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By F. Ahlborn.

The explanation of the Magnus effect published by the Göttingen Aerodynamic Institute** and the consequent invention of the Flettner rotor attracted much attention. Apparently this was a scientific discovery of the first magnitude, which would be of epoch-making importance in the utilization of the force of the wind. Wind-tunnel tests had demonstrated the possibility of extracting, by means of rotating cylinders, ten or more times as much energy as a sail of like length and width. The first ship "Buckau" equipped with such rotors was therefore logically called a "Windkraftschiff" (wind-power ship). The first trips of this ship were disappointing. It was demonstrated that the ship could move forward in a favorable side wind with the aid of the rotors, but the generally expected high speeds were not attained. Even the "Barbara," which was subsequently built at the suggestion of the Secretary of the Navy (Reichs-Marineleitung) by the firm of R. M. Sloman, Jr., at the Weser shipyard in Hamburg with careful attention to all

*"Der Magnuseffekt in Theorie und Wirklichkeit," From Zeitschrift für Flugtechnik und Motorluftschiffahrt, December 28, 1929, pp. 642-653.

**L. Prandtl, "Magnuseffekt und Windkraftschiff," Naturwissenschaften, 1925, p. 93.
A. Betz, "Der Magnuseffekt, die Grundlage der Flettner-Walze," V.D.I. 1925, p. 9.
J. Ackeret, "Das Rotorschiff," Göttingen, 1925.

the technical details, attained no results which at all justified the use of rotors for propelling ships.

Although the Flettner rotor must be regarded as a failure from the nautical and economical standpoints, it still holds true that the principle of the invention was based on experimental and theoretical inventions which cannot be disregarded and which require explanation in the interest of science. Considering the excellent technical construction of the rotor system, the cause of the failure must reside in the aerodynamic theories which Flettner had accepted in good faith from their authors.

The following critical and experimental investigation will show the relations and will also show the applicability, in the present case, of C. Maxwell's observation that one must avoid looking at things through the rose-colored glasses of an unwarranted optimism which blinds one to the facts and leads to false assumptions.

The Göttingen Theory of the Magnus Effect

If a cylinder is made to rotate rapidly in an air stream directed against its axis, it experiences, aside from the resistance or drag in the direction of the wind, a lateral force toward the side where the direction of the wind coincides with the direction of rotation. This force is called the Magnus effect, because it was observed and scientifically explained by Professor G. Magnus of Berlin in 1851. More accurate measurements of the

Magnus force were subsequently made by Lafay in Paris and repeated in Göttingen on a broader scale.*

The Göttingen scholars have now adopted the view that the explanation given by Magnus is inadequate for present-day requirements; that much progress was made by Lord Rayleigh in 1877 in a published solution of a related problem in theoretical hydrodynamics, but that the complete explanation is possible only with the aid of Prandtl's boundary-layer theory.

Lord Rayleigh's theorem, which is also the basis of the Kutta-Joukowsky wing theory, is explained in a brief treatise ("On the Irregular Flight of a Tennis Ball," Scientific Papers I, pages 344-346), which seems to have been only partially known in Göttingen and which is therefore briefly summarized here as follows.

It has long been known to tennis players that a rapidly rotating ball is deflected from its original direction. If such a ball bounces from a wall, it may even turn back and strike the wall again. This phenomenon has called forth all sorts of explanations, but the true scientific explanation, so writes Lord Rayleigh, had already been given by Professor Magnus in his treatise "On the Deflection of a Projectile" (Doings of the Berlin Academy, 1852, English translation in Taylor's scientific Memoirs, 1853, p. 210). Instead of assuming that

*Lafay, "Sur l'Inversion du Phénomène de Magnus," C.R. 1910, p. 867. Lafay, "Contribution Expérimentale à l'Aérodynamique du Cylindre," Rev. Mecanique, 30, 1912.

the ball is moving through still air, it is better to assume (as in the Magnus experiment) that the ball is exposed to a uniform air flow, which does not change the relative motion on which the forces depend.

Under these circumstances, when the ball is not rotating, force is exerted only in the direction of flow, without lateral components. When the ball is rotating, however, the friction between the fixed surface and the adjacent air forms a sort of eddy or vortex which modifies the force exerted by the flow. When the rotation is about an axis perpendicular to the flow, the superposing of the two motions on one side increases and on the other side decreases the speed and consequently produces a lateral force which drives the ball toward the side where the two motions are in the same direction. Lord Rayleigh expressly confirms the explanation of the phenomenon given by Magnus and shows that friction is the immediate cause of the curving motion and of the Magnus effect.

In the theoretical portion of his treatise Lord Rayleigh calls attention to the fact that no suitable physical representation is mathematically possible, since the theory presupposes a nonviscous fluid and excludes external forces, as here transmitted by friction from the rotating cylinder to the fluid. The theoretical problem can be solved only when the actual revolving motion about the rotating cylinder is replaced by the simpler form of a cyclic motion about the cylinder at rest, and the latter motion coincides with the simple potential flow about the cylinder.

The cyclic motion takes place in concentric circles about the cylinder and has the characteristic that, in all these circles, the product of the peripheral velocity, u times the length of the circumference has the same constant value $k = u \times 2 \pi r$. The velocity u is therefore inversely proportional to the radius of every circle and dies out in the distance. The cyclic constant k is also called the "circulation."

For theoretical reasons, such a motion is not possible in the simply continuous space about a sphere (tennis ball), whereby the actual production of physical phenomena in real fluids is naturally not denied. The theory is therefore obliged to make the space doubly connected by assuming, instead of a sphere or a cylinder of finite length, a cylinder of infinite length, about which the flow is then represented as two-dimensional in a cross-sectional plane.

The combined flow (which will be designated, for short, as the "Rayleigh Flow") and its two components are very well explained in the Göttingen publications. In Figure 1, diagram I represents the simple theoretical potential flow about the profile; II, the circulatory motion; diagrams III and IV, the Rayleigh flow for a weaker and for a stronger circulation, respectively. It is shown how the larger portion of the fluid is deflected upward by the rotating cylinder and flows away more rapidly than on the lower side. The fluid pressure is greater, therefore, on the lower side, and the pressure difference generates the force which pushes the cylinder upward. From the perfect symmetry of the flow, it follows that the force is a purely lateral force and perpendicular to the direction of flow.

From the theoretical standpoint, the Rayleigh force is a very interesting phenomenon, but Lord Rayleigh did not think for a moment that his theorem was a satisfactory scientific representation of the actual phenomena. In the concluding paragraph of his treatise, he states expressly that it must not be forgotten that the motion in a real fluid is quite different from that assumed in the calculation.

Hence it is quite obvious that the Rayleigh flow is only a pure mathematical construction and its author expressly states that it cannot be regarded as an exact scientifically correct representation of the Magnus experiment, which was correctly explained by Magnus himself and which, due to friction, lay outside the domain of hydrodynamic theory.

How it was possible that, in the Göttingen theory of the Magnus effect, this state of affairs was misunderstood and disregarded, is explainable only on the assumption that Lord Rayleigh's treatise was not at hand, since, otherwise, the attempt would have been made in one of the three Göttingen treatises to show the incorrectness of Lord Rayleigh's personal view.

It was found that the scientific explanation of the phenomenon by Magnus was not satisfactory, but it had to be acknowledged that it was hopeless to try to explain the occurrence of the circulation assumed in Lord Rayleigh's theorem without the aid of friction.

Instead of drawing the only correct conclusion, that the theoretical Rayleigh flow cannot agree with the actual flow, one hits upon the unfortunate idea of trying to demonstrate by argument that the phenomenon is not affected by the friction, excepting for a brief initial moment, and therefore the Rayleigh flow corresponds to the reality. It would therefore be too bad for this beautiful result of the theory to be disturbed by the friction. Hence the slogan "Fight for the retention of the potential flow" (J. Ackeret, "Das Rotorschiff," Göttingen, 1925). This is a dangerous tendency. The theoretical potential flows are so well grounded that they require no defense, excepting, as here, from being brought into discredit outside their field of application.

