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Nonclassical states of the second optical harmonic in the presence of self-action

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Abstract

The quantum theory of coherent radiation frequency doubling in crystals with quadratic and cubic optical nonlinearities is developed. The possibility to produce the quadrature - squeezed state of the second harmonic (SH) field is shown,the nonclassical SH states arising due to self-action effect.

1. Introduction

The quantum theory of the second and higher harmonic generation has been developed in a number of works (see, for instance, Refs. 1-3) in which the possibility of obtaining the squeezed states of electromagnetic field and photon antibunching has been analyzed. It has been established that the frequency doubling is accompanied with the generation of the squeezed states at the fundamental frequency whereas the second harmonic (SH) field turns out to be in the coherent state. At the same time the frequency doubling of the squeezed light, as it was shown in Ref. 1, causes a decrease in squeezing. From the practical point of view, the methods based on the quadratic and cubic medium nonlinearities with respect to the electric field are of considerable interest. It is known[4] that in the centrosymmetric nonlinear medium, i.e. the Kerr medium, the quadrature - squeezed field can be produced due to the self-action effect. In the media mentioned above the four frequency wave processes always occur in the presence of self-action. In the media with the induced quadratic optical susceptibility the three frequency wave interactions can also occur under the evident influence of self-action.

In the present paper the quantum theory of the SH generation (SHG) in the presence of self-action is developed. In the framework of the classical approach the problem under consideration has been solved in Refs. 5 and 6. The basic equations of the process which are of interest to us are presented in Sec. 2. In the Sec. 3 SHG is analyzed for the case of low efficiency of the fundamental radiation conversion into the SH; however, we do not take into account here the SH influence on the effective refractive index of the medium. The possibility of the SH quadrature - squeezed state generation is shown in Sec. 4.

2. Basic equations

Interaction of the fundamental wave of frequency ω and second harmonic wave of frequency 2ω in an optical medium with nonlinear susceptibilities of the second $\chi^{(2)}$ and third $\chi^{(3)}$ orders is described by the Hamiltonian:

$$\hat{H} = \hbar\omega\hat{a}\hat{a}^{+} + 2\hbar\omega\hat{b}\hat{b}^{+} + \hat{H}_{i} \quad , \tag{1}$$

$$\hat{H}_{i} = \hbar\beta(\hat{b}^{+}\hat{a}^{2} + \hat{b}\hat{a}^{+2}) + \hbar\gamma(\hat{a}^{+2}\hat{a}^{2} + \hat{b}^{+2}\hat{b}^{2} + 2\hat{a}^{+}\hat{a}\hat{b}^{+}\hat{b}) \; ,$$

where $\hat{a}^{+}(\hat{a})$ and $\hat{b}^{+}(\hat{b})$ are photon creation (annihilation) operators of the fundamental wave and SH which obey the commutation relations:

$$[\hat{a},\hat{a}^{+}] = 1 \; , \quad [\hat{b},\hat{b}^{+}] = 1, \tag{2a,b}$$

the nonlinear parameter β is proportional to $\chi^{(2)}$, and parameter γ to $\chi^{(3)}$. The operator evolution is given by the Heisenberg equations:

$$\frac{\partial\hat{a}}{\partial z} = -i2\beta\hat{a}^{+}\hat{b} - i\gamma\hat{a}^{+}\hat{a}^{2} - i2\gamma\hat{b}^{+}\hat{b}\hat{a} \; , \tag{3a}$$

$$\frac{\partial \hat{b}}{\partial z} = -i\beta\hat{a}^2 - i2\gamma\hat{a}^+\hat{a}\hat{b} - i\gamma\hat{b}^+\hat{b}^2 \qquad (3b)$$

where z is the length of the medium in the direction of wave propagation.

Let us discuss the terms on the right - hand side of Eqs. (3a) and (3b). The first terms are associated with the process of degenerated three-frequency interaction (the first term in Eq.(3b) describes the SH generation (SHG), whereas in Eq.(3a) the first term takes account of parametric interaction). The second terms in Eqs.(3a) and (3b) deal with the self-action and cross-action of the radiation of frequency ω. Finally the third terms in Eqs.(3a) and (3b) take into account the cross-action and self-action at frequency 2ω.

