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ONE DIMENSIONAL REPRESENTATIONS IN QUANTUM OPTICS

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Abstract

The possibility to represent the quantum states of a harmonic oscillator not on the whole α-plane but on its one dimensional manifolds is considered. It is shown that a simple Gaussian distribution along a straight line describes a quadrature squeezed state while a similar Gaussian distribution along a circle leads to the amplitude squeezed state. The connection between the one dimensional representations and the usual Glauber representation is discussed.

1 Introduction

There are several widely used representations to describe a state of a quantum oscillator in the Hilbert space. The most natural one is the expansion of the state into the number state

$$| c >= \sum_{n=0}^{\infty} c_n | n > . \tag{1}$$

Another well known possibility is the coherent state representation [1,2]

$$| f >= \frac{1}{\pi} \int f(\alpha^*) \exp(- | \alpha |^2 /2) | \alpha > d^2\alpha, \quad d^2\alpha = d(Re\alpha)d(Im\alpha), \tag{2}$$

$f(\alpha^*)$ being an analytical function of α^*. Here the state is represented by a superposition of nonorthogonal coherent states all over the complex α-plane.

As already Glauber pointed out, there is an infinite number of ways of expanding any state in terms of coherent states due to the overcompleteness of the latter states

$$| f >= \frac{1}{\pi} \int G(\alpha,\alpha^*) | \alpha > d^2\alpha, \tag{3}$$

here the expansion function $G(\alpha,\alpha^*)$ may be a rather general function of α and α^*. Being confined to some given class of functions the uncertainty in finding $G(\alpha,\alpha^*)$ can be reduced. In this paper we shall deal with such representations that correspond to kern functions $G(\alpha,\alpha^*)$ leading to integration over a one dimensional manifold of the α-plane in Eq. (3). The possibility to represent any state on a subspace of the complex plane comes from Cahill's theorem on overcompleteness [3,4]. We shall show that such nonclassical states as the quadrature and amplitude squeezed states can be represented very naturally by superposition of coherent states along a straight line or along a circle in the α-plane correspondingly.

2 Representation along a straight line

The most simple states emerging from superposition of coherent states are the even $| x,+>$ and the odd $| x,->$ states

$$| x,+>= c_+(| x> + | -x>), \tag{4}$$

$$| x,->= c_-(| x> - | -x>), \tag{5}$$

where $| x>$ is a usual coherent state with real eigenvalue of the annihilation operator $a \mid x>= x \mid x>$. It is remarkable that the even state $| x,+>$ being a superposition of two classical states is squeezed [5]

$$(\Delta a_2)^2 = \frac{1}{4} - x^2/[1+\exp(2x^2)]. \tag{6}$$

where a_1 and a_2 are the Hermitian quadratures of the annihilation operator.

The squeezing can be further enhanced by adding the vacuum state to $| x,+>$

$$| x,p>= c_p(| x> +p \,| 0> + | -x>). \tag{7}$$

This way one can achieve a variance $(\Delta a_2)^2 = 0.0651$ instead of 0.111 for $| x,+>$ or 0.25 for the vacuum state. Superposing more and more even states to it one can get even more squeezing

$$| f>= \int_{-\infty}^{\infty} f(x) \,| x> dx. \tag{8}$$

In fact for any positive even function $f(x)$, but for the $f(x) = \delta(x)$ describing the vacuum, the state defined by Eq. (8) is squeezed. A most important particular case is the Gaussian superposition function [5,6]

$$f(x) = c \,\exp(-x^2/\gamma^2), \quad c = \sqrt{\sqrt{(1+\gamma^2)}/\gamma^2\pi}, \tag{9}$$

describing the usual squeezed vacuum state with uncertainties of the quadratures

$$(\Delta a_1)^2 = (1+\gamma^2)/4, \quad (\Delta a_2)^2 = 1/4(1+\gamma^2). \tag{10}$$