The method now takes a very remarkable course. First the separation or boundary-layer theory of Prandtl is thoroughly explained, which, as I have recently demonstrated, does not correspond to the reality.* Then it is unanimously asserted that the friction is important only at the initial moment of the Magnus experiment. It then requires boundary-layer material in the space behind the cylinder, where it collects and changes into a "starting vortex." As a matter of fact, there is first formed, though in another way, a vortex trail, which develops

*F. Ahlborn, "Die Ablösungstheorie der Grenzschichten und die Wirbelbildung," Yearbook of the Wissenschaftliche Gesellschaft für Luftfahrt, 1927.

into a starting vortex and may be regarded as an indirect result of the friction.

Now, in order to show that the friction outside of this vortex has no appreciable effect and that, therefore, the circulation about the cylinder, as in Lord Rayleigh's theorem, has nothing to do with the friction, but occurs independently of it and continues permanently, one employs the following mistaken deduction, which can be found in the same form in Prandtl's wing theory.

The resultant circulation of a potential flow must be zero for theoretical reasons. In addition to the starting vortex there must, therefore, be still another circulation about the cylinder, which is equal and opposite to the vortex. "The vortex floats away with the current and the circulation continues around the cylinder."

This method of demonstration is indeed very simple, but fails to be convincing. It was not expected to prove that the circulation about the cylinder must exist, but how it is produced and that it is not produced by the friction. Of this there is no hint in the conclusion regarding the resultant circulation and it is left to the reader to overlook the omission or to imagine that the starting vortex behind the cylinder is produced by friction, but that the circulatory flow, around the cylinder and moving with it, has nothing to do with the friction. The starting vortex alone has the diabolical power to create the circulation about the cylinder not only during the first few moments, but even when it has floated along with the current to any distance and, finally, when it no longer exists! Even then it is still the cause which produces the circulation about the cylinder without friction in the continually arriving masses of air! With such inadequate arguments it was thought to transfer the combined potential flow of Lord Rayleigh across the forbidden limit to real fluids.

After the friction has thus been reasoned away with the aid of the starting vortex, it easily follows that "the friction does not affect the action of the forces on the cylinder." The lateral force is then developed, even in the frictionless or nonviscous fluid of the Rayleigh flow. The conclusion is inevitable that even the rotation of the cylinder is necessary only during the initial moments, in order to produce the starting vortex and its accompanying circulation. The engine can then be shut off, since the Magnus effect, after being once initiated, continues to act, even on the cylinder at rest, so long as the wind continues to blow.

We do not agree with this conclusion, however. The rotation of the cylinder must continue, it is said, because the wind fluctuates and the friction then causes the separation of vortices on one side or the other, in order to adapt the circulation to the force of the wind. In the research institute, however, the experimenters determined the Magnus effect in an artificial, homogeneous air stream. How was it possible that no one thought to explain the importance of the friction by an "experimentum crucis"? It was only necessary, in any experiment, to interrupt the rotation of the cylinder, which must then have shown immediately whether the Magnus effect conformed to the theory and continued in the air stream or immediately disappeared, which would have demonstrated the incorrectness of the theory. The theory was saved by the omission of this experiment.

After it had been thus decided, without experimental proof, that the friction had no influence on the Magnus effect, then the decisive question had to be answered "Whence comes the energy of the free lateral force?" The theoretical Rayleigh flow yields the pressure differences which produce the lateral forces, but neither the translatory potential flow nor the superposed circulation suffers any loss of energy thereby, since the system is and remains symmetrical, even when the directions of the two motions are opposite. Since, however, the Rayleigh force, excluding friction, can come only from the flow, it is contrary to the law of action and reaction. It is produced without the use of energy, from nothing.

In the wing theory, the Rayleigh flow is transferred to wing profiles by the purely formal method of orthomorphic transformation. The Rayleigh force here supplies the lift which carries the airplane through the air without demonstrable expenditure of energy and without drag. In the perfectly analogous case of the Rayleigh flow about a cylinder, it is regarded, however, in the Göttingen theory of the Magnus effect, as a self-evident fact that the lateral force comes from the translatory flow of the wind, without any foundation for this arbitrary departure from the theorem of Lord Rayleigh.

The evil can no longer be restrained. Since the friction is subordinate, the cylinder is further subjected to only the active wind forces. Consequently the cylinder, which theoretically does not need to revolve even once, abstracts from the wind the kinetic energy of the Magnus effect. Since the wind also acts on a sail, a sail and a rotor are comparable devices. The comparison had already been made by Lafay but, by mounting terminal disks of twice the diameter on the cylinder, the rotor was found to be tenfold superior to a sail of the same size. Thereby the sail was doomed. Since the friction of the air had no effect, the motor, which caused the cylinder to rotate and had to overcome hardly more than the friction of the ball bearings, was a subordinate affair, which could not place the success in doubt.

Thus definitely and authoritatively explained, the Flettner rotor could not fail to attract the attention of the world. A speed never yet attained by a sailing vessel was expected of the rotor ship "Buckau," because the reports emphasized the tenfold superiority of the rotor, but not the fact that only a tenth part of the sail area could be installed in the form of rotors. The first disappointments were attributed to "children's diseases" and it was decided that the motors for driving the rotors were too weak. Even the higher-powered "Barbara" demonstrated only that a ship could be propelled by rotors when the latter were turned by mechanical force and there was a sufficiently strong side wind.

Nevertheless the "Barbara" is still carrying its big rotors in Hamburg harbor (April, 1928). If it is thought possible to conclude from this circumstance that the theory may not be entirely wrong, the following experiments will destroy this last hope.

The Magnus Effect in Reality

For investigating the actual phenomena, use was made of my photographic method of flow analysis, which was also used in taking the Göttingen photographs and motion pictures. Among the latter, special attention is called to the excellent pictures of the flow about a rotating cylinder, published by O. Tietjens in the 1925 Yearbook of the W.G.L., page 100.

The experiments were made in quiet water with a moving object. Single photographs and motion pictures were taken of the streamlines and of the lines of force about the rotating and nonrotating cylinder of 5 cm (1.97 in.) diameter and 25 cm (9.84 in.) length. Stereoscopic pictures were also taken of the movements inside the water, as likewise of the surface flow, which show the pressure distribution. The use of "Osram-Nitra" lamps, instead of the earlier magnesium flashlights, as also of the new highly refractive lens with a focal-plane shutter, greatly facilitated and improved the process. In performing the experiments, the friendly and eager cooperation of Dr. Max Wagner, as in former years, was of great value to me. The investigations embraced:

1. The flow about a nonrotating cylinder;
2. The flow produced in still water by the friction of a rotating cylinder;
3. Comparison of the Magnus flow, produced by the translation and rotation of a cylinder, with the theoretical flow of Lord Rayleigh.

1. The streamline motion about a nonrotating cylinder is known from my previous experiments and from the pictures repeatedly published in the Göttingen reports. Figure 2 shows the shape of the streamlines in the first few moments of the motion, when it agrees externally with the theoretical flow of the perfect fluid. It already contains vortices, however, which are hardly visible in this picture, and the pressure is less on the downstream than on the upstream side. The fine vortices produced in ceaseless succession on the sides of the cylinder quickly develop into large vortex aggregates (Fig. 3) filling the space behind the cylinder and completely changing the original flow pattern. The dynamic effect is a resistance in the direc-

tion of translation, which is proportional to the square of the velocity and proceeds from the energy loss used for the production of the vortical motion and apparent in the latter.

The lines of force (absolute streamlines) of the same flow are shown in the first stage in Figure 4 and in a somewhat more advanced stage in Figure 5. It is seen how the vortices issue from the space bounded by the innermost streamlines, accumulate on the downstream side of the cylinder and thus transform the flow.