Assuming that at the input of the nonlinear medium the fundamental wave and second harmonic are in the coherent and vacuum states respectively, we have

$$\hat{a}(z=0) = \hat{a}_o , \quad \hat{a}_o|\alpha\rangle = \alpha_o|\alpha\rangle ;$$

$$\hat{b}(z=0) = \hat{b}_o , \quad \hat{b}_o|0\rangle = 0 . \qquad (4)$$

3. SH generation in the fixed photon number approximation

The analysis given below implies the low efficiency of the conversion of the fundamental radiation into the SH. Therefore, we can neglect the last terms in Eqs. (3a) and (3b). We thus take into account the refractive index variation due to the cubic nonlinearity caused only by the intensive fundamental wave. The SHG process is analyzed in the fixed photon number approximation. Using this approximation one neglects the photon number variation of the fundamental wave, i.e. we suppose that the operator of the photon number $\hat{n}(z)=\hat{a}^+(z)\hat{a}(z)$ remains unchanged during the process of the nonlinear interaction ($\hat{n}(z)=\hat{n}_o=\hat{a}_o^+\hat{a}_o$). It should be noted that this approximation is in fact the quantum analog of the fixed intensity approximation (see Ref. 7).

Let us introduce the new operators $\hat{c}(z)$ and $\hat{f}(z)$ for the fundamental radiation and SH respectively:

$$\hat{c}(z) = e^{i\gamma z\hat{n}_o}\,\hat{a}(z)\,, \qquad \hat{f}(z) = e^{2i\gamma z\hat{n}_o}\,\hat{b}(z. \qquad (5a,b)$$

These operators also obey the commutation relations similar to Eq.(2) and the initial conditions similar to Eq.(4). The evolution of the new operators is given by the equations:

$$\frac{d\hat{c}(z)}{dz} = -i2\beta\, e^{i\gamma z}\,\hat{c}^{+}(z)\hat{f}(z)\,, \qquad (6a)$$

$$\frac{d\hat{f}(z)}{dz} = -i\beta\, e^{-i\gamma z}\,\hat{c}^{2}(z)\,. \qquad (6b)$$

By differentiating Eq.(6b) and using Eq.(6a), we obtain the equation for the SH operator $\hat{f}(z)$

$$\frac{d^2}{dz^2}\hat{f}(z) + 4\beta^2(\,\hat{n}_o + \frac{1}{2}\,)\,\hat{f}(z) = 0 \qquad (7)$$

with the initial conditions

$$\hat{f}(z{=}0) = \hat{b}_o\,, \qquad \left.\frac{d\hat{f}}{dz}\right|_{z=o} = -i\beta\hat{a}_o^2. \qquad (8)$$

Below we make use of the operator $\hat{f}(z)$ expanded into the Taylor series to within χ^2 ($\chi=\beta z$):

$$f(z) = \hat{f}_o + \left.\frac{d\hat{f}}{dz}\right|_{z=o} z + \frac{1}{2}\left.\frac{d^2\hat{f}}{dz^2}\right|_{z=o} z^2 + \ \ldots \qquad (9)$$

Returning to the primary operators of the fundamental wave and SH ($\hat{a}$ and $\hat{b}$ respectively) we obtain the expression:

$$\hat{b}(z) = e^{-2i\gamma z\hat{n}_o}\,\{\hat{b}_o - i\beta z\hat{a}_o^2 - \frac{1}{2}\,\beta\gamma z^2\hat{a}_o^2 - 2(\beta z)^2(\hat{n}_o + \frac{1}{2})\hat{b}_o\}. \qquad (10)$$

The evolution of the SH field operator depends on the value of the nonlinear parameters βz and γz.

By averaging over initial states of the fields, we obtain the mean value for the operator $\hat{b}$ (10) :

$$\langle \hat{b}(z) \rangle = -(i\beta z + \frac{1}{2}\beta\gamma z^2)|\alpha_o|^2 e^{2i(\phi_o - \Phi)} \tag{11}$$

Here $\Phi = \gamma z|\alpha_o|^2$ is the nonlinear phase addition arising due to the self-action and $\phi_o = \arg|\alpha_o|$ is the fundamental wave phase. In the framework of the considered approximation the operators $\hat{b}^+$ and $\hat{b}$ satisfy the commutation relation (2b).

