TECHNICAL MEMORANDUMS

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No. 619

DETERMINATION OF RESISTANCE AND TRIMMING MOMENT OF PLANING WATER CRAFT

By P. Schröder

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I. Theory

Notation

$$\left.\begin{array}{lll} W & = & \text{resistance} \\ M & = & \text{trimming moment} \\ A & = & \text{lift} \end{array}\right\} \text{of water craft at planing speed } v.$$

$\epsilon = \frac{W}{A}$ = lift-drag ratio.

$\mu_o = \frac{M}{A}$ = trimming moment with respect to lift.

In conformity with previous reports**, ϵ = constant and μ_o = constant when $\kappa = \frac{v^2}{A}$ = constant.

Thus it becomes possible to interpret the resistance and the trimming moment for any loading of a planing aircraft when these values are given for one load. This application of the new theory forms the basis of the present paper.

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*"Über die Bestimmung von Widerstand und Trimmoment bei gleitenden Wasserfahrzeugen," Zeitschrift für Flugtechnik und Motorluftschiffahrt, Vol. 21, No. 22, November 28, 1930.

**P. Schröder, "Take-Off of Seaplanes Based on a New Theory of Reduction in Hydrodynamics," Zeitschrift für Flugtechnik und Motorluftschiffahrt, January 14, 1931 (to be issued as N.A.C.A. Technical Memorandum No. 621).

P. Schröder, "A Reduction Theory for Planing Water Aircraft and Its Experimental Verification," Bericht der Hamburgischen Schiffbauversuchsanstalt, July 14, 1930.

Derivation of Various Conversion Formulas

Case 1 - for different loads of hydrovanes

Given: W_1 and M_1 for a load A_1 = constant;

Find: W_2 and M_2 for a load A_2 = constant.

The equation defining the corresponding speeds now becomes:

$$\frac{v_1^2}{A_1} = \frac{v_2^2}{A_2} \quad (1)$$

or

$$v_2 = v_1 \sqrt{\frac{A_2}{A_1}} \quad (2)$$

By virtue of $W = \epsilon A$ and $M = \mu_0 A$, these speeds are

$$\frac{W_1}{A_1} = \frac{W_2}{A_2} = \epsilon \quad \text{and} \quad \frac{M_1}{A_1} = \frac{M_2}{A_2} = \mu_0$$

and W_2 and M_2 are now expressed as

$$W_2 = W_1 \frac{A_2}{A_1} \quad (3)$$

and

$$M_2 = M_1 \frac{A_2}{A_1} \quad (4)$$

These three equations (2), (3), and (4), now assume a general significance, inasmuch as they retain their validity even when A_1 and A_2 are arbitrary speed functions.

Case 2 - Conversion of resistance and trimming moment curves of a hydrovane for application as seaplane float

Given: W_1 and M_1 for a load A_1 = constant,

Find: W_2 and M_2 for a load $A_2 = A\,(v)$.

This case of A being a prescribed function of the speed is always applicable when the pertinent vane is to be used as seaplane float. If A_1 is the gross weight of the aircraft, the load formula by fixed trim is:

$$A_2 = A_1 \left(1 - \frac{v_2^2}{v_s^2}\right) \tag{5}$$

where v_s = getaway speed. Now we write the quotient A_2/A_1 in (2), resolve it according to v_2, and obtain the relation for computing the corresponding speeds

$$v_2 = \frac{v_1}{\sqrt{1 + \left(\frac{v_1}{v_s}\right)^2}} \tag{6}$$

Then we insert the quotient A_2/A_1 of (5) in (3) and (4), and use (6) for eliminating v_2 which yield the formulas for W_2 and M_2 as

$$W_2 = \frac{W_1}{1 + \left(\frac{v_1}{v_s}\right)^2} \tag{7}$$

$$M_2 = \frac{M_1}{1 + \left(\frac{v_1}{v_s}\right)^2} \tag{8}$$

C a s e 3 -

Given: W_1 and M_1 for a load $A_1 = A_1\ (v)$,

Find: W_2 and M_2 for a load $A_2 = A_2\ (v)$.