2. The effect of the friction of a cylinder rotating in a quiet fluid was discussed in my report of an experimental investigation on the production of great atmospheric circulations ("Beitrag z. Phys. d. freien Atmosphare," Vol. XI, pp. 117-153) and also in a lecture before the Göttingen Physical Society. While the view is expressed, in the Göttingen publications on the Magnus effect, that the effect of friction is restricted to the boundary layers, our investigations show that the rotary motion of the cylinder is transmitted through the boundary layers to all the surrounding fluid. After a short time the fluid, over a wide range, participates in a powerful circulatory motion, consisting of two large vortex rings. These are on opposite sides of the equatorial plane of symmetry and revolve in opposite directions. Their streamlines are space spirals which pass off tangentially from the surface of the cylinder. The velocities increase as the particles approach the equator. In Figure

7, according to the stereoscopic views, the streamlines are projected on an axial plane of the cylinder. Rotation of the diagram about the cylinder axis gives a spatial survey of the flow, though without the tangential components of the motion. The most imposing examples of this effect of friction are the trades and anti-trades which cover half the earth's surface on both sides of the equator. We shall therefore call these circulations "trade circulations."

The Göttingen measurements of the Magnus effect showed that the free force was considerably increased by providing the cylinder with terminal disks of twice its diameter. There was seen in this a confirmation of the assumption that the disks prevented the lateral flow of the air into the negative-pressure region about the cylinder. On the contrary, Figure 8 shows, again from stereoscopic photographs, that the effect is ascribable to the friction of the fluid on the disks rotating at a great peripheral velocity. On each disk there is developed a pair of powerful "trade vortices," which envelop the space around the cylinder and force the trade circulation of the cylinder envelope into a narrow equatorial space. It is obvious that, by a suitable distribution of such disks over the length of the cylinder, the friction and therefore the cause of the Magnus effect can be increased, just as by the use of a large cylinder diameter.

3. If we define the Magnus flow as a combination of the

phenomena referred to in sections 1 and 2, this does not mean a simple superposing as in the Rayleigh flow. It can easily be imagined that the comprehensive trade circulations cannot fully develop when the cylinder changes its location, or the whole fluid is in progressive motion. There then remain of this flow only the cores, the initial stages, in so far as they can develop during the passage of the fluid through the field, in the form of a particular kind of flow revolving with the cylinder.

Since this motion is produced and continuously maintained by friction in the air masses entering the field from without, it is entirely different from the Rayleigh circulation assumed as being produced without the expenditure of energy. The resultant motion must therefore differ considerably in the two cases. The differences will appear in the following comparison, but there are still a few preliminary remarks to be made regarding hydrodynamic fields of force.

When a nonrotating cylinder is moved through a still liquid, the whole body of liquid appears, in the camera moving with the cylinder, as if it were moving in the opposite direction along the streamlines shown in Figures 2 and 3. On the other hand, if the camera is stationary so that the cylinder passes under it, a snapshot then shows the phenomena in the form of lines, nearly coincident with the absolute streamlines of the hydrodynamic theory. Figure 9 shows the theoretical lines and Figure 4 the side of the moving body and end, in the beginning of the motion, on the rear side. In the immediate vicinity of the surface, in the region of the boundary or transition layers, they bend toward the surface in the direction of the motion, while the absolute theoretical streamlines end here at sharp angles. At their point of origin, therefore, the lines of force always take the direction in which the compressive forces are transmitted from the moving bodies to the (resting) liquid.

In each partial field bounded by the lines of force, a definite dynamic compressive force emanates from the front side of the body. This force is gradually transformed into kinetic energy, as it proceeds laterally. In the region of the retrogressive lines of force, at least in the initial moments of the motion, the kinetic energy is again converted into pressure and restored at the rear side of the body. The restoration is never complete, however, as in the ideal fluid, but always attended by losses, which reduce the pressure against the rear side and create the resistance of the fluid. The loss is complete when, soon after the beginning of the motion, the connection of these lines of force with the rear side of the body is fully dissolved.

The cause of this transformation lies in the development of lines of force of the second kind. These lines, which are geometrically similar to the lines of magnetic force in the field about charged electric conductors, are closed circuits lying free in the fluid and forming concentric vortices which cannot occur in the theoretical frictionless fluid. Figure 5 shows a somewhat later stage of development of the field of force. The lines of force of the second kind fill the vortex field, which is thrust in on every side between the cylinder and the potential flow.

I have already discussed the process of vortex formation in my treatise on the theory of discontinuous fluid motions (Phys. Zeitschrift, 1928, p. 34). The lines of force can be very irregular in the composite vortices, but always contain closed forms. The lines of force of the vortices lie in the negative-pressure region and have the properties of streamlines, in so far as the fluid particles of a vortex move along the lines of force.

The hydrodynamic fields of force of the theoretical Rayleigh flow and of the natural Magnus flow will now be compared. If the circulatory motion assumed in Lord Rayleigh's theorem is superposed on the theoretical field of the simple potential flow about a cylinder, the field assumes the form shown in Figure 10 (Lamb, "Hydrodynamics"). The upper half of the simple field (Fig. 9) is increased by the rotary motion at the expense of the lower half. After, as well as before, however, all the lines of force emanating from the front side of the cylinder return to the rear side and the motion is completed without the expenditure of energy. As in the representation by streamlines, the direction of the motion can be either from right to left or from left to right.

The comparison of this field with the natural field of force of the Magnus flow, as drawn from a photograph (Fig. 11), shows only a slight similarity between the two. Only in the hatched upper partial field do the lines of force return to the surface of the cylinder. Behind this, however, on the whole rear side, they are strengthened by the shearing forces of the friction and deflected over the lower side of the cylinder, so that, together with the lines of force emanating from the front side, they now encircle the cylinder spirally and finally escape into the fluid.

Thus the natural lines of force are described by the forces derived from the rotating cylinder by friction and maintain the peculiar circulation in its vicinity which, according to Magnus and Lord Rayleigh, is the immediate cause of the lateral force. Accordingly the assumption is also disproved experimentally, that the circulation is produced automatically without the direct effect of the friction, and that the natural Magnus flow is correctly represented by the theoretical potential flow of Lord Rayleigh.

The same result is obtained by comparing the theoretical and natural flows as represented by streamlines. Figure 12 represents an initial stage of the Magnus flow after three or four revolutions of the cylinder. It shows the initial series of vortices, which very soon develop into the starting vortex of

Prandtl. In the earlier stages there is no accumulation of boundary-layer material from which, according to Prandtl, this vortex develops; neither is there any thickening of the boundary layer, which, according to the last modification of the theory, is the cause of this vortex. On the other hand, it is shown how the fluid layers from the upper side of the cylinder are thrust wedge-shaped against the lower side. At the tip of the wedge, thrust far forward, these layers bend sharply backward toward the counterflow and enclose the rotational line R, from which the vortices of the layer successively proceed.

Bigger photographs show, after the disappearance of the large initial vortex, small vortices on the division line, where the flow from the upper side of the cylinder reunites with the flow on the lower side (Fig. 13). These constantly diminishing small vortices take their position according to the immediate continuation of the initial vortex sheet. Figure 13 shows the finished Magnus flow. In contrast with the symmetrical pattern of the theory, it is turned by the effect of friction through an angle α, so that the free Magnus force CM is now composed of a purely lateral force C_a and a resistance or drag C_w. One is here again referred to the above-mentioned beautiful picture of the Magnus flow taken by O. Tietjen.

The velocity difference between the surface of the rotating cylinder and the surrounding fluid, and hence also the amount of the friction, is greatest in the stream saddle on the lower side and reaches a minimum at a point A (Fig. 14) in the right upper quadrant. This is represented by a crescent whose greatest width is at the saddle S. This illustrates the unequal effect of the friction in contrast with its theoretical replacement by an all-round uniform circulation.

