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# FLAT SHEET METAL GIRDERS WITH VERY THIN METAL WEB*

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## PART I

### General Theories and Assumptions

#### P r e a m b l e

This treatise on sheet metal girders with very thin web is the result of my activities with the Rohrbach Metal Airplane Company.

My object was to develop the structural method of sheet metal girders and should for that reason be considered solely from this standpoint. The ensuing methods were based on the assumption of the infinitely low stiffness in bending of the metal web. This simplifies the basis of the calculations to such an extent that many questions of great practical importance can be examined which otherwise cannot be included in any analysis of the bending stiffness of the buckled plate. I refer here to such points as the safety in buckling of uprights to the effect of bending flexibility of spars, to spars not set parallel, etc.

The assumption of infinitely low resistance in bending of the plate produces errors whose intensity and effect on the

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stress in the plate will be discussed in Part II. It becomes apparent that the formation of wrinkles induces local bending stresses which are pronounced even in very thin plates and which also have a certain effect on the stress of the material. But primarily, it should be noted that the ultimate load of a sheet metal wall is always correctly interpreted by the subsequent theory, because after exceeding the yield limit the bending resistance (almost) disappears in comparatively thick metal plates.

Omission of the bending resistance of the plate has practically no effect on the calculation of the mean tension stress in the web and consequently on that of the spars and uprights.

The calculation methods in this report are confined to flat sheet metal girders because their derivation and application require no experimental data; the curved sheet metal girders are to be treated in a later report.

We begin with the simple, so to say, every-day applied calculations, following with explanatory considerations, and concluding with several rough technical computations.

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This report is quite voluminous, and it therefore seems desirable to start with the range of applicability of these calculation methods. Whether it is appropriate to build a sheet metal girder with a shear-resistant web or with a "thin-walled" web, so as to produce a diagonal tension field under stresses (our calculation methods refer to the latter), depends on the intensity of the cross stress Q with respect to the girder height h. These two quantities are best combined in an index value $K_W$ of the sheet metal girder

$$K_W = \frac{\sqrt{Q}}{h}$$

By small $K_W$ it is advantageous to use "thin-walled" webs (fields of tension diagonals), by high $K_W$ shear-resistant webs. For duralumin (there is no appreciable difference for steel or wood), the transition by $K_W$ values of about 2 or 3 kg, is $\frac{1}{2}$ $cm^{-1}$.

Let $K_W = 2$ correspond, for example, to a girder $h = 50$ cm high, and which is to be subjected to a cross stress $Q = 10,000$ kg (ultimate load). The sheet metal web of such a girder is of about 1.2 mm wall thickness; to make the web resistant to buckling the spacing of the reinforcements must not exceed forty times the wall thickness, that is, 50 mm. (The allowable shear stress then about equals the yield limit - See 1928 Yearbook of the Wissenschaftlichen Gesellschaft für Luftfahrt, page 119.) It becomes obvious that the reinforcements (uprights) must be already spaced close together, and that even with this high $K_W$ it is of no advantage to make the web resistant to shear.

The $K_W$ values of beams in bending in airplane construction are, however, almost without exception, markedly lower, and we are forced (unless we prefer dissatisfactory structures) to let the web form wrinkles.

Yes, even if we place the uprights close enough so that wrinkling is not very pronounced in normal flight stresses (to ensure a smooth skin, as of a wing) it nevertheless will be impossible to prevent the expressed forming of the diagonal tension field under higher stresses.

This is the reason that practically all sheet metal girders used in airplane construction are formed and calculated as diagonal tension fields (unless corrugated plates are used). Moreover, bearing in mind that such a sheet metal girder is generally lighter and less expensive than a lattice girder, it is entirely justifiable to subject these problems to an exhaustive investigation.

## Various Nonmathematical Considerations - Assumptions

Let us make the following experiments: Take a sheet of paper or a thin metal plate (Fig. 1a) which bends easily. Now fold it in parallel, uniform wrinkles or lobes, as in Figure 1b. While doing this the two edges A come closer together (by $\Delta a$); the ratio of the depth of the width of the wrinkles then depends on the amount of this approach. This process of bringing the edges A closer together is accomplished with practically no resistance, and the amount of compression exerted perpendicular to the edges A is zero (or very nearly so).

Now let a tension $\sigma$ be applied at the upper and lower edge of the lobed sheet (Fig. 1c) while the distance of the edges A is to remain the same as in Figure 1b. The sheet will withstand considerable tension stresses in this direction without any appreciable change in the shape of the wrinkles.

If we applied tension stresses obliquely to the wrinkles it would necessitate the presence of outside stresses perpendicular to the surface of the sheet in order to preserve equilibrium of the stresses and the stress components acting on a metal strip perpendicular to the surface of the sheet.

But since we disallow the presence of such stresses, the tension must be applied in the direction of the wrinkles. Besides, it is clear that only tension stresses can be applied in the direction of the wrinkles, but no compression stresses, because the infinitely thin sheet is not buckling resistant under compression.

Likewise it is not permissible to have shear stresses act at the edges in the direction of the wrinkles nor perpendicular to them, because of the inability of an infinitely thin sheet to take up such shear stresses (it would collapse obliquely to the shear stress).

It might be inferred that, due to the curvature of the sheet when wrinkling, it was nevertheless in the position to carry such shear, or, compression stresses acting in the direction of the wrinkles, but such is not the case. The depth of the wrinkles and the induced curvature of the sheet is infinitely small as long as $\frac{\Delta a}{\alpha}$ is of the order of an elongation, that is, infinitely small in the sense of the theory of elasticity.

We therefore repeat that the solely applying tension stress $\sigma$ is a principal stress and that the sheet in Figure 1c is subjected to a uni-axial stress attitude, that is, that the principal stress $\sigma$ alone assumes an appreciable value, while the other principal stresses are zero (or nearly so). The elongation $\epsilon$ induced by the tension is

$$\epsilon = \frac{\sigma}{E} \quad (E = \text{Young's modulus}). \tag{1}$$

It is in the direction of principal stress $\sigma$ (direction of lobes) and is the greatest positive elongation of the sheet.

It will be noted that no assumptions are made regarding the intensity of bending stress which this wrinkling of the sheet produces. Later on we shall show that this bending stress is very low for the considered very thin sheets.

If we had observed the sheet very closely while tension $\sigma$ was applied, we would have noticed a slight decrease in the lobe depth, because the contraction in area induced by the tension slightly lowers the lobe-forming effect of the approaching edges A. Finally, we can allow $\sigma$ to become so high that the formation of wrinkles stops (in which case $\frac{\sigma}{m\,E} \geqq \frac{\Delta a}{\alpha}$), but we shall disregard this in the present report. We further assume that the lobe-forming contraction in area

$$-\epsilon_q = \frac{\Delta a}{\alpha}$$

no matter from what cause (for example, deformation of edge profiles of sheet metal girder) is greater than the transverse contraction due to tension, so that

$$-\epsilon_q - \frac{\epsilon}{m} > 0 \quad (m = \text{transverse contraction figure}) \qquad (2)$$

We are always in a position to check these conditions which depend on the type of construction and on the applied stresses.

We now shall summarize the chief features of our discussion thus far:

If a very thin plate forms wrinkles during deformation, there is no normal stress perpendicular to the run of the wrinkles, no matter what value $-\epsilon_q$ may assume (provided inequation (2) is complied with); consequently, elongation $\epsilon$ falling in the direction of the wrinkles is affected by $\sigma$ (equation 1) but unaffected by $-\epsilon_q$. There is no shear stress in a section perpendicular or parallel to the wrinkles; $\sigma$ being a principal stress the direction of the wrinkles is in that of the greatest positive elongation $\epsilon$.

Before proceeding to more general cases we examine the disturbing effect of the edge profiles. Assuming our plate with area a a to be bounded by four-edge strips, which we load and deform conformal to Figure 2 (while the strips remain straight), the plate edges become deflected as in Figure 1c. But since the plate edges must now remain flat, while the center of the plate is pushed out of its original plane (at the highest half the depth of the wrinkle $\zeta$), $\sigma$ now must be slightly higher (by $\Delta\sigma$) than $E\epsilon$ ($\epsilon$ = elongation as seen from above).

Now we shall indicate that this increase $\Delta\sigma$ can be disregarded in our considerations.

By given $-\epsilon_q = \frac{\Delta a}{\alpha}$ and given $\sigma$ we know the ratio of lobe width to lobe depth. And it can be shown that the number of lobes by given edge deformation is higher as the plate is thinner, and consequently, that the width and the depth of the lobes $\zeta$ is small in thin plates and become infinitely small $(\zeta \rightarrow 0)$ when the plate becomes infinitely thin $(s \rightarrow 0)$; but in the limiting case of s 0 with diminishing lobe depth, $\Delta\sigma$ equally approaches a zero value.

