N94-10576

SQUEEZING IN PHASE-CONJUGATED RESONANCE FLUORESCENCE

Henk F. Arnoldus

Department of Physics, Mendel Hall, Villanova University, Villanova, Pennsylvania, USA, FAX: 215 - 645 - 7465, bitnet: arnoldus@vuvaxcom

Emission of resonance fluorescence by an atom near the surface of a four-wave mixing phase conjugator is considered. The dipole radiation field, regarded as a Heisenberg-operator field, is decomposed into plane waves with the aid of Weyl's representation of the Green's function for the wave equation. Each plane-wave component which is incident on the surface of the nonlinear medium, is reflected as its phase-conjugate image. Summation of all reflected plane waves then yields the phase-conjugate replica of the incident dipole radiation. This field adds to the radiation which is emitted by the atom into the direction away from the medium. The condition under which squeezing occurs in the emitted resonance fluorescence is investigated.

I. INTRODUCTION

Squeezing in resonance fluorescence from a two-state atom was first considered by Walls and Zoller.[1] They derived conditions on the optical parameters for which the emitted radiation would exhibit squeezing, and it appeared that only for a very limited range of the parameters squeezing could occur. On the other hand, squeezed states of the free electromagnetic field can be generated through four-wave mixing as two-photon coherent states.[2] In this paper we consider a combination of these two processes: a two-state atom with transition frequency ω_o is close to the surface of a four-wave mixer in the phase conjugation setup. The nonlinear transparent crystal is pumped by two counterpropagating laser beams with frequency ω_p, as shown in Fig. 1. Then, an incident plane wave with frequency ω is reflected as a wave with frequency $2\omega_p - \omega$, and this wave counterpropagates the incident wave. This device will be referred to as a phase conjugator (PC). When an atom in the neighborhood of this PC emits fluorescence, then part of this radiation will be incident on the PC, and will be reflected as its phase-conjugate replica. The total radiation field then is the sum of regular fluorescence, which is emitted directly into the direction of the detector, and the phase-conjugate image of the incident field. In addition, we shall assume that the atom is driven by a laser with frequency ω_L, and this field propagates parallel to the surface of the crystal.