The mechanism of the Magnus force is accordingly as follows. If no rotation and friction were present, the two lateral currents flowing around the cylinder at the beginning of the translatory motion would unite again in the middle of the rear side. Here in the field of the increasing friction, the friction layers are thrust toward the lower side by the rotation. By the resistance of the opposed lateral current its kinetic energy is partially transformed into pressure. This pressure reaches its maximum value in the stream saddle forward under the cylinder where the two motions, diagonal to each other, maintain the equilibrium and come to rest at a point. Thereby the whole stream, meeting the cylinder in front of the saddle, is forced to flow over the cylinder. On this path of the diminishing friction there is no obstructing counterflow. The frictional forces therefore have only an accelerating effect and thus increase the effect of the overpressure in the stream saddle, which dominates this portion of the circulation. The resultant effect of the friction therefore appears to be an increase in the velocity of the flow on the upper side of the cylinder.

The centrifugal force, which is proportional to the square of this velocity, finally produces the low negative pressure on the upper side, which, together with the overpressure in the stream saddle, produces the Magnus force.

Regarding the problem of the ratio of the positive and negative pressures, Prandtl first found the magnitude of the positive pressure on the under side of the cylinder to be equal to the dynamic pressure $\left(p = \frac{\rho}{2} \times V^2\right)$ of the simple wind velocity V, where ρ represents the density of the fluid. The negative pressure depends on the velocity of the flow on the upper side of the cylinder. On the nonrotating cylinder this is theoretically $u = 2\ V$ both above and below. Now, in order that this velocity on the under side may be zero at the center of dynamic pressure, it is assumed that the counter-circulation must also have the velocity $2V$. Therefore the velocity on the upper side is $4V$. From this it follows, on the assumption that the friction does not here come into consideration, that there must be at this point a pressure decrease of $\frac{\rho}{2}(4V)^2 = 16\frac{\rho}{2}V^2$, equal to 16 times the amount of the simple dynamic pressure on the lower side. This produces a negative pressure 15 times the dynamic pressure.

Against this method of calculation, it may be first objected that the peripheral velocity of the supplementary circulation does not need to be $u = 2V$, in order to produce the dynamic pressure of the simple wind velocity at the center of dynamic pressure, For this $u = V$ suffices, since the circulation is expected to counterbalance, at the center of dynamic pressure, only the simple wind strength, not its double, which is here no longer the case, due to the displacement of the center of dynamic pressure. However, if $u = V$, there is then, on the upper side, a resultant velocity of $U = 3V$ and consequently a theoretical pressure diminution of ninefold the amount of the simple dynamic pressure, so that the negative pressure would produce eight times, instead of fifteen times the dynamic pressure. Simultaneously the lift coefficient $(c_a)max = 4\pi = 12.57$ calculated by Prandtl would drop to $2\pi = 6.28$.

In reality, as we have seen, the circulatory motion produced by the decreasing friction cannot have the simple form like the theoretically uniform supplemental circulation. It was found, however, that it produces a motion over the after symmetrical half of the cylinder which is equal and opposite to the wind force V and which produces the overpressure in the stream saddle. Since the same acceleration from the friction must be assumed over the forward symmetrical half of the cylinder, there is produced, together with the velocity of the simple potential flow on the upper side of the cylinder, the velocity $3V$ and therefrom, according to Prandtl's calculation, as above, a negative pressure eight times as large as the positive pressure on the lower side. Along with this summary of the results, however, the following observations should not be overlooked.

According to the Göttingen theory, the circulation, after it has once been established as the counterpart of the initial vortex, remains constant without further effect, as if it were a property of the space about the cylinder, which is instantaneously transmitted to passing masses of air. The actual circulation is quite different and is produced only by the continuous action of the friction; is not instantaneously transmitted to the passing fluid; does not pass off in concentric but in eccentric approximate circles on the lower side; and does not have the same constant value $T = 2 \Gamma 2 \pi r u$ in all these circles, but decreases from within outwardly. This explains the discrepancies between the actual phenomena and the theoretical.

The peripheral velocity of the rotating cylinder was fixed at $U = 4 V$, because no retardation of the boundary layers and of the formation of vortices could then take place at any point. In fact, O. Tietjen obtained his excellent picture of the vortex-free Magnus flow at $U = 4 V$ with a cylinder of 4 cm (1.57 in.) diameter and 5 cm (1.97 in.) per second velocity. On the other hand, my own experiments yielded the vortex-free flow first at $U = 7 V$ with a cylinder of 5 cm (1.97 in.) diameter and about 10-15 cm (3.94-5.91 in.) per second velocity. This observation indicates that the ratio $U = 4 V$ would be much too small in a strong wind to produce the vortex-free flow and the maximum lateral force. The problem is not to reduce the friction so it would have no retarding effect on the boundary layers, as Prandtl believes, but to make it so great that it can generate in the fluid a circulation corresponding to any wind velocity and thus prevent the formation of vortices. Since the energy of the vortices is proportional to V^2, while that of the friction is only proportional to U, obviously, if no vortices are to be developed, U must increase quadratically and V only linearly.

In a vortex-free Magnus flow, when the wind velocity or the rotational speed of the cylinder is changed, the otherwise only disturbing friction, according to Prandtl, has the important task of restoring the disturbed condition. It then produces "more vortices in one direction than in the other until a circulation is produced corresponding to the momentary condition." Such a "releasing effect of the friction" would obviously have to control the Magnus flow automatically, without its being necessary for anyone to look after the maintenance of the theoretical rotational speed. A ll this, however, is only an inconclusive result of the fundamental error that the circulation about the cylinder continues without the aid of the friction, as in the potential flow of Lord Rayleigh.

The stereoscopic views of the Magnus flow on the surface of the water are particularly instructive. The dynamic pressures in the water produce elevations of the water level in opposition to gravity at points of positive pressure and depressions at points of negative pressure. Thus the internal pressure distribution is represented in relief by the elevations and depressions in the surface of the water, and the relation between the flow and the pressure distribution can be seen at a glance.

In the vortex-free Magnus flow (Fig. 6), the over-pressure of the liquid appears in the stereoscope as a slight elevation of the water surface on the lower side of the cylinder. On the upper side, however, there is a deep funnel-shaped depression, which indicates the negative pressure. According to the appearance, the negative pressure may be about four times the positive pressure, but not 16 times or nine times, as it would have to be according to Prandtl's calculation. There is the possibility, however, that the lowest negative pressure is limited to a very narrow space against the wall of the cylinder and thus escapes observation.

The Magnus flow with vortices is developed from the vortex-free flow, when the velocity exceeds a certain limit, although the ratio U : V is automatically maintained. At a critical point P (Fig. 15) of the upper rear quadrant there are produced close together two small, oppositely rotating vortices, which form the beginning of two vortex sheets. One sheet pushes forward over the upper side of the cylinder and the other down the back side of the cylinder and forward on the lower side, so that finally 2/3 to 3/4 of the circumference is covered with vortices (Figs. 16 and 17). In the sheets the vortices form large aggregates which finally break off alternately at the rear.

Here there are obviously very strong resistance forces, which greatly obstruct the development of the lateral force and even appear to be able to make them oscillate toward the opposite side, when the lower side of the cylinder is covered with the deepest depressions visible in the stereoscope. The observations of Lafay on the inversion of the Magnus effect are thus explained.

On the other hand, it is evident that the too-slow rotation for producing the vortex-free Magnus flow must greatly increase the quadratic resistance of the vortices, since the energy, transmitted by friction from the cylinder to the liquid, is entirely absorbed by the vortices. This is confirmed by the resistance measurements in comparison with a nonrotating cylinder.

When, therefore, the "Barbara" with her three rotors of 4 m (13 ft.) diameter and 17 m (56 ft.) height sails with a quartering after wind, and the flow system has the vortical form at an inadequate rotational speed, a stronger sail effect than with a nonrotating rotor can be obtained only through the resistance component lying in the direction of motion of the ship. The converse is true with the wind more from the front than from the rear. In both cases, therefore, great variations in the speed of travel are to be expected. There is also the further consideration that the three rotors, when they stand in line obliquely to the direction of the wind, mutually exert a strong effect on one another and offer the wind a greater surface of attack than when standing alone. All this, however, does not concern the Magnus effect, because it no longer has to do with the lateral force, but only with increasing the resistance through rotation and friction, as can also be done without rotors and the expenditure of energy by the use of sails. Whether sailing with a following wind with rotors or against a head wind with a propeller, it is obvious that the trips made can form no criterion for comparing the two kinds of propulsion, since the passive sail effect (vortex formation) acts in the direction of the rotors, but opposite to the thrust of the screw propeller.

