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NON CLASSICAL EFFECTS IN PLANAR WAVEGUIDES

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Abstract

The quantum description of ligth propagation inside a planar waveguide is given , looking in particular at the behaviour of the field inside a directional coupler . Nonclassical effects are presented and discussed.

Introduction

Electromagnetic fields in optical guided wave systems are usually described simply by using classical Maxwell's equations, but there are cases in which a quantum treatment is necessary.Three purely quantum phenomena are known having no classical analogous; namely photon antibunching, sub-poissonian photon statistics, and squeezing of optical fields. If problems connected with these phenomena or the evolution of photon statistics are to be dealt with,a quantum mechanical treatment must be used.

Of course a linear system is not able to produce or change these properties but a nonlinear one is. For this reason,in the following the Hamiltonian for a nonlinear optical waveguide will be derived,and its application to some propagation problems will be considered.Although the Hamiltonian is quite general,emphasis is given to planar structures only and a more suitable approach to describe propagation fenomena is discussed.

One of the results of having the propagation problem treated in quantum mechanical form is to allow for the possibility of studying how purely quantum effects propagate in linear systems.We will show ,for example,that a quantum effect as squeezing is affected by the operation of switching in a linear structure because of the phase changes involved in the operation.

2. Quantization of the radiation field

The recent experiments on nonclassical states of light have called for a full quantum analysis of the electromagnetic field [1] especially in the cases of propagation of the fields inside dipersive media.

We remember that the standard quantization method consists of writing the Hamiltonian in a given volume V , demanding periodicity in space. For propagating fields, the space evolution is then replaced by a time evolution, by linking the space and time variables by the equation $z= ct$. The length of the nonlinear medium is then replaced by an effective interaction time. Of course this method has two main limitations. The firt one is that , by identifying the space evolution with time evolution we lose one variable and this formalism can describe only c.w. operation : the second problem is that this procedure cannot be applied rigorously to a dispersive medium, where each frequency propagates with different velocity.

However we remember that by using the Hamiltonian formalism and working in the Heisenberg picture , the time evolution of $\hat{E}(z,t)$ operator is given by

$$i\hbar\frac{\partial\hat{E}(z,t)}{\partial t} = [\hat{E}(z,t),\hat{H}] \quad , \tag{1}$$

so that the generator for time evolution is the Hamiltonian $\hat{H}$, while the generator for space propagation is the momentum operator $\hat{G}$:

$$-i\hbar\frac{\partial\hat{E}(z,t)}{\partial z} = [\hat{E}(z,t),\hat{G}] \tag{2}$$

The $\hat{G}$ operator is related to the wave flux of the Poynting vector [2].

Therefore a suitable way to quantize the radiation field to describe the propagation fenomena is the one starting from the flux of the Poynting vector. This leads us to the realization that the important quantity is the flux and not , as usually is assumed with the Hamiltonian formalism , the energy density.

In this way instead of quantizing the field in a large volume and demanding for spatial periodicity , it is necessary to assume a time periodicity T of the field , with the requirement that T must be large with respect to any relevant time . Then instead of writing the field in term of spatial modes (thus performing a Fourier analysis of the space variable z into the wave vector K_m) it is possible to write it in term of temporal modes (thus performing a Fourier analysis of the time variable t into discrete frequencies ω_m) and space dependent operators. The advantage is that the temporal modes remain the same inside and outside the dielectric medium [3] and the space evolution of the mode operator can now be obtained by means of the momentum operator; moreover dispersion of the material can be included.

By the help of this formulation the expression of the electric field is (inside a dielectric)

$$\hat{E}(z,t) = \hat{E}^{+}(z,t) + \hat{E}(z,t),$$

where the cross stands for c.c. and

$$\hat{E}^{+}(z,t) = \sum_{m}\left[\frac{h\,\omega_m}{2\varepsilon_0 c T n(\omega)}\right]^{1/2}\left[\hat{a}(z,\omega_m)\exp(-i\omega_m t)\right] \qquad (3)$$

being $\hat{a}(z,\omega_m)$ and their conjugates form a set of localized creation and the annihilation operators ,ω_m the field frequency , $n(\omega)$ the refractive index at the ω frequency , ε_0 the dielectric constant and c the ligth velocity.