Thus we assume (the extent of these assumptions to practical cases is given in Part II - N.A.C.A. Technical Memorandum No. 605), the plate thickness s to be such that lobe depth $\zeta$ is small enough to make $\Delta\sigma$ negligible with respect to $\sigma$. This eliminates the width and depth of the wrinkles from the consideration, leaving, however, the direction of the wrinkles $\sigma$, $\epsilon$ and $-\epsilon_q$, respectively. The connections between these quantities and the deformation of the plate edges must be explained.

## Example

Let us take a square panel a a formed of four perfectly rigid members but with flexibly connected corners (Fig. 3a). To make this panel suitable for taking up transverse stresses, we reinforce it with cross diagonals $D_1$ and $D_2$. The stresses in the diagonals (as long as $D_2$ does not buckle) are inversely equivalent; $D_2$ is stressed in compression and $D_1$ in tension. But if the diagonals consist of low bending resistant sections any further increase in cross stress P induces $D_2$ to buckle long before stress in it reaches the yield limit of the material. By further increase in P, we can assume that the stress in the buckled $D_2$ remains constant but that thereby the tension in $D_1$ raises twice as fast, so that finally the tension diagonal $D_1$ transmits the principal portion of the cross stress. During this deformation the angle formed by the two diagonals remains rectangular.

But instead of the cross diagonals we can use a solid web plate (wall thickness = s) (Fig. 3b). The principal stresses $\sigma_1$ and $\sigma_2$ slope, as we know, at 45° toward the direction of the edge strips, and are of the order of $\pm \tau$.

Now a further increase in stress P forces a comparatively thin plate to buckle with respect to compression stress $\sigma_2$ (this buckling is of course somewhat delayed by the contemporary tension stress $\sigma_1$); that is, the plate wrinkles in the direction of principal stress $\sigma_1$ (Fig. 3c). Under continual rise of P only $\sigma_1$ becomes materially larger and assumes, in the limiting case, twice the value of the plate in shear. This means that $\sigma_2$ may be disregarded relative to $\sigma_1$ for very thin plates and correspondingly high stress P. We say: the plate is under tension; it forms a diagonal tension field.

It is easily shown that by the described deformation of the plate edges all fibers running in the direction of the wrinkles undergo the same elongation $\epsilon$ (the edge sections to be perfectly rigid, hence resistant to bending) so that $\sigma_1$ has a constant value in the whole field. The vertical component of the stress transmitted by the plate in cross section 1 (Fig. 3c) is calculated as*

$$\sigma_1 \frac{a}{\sqrt{2}} s \sin 45^\circ = \sigma_1 \frac{a\ s}{2} = P,$$

hence

$$\sigma_1 = 2 \frac{P}{a\ s} = 2\tau.$$

Now we assume the plate, having formed a diagonal tension field, to be cut into numerous strips parallel to the wrinkles so as to form nothing but individual diagonals. This does not change the stress attitude in the plate. Supposing that this cut was made prior to loading and a straight line was drawn on the plate over these separate diagonals; we find that this line also remains a straight one after the deformation. This, however, implies that a straight member, if flexibly attached to the edge strips can remain straight after the deformation and is not stressed laterally. If we place the plate parallel to the rigid edge strips (Fig. 3d), so that the distance of the ends of the added plate does not change during the deformation, this plate is not subjected to deformation at all, the plate exerting

*It can be proved that the transverse stresses in the two edge strips at section 1, induced by the stressed skin, are inversely equivalent, so that the cross stress transmitted by the skin must = P.

no effect on the panel and vice versa. It has no effect on the deformation attitude of the sheet metal, nor on the direction of the wrinkles and tension stresses, respectively.

When the edge strips are rigidly connected the direction of the tension stress in the skin is unaffected by the distance of the members. If the four rigid edge strips form a rectangular panel or field the tension stress slopes at 45° toward these strips.

## Differential Equation of the Diagonal Tension Field - The Uni-Axial Plane Stress Attitude

In a plane stress attitude we generally find in every point of the plane two principal stresses at right angles to each other and having a finite intensity. Accordingly, a stress attitude is uni-axial when one of these principal stresses is zero in every point of the considered part of the plane of the plate. The stress trajectories form that system of lines which pertain to the principal stresses which are not zero.

Now we prove:

Theorem 1, that the stress trajectories in the uni-axial plane stress attitude are straight lines;

Theorem 2, that the field of the principal stresses is free of sources, i.e., that the distance of two adjacent stress trajectories is inversely proportional to the intensity of the principal stress.

Before attempting the mathematical proof we check its accuracy in a simpler manner. We exhibit in Figure 4 several stress trajectories. In conformity with our assumption the second principal stress is zero and, owing to the absence of shear stresses in the planes of intersection along the stress trajectories, there are no stresses in the shaded part of the plate marked II. From the equation of equilibrium for this part of the plate it follows that the two stresses acting on area I must be inversely equivalent, that is, in the same direction, and that the tension stresses $\sigma$ (principal stresses) must be higher as the two adjacent stress trajectories come closer together. This proves the accuracy of the above two statements.

We see on Figure 5 a very small triangular piece of the plate. Now let $\sigma$ represent the principal stress which is not zero; the second principal stress is to be zero; $\alpha$ to denote the direction of the principal stress. Under these assumptions we can compute the stresses $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ acting on the planes of intersection in direction of axes x and y, as

$$\sigma_x = \sigma \cos^2 \alpha$$

$$\sigma_y = \sigma \sin^2 \alpha$$

$$\tau_{xy} = \sigma \sin \alpha \cos \alpha$$

Now we consider the principal stress $\sigma$ and its directional angle $\alpha$ as steady function of coordinates x and y. Noting that the equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0; \qquad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

are valid in general for the plane stress attitude, and introducing the above values in these equations for $\sigma_x$, $\sigma_y$, and $\tau_{xy}$, we obtain the differential equations for the uni-axial plane stress attitude. A partial execution of the differentiations indicated in these equations, followed by transformation yields

$$\frac{\partial \alpha}{\partial x} \cos \alpha + \frac{\partial \alpha}{\partial y} \sin \alpha = 0 = \frac{\partial \alpha}{\partial x}\frac{dx}{dz} + \frac{\partial \alpha}{\partial y}\frac{dy}{dz} = \frac{\partial \alpha}{\partial z}$$

$$\frac{\partial(\sigma \cos \alpha)}{\partial x} + \frac{\partial(\sigma \sin \alpha)}{\partial y} = 0$$

($dz$ being a linear component of the stress trajectory in the direction of $\alpha$).

The first equation implies that $\alpha$ does not change from $z$ in direction, that is, the stress trajectories are straight lines. The second equation contains the derivatives of the principal stress components $\sigma$ and expresses the nonexistence of sources in the field of the principal stress $\sigma$.

## The Attitude of Deformation

Figure 6 shows various straight stress trajectories. As a rule the stresses of two adjacent stress trajectories differ; the shaded portion of the plate lying between both stress trajectories is therefore more elongated by one trajectory than by the one adjacent to it, and its effect is to cause a slight curvature in the originally straight plate fiber $g$ (as in an arched girder). The radius of this curvature can be calculated from

$$\frac{1}{\rho} = -\frac{1}{E}\frac{\partial \sigma}{\partial n} = -\frac{\partial \epsilon}{\partial n} \tag{2a}$$

where $n$ is the direction perpendicular to the stress trajectory. This curvature $1/\rho$ is inversely proportional to the modulus of elasticity $E$ of the material and, accordingly, to be considered as infinitely small of the first order, like all deformations of an elastic body. The plate fiber parallel to the stress trajectory $g$, therefore, is straight even after the loading (aside from just this infinitely small quantity).

The next step would be to examine the properties of the field of the transverse contractions $-\epsilon_q$, in order to ensure the properties of the field of elongations $\epsilon$, but for the time being, we are only concerned with deformation attitudes in which the contraction in area is constant over the whole range of the plate and leave these questions for later discussion. We merely state here that $-\epsilon_q$, as shown, does not represent the contraction in plate area due to principal stress $\sigma$ but pertains to the formation of wrinkles. Thus, when the two plate portions, 1 and 2 (see Fig. 7) after loading and wrinkling are in the highest points (points of culmination) of two adjacent wrinkles, $-\epsilon_q$ represents the ratio of the distance of approach (shown as $-b\,\epsilon_q$ in Fig. 7) to width of wrinkle $b$; $-\epsilon_q$ denotes the comparative (specific) approach of two culmination points.

The comparative approach of two other points is generally different from $-\epsilon_q$. But the farther the space of the two points for a given width of wrinkle, the closer their specific approach to $-\epsilon_q$. Consequently, the transverse contraction $-\epsilon_q$ can be expressed as the specific transverse approach of two points, provided the two points are far enough apart to be crossed by a large number of wrinkles. If, in the limiting case of the infinitely thin plate, the width of the wrinkle is infinitely small, then $-\epsilon_q$ yields precisely the transverse approach of two points which are a finite space from each other.