Similar distributions can be constructed not only along the real axis but along any straight line. For example the squeezed coherent state with coherent signal α and squeezing parameter $\varsigma = r\exp(i\theta)$ can be written in the form [7,8]

$$|\alpha,\varsigma> = \int_{-\infty}^{\infty} f(x,\alpha,\varsigma)\,|\,\alpha + \exp(i\theta/2)x > dx, \qquad (11)$$

$$f(x,\alpha,\varsigma) = c\ \exp(-x^2/\gamma^2 - i\delta x), \quad \delta = Im[\alpha^* \exp(i\theta/2)], \quad \gamma = \sqrt{e^{2r}-1}. \qquad (12)$$

As the Gaussian superposition of coherent states of Eq. (8,9) was a useful generalisation of the even states of Eqs. (4,7) analogously one can build an odd state $|\gamma,1>$ resembling Eq. (5)

$$|\gamma,1> = \int_{-\infty}^{\infty} G(x,\gamma,1)\,|\,x > dx, \qquad (13)$$

$$G(x,\gamma,1) = c_1 x \exp(-x^2/\gamma^2), \quad c_1 = \frac{2}{\gamma^3}\sqrt{\sqrt{(1+\gamma^2)^3}/\pi}. \qquad (14)$$

The mean photon number and the uncertainty of the quadrature a_2 in this state are

$$<\gamma,1\,|\,a^\dagger a\,|\,\gamma,1> = 1 + \frac{3\gamma^4}{4(1+\gamma^2)}, \qquad (15)$$

and

$$(\Delta a_2)^2 = \frac{3}{4(1+\gamma^2)}. \qquad (16)$$

We can see that the state $|\gamma,1>$ coincides with the one photon state $|1>$ in the limit $\gamma = 0$ and with increasing γ at $<\gamma,1\,|\,a^\dagger a\,|\,\gamma,1> = 2$, $\gamma = \sqrt{2}$ it becomes squeezed.

Similarlyone can define states $|\gamma,n>$ with x^n instead of x in their weight function $G(x,\gamma,n)$. Superpositions of such states leading to Hermite polynomial weight functions are rather remarkable

$$|h_n> = \int_{-\infty}^{\infty} h_n(x)\,|\,x > dx, \qquad (17)$$

$$h_n(x) = \sqrt{\sqrt{3}/(2\pi n!)}H_n(\frac{\sqrt{3}x}{2})\exp(-x^2/2). \qquad (18)$$

The states $|h_n>$ are orthonormalized ($<h_n\,|\,h_m> = \delta_{nm}$), satisfy the relation

$$a\,|\,h_n> = \sqrt{\frac{n+1}{3}}\,|\,h_{n+1}> + \sqrt{\frac{4n}{3}}\,|\,h_{n-1}>, \qquad (19)$$

and correspondingly

$$< h_n \mid a \mid h_n >= 0, \quad < h_n \mid a^2 \mid h_n >= \frac{2}{3}(2n+1), \quad < h_n \mid a^\dagger a \mid h_n >= \frac{5n+1}{3}. \tag{20}$$

The projection operator constructed from the Hermite states

$$P_h \equiv \sum_{n=0}^{\infty} \mid h_n >< h_n \mid, \tag{21}$$

is a unity operator both for the coherent and photon number states

$$< x \mid P_h \mid y >=< x \mid y >= \exp[-(x-y)^2/2], \tag{22}$$

$$< n \mid P_h \mid m >= \delta_{nm}, \tag{23}$$

which shows that any state can be represented by them. For example one can expand a $\mid f >= \int_{-\infty}^{\infty} f(x) \mid x > dx$ state into the $\mid h_n >$ states

$$\mid f >= \sum_{n=0}^{\infty} f_n \mid h_n >, \tag{24}$$

where

$$f_n = \int \int_{-\infty}^{\infty} f(x) h_n(y) \exp[-(x-y)^2/2] dx dy. \tag{25}$$

3 Representation along a circle

Let us now consider a state emerging from superposition of coherent states with the same amplitude $\mid \alpha \mid = R$ i. e. we choose only those coherent states which lie on the same circle in the α-plane [5].

$$\mid F, R >= \frac{\exp(R^2/2)}{2\pi} \oint F(\phi) \mid R\exp(i\phi) > d\phi. \tag{26}$$