This case has practical significance when resistance and moment of a seaplane flotation gear have been measured for a certain wing and it is subsequently desired to apply these values to some other gross weight and other wing. In addition, it contains the possibility of embodying the effect of the wind in take-off investigations of seaplanes.

α) Conversion of the take-off resistance curves and trimming moment curves of a seaplane flotation gear to different wings and gross weights:

We assume W_1 and M_1 measured for

$$A_1 = G_1 \left(1 - \frac{v_1^2}{v_{s_1}^2}\right) \tag{9}$$

where G_1 = gross weight of aircraft upon which the measurement was based, and v_{s_1} = relevant getaway speed. The new gross weight is to be G_2 and the getaway speed with the new wing is to be v_{s_2}. Then the loading

$$A_2 = G_2 \left(1 - \frac{v_2^2}{v_{s_2}^2}\right) \text{ is requested} \tag{10}$$

A calculation analogical to the preceding case yields with abbreviation

$$N^2 = \frac{G_1}{G_2} \quad 1 - \left(\frac{v_1}{v_{s_1}}\right)^2 + \left(\frac{v_1}{v_{s_2}}\right)^2 \tag{11}$$

the following results:

$$v_2 = v_1 \, N^{-1} \tag{12}$$

$$W_2 = W_1 \, N^{-2} \tag{13}$$

$$M_2 = M_1 \, N^{-2} \tag{14}$$

β) Effect of wind in take-off studies of seaplanes:

Assume as measured, W_1 and M_1 for take-off with zero wind by lift

$$A_1 = G \left(1 - \frac{v_1^2}{v_s^2}\right) \tag{15}$$

where G = gross weight and v_s = getaway speed in zero wind. W_2 and M_2 are to be computed for w m/s head wind. Because of the greater unloading by the wings the loading of the flotation gear remains as

$$A_2 = G \left[1 - \left(\frac{v_2 + w}{v_s}\right)^2\right] \tag{16}$$

The calculation of v_2 reveals

$$v_2 = -\left(\frac{v_1}{v_s}\right)^2 w + v_1 \sqrt{1 - \left(\frac{w}{v_s}\right)^2 \left[1 - \left(\frac{v_1}{v_s}\right)^2\right]} \tag{17}$$

and the elimination of v_2 sets forth the quotient A_2/A_1 as:

$$\frac{A_2}{A_1} = 1 + \frac{v_1^2 \, w^2}{v_s^4} - \frac{2 v_1 \, w}{v_s^2} \sqrt{1 - \left(\frac{w}{v_s}\right)^2 \left[1 - \left(\frac{v_1}{v_s}\right)^2\right]} \tag{18}$$

which, substituted in (3) and (4), supplies the desired values for W_2 and M_2.

Graphic Method

The preceding analytical problems can equally be solved by graphical means. Given curve W_1 or M_1 under the secondary assumption that $A_1 = A_1\ (v)$, find W_2 or M_2, respectively, for the assumption $A_2 = A_2\ (v)$. Equations (2), (3), and (4) may be written as

$$\frac{W_2}{W_1} = \frac{M_2}{M_1} = \frac{A_2}{A_1} = \frac{v_2{}^2}{v_1{}^2}$$

on which the following method can be based:

Plot A_1 and A_2, as well as W_1 and M_1 over v^2. For the arbitrarily chosen speed v_1 on the W_1, M_1, and A_1 curves, we have point P_1, which we connect with the origin of the coordinates O. The connecting line intersects the prescribed lift A_2 in point P_2. The ratio of the distances OP_2 to OP_1 is the same in all three diagrams, namely, W/v^2, M/v^2, and A/v^2, thus revealing P_2 as the point of the desired resistance and moment curve. This representation yields W_2 and M_2 in simple fashion for any stage of the prescribed lift.