One more remark on wrinkling, while ignoring for the time being the appearance of a tension stress in the plate. A very thin plate can only be brought into such a form (we ignore the very slight stress in bending) which is redevelopable into a plane without producing stresses. So the area produced by wrinkling must contain a sheaf of straight lines; the wrinkles must be straight.

The simplicity of these considerations is based on the formation of straight wrinkles without resistance, and on the fact that wrinkling in the very thin plate calls for a uni-axial stress attitude, which yields straight stress trajectories, so that the principal stresses are able to follow the shape of the wrinkle at every point.

Our next problem will be to define the deformation attitude of the plate when the deformation of the edges is given. Thus, with equation (1), theorem 1, and theorem 2, we so choose a deformation $\epsilon$ due to $\sigma$ and deformation $-\epsilon_q$ that certain limiting conditions are complied with.

## Rectangular Panel with Edge Strips Rigid in Bending - Given Deformation of Edge Strips

Now let us consider a rectangular panel with four flexibly connected edge strips (Fig. 8) and covered with a very thin skin. Due to the interaction of outside forces the edge strips are subjected to length changes. But we assume that the length changes $\lambda_x$ in the two strips parallel to axis $x$ are equal, and likewise that the length changes $\lambda_y$ in the strips parallel to axis $y$ are equal. Now, if $l_x$ and $l_y$ is the length of these strips, the elongation in the direction of axes $x$ and $y$ becomes

$$\epsilon_x = \frac{\lambda_x}{l_x} \qquad \epsilon_y = \frac{\lambda_y}{l_y} .$$

We further presume that the strips, due to the action of the outside forces, are subjected to a direction of change at angle $\gamma_x$ and $\gamma_y$. The mutual angle formed by both strips after the deformation differs by $\gamma = \gamma_x + \gamma_y$ from $90^\circ$. In addition, let us suppose that the strips remain straight by the deformation.

To calculate the direction of the ensuing wrinkles and the tension stresses in the skin, we proceed as follows.

We assume the plate is resistant to buckling; then we draw a circle with radius 1 around a point O while the plate is as yet undeformed. Now when the edge strips are deformed the plate does likewise, and the circle becomes an ellipse. The shape and position of this ellipse is unaffected by the selection of point O; the attitude of deformation is the same in all points. The directions of the principal axes presage the directions of the principal normal stresses. Let us suppose the deformation of the edge strips to be such that one of these stresses (the one falling in the direction of the great main axis) is in tension, the other in compression. Now, if the plate is not resistant to buckling it collapses under this compression and wrinkles in the direction of the principal tension stress. The position of the elongation ellipse and the amount of the elongation are kept perfectly intact, provided the elongation ellipse was drawn large enough to extend over several wrinkles. (Strictly speaking, this statement applies only to the limiting case of the infinitely thin plate, i.e., for infinitely small wrinkles.) The elongation in direction of the large axis of the ellipse (this elongation is called $\epsilon$) now corresponds to the principal tension stress $\sigma$, which is the direction of the wrinkles. The elongation perpendicular to it is our transverse contraction $-\epsilon_q$.

This proves that the geometrical relations between the elongations in a buckled plate are exactly like those in a nonbuckled plate.

Now we would like to propose a somewhat clearer method for defining the direction of the wrinkles (Fig. 8a). Beginning with the deformation of a thin plate, we assume an originally flat nonstressed thin plate to be evenly deformed through some outside forces so that the parallel tension stresses $\sigma$, which form an angle $\alpha$ with axis $x$, elongate the plate evenly in the direction of $\sigma$ to the amount of $\epsilon$, and that owing to the uniform wrinkling in the direction of $\sigma$, the plate is subjected to an equivalent transverse contraction $-\epsilon_q$ perpendicular to this direction. The quantities $\alpha$, $\epsilon$, and $-\epsilon_q$, which are constant in the whole field, are assumedly given.

Thus, when we draw two straight lines on the plate prior to deformation, one in the direction of axis $x$, and the other in that of axis $y$, these lines undergo changes in length and direction by this deformation. For example, point $P_x$, originally on axis $x$ and having $l_x$ as abscissa, changes its position. The components of its displacement are:

$$\left\{\begin{array}{l} \lambda_x = l_x\,\epsilon_x = l_x(\epsilon\cos^2\alpha + \epsilon_q\sin^2\alpha) \text{ in the direction of } x, \\ l_x\,\gamma_x = l_x\sin\alpha\cos\alpha\,(\epsilon - \epsilon_q) \text{ in the direction of } y. \\ \text{Likewise, for point } P_y \text{ on axis } y \\ l_y\,\gamma_y = l_y\sin\alpha\cos\alpha\,(\epsilon - \epsilon_q) \text{ in the direction of } x, \\ \lambda_y = l_y\,\epsilon_y = l_y\,(\epsilon\sin^2\alpha + \epsilon_q\cos^2\alpha) \text{ in the direction of } y. \end{array}\right. \qquad (3a)$$

Consequently,

$$\left\{\begin{array}{r} \epsilon_x = \epsilon\cos^2\alpha + \epsilon_q\sin^2\alpha \\ \epsilon_y = \epsilon\sin^2\alpha + \epsilon_q\cos^2\alpha \\ \gamma_x + \gamma_y = \gamma = 2\sin\alpha\cos\alpha\,(\epsilon - \epsilon_q) \end{array}\right. \qquad (3b)$$

Now let us arrange on the plate, prior to deformation, four flexibly connected members, forming a rectangle and parallel to the axes and then subject the plate again to the previously discussed deformations $\alpha$, $\epsilon$, $-\epsilon_q$ and at the same time change the length and direction of the four members as outlined in (3a) and (3b). The connection between sheet and members remains intact over the entire sheet length, providing we force the plate, which likewise has moved on both members to the depth of the wrinkles from its original plane, back to its initial plane on the members, as in Figure 2. If the plate is very thin and the wrinkles are accordingly of very small width and depth, this forcing does not change the stress attitude of the plate appreciably. Equations (3) also indicate what the deformation attitude the edge members running in the direction of the axis must be, if the plate deformation $(\alpha, \epsilon, -\epsilon_q)$ is given. Resolving these equations according to $\alpha$, $\epsilon$, $\epsilon_q$, that is

$$\tan 2\alpha = \frac{\gamma}{\epsilon_x - \epsilon_y} \tag{4a}$$

$$\epsilon = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma^2} \tag{4b}$$

$$\epsilon_q = \frac{1}{2}(\epsilon_x + \epsilon_y) - \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma^2} \tag{4c}$$

these equations yield for given deformation of edge members $(\epsilon_x, \epsilon_y, \gamma)$ at least a feasible deformation attitude of the skin.

Equation (4a) is quite simple. It shows that the direction of the wrinkles is $45^\circ$ in a rectangular field when the edge members are not elongated or else have the same elongation by an angular change $\gamma$ between these members. In case the vertical edge members are under a slight tension or even compression the wrinkles run at a slightly smaller angle to the horizontal. If the compression, for example, in the vertical edge members is lower than in the horizontal, the wrinkles are in more of a vertical direction.

Equations (4b) and (4c) are out of the discussion inasmuch as they are not used, at least for the conventional calculations of sheet metal girders. After defining angle $\alpha$ the tension stress $\sigma$ is more easily determined from the outside stresses of a girder.

Now we must prove that $-\epsilon_q - \frac{\epsilon}{m} > 0$, otherwise our considerations have no real meaning.

Adding (4b) and (4c) we obtain:

$$\epsilon + \epsilon_q = \epsilon_x + \epsilon_y$$

For $-\epsilon_q - \frac{\epsilon}{m} > 0$ it follows from the last equation that this condition can also be expressed by

$$\epsilon - \frac{m}{m-1}(\epsilon_x + \epsilon_y) > 0. \qquad (4d)$$

Since in sheet metal girders it is exclusively the case of $\epsilon > 0$, and $\epsilon_x + \epsilon_y < 0$, our method of calculation is always applicable in normal cases. In abnormal cases the validity of (4d) must be proved, which generally is easily accomplished.

Lastly, by certain deformations of the edge strips, that is, by large negative $\epsilon_x$ and $\epsilon_y$ and by small angle of displacement $\gamma$ (Compare equation 4b), it may happen that both $\epsilon_q$ and $\epsilon$ are negative. In that case the whole infinitely thin plate is without stress, for compression in both directions is impossible. However, such cases do not occur when the field of bending resistant edge members have to take up and transmit cross stresses; consequently, in sheet metal girders with bending resistant spars, $\sigma$ is always a tension stress, when cross stresses occur.