The number operator for the field becomes

$$\hat{N}(z_0,\omega_m) = \hat{a}^{+}(z_0,\omega_m)\,\hat{a}(z_0,\omega_m) \qquad (4)$$

which represents the number operator of the photons of frequency ω_m passing through the plane $z = z_0$ during a period T , and the commutation rules now become commutation at "equal space":

$$[\hat{a}(z,\omega_i),\hat{a}^{+}(z',\omega_j)] = \delta_{i,j}e^{ik_i(z-z')} \qquad (5)$$

and the $\hat{G}$ operator is defined as

$$\hat{G}(z) = \sum_{m}(h\,k_m)\;\hat{a}^{+}(z,\omega_m)\hat{a}(z,\omega_m) \qquad (6)$$

where $k_m = n(\omega_m)\dfrac{\omega_m}{c}$ is the wavevector of the field.

3. Quantum mechanical description of propagation in a planar waveguide

A planar dielectric waveguide is a medium whose dielectric permittivity depends on one direction,parallel to which we shall assume the x-axis (s.Fig.1).

If this medium does not contain absorbing centers,if there is no amplification of radiation,and if the permittivity is weakly dependent on the field frequency ω_i ,the electromagnetic field inside the guide is expressed in terms of normal modes in the following form [4,5]

$$A(r) = \sum_j A_0 f(x) \exp(i \beta_j . r), \tag{6}$$

where β_j is the wave-vector with components y and z ($\beta_j . r = k_y y + k_z z$) of the j-th mode propagating inside the waveguide and f (x) is a function dependent only on x, defined over all space, and determined by the waveguide structure. Therefore each guided mode is defined by a β_j vector at each ω_j frequency.

From the quantum thery point of view (as pointed out in the previous paragraph ,if the operator describing the field mode in a free space is given by (note the operator is the one that obeys at the equal space commutation rules)

$$\hat{a}(z,\omega_i) = \hat{a}_i(z) e^{-i\omega_i t} = \hat{a}_i(0)\, e^{-ik_i z} e^{-i\omega_i t} \tag{7}$$

in a guided strucure it can be described as

$$\hat{a}(z,\omega_i) = \hat{a}_i(z) e^{-i\omega_i t} = \hat{a}_i(0)\, e^{-\beta_i z} e^{-i\omega_i t} \tag{8}$$

and eq. (6) written in operatorial form becomes

$$\hat{A}(r,t) = \sum_j A_0 \left[\hat{a}(z,\omega_j) \exp(-i\omega_j t) + c.c.\right] \tag{9}$$

where A_0 is a constant.

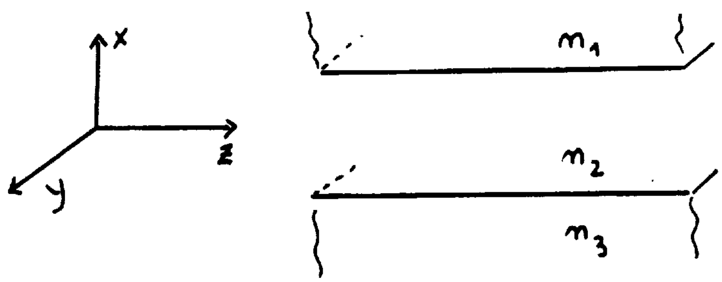

Fig.1 A planar waveguide

We would like to remark that in the case of dispersionless material the time evolution of the field operators(Heisenberg equation) is the same as the one in space , i.e. the Hamiltonian operator and the momentum operator approach provide the same results. This remark is particularly important when we consider the quantum treatmen of a guided mode inside a guiding structure, due to the fact that in general we propagate different spatial modes of equal frequency and we are not obliged to take into account dispersion of the medium if we assume a c.w. propagation.

The same kind of consideration is still valid if we study the propagation of single or more modes inside a nonlinear planar waveguide with third order nonlinearity. These cases have been extensively studied in the paper [5].

In the following we analyze the case of propagation in a directional coupler , which is one of the most interesting guiding devices , very important from the point of view of its switch properties.

4. Behaviour of a linear directional coupler when nonclassical states are involved (squeezing)

The directional linear coupler consists in two adjacent and parallel waveguides [5,6](channels).When radiation goes through the structure,exchange of power between the channels is possible because of the evanescent field which is present in the region between them.