II. Examples of Application

The examples have been selected so as to allow the comparison of the mathematical figures with the test data. This applies to all but the last example where the lack of suitable data makes this impossible.

The solutions are given in graphical form. The points defined by calculation and graphic construction, respectively, are indicated by small circles in the first four examples. The pertinent curve in question was included for comparison in conformity with measurements made prior to the development of the conversion method.

1. Example - Conversion of the resistance of a hydrovane:

Given resistance curve W_1 (Fig. 1) of a 1 : 8 scale model of a flat-bottom hydrovane (designed by Engineer Ellinghausen, Bremen), pertaining to loading $A_1 = 6.5$ tons;

Find the resistance curve W_2 for loading $A_2 = 5.5$ tons. The lines emanating from origin O define the corresponding speeds. Each intersection point P_1 of such a line with curve W_1 has a relevant point P_2 on curve W_2, so that

$$OP_2 : OP_1 = 5.5 : 6.5 = 0.847.$$

This defines W_2. The portion of the W_2 curve from 2.5 to 4.5 m/s was measured direct (Test No. 2681 of the Hamburg seaplane channel laboratory, January, 1928).

2. Example - Compute the resistance curve of twin floats:

Given resistance curve W_1 (Fig. 3) of a 1 : 6 scale model of twin floats for a seaplane with constant loading $A_1 = 2920$ kg;

Find the resistance curve for loading

$$A_2 = A_1 \left(1 - \frac{v^2}{{v_s}^2}\right)$$

with 100 km/h getaway speed. For the model we obtain

$$A_1 = 2920 : 6^3 = 13.52 \text{ kg}$$

$$v_s = \frac{100}{3.6 \times \sqrt{6}} = 11.34 \text{ m/s}$$

Figure 2 exhibits the construction of the lift ratio A_2/A_1. The formula $OP_2 : OP_1 = A_2 : A_1$ reveals a point P_2 of the desired resistance curve on each line emanating from O. The plotted curve W_2 was measured direct. (Report Jf 39/2 of the D.V.L. (Deutsche Versuchsanstalt fur Luftfahrt), January 21, 1929, page 39).

3. Example - Compute the trimming moments of twin floats:

Given trimming moment curve M_1 (Fig. 5) of a 1 : 6 scale model of the twin floats for a seaplane with constant loading $A_1 = 2400$ kg;

Find the moment curve for the same trim for loading A_2 of the floats

$$A_2 = A_1 \left(1 - \frac{v^2}{{v_s}^2}\right)$$

for a seaplane with 84 km/h getaway speed. We find:

$$A_1 = 2400 : 6^3 = 11.12 \text{ kg},$$

$$v_s = \frac{84}{3.6 \sqrt{6}} = 9.53 \text{ m/s}$$

Figure 4 represents the lift ratio A_2/A_1. The points P_2, which in the moment diagram (Fig. 5) reveal the desired moment M_2, are defined by

$$OP_2 : OP_1 = A_2 : A_1.$$

The M_2 curve was measured direct. (Report Jf 63/1 of the D.V.L., June 12, 1929, pages 38 and 39 for 5° trim run).

4. Example - Conversion of resistance curve of an aircraft to a different gross weight:

Given resistance curve W_1 of a 1 : 6 scale model of twin floats for a prescribed loading

$$A_1 = G_1 \left(1 - \frac{v^2}{v_s^2}\right).$$

The gross weight is $G_1' = 2400$ kg and the getaway speed is 84 km/h, so that, as in the preceding example, $G_1 = 11.12$ kg, and $v_s = 9.53$ m/s.

Find the resistance curve for the same twin floats with a smaller wing and a lower gross weight, that is,

$$G_2' = 0.75 \; G_1' = 1800 \text{ kg}$$

by identical wing loading.