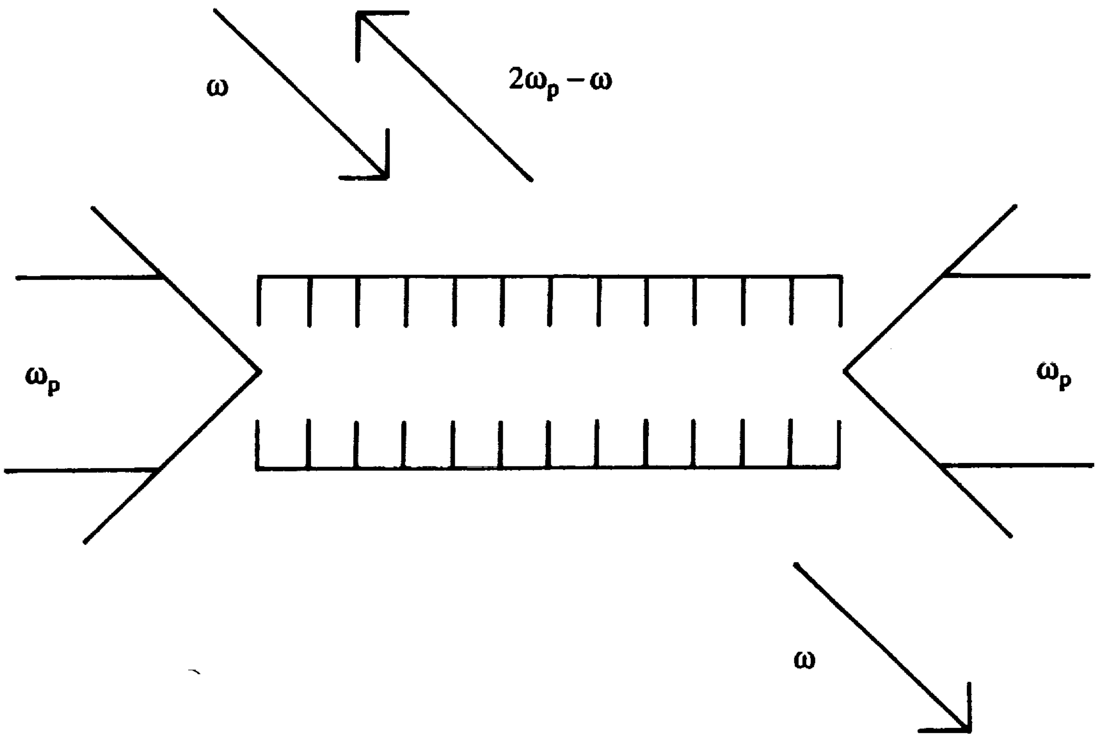

Fig. 1. Geometry of a four-wave mixing phase conjugator.

II. DIPOLE RADIATION

An electric field $\vec{E}(\vec{r},t)$ has a Fourier transform, defined as

$$\hat{\vec{E}}(\vec{r},\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\, \vec{E}(\vec{r},t) \tag{1}$$

In terms of this transform, the positive-frequency part of $\vec{E}(\vec{r},t)$ is defined as

$$\vec{E}(\vec{r},t)^{(+)} = \frac{1}{2\pi}\int_{0}^{\infty} d\omega\, e^{-i\omega t}\, \hat{\vec{E}}(\vec{r},\omega) \tag{2}$$

and the total field can then be written as

$$\vec{E}(\vec{r},t) = \vec{E}(\vec{r},t)^{(+)} + \mathrm{H.c.} \tag{3}$$

Here, the field is a quantum operator field, and the t-dependence signifies the Heisenberg picture.

For a (quantum) dipole $\vec{\mu}(t)$, with Fourier transform $\hat{\vec{\mu}}(\omega)$, which is located at position $\vec{h}$, the Fourier transform of its electric field is given by

$$\hat{\vec{E}}_p(\vec{r},\omega) = \frac{1}{4\pi\varepsilon_o}\{k^2\hat{\vec{\mu}}(\omega) + [\hat{\vec{\mu}}(\omega)\bullet\nabla]\nabla\}\frac{e^{ik|\vec{r}-\vec{h}|}}{|\vec{r}-\vec{h}|} \quad , \tag{4}$$

with $k = \omega / c > 0$. The subscript *p* indicates that this field is the particular solution for a dipole in empty space. We shall assume that the plane z = 0 is the surface of the medium, and that the atomic dipole position vector is given by $\vec{h} = h\vec{e}_z, h > 0$. In order to obtain the field reflected by the PC, we expand the dipole field into plane waves. Then for each wave its phase-conjugate image is a counterpropagating wave, multiplied by the appropriate Fresnel coefficient, and shifted in frequency according to the rule of Fig. 1. The decomposition of the field $\hat{\vec{E}}_p(\vec{r},\omega)$ is accomplished by using Weyl's representation of the Green's function for the scalar wave equation:

$$\frac{e^{ik|\vec{r}-\vec{h}|}}{|\vec{r}-\vec{h}|} = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \frac{1}{\gamma} e^{i\alpha x + i\beta y + i\gamma|z-h|} \quad , \tag{5}$$

where γ is given by

$$\gamma = \begin{cases} \sqrt{k^2 - \alpha^2 - \beta^2} \\ i\sqrt{\alpha^2 + \beta^2 - k^2} \end{cases} \tag{6}$$

It is understood that we take the form for which the argument of the square root is positive. When we substitute (5) into (4) and carry out the ∇ operations, then the result is the desired expansion into plane waves. The polarization of the waves is determined by the dipole operator, and this has to be decomposed into surface- and plane polarization components. The details of this lengthy calculation can be found in Refs. 3 and 4. Furthermore, we have to make an asymptotic expansion in order to find the field in the radiation zone. This was done with the method of stationary phase.[5] Subsequently, the inverse Fourier transform has to be calculated, to obtain the positive-frequency part of the field. The final result for the radiation field, evaluated at the position of a detector, located under an angle θ with the normal to the surface, is

$$\vec{E}(\vec{r},t)^{(+)} = \frac{\omega_o{}^2\, e^{-i\omega_o\tau}}{4\pi\varepsilon_o r c^2}\{\vec{M} - \hat{r}(\hat{r}\bullet\vec{M})\} \tag{7}$$