## Given Stresses

We rewrite equations (4) but in different form, because a sheet metal girder is usually given with the allowable compression stress $\sigma_y$ of the uprights and the allowable tension stress $\sigma$ in the metal skin. To save weight, we dimension the members and the metal skin so that the stresses at failure under loading always reach the maximum safe value (for example, the yield limit). We will also show that the stresses existing in the two spars in practical cases have no appreciable effect on our problem, and that this stress $\sigma_x$ on the average can be expressed by a small compression stress, namely, the mean stress, of both spars. The angle of displacement $\gamma$ is usually not given and must be computed from the given quantities. So we assume that $\sigma_x$, $\sigma_y$, and $\sigma$, respectively,

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \frac{\sigma_y}{E} \qquad \epsilon = \frac{\sigma}{E}$$

are given, and we obtain from equation (4) by transformation

$$\tan^2 \alpha = \frac{\sigma - \sigma_x}{\sigma - \sigma_y} = \frac{\epsilon - \epsilon_x}{\epsilon - \epsilon_y} \tag{5a}$$

$$\gamma = \frac{2}{E} \sqrt{(\sigma - \sigma_x)\ (\sigma - \sigma_y)} = 2 \sqrt{(\epsilon - \epsilon_x)\ (\epsilon - \epsilon_y)} \tag{5b}$$

$$- \epsilon_q = \epsilon - \epsilon_x - \epsilon_y \tag{5c}$$

Lastly, we compute $\gamma$ by given $\alpha\ \epsilon,\ \epsilon_x$. We obtain

$$\gamma = 2 \cot \alpha\ (\epsilon - \epsilon_x) \tag{6}$$

Now we work out a simple example. The compression $\sigma_y$ in the uprights is (in order to avoid buckling or wrinkling) to be slightly lower than the tension in the metal skin, and we carry our example through for three values

$$\frac{\sigma_y}{\sigma} = -\ 0.6, \quad -\ 0.7, \quad -\ 0.8$$

For $\sigma_x$ we write $-\ 0.2\ \sigma$, and for $\alpha$ we find

$$\alpha = 40^\circ\ 50'$$
$$\alpha = 40^\circ\ 00'$$
$$\alpha = 39^\circ\ 15'$$

From this we can conclude that for practical cases angle $\alpha$ can nearly always be assumed at

$$\alpha = 40^\circ, \tag{7}$$

unless it is a question of absolutely correct calculation. This simple result is the most important feature of our considerations so far.

For $\gamma$ we obtain

$$\gamma = 2.76\ \epsilon, \quad 2.84\ \epsilon, \quad 2.94\ \epsilon .$$

The angle of displacement is approximately correct if expressed at

$$\gamma = 2.8\ \epsilon = 2.8\ \frac{\sigma}{E}. \tag{8}$$

For $\frac{1}{\epsilon}\left(-\ \epsilon_q - \frac{\epsilon}{m}\right)$ (Compare equation 2) we obtain with $m = 3.3$

$$\frac{1}{\epsilon}\left(-\ \epsilon_q - \frac{\epsilon}{m}\right) = +\ 1.5, \quad +\ 1.6, \quad +\ 1.7 > 0$$

Wrinkling actually occurs in all these cases.

The conditions existing after the yield limit has been exceeded will be discussed in Part III (N.A.C.A. Technical Memorandum No. 606).

## Sheet Metal Girder with Spars Resistant in Bending - Stress Calculation

Figure 9 shows a sheet metal girder pin-ended at the right side. We assume its spars to be continuous and very (infinitely) rigid in bending, and all uprights pin-jointed to the spars.

We use the following symbols:

$s$ = wall thickness of web plate.

$h$ = girder height from C.G. to C.G. of spar.

$F_{Ho}$ = cross-sectional area of upper spar.

$F_{Hu}$ = cross-sectional area of lower spar.

$t$ = spacing of two uprights.

$F_v$, $J_v$, $i_v$ = cross-sectional area of an upright and inertia moment and radius of this area.

$e$ = distance of the C.G. from the cross section of the upright to the web plate.

$\beta$ = angle of upright and spar.

$x$ = distance of a point on the plate wall from the line of action of stress $P$.

$Q$ = cross stress to be transmitted by the plate wall.

$\sigma$ = principal web tension.

$\alpha$ = angle of direction of this tension with the spars.

$\epsilon = \frac{\sigma}{E}$ = principal elongation in the plate web.

$H_O$, $H_U$ = longitudinal stresses in upper and lower spars.

$Q_{HO}$, $Q_{HU}$ = (local) cross stress in spar.

$M_{HO}$, $M_{HU}$ = (local) bending moments in spar.

$\sigma_{xo} = \frac{H_o}{F_{HO}}$ ; $\sigma_{xu} = \frac{H_u}{F_{HU}}$ ; $\epsilon_{xo} = \frac{\sigma_{xu}}{E}$ ; $\epsilon_{xu} = \frac{\sigma_{xu}}{E}$, longitudinal stresses and elongations in the spars (spars parallel).

$\epsilon_x = \frac{1}{2} (\epsilon_{xo} + \epsilon_{xu})$ mean elongation of both spars,

$- V$ = compression in upright.

$\sigma_v = \frac{V}{F_v}$; $\epsilon_v = \frac{\sigma_v}{E}$ = stress and elongation in an upright.

$\epsilon_y$ = elongation perpendicular to spars (in uprights $\epsilon_y = \epsilon_v$).

Now we subject the girder on the left side to cross shear $Q$, so that the web plate forms a diagonal tension field. We assume the dimensions of the sheet metal girder to be such that the direction of the tension stresses $\sigma$ is constant in the whole range of the web plate ($\alpha$ = constant). Then we compute $\sigma$.

The tension stresses $\sigma$ of the web plate act at angle $\alpha$ on the spars, and the ensuing stress $p$ per unit spar length is (Compare Fig. 9)

$$p = 1 \sin \alpha \, s \, \sigma$$

Now we divide this stress into horizontal and vertical components. It is

$$p_x = p \cos \alpha = \sigma \, s \sin \alpha \cos \alpha$$

$$p_y = p \sin \alpha = \sigma \, s \sin^2 \alpha$$

Stress $p_x$ acting in the spar direction effects, for example, an increase in its longitudinal stress $H_o$ on the upper spar. It is

$$\frac{d\,H_o}{d\,x} = p_x$$

Due to cross stress $Q$ at a point $x$, the whole girder is subjected to a bending moment $Q\,x$. As a result, the longitudinal stress $H_o$ in the upper spar must raise when $x$ increases, that is

$$\frac{d\,H_o}{d\,x} = \frac{d\left(\frac{Q\,x}{h}\right)}{d\,x} = \frac{Q}{h}\,.$$

A comparison with the above equation yields

$$\frac{d\,H_o}{d\,x_o} = \frac{Q}{h} = p_x = \sigma \, s \sin \alpha \cdot \cos \alpha,$$

consequently,

$$\sigma = \frac{Q}{h\,s} \, \frac{1}{\sin \alpha \cos \alpha} = \frac{2\,Q}{h\,s} \, \frac{1}{\sin 2\,\alpha} \qquad (9)$$

The stress $p_y$ pulls both spars toward the web; the uprights prevent both from approach, but in doing so, must take up a compression of the order

$$- V = p_y\ t = \sigma\ s\ t\ \sin^2 \alpha.$$

Compared with equation (9), we have

$$- V = Q \frac{t}{h} \tan \alpha \qquad (10)$$

and the compression stress in the upright is

$$\sigma_v = - \frac{Q}{F_v}\ \frac{t}{h} \tan \alpha = \sigma_y \qquad (10a)$$

The stress $p_y$ produces local cross stresses $Q_{HO}$ and $Q_{HU}$ and local bending moments $M_{HO}$ and $M_{HU}$ in upper and lower spars. The spars being continuous, the bending moments are highest at the point of attachment to the uprights, and this applies to both upper and lower spar

$$M_{H\ max} = \frac{V\ t}{12} = \frac{Q}{h} \tan \alpha \frac{t^2}{12} \qquad (11)$$

Now let us imagine at point $x$ a cut parallel to the uprights (parallel to Q) through the sheet metal girder and bring the outside stress $Q$ into balance with the inside stresses transmitted at the intersection. The latter stresses have been included in Figure 9.

The tension

$$Z_s = h \cos \alpha\ s\ \sigma$$

transmitted by the web in the direction of $\alpha$ is now divided into two components

$$X_s = h \cos \alpha\ \sigma\ s \cos \alpha$$

$$Q_s = h \cos \alpha\ \sigma\ s \sin \alpha$$

Noting that the cross stresses $Q_{HO}$, $Q_{HU}$, and the bending moments $M_{HO}$ and $M_{HU}$ caused by the spars are inversely equivalent on upper and lower spar, that is, in balance, the equilibrium of the vertical stress components yields $Q_s = F$, which brings us back to equation (9).