In the frame of classical theory the coupler is studied by using the <u>coupled mode theory</u> [5,6],in which a perturbation polarization responsible for the coupling contains the refractive index of the guides. Complete power transfer occurs in a distance $L = (\pi/2)K$, where K is the coupling constant determined by the refractive indices of the structure; if the detuning parameter δ is zero,that is in the case of complete phase matching [6] ,being

$$\delta = \frac{1}{2}(\beta_a - \beta_b) \tag{10}$$

where β_a and β_b are the wavevectors of two modes of equal frequency propagating in channel a and b respectively .If δ is not zero the maximum fraction of power that can be trasferred is proportional to $\frac{K^2}{K^2+\delta^2}$.

From the classical equations for the complex amplitude for the directional coupler,in the frame of the coupled-mode theory,with obvious generalization we have the following Heisenberg equations for the operators,

$$\begin{cases} d\hat{a}/dz = -iK\hat{b}\exp(i2\delta z), \\ d\hat{b}/dz = -iK\hat{a}\exp(-2i\delta z), \end{cases} \tag{11}$$

where $\hat{a}$ and $\hat{b}$ are the field annihilation operators in channel a and b respectively .For the sake of simplicity we neglect damping terms because we are interested in the coupling effect only;this is a good approximation in the region of low temperature and optical frequencies. Without taking into account dispersion the set of eqs. (11) is the same that we can write startimg from the momentum operator with the substitutions t -> z/c and $\hat{H}$ -> c $\hat{G}$.

In this way we get the following solutions of eqs.(11)

$$\hat{a} = C_a \hat{a}_0 + G_a \hat{b}_0 \qquad \hat{b} = C_b \hat{b}_0 + G_b \hat{a}_0 \tag{12}$$

where $\hat{a}_0$ and $\hat{b}_0$ are the input annihilation operators and

$$C_a = e^{i\delta z}[\cos(\gamma z) - i\,\delta/\gamma \sin(\gamma z)]$$

$$C_b = e^{-i\delta z}[\cos(\gamma z) + i\,\delta/\gamma \sin(\gamma z)]$$

$$G_a = -i\,K/\gamma \sin(\gamma z)\, e^{i\delta z}$$

$$G_b = -i\,K/\gamma \sin(\gamma z)\, e^{-i\delta z}$$

where $\gamma^2 = K^2 + \delta^2$.

To study the propagation of nonclassical field through the structure we use the following characteristic function

$$C_N(\beta) = Tr[\rho \exp[\beta \hat{a}^+]\, \exp[-\beta^* \hat{a}] = \exp[-M|\beta|^2 + ½(S^*\beta^2 + S\beta^{*2}) + \beta W^* - \beta^* W] \tag{13}$$

which is able to describe a field which is not a pure coherent or squeezed state,but has simultaneously squeezed ,coherent and chaotic features [7].In eq.(13) W = W exp(i ϕ) is the coherent signal,and M and S are related to the noncoherent part of the field.So for the vacuum state we have M= S= W= 0 ;in the pure coherent state M=S=0, and for the chaotic field W=S=0.

The state is a pure squeezed state if

$$M= Q= 0.5((4 S + 1) -1)$$

$$S= \exp(i \Phi) \cosh(r) \sinh(r) \quad (14)$$

r being the squeezing parameter [13].

A mixed state is given from a superposition of a pure squeezed state with coherent signal W and a cahotic field described by the normally ordered characteristic function given by eq.(13) if

$$M = Q + N, \quad (15)$$

where N is the noise photon number.

We shall suppose that the input statistics of light in both modes can be described by the normally ordered characteristic function (13).Putting solutions (12) into eq.(13) we can see that the truncated normally ordered output characteri- stic functions will have the same functional form as the input ones with new terms

$$M_a = M_a^{(0)} |C_a|^2 + M_b^{(0)} |G_a|^2$$

$$S_a = S_a^{(0)} C_a^2 + S_b^{(0)} G_a^2$$

$$W_a = W_a^{(0)} C_a + W_b^{(0)} G_a \quad (16)$$

where the superscript (o) labels the input quantities,and similar expressions can be found for the b-mode,by interchanging the subscripts a and b.