Inside the water I have investigated in particular the phenomena at the ends of the rotating cylinder by means of stereoscopic pictures. The Magnus flow here goes into strong spiral vortex trails similar to those previously observed at the edges of oblique plates, wings and propeller blades. At the beginning of the motion, these vortex trails are connected, behind the cylinder, by the initial vortex sheet, which is parallel to the cylinder axis and develops, at the free end, into a starting vortex. The system then has the form of a closed vortex ring whose front portion is formed by the cylinder, which produces an artificial vortex by its rotation. On cylinders with large end disks the vortex trails have the diameter of the disks, and the flow appears as though the whole cylinder had the greater diameter. The flow is not therefore restricted laterally by the end disks, but is strengthened by the friction of the large surfaces.

The practical application of the Magnus effect loses its importance through the proof that the force does not come from the wind, but from the engine which rotates the cylinder and produces the friction. Without the theoretical error that the Magnus effect is a sort of powered sail effect, one would hardly have thought of applying the rotor to the propulsion of ships and other uses.

The rotor is not a rival of the sail, but of the screw propeller. I hold it impossible for the efficiency of the rotor, through the Magnus effect, to equal or exceed that of a good screw, on account of the difficulty of maintaining the right revolution speed. Since the rotor is dependent on the wind and can work only part of the time, and since, moreover, it cannot replace the propeller drive, needlessly makes the control of the ship more difficult and renders its operation uneconomical, it has no excuse for existence.

Supplement.- In a lecture delivered before the Congres International de la Navigation Aerienne, 1925, D. Riabouchinsky calls attention to a short paper of Maxwell's (Cambridge and Dublin Mathematical Journal, Vol. IX), which appeared in the same year (1853) as the treatise of Magnus on the deflection of projectiles in "Poggend. Annalen" Vol. 88, I). Unfortunately, Maxwell's note is not available to me, but I gather from Riabouchinsky's paper that Maxwell already knew of the occurrence of a lateral force in the similar case of a freely falling rotating narrow rectangular plate. The falling motion of such a body is known to pass very quickly from the initial horizontal position into rotation and then follows a straight line deviating from the vertical toward the side on which the rotation coincides with the direction of the opposed relative wind.

I instituted experiments long ago on this and other forms of the falling motion of plates ("Der Schwebflug und die Fallbewegung ebener Tafeln in der Luft," Abh. d. Naturw. Ver. Hamburg, Vol. XV, 1897) and subsequently, motion with rotation, designated as "rolling flight" (Rollflug). With Riabouchinsky I would also call attention to the four-winged rolling flier (Rollflieger) which the universally honored Nestor of meteorologists, Dr. W. Koeppen, made and tested in the court of the German Naval Observatory. As in the Magnus experiment, there is also developed in rolling flight a lateral force in connection with the simple resistance to the falling motion. Riabouchinsky speaks therefore of the "Maxwell-Magnus" phenomenon, a designation which is without historical foundation, since Magnus, previously to his article in Poggend. Annalen, had published the results of his investigations in 1852 in the Abhandl. d. Akad. d. Wissensch. zu Berlin, the English translation of which then appeared in Taylor's Scientific Magazine in 1853.

Naturally the flow in the rolling flight of flat or prismatic bodies is always accompanied by periodical vortex formations and strongly fluctuating resistances. The vortices, according to my motion-picture investigations with the Savonius rotor, may be of a very complex nature. The circulatory motion, however, is always easily recognizable from the fact that the separation point of the flow is unsymmetrically located toward the side of the opposing wind and consequently the larger portion of the fluid passes off on the side of the accompanying wind. On this side, therefore, the velocity and the centrifugal-pressure reduction must be greater than on the opposite side, which explains the lateral force. The Savonius rotor is of the nature of a Robinson cup anemometer on which the cups are replaced by hollow semicylinders.

Riabouchinsky had already in 1909, in the wind tunnel of his laboratory in Koutchino near Moscow, measured the resistance forces produced on single-vaned, three-vaned, and four-vaned models (called wind vanes), when set in rotation by a uniform air stream. He found the maximum coefficients K_y of the lateral force on an experimental vane consisting of only one rectangular surface, whose axis of rotation coincided with the longer middle line.

The motion-picture analysis of the flows offers no special difficulty, when the experimental body in water goes into autorotation at a sufficiently low velocity of the water.

A narrow strip of paper, falling with the rotation, is the simplest conceivable means for the demonstration of the Magnus effect, since it shows very pronounced deviations from the expected vertical falling path.

C o n c l u s i o n s

1. The view held by Professor Prandtl and his coworkers, that the Magnus effect is derived from the wind, without the direct aid of friction, is not applicable and is due to the endeavor to substitute for the reality a theorem of Lord Rayleigh based on the ideal frictionless fluid, although Rayleigh himself had uttered a sufficient warning against such a course.

2. The attempts to make the separation theory of Prandtl serve this purpose have been shown to be untenable.

3. In agreement with the physical explanation of the Magnus effect given by G. Magnus and confirmed by Lord Rayleigh, it has been shown by photographic analysis that the immediate cause of the Magnus effect is the friction of the air on the rotating cylinder.

4. The Magnus effect is the reaction of the wind against a one-sided displacement of the air masses by active mechanical forces, which are transmitted to the air by the friction of the rotating cylinder.

5. Since, on the contrary, only wind forces act on a sail, the Magnus effect is not comparable with the action of a sail. The assumption of Lafay that a rotating cylinder extracts energy from the wind, which, according to the Göttingen tests, amounts to ten or more times the propelling force of a sail of like height and width, is therefore based on an error.

6. The Flettner rotor is no wind-force machine, but is driven by mechanical energy like a ship's propeller. Since it can work only in a favorable wind, it cannot be considered as a rival of the screw propeller.

Exceptions to the Above Treatise "The Magnus Effect in Theory and in Reality"

By W. Hoff

The scientific editorial staff of the Z.F.M., in this special case represented only by myself, felt constrained to publish the above treatise of Dr. Ahlborn, although not agreeing with the ideas therein enunciated. The editorship therefore takes the following exceptions to the conclusions of the above treatise.

1. Friction is essential for the development of any kind of circulation. This is stated by Prandtl in his boundary-layer theory, in his wing theory and also in the Göttingen papers on the Magnus effect. After the circulation has been started, however, the assumption of a frictionless or nonvis-

cous fluid suffices for explaining the Magnus effect. Since energy is always absorbed in a viscous fluid, whenever velocity differences exist in adjacent layers, a certain small amount of energy must naturally be transmitted by means of friction from the rotating cylinder to the fluid, in order to prevent the circulation from gradually disappearing. If, in a viscous fluid, the friction between cylinder and fluid could suddenly be reduced to zero (whether the cylinder stood still or rotated would then make no difference), the circulation would, nevertheless, gradually disappear, due to the friction between the fluid layers.

If, in a viscous fluid, the rotation of the cylinder is suddenly stopped, a vortex in the negative direction is then formed as a result of the friction. In Prandtl's hydrodynamic motion-picture film, displayed in Wiesbaden, at the 1927 session of the Wissenschaftliche Gesellschaft für Luftfahrt (1927 W.G.L. Yearbook, p. 133), the positive and negative flows can be clearly discerned. This experiment (the omission of which, according to Mr. Ahlborn, saved the theory) was, it is true, not made with a rotating cylinder, but was made with a supporting wing, where the relations are similar. The experimental result is in no way contrary to the Prandtl theory.