The latter implies that v_s does not change. The new lift is

$$A_2 = G_2 \left(1 - \frac{v^2}{v_s^2}\right),$$

where $G_2 = 1800 : 6^3 = 8.34$ kg, as illustrated in Figure 6 for A_2/A_1. The small circles on curve W_2 of Figure 7 again conform to the condition

$$OP_2 : OP_1 = A_2 : A_1.$$

The part curves shown are from direct measurements (Report Jf

63/1 of the D.V.L., June 12, 1929, page 36, and Report Jf 63/1 of the D.V.L., June 14, 1929, page 30, 5° trim run).

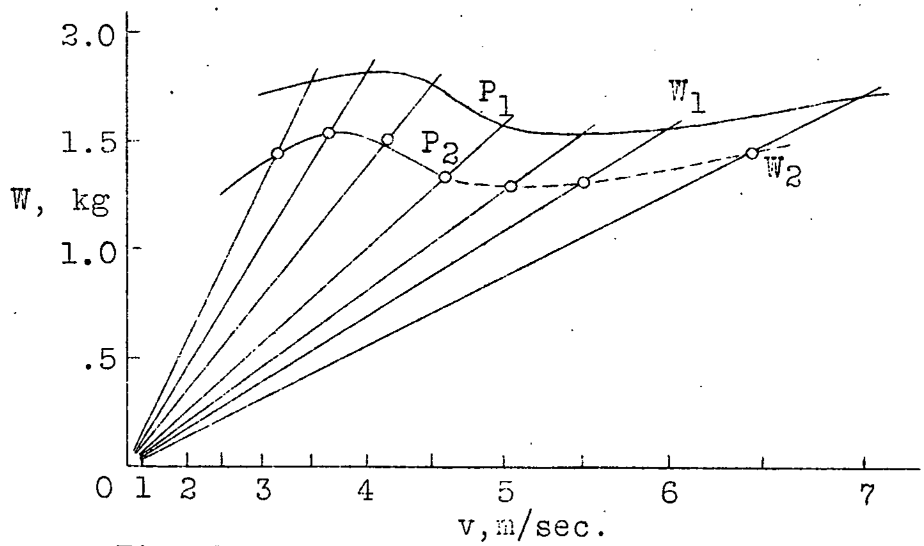
5. Example - Including the effect of the wind in take-off studies of seaplanes:

Given resistance curve W_1 (Fig. 9) of twin floats for take-off with no wind;

Find therefrom the resistance curve for take-off at $w_2 = 5$ m/s, and $w_3 = 10$ m/s head wind.

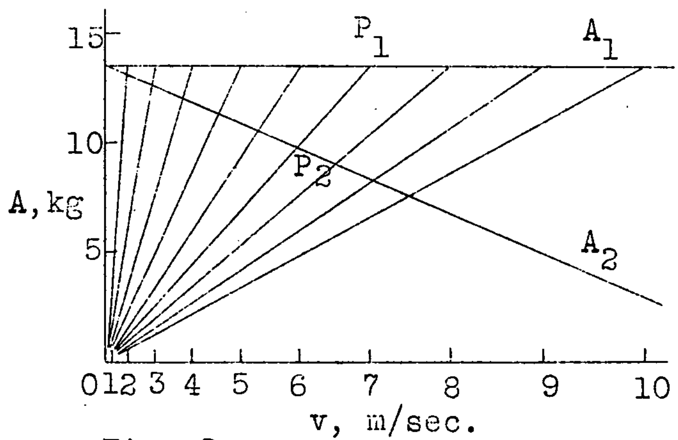
Figure 8 contains the prescribed loading A_1 for $w_1 = 0$ and the construction of the lift ratios A_2/A and A_3/A_1. Figure 9 reveals the desired resistance W_2 and W_3. The twin floats selected produced unfavorable resistances at high speeds, and the lowered resistance, owing to the wind, is particularly noticeable.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.