We are interested in finding expressions for the variances $<(\Delta Q)^2>$ and $<(\Delta P)^2>$, (where $\hat{Q} = \hat{a} + \hat{a}^+$ and $\hat{P} = -i(\hat{a} - \hat{a}^+)$). In ref.(14) it is shown that

$$<(\Delta Q)^2> = 1 + 2M + S + S^*$$
$$<(\Delta P)^2> = 1 + 2M - S - S^* \qquad (17)$$

Several interesting cases can be considered which depend on the way the coupler is feeded.

Let us suppose first that a pure squeezed state enters channel b and a coherent state (or vacuum) channel a. It can be shown in this case that for $L = \pi/2K$ and $\delta = 0$ we have

$$<(\Delta\hat{Q})^2>_a = <(\Delta\hat{P})^2>_{b0}, \; <(\Delta\hat{P})^2>_a = <(\Delta\hat{Q})^2>_{b0},$$
$$<(\Delta\hat{Q})^2>_a = <(\Delta\hat{P})^2>_b = 1. \qquad (18)$$

This means that at the output of channel a we have an opposite squeezing than at the input of channel b, while the output in channel b shows no squeezing.

A related situation is obtained when two opposite squeezed fields enter the two channels in the same conditions as in the previous case. In this case squeezing is preserved in both channels because the field entering channel a comes out of channel b with opposite squeezing and the same happens with field entering channel b. At intermediate lengths of the coupler the squeezing is not completely preserved.

A very interesting role is played by the detuning parameter. In general [14] if $\delta \neq 0$ some noise is added to both channels and squeezing is reduced, and for some special values of δ, noise is absent. Let us consider for example the case in which $\delta = \sqrt{3}$ K. . In this case $\gamma = 2k$ and for the same coupler length $L = \pi/2k$ if a squeezed field entered channel b and a coherent or vacuum field channel a then we have at the output

$$< (\Delta\hat{Q})^2 >_a = < (\Delta\hat{P})^2 >_a = 1.$$
$$< (\Delta\hat{Q})^2 >_b = < (\Delta\hat{Q})^2 >_{b0}.$$
$$< (\Delta\hat{P})^2 >_b = < (\Delta\hat{P})^2 >_{b0}. \qquad (19)$$

We can see that changing the detuning parameter from zero to $\sqrt{3}K$ switches from one channel to another.This result is rather interesting for the purpose of measurement.The squeezed state is detected by interfering it with a coherent reference light and looking at fluctuations.The switching behaviour just described allow to preserve both the squeezed state and its reference beam.

5 Directional and contradirectional coupler with modes with small different frequency propagating inside

We have studied the problem of propagation of radiation in a coupler assuming two different frequencies inside the channels , with the hypothesis that each channel can support one only guided mode : this is possible if the two frequencies are quite similar. In general for a coupler the more realistic description of the field propagating inside all the structure is the one which takes into account the superposition of the single modes propagating in each channel (so called supermodes); in the case of different frequencies this approach is particularly convenient and it is the one that we have adopted but in its quantum analogous, i.e. introducing this concept in the statistical dependence of the modes supported by the structure.

The motion equation of the operator describing the propagation inside the structure is given by (Heisenberg form)

$$\frac{\partial \hat{a}_j(z)}{\partial z} = \frac{i}{h}[\hat{a}_j(z), \hat{G}(z)] , \tag{20}$$

where j is the index corresponding to the mode (j= 1,2) and this equation is related to the momentum operator $\hat{G}$, that for this case is given by

$$\hat{G}(z) = h\sum_{j=1}^{2} k_j \hat{a}_j^{\dagger}(z)\, \hat{a}_j(z) + h(\chi^{*}\, \hat{a}_1^{+}(z)\, \hat{a}_2(z) + h.c.), \tag{21}$$

where $k_j = \beta_j = \frac{n_{eff}}{\lambda_j}$, is the mode wave vector , χ is the coupling constant , which depends on the refractive index $n(\omega_j)$ distribution inside the coupler . It is very interesting to observe that the $\hat{G}$ operator looks like the one of a second order nonlinearity for a bulk material.