2. There can be no question of any proof of the untenability of the Prandtl theory through the statements of Mr. Ahlborn. In principle it may be remarked that a theory does

not need to agree with the reality. When the theory represents correctly the greatest possible number of observed facts, it is already useful. It can then be abandoned only when a new theory correctly represents, in addition to the already explained phenomena, still others not explainable by the previous theory. In this sense the Prandtl theory has explained the separation phenomena, the production of the circulation about airfoils and rotating cylinders, etc. The calculations based on the theory agree well with the results of practical tests. No such claim can be made for Mr. Ahlborn's theory. It would be very difficult to construct any mathematical theory on his arguments.

3. Friction is the cause of the Magnus force in the sense that there is no circulation without friction. The energy employed to turn the rotors, however, does not represent the maximum amount of energy which can be obtained by the practical use of the rotors (somewhat as a sailing vessel).

4. The Magnus force (like the lift on a wing) is the reaction of the air masses deflected by the rotating cylinder (or by the wing).

5. The Magnus force on a rotating cylinder corresponds exactly to the lifting force on an airfoil (or on a sail). The Lafay and Göttingen tests prove this conclusively.

6. The Flettner rotor does not work like a ship's propeller, but like a sail. This statement is proved by the results of the trial trips of the "Buckau" (published by Tradt in Werft Reederei Hafen, 1925, p. 160). In the propulsion of a ship by a screw propeller (i.e., without rotors), the ship acquired a maximum speed of 7.85 knots with 134 hp (trip No. 4 of November 24, 1924). When the same ship was operated as a rotor ship (without using the screw propeller), it acquired a maximum speed of 8.2 knots, 33.4 hp being used to turn the rotors (trip No. 7 of January 6, 1925).

If we designate the power, efficiency and speed of the "Buckau" as a motor ship by N_o, η_o and v_o respectively, and as a rotor ship by the corresponding values N, η and b, we obtain, in the customary manner,

$$\eta = \left(\frac{v}{v_o}\right)^3 \frac{N_o}{N} \eta_o .^*$$

On substituting the above-mentioned values, we obtain

$$\eta = \left(\frac{8.2}{7.85}\right)^3 \frac{134}{33.4} \eta_o = 4.58\ \eta_o .$$

If we assume for the motor ship $\eta_o = 0.50$ (which is certainly very small), we then obtain for the rotor ship

$$\eta = 2.29\ (!)$$

From this it follows that the rotor ship must have another source of energy, which can only be the wind as in the case of

*Subsequently I found that, in trip 2 of May 1, 1925, in which the wind blew almost exactly from the side, $\eta = 435\ \eta_o$, i.e., almost the same value as given above.

a sailing ship. This disposes of the point upon which Mr. Ahlborn has built his whole argument.

Mr. Ahlborn's Reply.- The above six points of the editorship compel me to make the following remarks.

1. The theory of the frictionless or nonviscous fluid is not applicable to a phenomenon which depends so completely on friction as the Magnus effect does. If Lord Rayleigh's warning had been heeded, modern aerodynamics would have been spared the depressing defeat of the rotor theory, which cannot be brushed aside by any argument. The "certain small amount of energy" continuously requires the full output of the driving engine and permanently has the same value as required at the beginning for the production and maintenance of the Magnus effect.

The deciding "cross experiment" (missed by me in the Göttingen researches) of measuring the Magnus force with the engine stopped was not performed, as a matter of fact, and cannot be replaced, either by the motion-picture film or by the photograph of the "starting vortex."

2. Of course all theories are only approximations of the reality, but a theory is wrong when it makes assumptions which alternately, according to circumstances, contradicts first the reality and then the theory itself and when it seeks in this way to replace an incontestable scientific explanation of the

phenomena (Magnus, Lord Rayleigh).

4. With the sentence, "The Magnus force is the reaction of the air masses deflected by the rotating cylinder" the editorship admits that a theory cannot be maintained which denies to the rotation and friction their deciding influence on the action of the forces on the cylinder and tries to derive the Magnus force from the kinetic energy of the wind. Notwithstanding this recognition and confirmation of my conclusion, Mr. Hoff still believes he can revive the Göttingen theory by a calculation.

3. If the energy employed to turn the rotors does not represent the maximum amount of energy obtainable by the practical use of the rotors, this can, of course, only mean that an experienced seaman, without the benefit of the Magnus effect, can still use the direct thrust of the wind on the ship for increasing its speed, as, conversely, the speed would be reduced in sailing against the wind and waves. Mr. Hoff disregards this last possibility and utilizes only the pushing effect of the wind most favorable to rotor propulsion, in order to prove that the rotor works passively like a sail and not actively like a screw propeller.

6. For this purpose he makes use of the not verifiable statement regarding the speed of a single trip (No. 7) of the "Buckau" especially favorable for the rotor, for which the

direction and velocity of the following wind is not given. Then he ascribes the total wind pressure to the Magnus effect on the rotor, giving the latter an efficiency of $\eta = 2.29$. The rotor must therefore give out 2.29 as much energy as it has and, since this is impossible, the rotor ship (not the rotor) must have another source of energy, the wind.

This is correct, but the calculation was not necessary for this conclusion. The thrust of the wind acts on every ship, even on the high structures of a steamer, without anyone having hitherto entertained the thought of ascribing more than 100% efficiency to the screw. The proof for the passive rotor therefore rests on a vicious circle.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

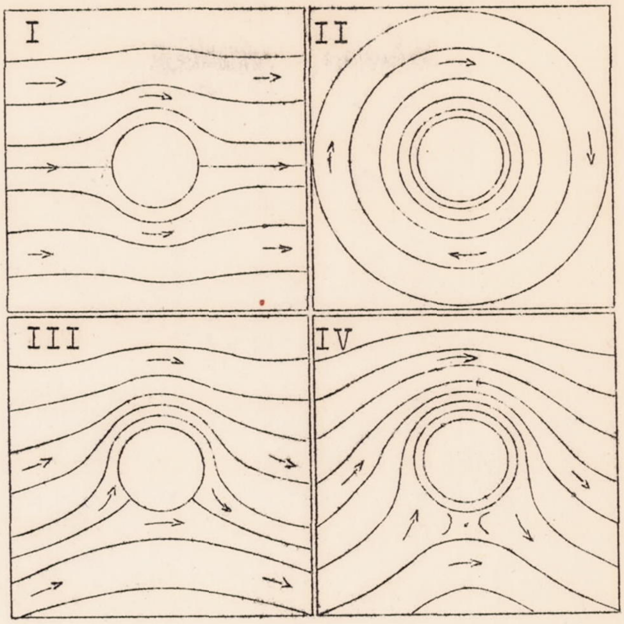

Fig.1 From the simple potential flow I and the simultaneous circulatory motion II there is developed the Rayleigh flow III for a weaker and IV for a stronger circulation.

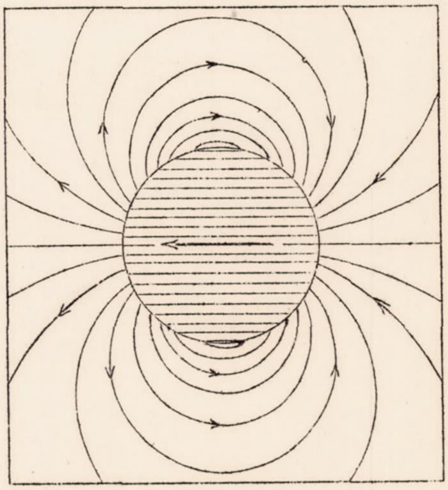

Fig.9 Absolute streamlines or lines of force of the simple potential flow about a cylinder.