Here, $\tau = (h/c)\cos\theta$, ω_o is the atomic transition frequency, and the Heisenberg operator $\vec{M}(t)$ is given by

$$\vec{M}(t) = \vec{\mu}(t)^{(+)} - P^* e^{-2i\omega_p t} \vec{\mu}(t)^{(-)} \tag{8}$$

The positive-frequency part of the dipole operator is proportional to the atomic lowering operator, and the negative-frequency part is proportional to the raising operator.

III. DRIVEN ATOM

Now assume that the atom is irradiated by a nearly-monochromatic laser beam, with an electric field of the form

$$\vec{E}_L(t) = E_o \,\mathrm{Re}\, \vec{\varepsilon}_L \, e^{-i(\omega_L t + \phi(t))} \tag{9}$$

The phase $\phi(t)$ is a random process, which accounts for the laser linewidth. We take the phase to be the independent-increment process, leading to a Lorentzian laser lineshape with a width equal to λ. This field couples to the atomic dipole as $-\vec{\mu} \bullet \vec{E}_L$ in the Hamiltonian, giving rise to stimulated transitions between the two levels. The equation of motion for the atomic density operator σ in the rotating frame, and averaged over the stochastic laser phase, can readily be solved. For the matrix elements we obtain:

$$<e|\sigma|e> = \frac{1}{2} \frac{\Omega_o^2 \eta + A P_o^2 (\Delta^2 + \eta^2)}{\Omega_o^2 \eta + A(1 + P_o^2)(\Delta^2 + \eta^2)} \quad , \tag{10}$$

$$<e|\sigma|g> = -\frac{1}{2} \Omega \frac{A(\Delta - i\eta)}{\Omega_o^2 \eta + A(1 + P_o^2)(\Delta^2 + \eta^2)} \tag{11}$$

Here we introduced the notations: $\Delta = \omega_L - \omega_o$, $\eta = \lambda + A(1 + P_o^2)/2$, and $\Omega_o = |\Omega|$, with Ω the (complex) Rabi frequency of the transition, A the Einstein coefficient for spontaneous decay, and P_o the absolute value of the Fresnel reflection coefficient.

IV. DEFINITION OF SQUEEZING

The electric field of the emitted radiation is given by Eq. (7). The slowly-varying amplitude of the resonance fluorescence, with respect to the incident field, is given by[6]

$$E_\alpha(t) = E(t)^{(+)} e^{i(\omega_L t + \phi(t) - \alpha)} + \mathrm{H.c.} \tag{12}$$

with $E(t)^{(+)}$ the projection of the field from Eq. (7) onto a fixed polarization direction. Angle α can be varied in an experiment. For $\alpha = 0$ or $\alpha = \pi / 2$ this corresponds to the in-phase and out-of-phase quadrature component of the field, respectively. The Heisenberg uncertainty relation for quadrature fields with different values of α is

$$\Delta E_\alpha(t) \Delta E_{\alpha'}(t) \geq \frac{1}{2} | < [E_\alpha(t), E_{\alpha'}(t)] > | \quad , \tag{13}$$

and with Eq. (12) this becomes

$$(\Delta E_\alpha(t))^2 (\Delta E_{\alpha'}(t))^2 \geq < [E(t)^{(+)}, E(t)^{(-)}] >^2 \sin^2(\alpha - \alpha') \quad . \tag{14}$$

Then we define the field $E_\alpha(t)$ squeezed, if

$$(\Delta E_\alpha(t))^2 < | < [E(t)^{(+)}, E(t)^{(-)}] > | \quad , \tag{15}$$

holds. From Eq. (14) it follows that when $E_\alpha(t)$ is squeezed for a certain value of α, then the quadrature component of the field which is 90^o out of phase with this $E_\alpha(t)$ must have enhanced fluctuations.