3.0
1.5
W, kg
1.0
.5
0 1 2 3 4 5 6 7
v, m/sec.
P1
W1
P2
W2

Fig. 1



15
10
A, kg
5
0 1 2 3 4 5 6 7 8 9 10
v, m/sec.
P1
A1
P2
A2

Fig. 2

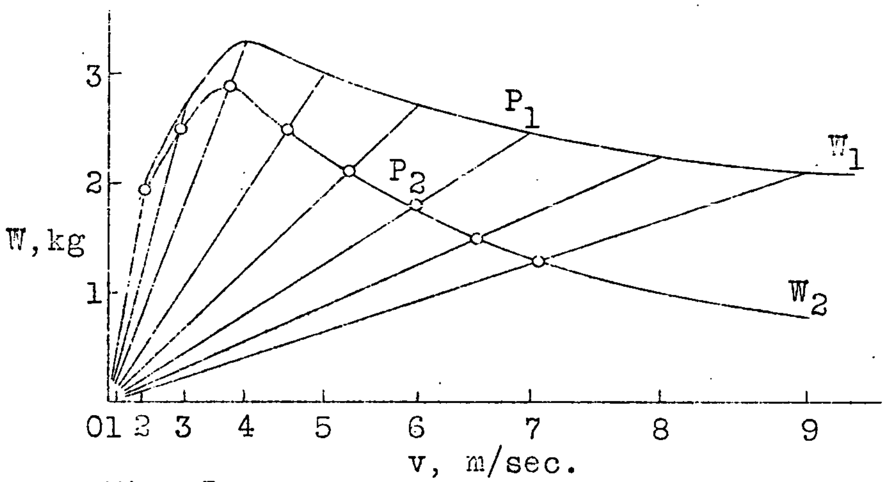


Fig. 3

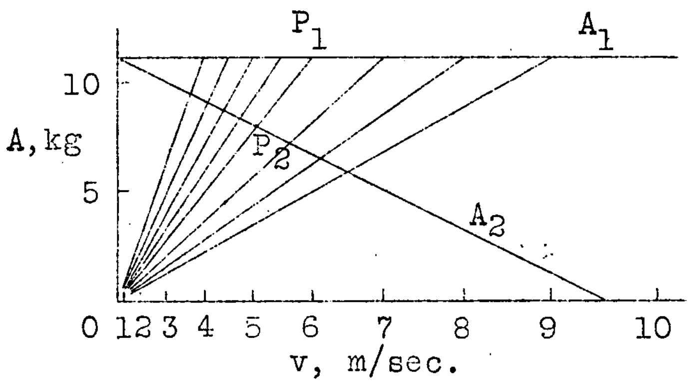


Fig. 4