The equilibrium of the horizontal stress components and the moments yield

$$\left.\begin{aligned} H_o &= \frac{Q\,x}{h} - \frac{Q}{2} \cot \alpha \\ H_u &= -\frac{Q\,x}{h} - \frac{Q}{2} \cot \alpha \end{aligned}\right\} \qquad (12)$$

and $\sigma_{HO}$ and $\sigma_{HU}$. If, for example, the cross-sectional areas $F_{HO}$ and $F_{HU}$ of both spars are the same, that is $F_{HO} = F_{HU} = F_H$, then the mean stress of both spars is

$$\sigma_{H\ mean} = \frac{\sigma_{HO} + \sigma_{HU}}{2} = -\frac{Q}{2\,F_H} \cot \alpha \qquad (12a)$$

This stress is usually very low, because the spars must have a large cross-sectional area for taking up the outside bending moment $Q\,x$.

So, when $\alpha$ is known, all stresses are known; $\alpha$ can either be calculated, or, estimated at about 40–42°, which suffices for structural purposes. With these figures the equations advisable for structural purposes are

$$\sigma = 2\,\frac{Q}{hs} \qquad (13a)$$

$$-V = -\sigma_V\,F_V = 0.9\,Q\,\frac{t}{h} \qquad (13b)$$

$$X_{o,u} = \pm\,\frac{Q\,x}{h} - 0.6\,Q \qquad (13c)$$

When the designer builds according to these formulas, he is almost always on the safe side.

We have seen that the tension $(p_y)$ in the metal skin can produce bending moments in the spars with a resulting deflection between two uprights. This is followed by irregularities in the stress distribution over the web plate, $\sigma$ and even $\alpha$ are no longer perfectly constant. But an approximate calculation of the effect of these deflections proves it to be only very slight in correctly chosen construction, so that $\sigma$ and $\alpha$ are actually nearly constant.

Lastly, we discuss the case of uprights eccentrically arranged with respect to the plate wall. If $i_V$ is the inertia radius of the cross section of an upright, e the distance of the C.G. of the cross section of the upright from the plate of the web plate, and when we note that stress V, to be taken up by the upright, acts in the plane of the plate wall (This is valid for a case when the wall has a pronounced curvature due to bending in the eccentrically arranged upright); the stress $\sigma_V$ in the upright in the plane of the plate wall is

$$\sigma_V = \frac{V}{F_V} + \frac{V\ e}{\frac{F\ i_V^2}{e}} = \frac{V}{F_V}\left(1 + \frac{e^2}{i^2}\right) = \frac{V}{F_{V\ red}} \qquad (14a)$$

whereby

$$F_{V\ red} = \frac{F_V}{1 + \frac{e^2}{i^2}} \qquad (14b)$$

## Direction of Wrinkles by Given Dimensions

To establish the dimensions of a sheet metal wall we usually proceed as follows: First, we estimate angle $\alpha$ at about $41^\circ$; next, make a first estimate of the allowable stresses in the uprights; then get the dimensions of spars, uprights and sheet metal wall according to formula (13).

Then we ascertain whether the estimated stresses are really conformal to the chosen dimensions or whether dimensional changes are necessary. Under certain circumstances it may be advisable to check so that the estimated direction of the tension stresses, that is, angle $\alpha$, actually corresponds to the dimensions. For this purpose we now calculate $\alpha$ with respect to the dimensions, but assume that $F_{HO} = F_{HU} = F_H$.

We introduce the values according to (9), (12a), and (10a) for $\sigma$, $\sigma_x$, $\sigma_y$, $= \sigma_V$ into equation (5a), so

$$\tan^2 \alpha = \frac{\frac{Q}{h\ s} \frac{1}{\sin \alpha \cos \alpha} + \frac{Q}{2\ F_H} \cot \alpha}{\frac{Q}{h\ s} \frac{1}{\sin \alpha \cos \alpha} + \frac{Q}{F_V} \frac{t}{h} \tan \alpha}$$

Now that $Q$ is eliminated, we resolve according to $\sin \alpha$ and find

$$\sin^2 \alpha = + \sqrt{a^2 + a} - a \tag{15a}$$

whereby

$$a = \frac{1 + \frac{hs}{2\ F_H}}{\frac{ts}{F_V} - \frac{hs}{2\ F_H}} \tag{15b}$$

If the uprights are eccentrically arranged, we substitute $F_{V\ red}$ for $F_V$ conformal to (14b). We wish to point out that the direction of the wrinkles is unaffected by the stress.

It also should be noted that we made $\sigma_x = \sigma_{H\ mean}$, that is, the mean stress of the spars. The accuracy of this is proved in equation (55c) (Part III - Technical Memorandum No. 606).

Example

Let $Q = 8000$ kg, $h = 60$ cm, and $t = 25$ cm - which is the spacing of the uprights centrical to the plate wall. We further assume the spar dimensions established: $F_{HO} = F_{HU} = F_H = 23\ cm^2$.

Now we estimate the allowable stresses in the plate wall. We take, at random, $\sigma = 2800$, $\sigma_V = 1700$; then, with the above figures, equation (13) yields

$$s = \frac{1}{\sigma}\frac{2Q}{h} = \frac{1}{2800} \times \frac{5 \times 8000}{60} = 0.095\ \text{cm} = \text{about } 0.1\ \text{cm}$$

$$F_V = \frac{1}{\sigma_V}\ 0.9\ Q\ \frac{t}{h} = \frac{1}{1700} \times 0.9 \times 8000 \times \frac{25}{60} = 1.76\ \text{cm}^2$$

Now we dimension the uprights. We assume $F_V = 1.76\ cm^2$, and compute the direction of the actual wrinkles as

$$\frac{hs}{2\ F_H} = \frac{60 \times 0.1}{2 \times 23} = 0.13 \qquad \frac{ts}{F_V} = \frac{0.1 \times 25}{1.76} = 1.42$$

so, according to (15)

$$a = \frac{1 + 0.13}{1.42 - 0.13} = 0.875$$

$$\sin^2 \alpha = \sqrt{a^2 + a} - a = 0.636$$

$$\alpha = 39.5^\circ.$$

Since $\sin 2\alpha = \sin 79^\circ = 0.98$ (instead of 1 as assumed) the skin stress is about 2% higher than equation (13) calls for; but, having rounded off the wall thickness from 0.095 to 0.1 cm and the calculation not being very exact, it is of no particular importance. Since $\tan 39.5^\circ = 0.81$ (instead of 0.9, assumed in (13b) ) the uprights can be lightened about 9 per cent.

Having selected the allowable stresses at the beginning, we naturally are able to compute angle $\alpha$ at $39^\circ$ as soon as we estimate $\sigma_x$ according to equation (5a).

## Oblique "Uprights"; Stiffness

Structural requirements may make it necessary to set the uprights obliquely to the spars (Fig. 10). The angle formed by the uprights and the spars ($90^\circ$ if the former are perpendicular), is designated by $\beta$.

We begin by calculating the direction of the stresses in tension, that is, angle $\alpha$. We assume as known the allowable tension stress $\sigma$ in the plate, the allowable compression stress $\sigma_v$ in the uprights and the mean stress of both spars; $\epsilon$ and the respective subscripts to denote the corresponding elongations.

As previously shown, we take one component of the web plate (Fig. 11) which wrinkled, and calculate the respective elongations $\epsilon_x$ and $\epsilon_v$ for $\alpha$, $\epsilon$, and $\epsilon_q$. As will be seen from Figure 11 (Compare also Fig. 8a), we obtain for:

$$l_x \epsilon_x = l_x [\epsilon \cos^2 \alpha + \epsilon_q \sin^2 \alpha]$$

$$l_v \epsilon_v = l_v [\epsilon \cos^2 (\beta - \alpha) + \epsilon_q \sin^2 (\beta - \alpha)].$$

Now abbreviate by $l_x$ and $l_v$, eliminate $\epsilon_q$ and resolve according to $\alpha$, yielding

$$\cot \alpha = \frac{1}{\sin \beta}\left(\sqrt{\frac{\epsilon - \epsilon_v}{\epsilon - \epsilon_x}} + \cos \beta\right) \tag{16}$$

Instead of the elongations we can also insert the corresponding stresses. For $\beta = 90^\circ$ this equation becomes (5a).

In the special case $\epsilon_v = \epsilon_x$, or, for infinitely stiff uprights and spars $(\epsilon_v = \epsilon_x = 0)$ equation (16) yields

$$\alpha = \beta/2 \tag{16a}$$

If the uprights are obliquely arranged the tension stresses even in infinitely rigid uprights are no longer at 45° but halfway between uprights and spars.