Using the approach of the supermodes we can describe the two fields of different frequency supported by the strucure as

$$|1\rangle = 2^{-1/2}(|1_{in}\rangle + u_{11}|1_{out}\rangle + u_{21}|2_{out}\rangle)$$

$$|2\rangle = 2^{-1/2}(|2_{in}\rangle + u_{12}|1_{out}\rangle + u_{22}|2_{out}\rangle) \tag{22}$$

being $u_{j,k}$ a function related to the transformation law of the coupler , containing all the informations about the structure , such as the coupling constant , the detuning parameter , etc. (see functions $C_{a,b}$ and $G_{a,b}$ of the provious paragraph).

To follow the statistics of the field we start from the characteristic function (the antinormal one) from which it is possible to derive all the factorial moments and the photon counting distribution. As in the previous paragraph we suppose the input state is a superposition of a coherent state and noise , including squeezing .

Due to to the hypothesis of the supermode we can write the charateristic function for all the fields β_1 and β_2, $C_A(\beta_1,\beta_2)$, but we can follow also the behaviour of each separate mode $C_A(\beta_j)$ [8] :

$$C_{A_{in}}(\beta_1,\beta_2) = \exp\Big\{\sum_{j=1}^{2}[-B_{j_{in}}|\beta_j|^2 + \frac{1}{2}(C^*_{j_{in}}\beta_j^2 + c.c.) + (\xi^*_{j_{in}}\beta_j - c.c.) +$$

(noise) (squeezing) (coherent)

$$+ (-B_{12_{in}}\beta_1^*\beta_2 + C^*_{12_{in}}\beta_1\beta_2 + c.c.)]\Big\}.$$

(interference of noise) (23)

The output characteristic function is of the same form as the input, where all the features of the coupler are inside the B and C coefficients of the eq.(23).
Several cases of inputs states have been studied, such as coherent , two-photon coherent , two-mode squeezed states and all factorial moments have been calculated [8] finding as the detuning parametr plays a very important role on the evolution of the fileds : it adds addidional noise if it is non zero [8].
It is interesting also to follow the photon counting distribution which put into evidence the switch properties of the structure always starting from the hypotesis of supermode supported by the coupler . An example is shown in Fig.2 , where the detuning parameter δ is zero, the input state in the first channel is a two photon coherent state in the first mode , with a " small amount of squeezing" ,and a coherent state in the other mode . The picture shows the marginal photon number distribution in the channel 2 ; at a suitably distances the sub-Poissonian behaviour turns super-Poissonian, which characterizes the field in the squeezed vacuum state: this confirms the switching of ligth of certain photon statistics from one mode to the other one.

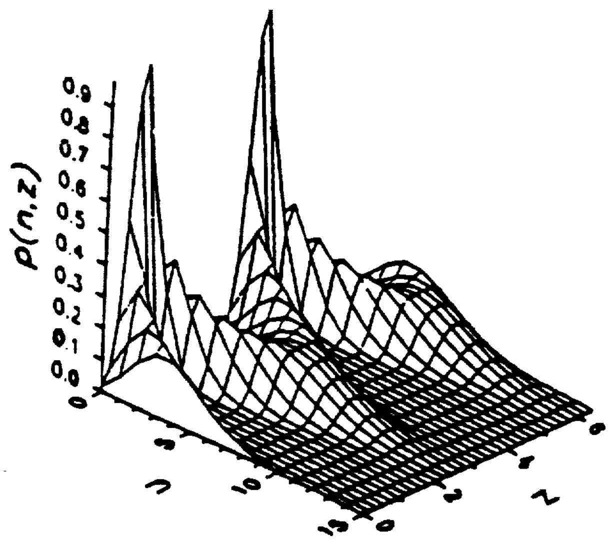

Fig.2 - The marginal photon number distribution for $\delta = 0$ and $K = i$ for the channel 2

Conclusions

The propagation of ligth in a linear directional coupler can be studied without taking into account the dispersion of the dielectric constant until c.w. field propagation is considered; of course dispersion must be taken into account in non stationary cases and when the structure of the propagating device supports different frequencies.

The ability of the coupler to switch from one channel to the other by introducing a phase lag allows to change the squeezing directions, until the δ parameter is of suitably values; in general a detuning different from zero reduces the switch properties of the coupler and adds additional noise to the propagating fields. This effect is in turn evident also on the photon counting distribution.

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