As a measure for the amount of squeezing we introduce the normalized quantity

$$s = \frac{(\Delta E_\alpha)^2 - | < [E(t)^{(+)}, E(t)^{(-)}] > |}{< E_\alpha^2 >} \quad , \tag{16}$$

so that squeezing occurs under condition

$$s < 0 \tag{17}$$

V. CONDITION FOR SQUEEZING

The squeezing parameter s can readily be evaluated, given the solution for the atomic density operator σ. It appears that parameter α can be chosen, such that it minimizes s, but this choice depends in a complicated way on the phase of the atomic transition dipole moment, the phase of the Rabi frequency, and the normal distance between the atom and the surface of the medium.[7] For this optimum value of α, parameter s is found to be

$$s = 1 - \frac{A(\Delta^2 + \eta^2)}{(1 + P_o^2)[\Omega_o^2 \eta + A(1 + P_o^2)(\Delta^2 + \eta^2)]^2}$$

$$\times\left[\Omega_o^2(A+|1-P_o^2|\eta)+A|1-P_o^4|(\Delta^2+\eta^2)\right] \tag{18}$$

Therefore, squeezing occurs when the following condition on the optical parameters holds:

$$\begin{aligned}&(1+P_o^2)[\Omega_o^2\eta+A(1+P_o^2)(\Delta^2+\eta^2)]^2<\\&\quad<A(\Delta^2+\eta^2)\left[\Omega_o^2(A+|1-P_o^2|\eta)+A|1-P_o^4|(\Delta^2+\eta^2)\right]\end{aligned} \tag{19}$$

If we set $P_o^2=0$ in Eq. (19), then we recover the result for a free atom.[8] When we set $\Omega_o^2=0$, which corresponds to the case without the driving laser, then it is easy to verify that in this situation squeezing never occurs. Figure 2 shows the region were squeezing occurs, as a function of the laser power and the phase-conjugate reflectivity, and for zero detuning Δ and laser linewidth λ.

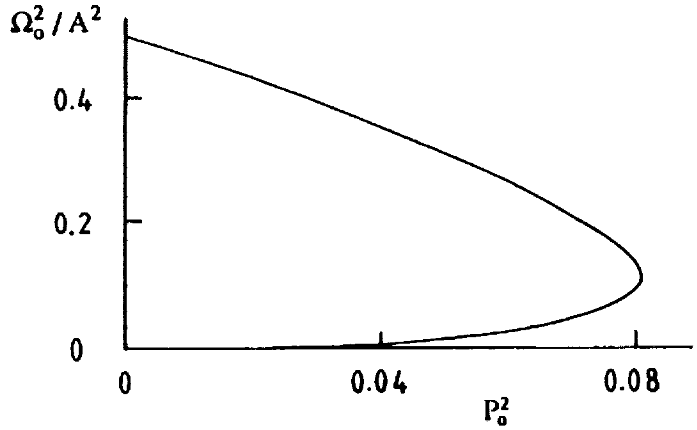

Fig. 2. Squeezing occurs when the reflectivity and the laser power are such that the corresponding point in this plane is within the loop.

REFERENCES

1. D. F. Walls and P. Zoller, *Phys. Rev. Lett.* **47**, 709 (1981).
2. H. P. Yuen and J. H. Shapiro, *Opt. Lett.* **4**, 334 (1979).
3. H. F. Arnoldus and T. F. George, *Phys. Rev. A* **43**, 3675 (1991).
4. H. F. Arnoldus and T. F. George, *Phys. Rev. A* **43**, 6156 (1991).
5. M. Born and E. Wolf, Principles of Optics (Pergamon, Oxford, 1980), p. 752.
6. M. J. Collett, D. F. Walls and P. Zoller, *Opt. Commun.* **52**, 145 (1984).
7. H. F. Arnoldus and T. F. George, *Phys. Rev. A* **46** (1992), in press.
8. H. F. Arnoldus and G. Nienhuis, *Optica Acta* **30**, 1573 (1983).

II. UNCERTAINTY RELATIONS