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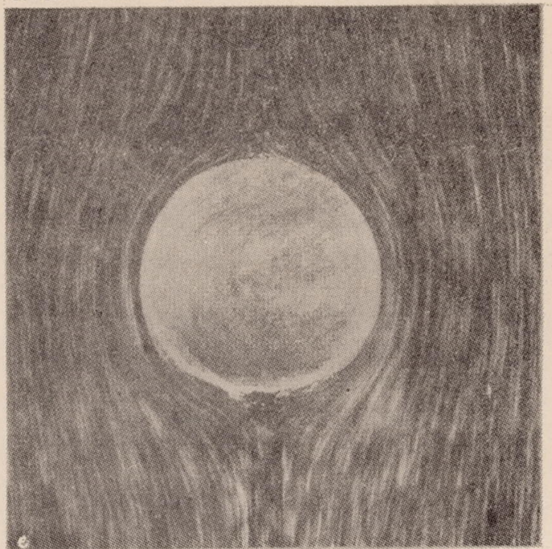


Fig.2 Incipient streamline about a cylinder.

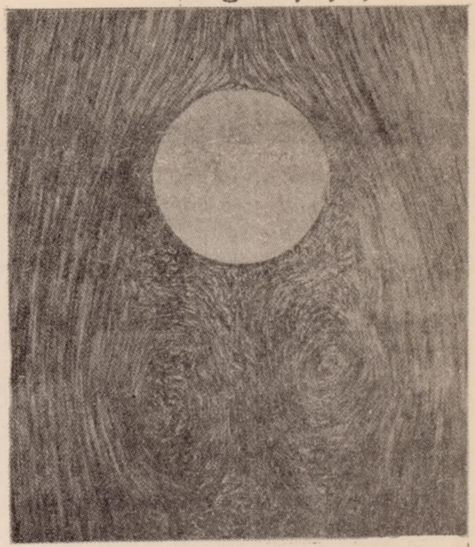


Fig.3 Established flow about a cylinder.

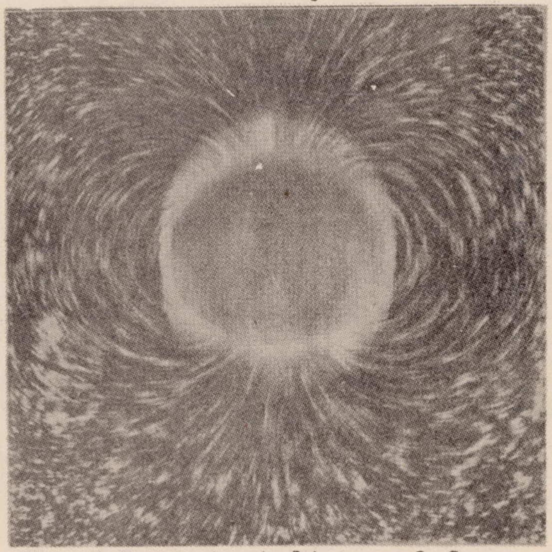


Fig.4 Incipient lines of force about a cylinder.

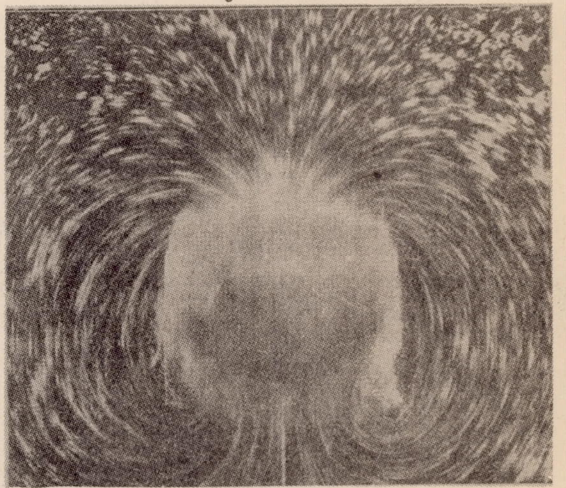


Fig.5 Force field about a cylinder (without rotation) with the first visible vortices.

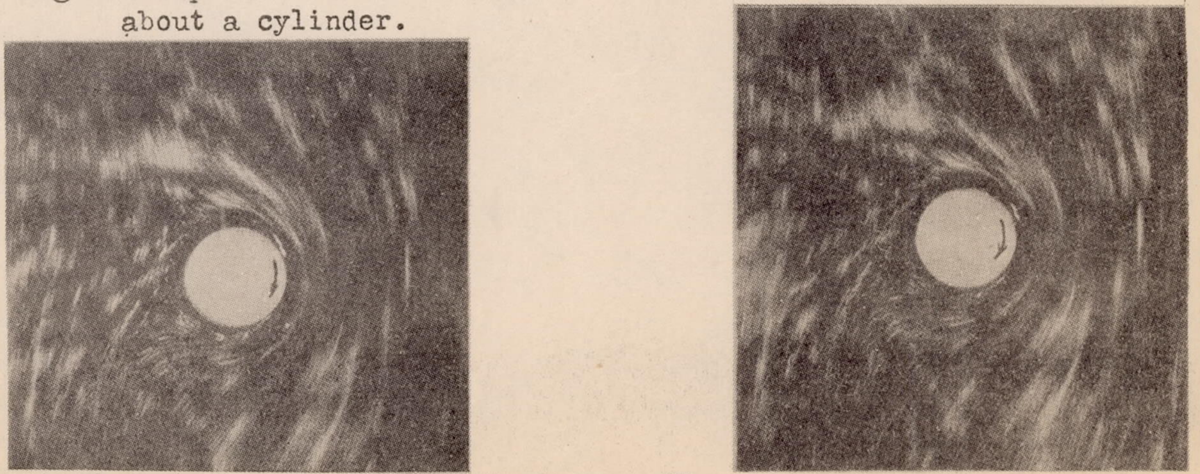


Fig.6 Stereoscopic view of the Magnus flow. The stereoscope shows in relief the pressure distribution which produces the Magnus effect.

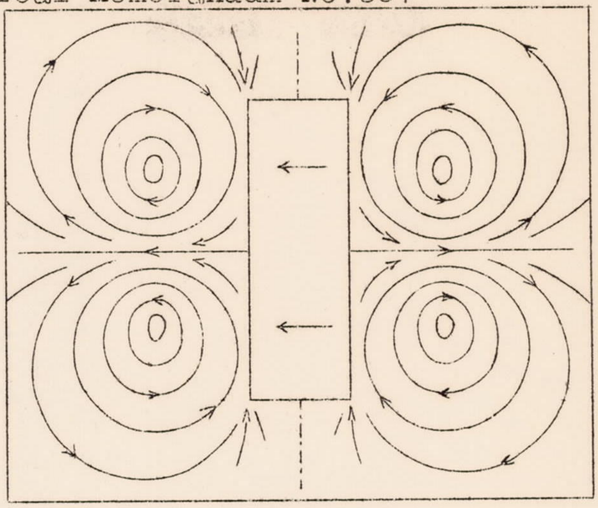


Fig.7 The opposite spiral vortex rings of the "trade circulations" produced by the rotation of a cylinder in still water.

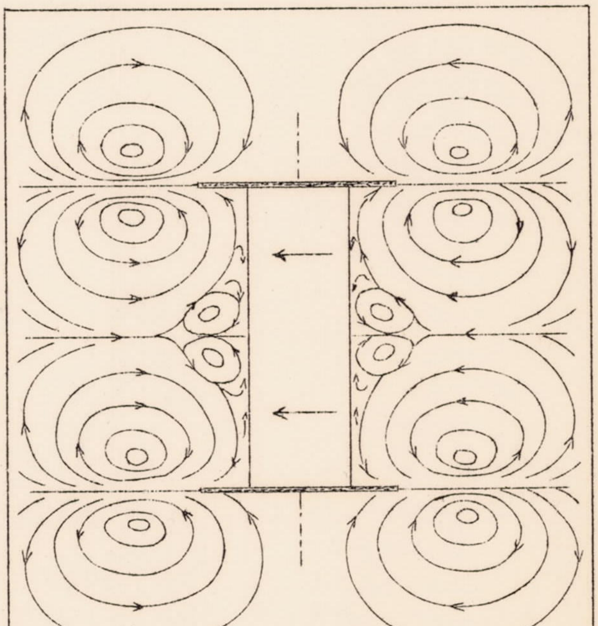


Fig.8 A cylinder provided with end disks develops by rotation in still water four pairs of opposite "trade" vortices. The vortex pairs at the end disks are much stronger than those around the cylinder.