With the direction of the tension stresses known we calculate the angle of displacement $\gamma$ (according to equation (6)) in which the elongation $\epsilon_v$ of the uprights does not appear. Thus

$$\gamma = 2 \cot \alpha (\epsilon - \epsilon_x) \tag{17}$$

which yields $\alpha$ with respect to the stresses in the individual components of the sheet metal girder. Now we must calculate the stresses in these components (uprights, web and spars), if cross stress Q is given and angle $\alpha$ is known. This is best done as follows: Figure 12a depicts a system of members having two parallel spars and oblique uprights. In addition, the panel points between the spars and uprights are connected by diagonals which form an angle $\alpha$ with the spars. Now we compare this system with that on Figure 12b, which has perpendicular uprights, and compute the spacing, as

$$t = h(\cot\alpha - \cot\beta) \quad \text{for Figure 12a,}$$
$$t_1 = h\cot\alpha \quad \text{" " 12b.}$$

Both systems are to be loaded with the same cross stress $Q$, so the stress in the diagonals is identical in both systems.

$$D = \frac{Q}{\sin\alpha}.$$

Now we consider the diagonals as relatively thin-walled but wide bands or strips, until at last we can choose the diagonals so wide that two adjacent strips touch each other, in which case the width of the diagonal strips is

$$b = t \sin\alpha, \quad \text{for system Figure 12a,}$$
$$b_1 = t_1 \sin\alpha, \quad \text{" " " 12b.}$$

With $s$ as wall thickness of the diagonal bands, their stress is

$$\sigma = \frac{Q}{\sin\alpha}\,\frac{1}{s\,t\,\sin\alpha}, \quad \text{Figure 12a}$$
$$\sigma_1 = \frac{Q}{\sin\alpha}\,\frac{1}{s\,t_1\,\sin\alpha}, \quad \text{" 12b}$$

If we make these diagonals wide enough so that two adjacent bands touch each other, we consider them simply as a continuous skin. Thus, the skin stress by oblique uprights is, according to equation (9)

$$\sigma = \sigma_1 \frac{t_1}{t} = \frac{2Q}{hs} \frac{1}{\sin 2\alpha} \frac{t_1}{t} =$$

$$= \frac{2Q}{hs} \frac{1}{\sin 2\alpha} \frac{1}{1 - \tan\alpha \cot\beta} \qquad (18)$$

Now, of course, we can select the spacing of the uprights at random without interfering with the skin stress; and equation (18) is valid for any spacing of uprights, provided the effect of the spar deflection is negligible.

The stress V in an upright is

$$V = \frac{1}{\sin\beta} Q \frac{t}{h} \tan\alpha \frac{t_1}{t}$$

hence

$$V = \frac{1}{\sin\beta} Q \frac{t}{h} \tan\alpha \frac{1}{1 - \tan\alpha \cot\beta} \qquad (18a)$$

The spar stresses are

$$H_{OU} = \pm \frac{Q x}{h} - \frac{Q}{2} (\cot\alpha + \cot\beta) \qquad (18b)$$

The last equation is valid only for very (infinitely) closely spaced uprights. If spaced farther apart they indicate the spar stress in the center between two uprights very accurately; the slight discrepancies at other points are easily calculated, but are as a rule ignored. These discrepancies are due to the fact that the upright at its point of attachment to the spar exerts a stress on it in the direction of the spar, so that the spar stress makes a jump at that point.

We can calculate the direction of the tension stresses by given size of the oblique uprights, but the calculation yields an equation of the fourth order for $\sin^2 \alpha$, so that $\alpha$ cannot be shown in precise form. To be sure we can draw curves from which $\alpha$ can be read off, but we shall forego this step, because the allowable stress is given, as a rule.

Now we discuss the effect of oblique uprights on the required amount of structural material and on the stiffness of a girder. Comparison of material requirement is confined to uprights and web, since it is not feasible to include the spars whose requirements are materially affected by the amount and directional change in bending moment with respect to the cross stress.

With $\sigma$ as allowable tension stress in the web plate, its cross-sectional area $f_s = h\ s$ becomes, according to (18)

$$f_s = h\ s = \frac{Q}{\sigma/2}\ \frac{1}{2 \sin^2 \alpha}\ \frac{1}{\cot \alpha - \cot \beta}.$$

As average structural material per unit girder length, we have for the uprights (according to 18a)

$$f_V = \frac{\text{volume of one upright}}{\text{spacing of uprights}} =$$

$$= \frac{\frac{h}{\sin \beta}\ \frac{V}{\sigma_V}}{t} = \frac{Q}{\frac{\sigma_V}{2}}\ \frac{1}{2 \sin^2 \beta}\ \frac{1}{\cot \alpha - \cot \beta}.$$

The total average structural material required per unit girder length is then $f = f_s + f_V$.

Now let us compare the amount of material for a plate wall girder (spars excepted) with that for a web plate which transmits the cross stress to shear and whose allowable shear stress is $\tau = \frac{\sigma}{2}$. The materials required are $\frac{Q}{\sigma/2}$. The comparative requirements for the plate wall girder then are

$$\varphi = f \Big/ \frac{Q}{\frac{\sigma}{2}} = f_s \Big/ \frac{Q}{\frac{\sigma}{2}} + f_V \Big/ \frac{Q}{\frac{\sigma}{2}} = \varphi_s + \varphi_V$$

Here $\varphi_s$ and $\varphi_V$ denote the relative requirements for web plate and uprights.

Before making any mathematical comparisons regarding the stiffness of such plate wall girders, we wish to state that the stiffness of such girders which form a diagonal tension field, is in every case just as high as that of a conventional structure with upright and tension diagonals, when both are in the same direction and the materials used in both are the same. Moreover, we want it to be noted that the comparisons which follow pertain primarily to girders used as wall of a torsion box, for in such the web stiffness is paramount; in long flexural girders the deflection (that is, the spars) takes preference over the displacement due to cross stress (the webs).

The comparative stiffness of a plate wall girder under shear stresses is expressed by the angle of displacement $\gamma$ with respect to the angle of displacement $\tau/G$ of a web plate subjected to shear, $\tau = \frac{\sigma}{2}$. With (17) we have:

$$\frac{\gamma}{\frac{\sigma}{2}/G} = \frac{4\ G}{E} \cot\alpha \left(1 - \frac{\epsilon_x}{\epsilon}\right),$$

$\cot\alpha$ to be defined according to (16).

Figure 13 shows the comparative material requirements for plate wall girders with different settings of the uprights; we assumed

$$\epsilon_V = -\ 0.5\ \epsilon \qquad \epsilon_x = -\ 0.2\epsilon.$$

Ordinarily the allowable compression stress in the uprights is higher than half the allowable tension stress in the web plate, so to keep on the safe side the weight estimates should follow Figure 13. The selection of $\epsilon_x$ has no appreciable effect on the weight calculation and stiffness.

It becomes apparent from Figure 13 that - by given cross stress - a minimum amount of material is needed when the uprights are set at about $\beta = 90^\circ$ (that is, perpendicular uprights); it represents 90% of the web materials. At other angles $\beta$ the material requirements for the uprights are higher, although not very markedly so. The effect of $\beta$ on the amount of material for the web is very considerable; it raises when $\beta$ increases; for $\beta = 90^\circ$ it equals the amount used in the shear plate. The total minimum requirement is at $\beta = 120^\circ$; in round numbers it is 1.63 times as high as the materials required for the shear plate. For $\beta = 90^\circ$ the total requirements are about 1.9 times that of the shear plate.

Another feature of Figure 13 is that angle $\beta$ must be changed in $180 - \beta$ by a reversal of the cross stress directions. If, for example, the web is given the dimensions for $\beta = 120^o$ and a cross stress $Q$, it carries in the opposite direction (that is, for $\beta = 60^o$) only a cross stress $(- Q)$ of the order $(- Q) = \sim - 0.34\ Q$, because the material used for the web plate at $60^o$ is, according to Figure 13, $1.72\ Q/\frac{\sigma}{2}$, but only $0.58\ Q/\frac{\sigma}{2}$ for $120^o$. The ratio of both then yields 0.34. The $\frac{(- Q)}{Q}$ values for different $\beta$ are also shown on Figure 13.