Let us turn to the analysis of the SH field photon statistics. Calculations of the Fano factor $F = \sigma_N^2/\langle \hat{N} \rangle$ (where $\hat{N} = \hat{b}^+(z)\hat{b}(z)$) result in the following expression:

$$F(z) = 1 + (\beta z)^2 (4|\alpha_o|^2 + 2) . \tag{12}$$

Thus, as one can see from Eq.(12), the photon statistics of the SH field becomes super-Poissonian.

4. Quadrature components of the SH field

In this section we dwell upon the fluctuations of the SH quadrature components described by the operators:

$$\hat{X}(z) = \frac{1}{2}(\hat{b}(z) + \hat{b}^+(z)), \qquad \hat{Y}(z) = \frac{1}{2i}(\hat{b}(z) - \hat{b}^+(z)). \tag{13}$$

The quadrature components (13) are registered by the balanced homodyne detection (see Figure). The SH field being under investigation is mixed with the coherent one at the same frequency, generated in the absence of the self-action and cross-action effects. The mixed radiation of the both reference coherent wave and that of the analyzed SH is fed to the balanced detector input. Thus, we have the possibility to record one of the SH quadrature components for the field under consideration.

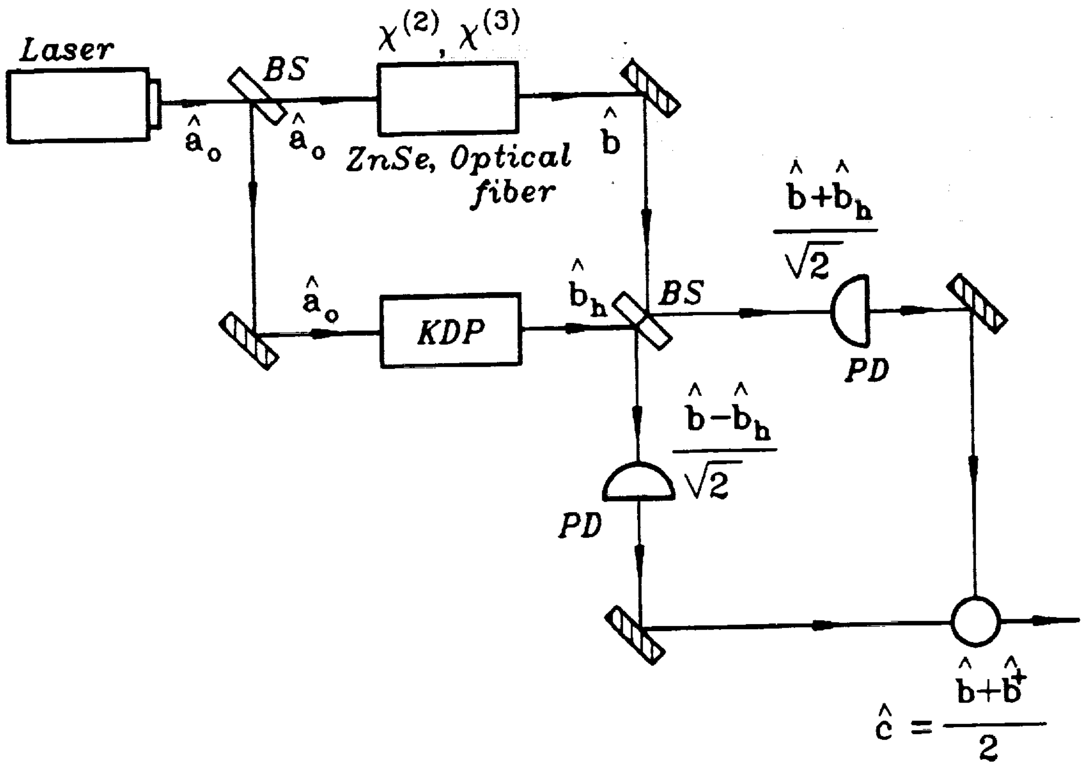

Figure: Layout of measurement of SH quadrature components.