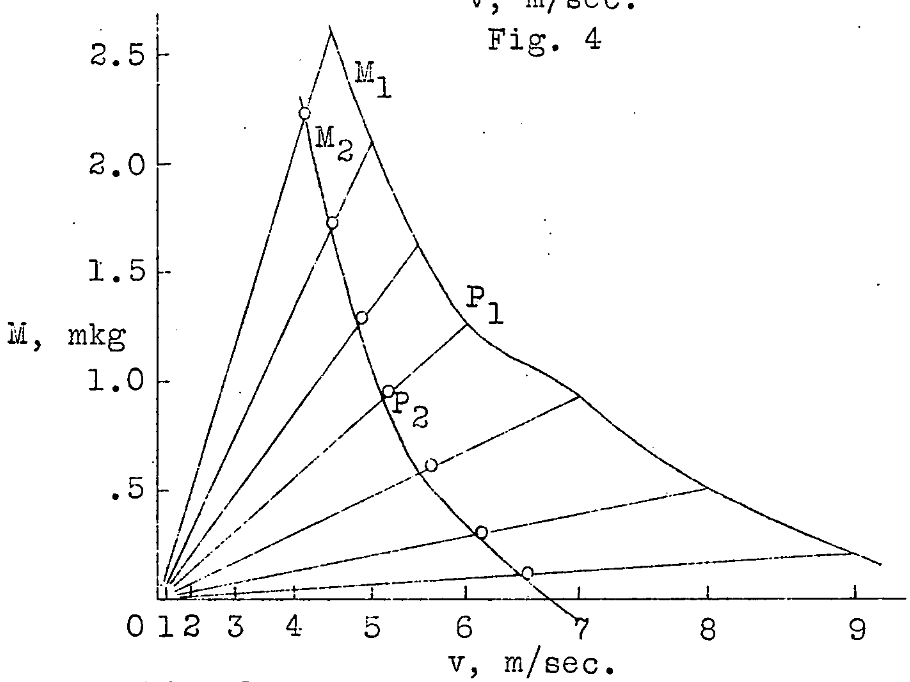


Fig. 5

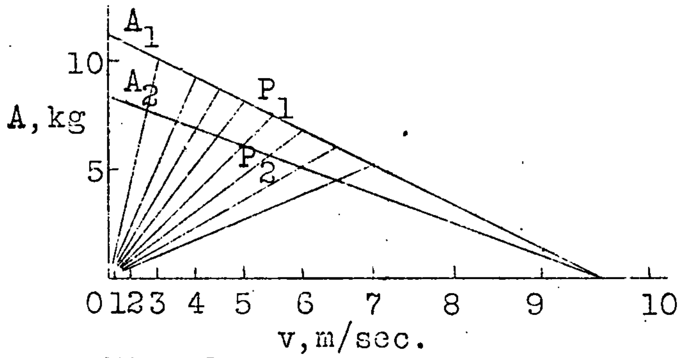

A, kg
A_1
A_2
P_1
P_2
10
5
0 1 2 3 4 5 6 7 8 9 10
v, m/sec.

Fig. 6

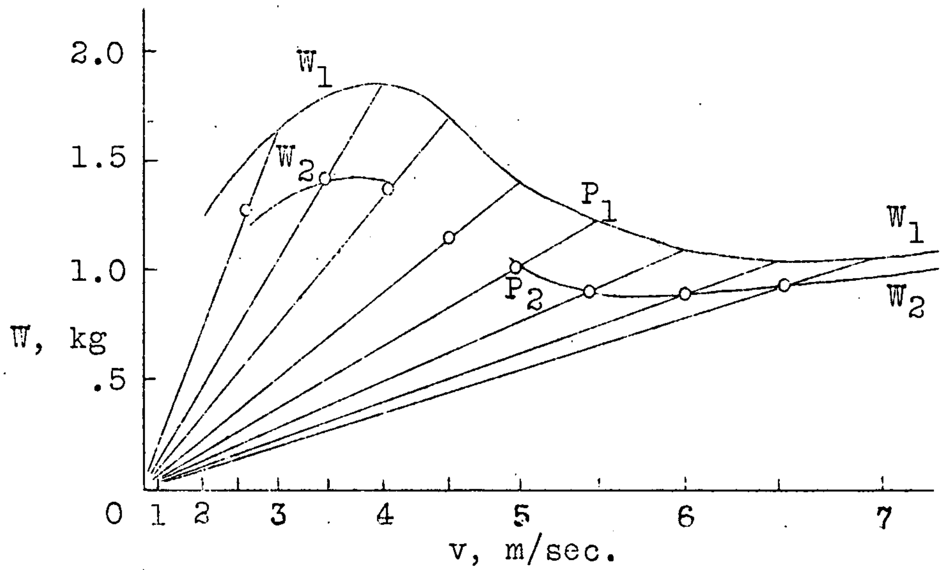

W, kg
2.0
1.5
1.0
.5
W_1
W_2
P_1
P_2
W_1
W_2
0 1 2 3 4 5 6 7
v, m/sec.