For Part II, see N.A.C.A. Technical Memorandum No. 605, which follows.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

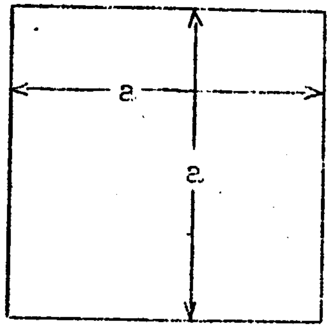



Fig.1a

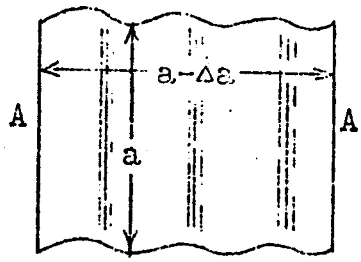



Fig.1b

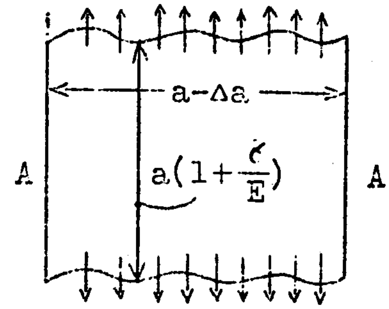



Fig.1c

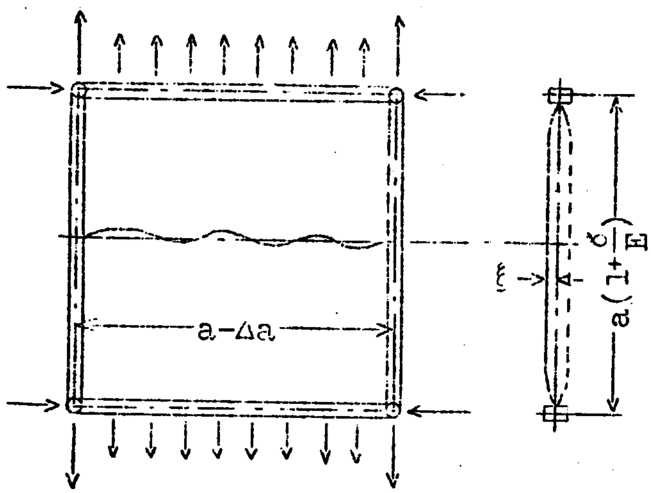



Fig.2

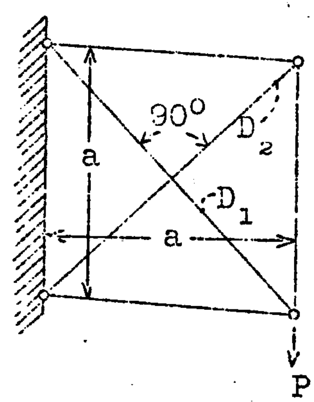



Fig.3a

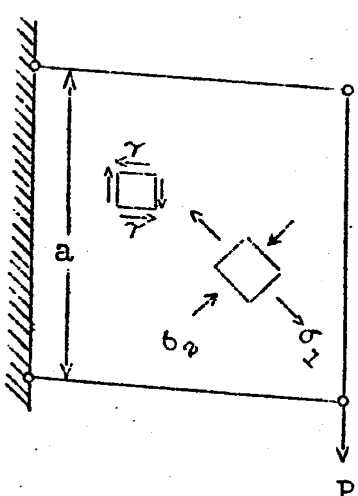



Fig.3b

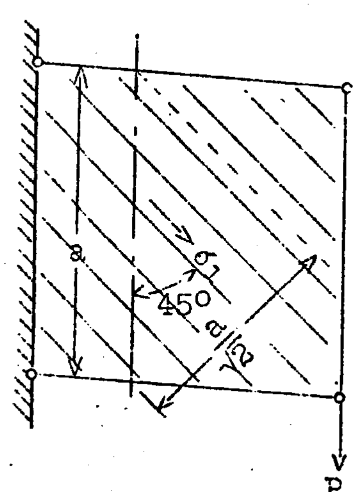



Fig.3c

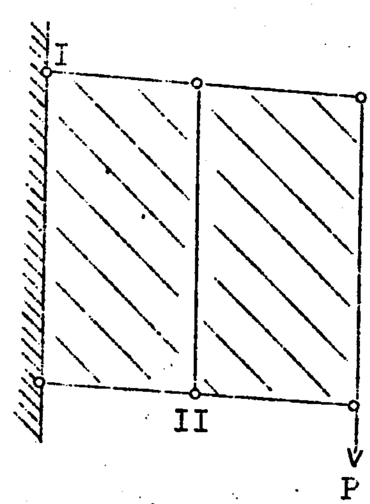



Fig.3d

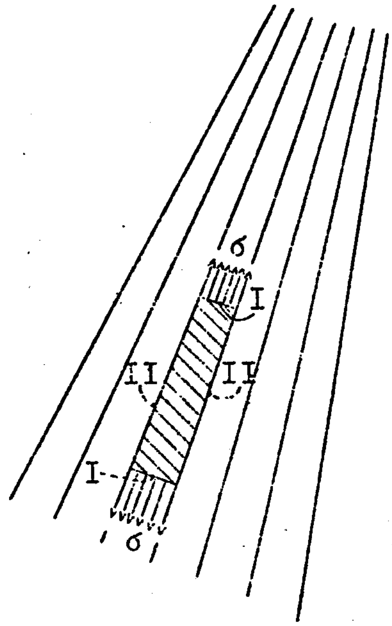



Fig.4

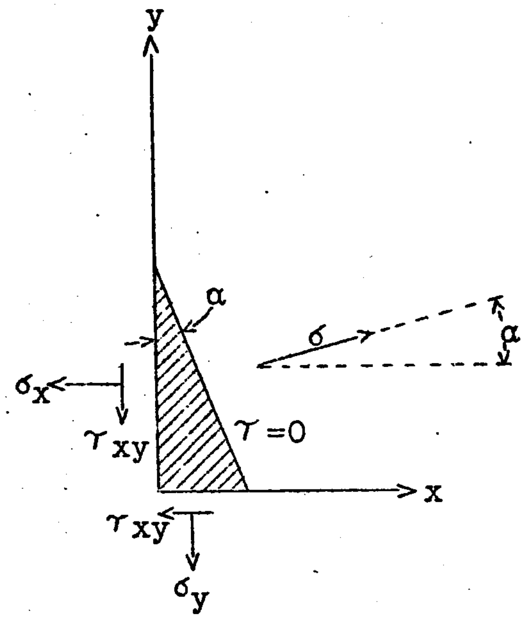



Fig.5

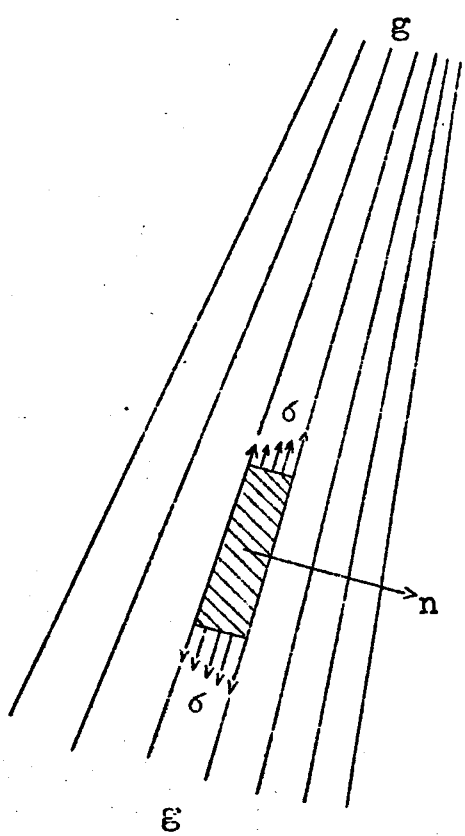



Fig.6

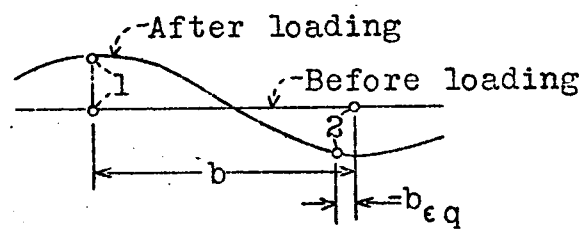



