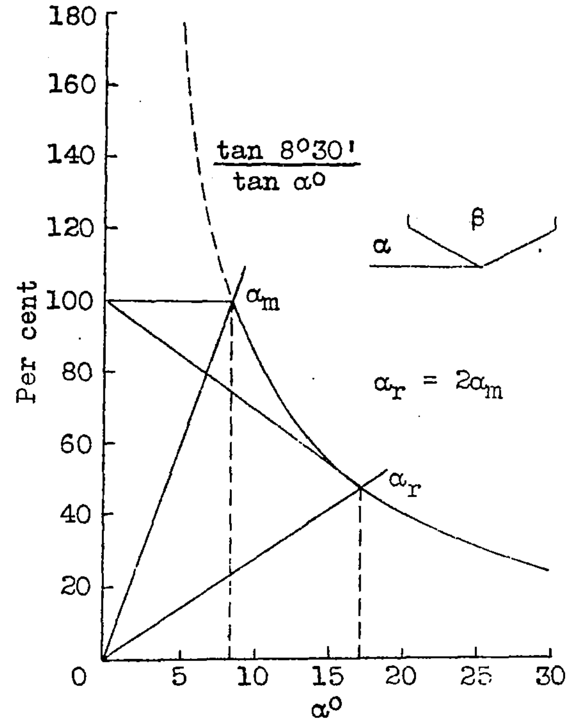


Fig. 49 Variations in impact force with bottom angle and elasticity.

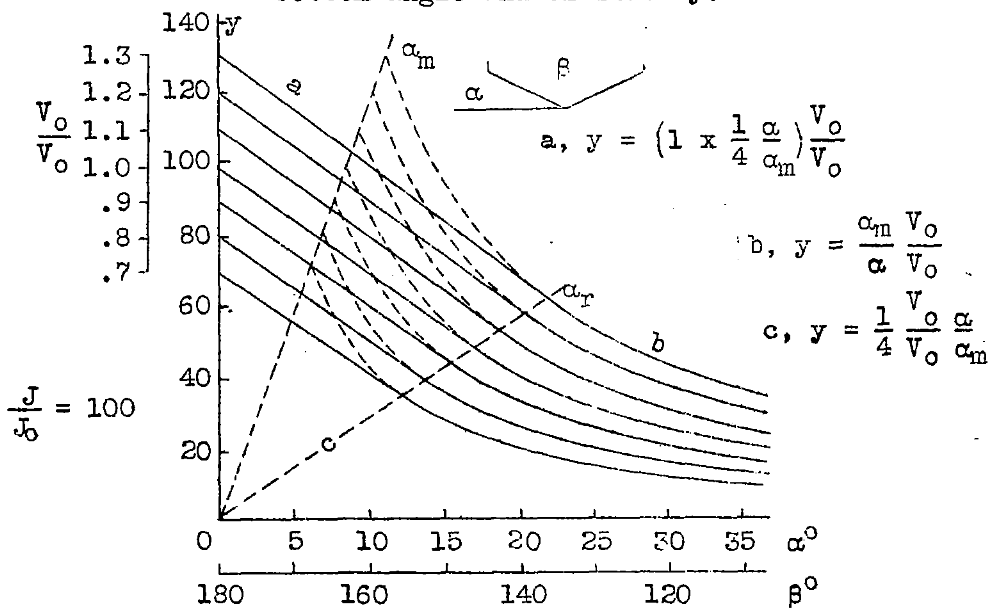


Fig. 50 Variations in impact force.