According to Eqs.(10) and (13) the mean values of the quadrature components (13) are equal to

$$\langle \hat{X}(z)\rangle = \beta z|\alpha_o|^2 \sin[2(\phi_o-\Phi)] - \frac{1}{2}\beta z\Phi \cos[2(\phi_o-\Phi)],$$
$$\langle \hat{Y}(z)\rangle = -\beta z|\alpha_o|^2 \cos[2(\phi_o-\Phi)] - \frac{1}{2}\beta z\Phi \sin[2(\phi_o-\Phi)] \tag{14}$$

and determined by the value of the nonlinear parameter βz and nonlinear phase addition Φ.

Calculations of the variances of the SH quadrature component yeild in the expressions:

$$\sigma^2_{x,y} = \frac{1}{4} + \frac{|\alpha_o|^2}{4}\{(\frac{1}{2}(\beta\gamma)^2z^4 -2(\beta z)^2)(\cos[4(\phi_o-\Phi)-2\gamma z]-\cos[4(\phi_o-\Phi)])$$
$$\mp 2\beta^2\gamma z^3(\sin[4(\phi_o-\Phi)-2\gamma z]-\sin[4(\phi_o-\Phi)])\}, \tag{15}$$

where the upper sign is for the $\hat{X}$ quadrature and the lower is for $\hat{Y}$. Let us transform Eq.(15) by retaining only the terms of order $(\beta z)^2$ and smaller. As a result we have to within $(\beta z)^2$

$$\sigma^2_x = \frac{1}{4} - K^2\Phi \sin[4(\phi_o-\Phi)] + 2(\beta z\Phi)^2 \cos[4(\phi_o-\Phi)],$$
$$\sigma^2_y = \frac{1}{4} + K^2\Phi \sin[4(\phi_o-\Phi)] - 2(\beta z\Phi)^2 \cos[4(\phi_o-\Phi)], \tag{16}$$

where the coefficient $K^2=(\beta|\alpha_o|z)^2$ characterizes the efficiency of the SH conversion. It follows from Eq.(16) that the variances are the oscillatory functions of the parameter Φ due to the self-action. The oscillation amplitude depends on the SHG efficiency and the value of the phase Φ. It is evident that the variations of the variances have the opposite tendency. The analysis of Eq.(16) is more clear provided the initial radiation phase ϕ_o is optimized:

$$\phi_o = \Phi + \frac{1}{4}\arg(-\frac{1}{2\gamma z}) \cong \Phi. \tag{17}$$

The extremal values of the variances Eq.(16) are equal to:

$$\sigma^2_{x,y} = \frac{1}{4} \pm K^2\Phi \ . \qquad (18)$$

One can see from Eq.(18) that it is possible to obtain the quadrature - squeezed states of SH field. In this case the predominant role is played by the self-action effect. In the absence of the self-action (Φ=0) the SH field is in the coherent state ($\sigma_x^2=\sigma_y^2=1/4$). It is obvious from Eq.(18) that the degree of squeezing can be arbitrary high and is determined by the efficiency of the SH conversion K^2 and phase Φ.

It follows from the calculation of the uncertainty relation for the SH quadrature components that we have the ideal quadrature squeezing to within $(\beta z)^2$.

6. Conclusions

It follows from the analysis given above that SH quadrature - squeezed states are produced by frequency doubling in the presence of the self-action phenomenon which plays a predominant role. The degree of squeezing is determined by both the SHG efficiency and nonlinear phase induced by self-action. The nonlinear medium where the considered process is likely to occur can be realized in noncentrosymmetric nonlinear crystal (for example ZnSe) or centrosymmetric medium in a static electric field. It seems to be promising to use optical fibers, in which the SHG efficiency can reach 1-5% (Ref.7).

As it was mentioned above the possibility to produce the squeezed states of the fundamental radiation isusually studied in SHG process occurring in the absence of the self-action. Outside the framework of the fixed photon number approximation we also considered the fundamental field statistics, taking into account the self-action effect. We have found that the degree of the fundamental radiation squeezing depends on the SHG efficiency and nonlinear phase as in the case of the second harmonic field.

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