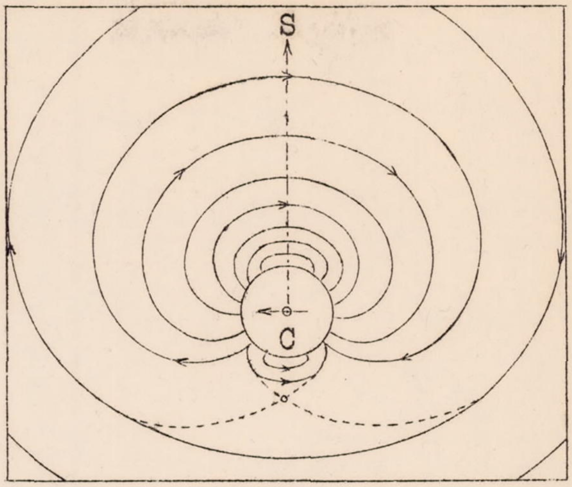


Fig.10 Absolute streamlines of the Rayleigh flow (according to Lamb's "Hydrodynamics". C-S is direction of lateral force.

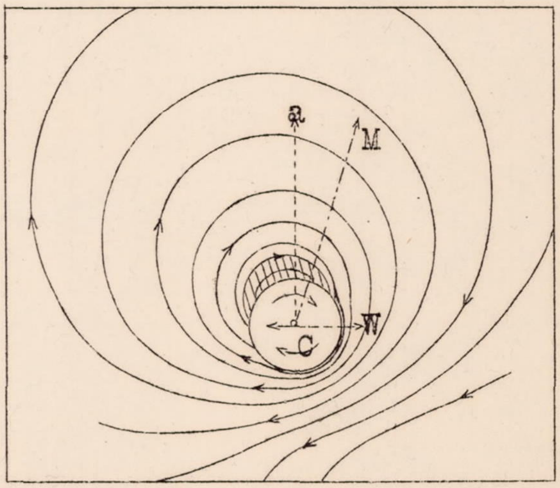


Fig.11 Field of force of Magnus flow with the spiral lines of force, according to motion pictures. C - M is the Magnus force.

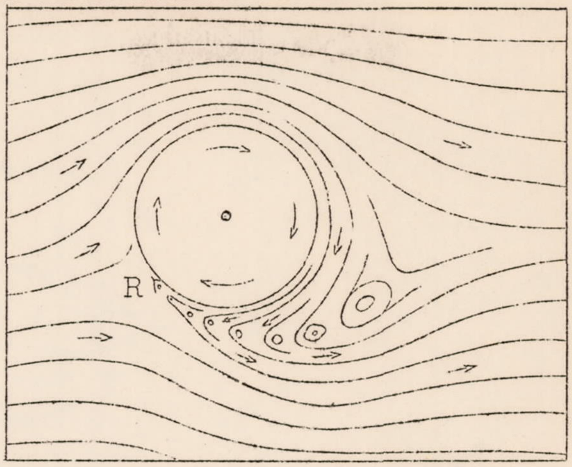


Fig.12 Magnus flow with initial vortex sheet.

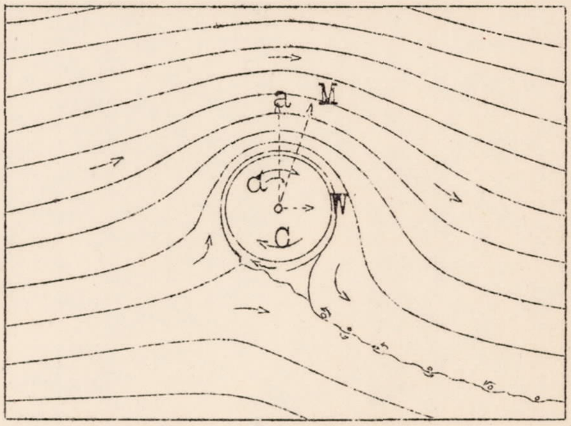


Fig.13 Completed Magnus flow. C-M is the Magnus force; C_a, the lateral component; C_w, the resistance component.

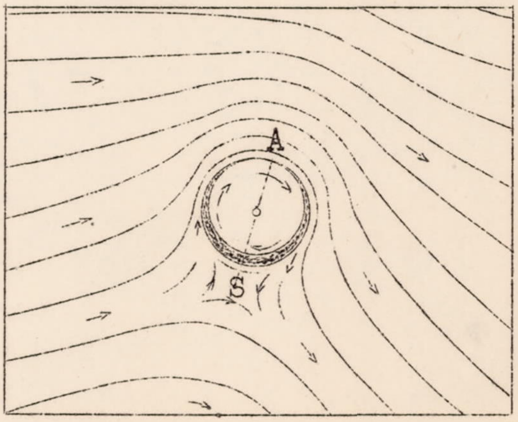


Fig.14 Magnus flow, schematic. S, center of dynamic pressurs in flow saddle. The increasing and decreasing friction is indicated by the crescent.

Fig.15

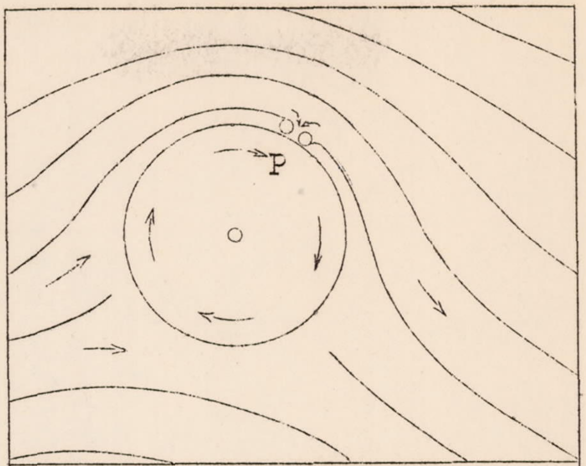


Fig.16

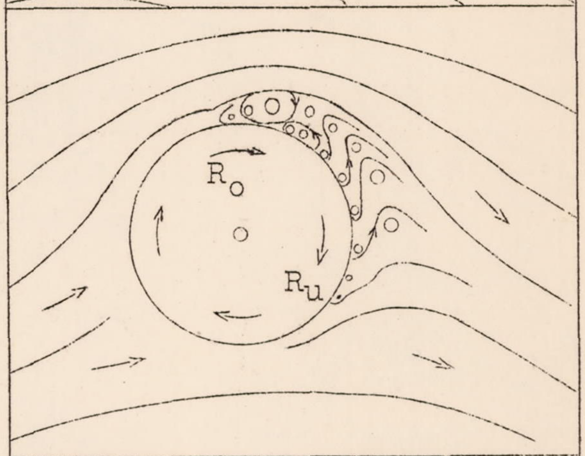
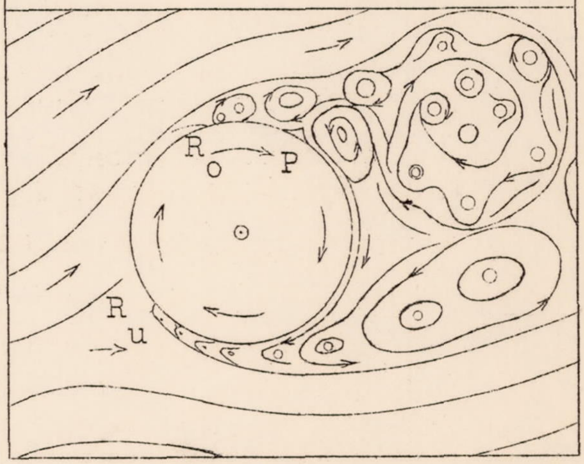


Fig.17



Figs.15,16,17 Magnus flow with vortex formation in three stages according to motion pictures. P, critical point at which the vortex formation begins. R_o, upper line of rotation. R_u, lower line of rotation.