Fig. 7

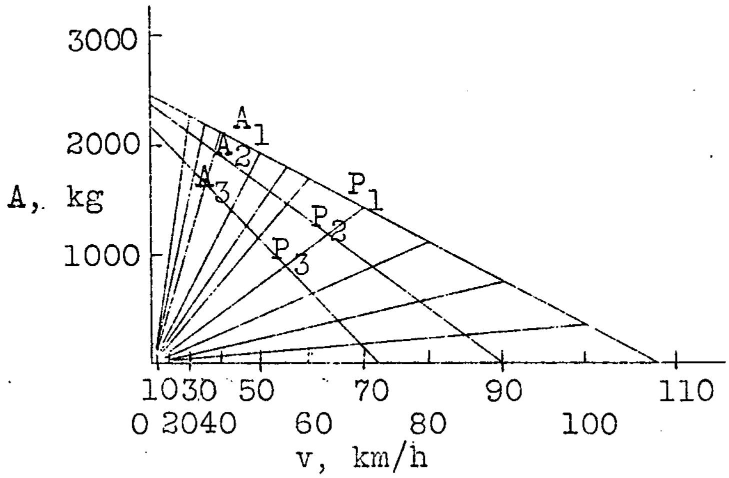

3000
2000
1000
A, kg
A_1
A_2
A_3
P_1
P_2
P_3
10 30 50 70 90 110
0 20 40 60 80 100
v, km/h

Fig. 8

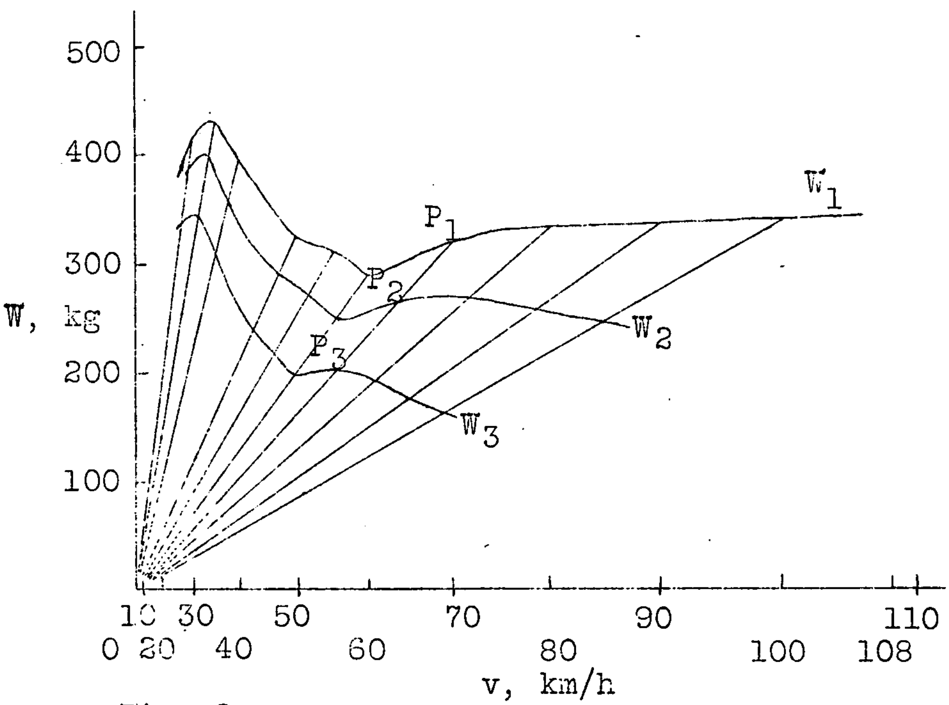

500
400
300
200
100
W, kg
W_1
W_2
W_3
P_1
P_2
P_3
10 30 50 70 90 110
0 20 40 60 80 100 108
v, km/h

Fig. 9