Fig.7

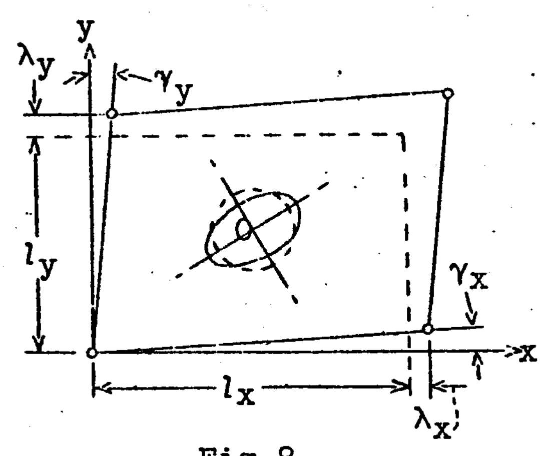



Fig.8

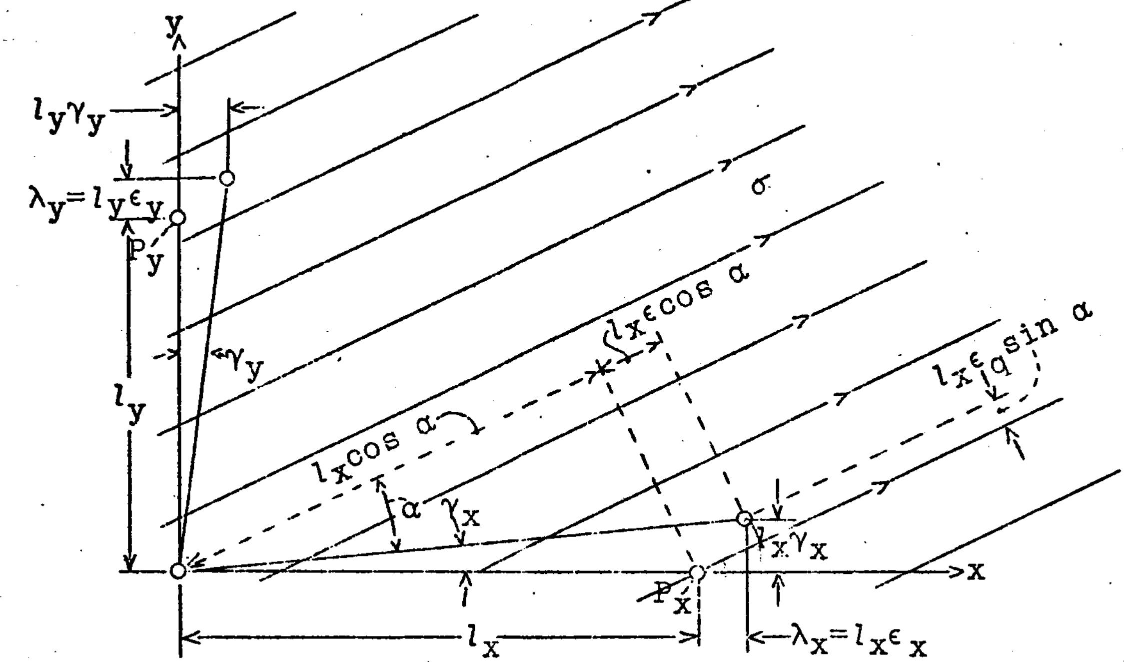



Fig.8a

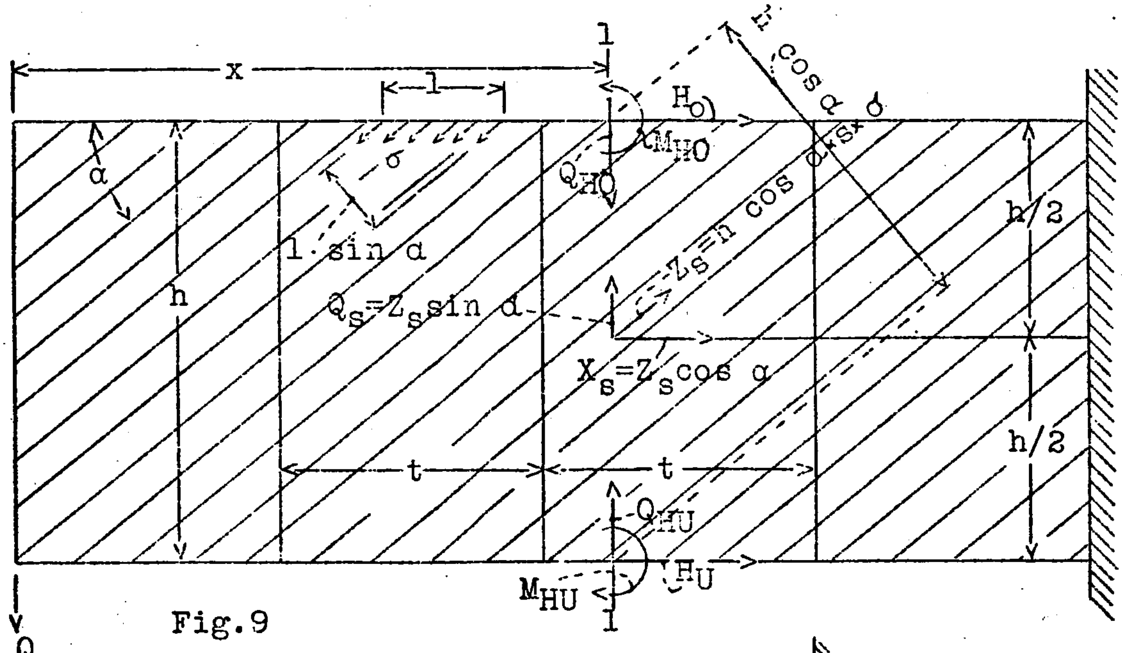


Fig.9

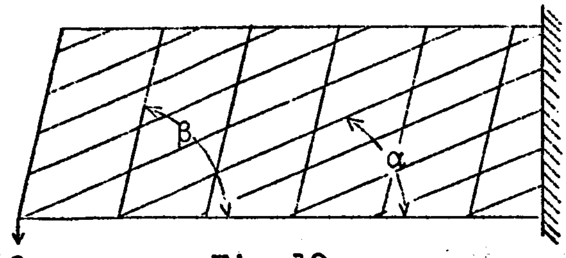


Fig.10

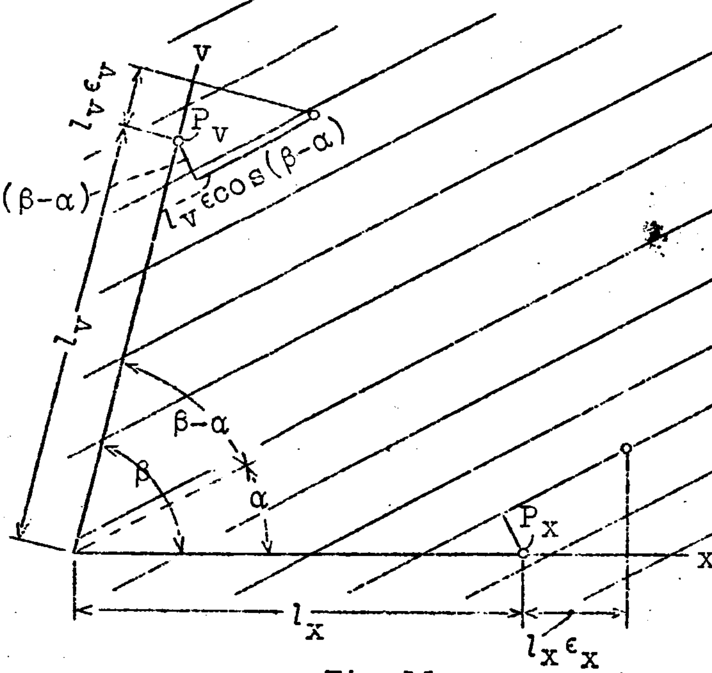



Fig.11

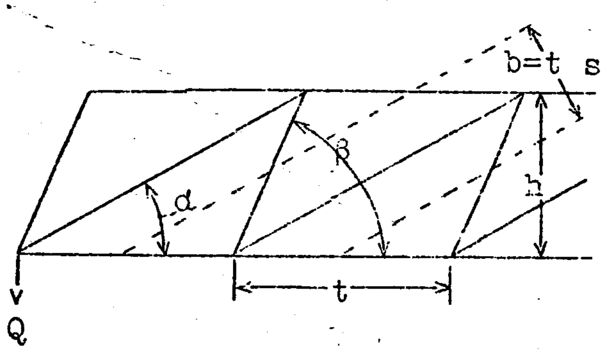



Fig.12a

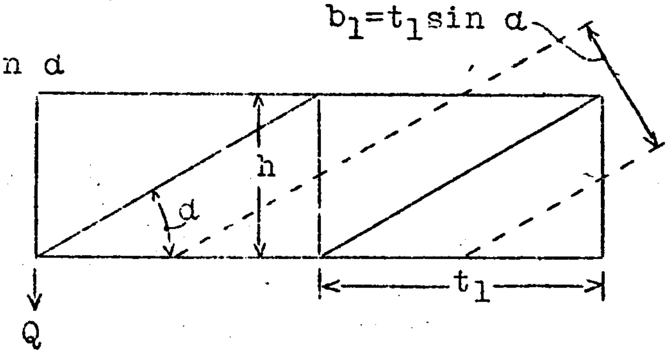
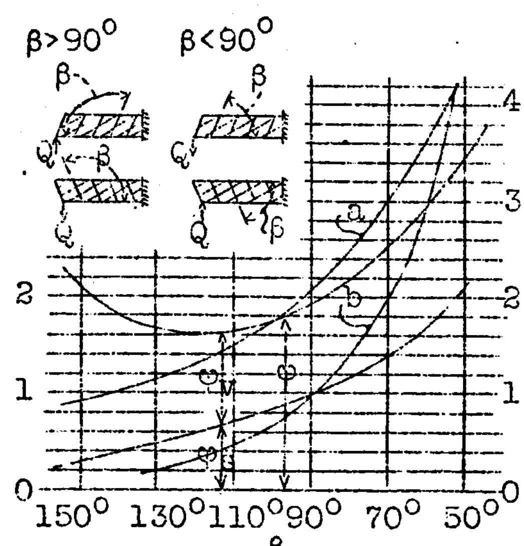



Fig.12b




$a= \frac{\gamma}{\frac{\sigma}{2}/G}$

$b=\frac{-Q}{Q}$

Fig.13

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NACA-TM-604

# FLAT SHEET METAL FIRDLES WITH VERY THIN METAL WEB

National Advisory Committee for Aeronautics
Washington, D. C.

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