If the radius of the circle is chosen big enough so that Eq. (2) can be replaced by

$$\mid f >= \frac{1}{\pi} \int_{\mid \alpha \mid < R} f(\alpha^*) \exp(- \mid \alpha \mid^2 /2) \mid \alpha > d^2\alpha, \tag{27}$$

then we can find connections between the distribution function $F(\phi)$ and Glauber's weight function $f(\alpha^*)$

$$F(\phi) = \frac{R\exp(i\phi)}{\pi} \int_{\mid \alpha \mid < R} f(\alpha^*) \frac{\exp(- \mid \alpha \mid^2)}{z - \alpha} d^2\alpha, \quad z = R\exp(i\phi) \tag{28}$$

and

$$f(\alpha^*) = \oint F(\phi) exp(\alpha^* z) d\phi. \quad (29)$$

We note that if one knows the time behaviour of the annihilation operator $a(t)$ the analytic expansion function $f(\alpha^*, t)$ can be found from the expression [9]

$$f(\alpha^*, t) = \int d^2\eta \chi(\eta, t) \exp(-|\eta|^2 - \eta\alpha^*) / \sqrt{\pi \int d^2\eta \chi(\eta, t) \exp(-|\eta|^2)}, \quad (30)$$

where $\chi(\eta, t)$ is the normally ordered characteristic function (ρ being the density operator)

$$\chi(\eta, t) = Tr[\rho \exp(\eta a^\dagger(t)) \exp(-\eta^* a(t))]. \quad (31)$$

Using Eq. (28) we find for the n-photon state and the coherent state correspondingly

$$F(\phi, n) = \sqrt{n!} R^{-n} \exp(-in\phi), \quad (32)$$

$$F(\phi, \alpha) = \frac{z \exp(-|\alpha|^2/2)}{z - \alpha}, \quad |\alpha| < R. \quad (33)$$

According to Eq. (32) we can obtain the coefficients of the n-photon representation c_n of Eq. (1) if we know the distribution function $F(\phi)$

$$c_n = R^n F_n / \sqrt{n!}, \quad (34)$$

where F_n are Fourier coefficients of $F(\phi)$

$$F(\phi) = \sum_{n=0}^{\infty} \exp(-in\phi) F_n. \quad (35)$$

An interesting state is the state with Gaussian distribution function $|u>$

$$F(\phi, u) = c_u \exp(-i\delta\phi - \frac{u^2}{2}\phi^2). \quad (36)$$

In case of extremely large u it describes the usual coherent state while in the opposite limit it is the n-photon state ($n = \delta$). Between these states it will be an amplitude squeezed banana state. Graphically it can be understood if one visualizes how with decreasing u the muffin-like coherent state going through a squeezed crescent-like state deforms along the circle into the donut-like number state.

It is also worth mentioning that the Gaussian superposition of coherent states along an arc are not only describe amplitude squeezing [10,11] but they are also approximate number-phase intelligent states [10] associated with the Pegg-Barnett phase operator [12].

Remarkable feature of this state is the complete analogy with the usual quadrature squeezed state discussed in the previous Section, as the Gaussian arc distribution is amplitude squeezed while the Gaussian straight line distribution is quadrature squeezed. Moreover, as the even superposition of two coherent states from Eq. (4) can be derived by truncation from the straight line Gaussian state of Eqs. (5,9) so Schleich's superposition state [13]

$$|\psi>= c_{\alpha\phi}(|\alpha e^{i\phi/2}> + |\alpha e^{-i\phi/2}>), \tag{37}$$

similarly can be considered as a truncated arc Gaussian state of Eqs. (26,36).

A physical example, the so called phonon squeezing [14], where an arc distributed state occurs is the Franck-Condon transition induced by short coherent light pulse in a molecule [5,14,15]. It is worth mentioning that using Eq. (28) one can to some extent purposefully shape the molecular vibrational state by special choice of the characteristics of the exiting light pulse. For example we showed that by appropriate linear chirp the vibrational state can be turned in the α -plane while using nonlinear chirp the amplitude squeezed vibrational state can be deformed into a typical quadrature squeezed form.

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