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DISSIPATION IN A SQUEEZED-STATE ENVIRONMENT

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Abstract

The problem of a quantum particle coupled to a quantum-mechanical heat bath has a broad and general description in terms of a generalized quantum Langevin equation, as described in a series of papers by Ford, Lewis and O'Connell. Here we show how a squeezed-state environment may be incorporated in this general framework.

1 INTRODUCTION

In a paper entitled "Quantum Langevin Equation", Ford, Lewis and O'Connell [1] gave a broad and general description, in terms of a generalized quantum Langevin equation (GLE), of a quantum particle, moving in an arbitrarily external potential and coupled to a quantum-mechanical heat bath. Related papers included an extension incorporating the presence of an external time-dependent field [2]. In Ref. 1, we presented the general form of this equation consistent with fundamental physical requirements, in particular causality and the second law of thermodynamics. Next, we discussed an independent-oscillator (IO) model of the heat bath and we showed that, in addition to being a simple and convenient model with which to calculate, the most general GLE can be realized with an IO model. In addition, the IO model incorporates many other models that have appeared in the literature, in particular the blackbody radiation heat bath.

In the IO model, the quantum particle is surrounded by an infinitely large number of heat-bath particles, each attached to it by a spring. In Ref .1, the heat-bath is taken to be at temperature T. Here, we assume that the modes of the bath are squeezed and our purpose is to outline what aspects of Ref. 1 need to be modified as a result. As it turns out, the only changes occur in expressions involving ensemble averages, specifically the autocorrelations of the random (noise) force F(t) and the oscillator position x(t).

2 DISCUSSION

As before, the Hamiltonian of the IO system is

$$H = \frac{p^2}{2m} + V(x) + \sum_j \left(\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 (q_j - x)^2 \right). \tag{1}$$

Here m is the mass of the quantum particle while m_j and ω_j refer to the mass and oscillator frequency of heat-bath oscillator j. In addition, x and p are the coordinate and momentum operators for the quantum particle and q_j and p_j are the corresponding quantities for the heat-bath oscillators. Also, V(x) is a one-dimensional potential (but generalization to three dimensions is straightforward[1]). Use of the Heisenberg equations of motion lead to the GLE describing the time development of the particle motion:

$$m\ddot{x} + \int_{-\infty}^{t} dt' \mu(t - t')\dot{x}(t') + V'(x) = F(t), \tag{2}$$

where the dot and prime denote, respectively, the derivative with respect to t and x. In addition, $\mu(t)$ is the memory function:

$$\mu(t) = \sum_j m_j \omega_j^2 \cos(\omega_j t)\theta(t), \tag{3}$$

where $\theta(t)$ is the Heaviside step function. Also

$$F(t) = \sum_j m_j \omega_j^2 q_j^h(t) \tag{4}$$

is the random (fluctuation) force, where $q_j^h(t)$ denotes the general solution of the homogeneous equation (corresponding to no interaction). In Ref. 1, to find the expression for the (symmetric) autocorrelation of F(t), we assumed that in the distant past the oscillators are in equilibrium at temperature T and with respect to the heat-bath Hamiltonian. This led to the result

$$\begin{aligned}&\frac{1}{2}\langle F(t)\, F(t') + F(t')\, F(t)\rangle \\ &= \frac{1}{\pi}\int_0^\infty d\omega\, \mathrm{Re}\big[\tilde{\mu}(\omega + i0^+)\big]\hbar\omega \\ &\times \coth(\hbar\omega/2kT)\cos[\omega(t - t')],\end{aligned} \tag{5}$$

where $\tilde{\mu}(\omega)$ is the Fourier transform of the memory function $\mu(t)$. To get the corresponding result in the case of a squeezed bath, we essentially have to generalize the expressions for $<q_j\, q_k>$ etc. appearing in Eq.(4.12) of Ref. 1. To this end, it is convenient to use the familiar oscillator operators a, a^+ and a_j, a_j^+. As a result, using the procedure of Ref. 1, we obtain

$$\begin{aligned}&\frac{1}{2} <F(t)\, F(t') + F(t')\, F(t)> \\ &= \sum_j \hbar m_j\, \omega_j^3\{(<a_j^+\, a_j> + 1/2)\cos\omega_j(t - t') \\ &\qquad + \mathrm{Re} <a_j\, a_j> \cos\omega_j\,(t + t') \\ &\qquad + \mathrm{Im} <a_j\, a_j> \sin\omega_j\,(t - t')\}\end{aligned}$$

$$\begin{aligned}&= \frac{2}{\pi}\int_0^\infty d\omega\, \mathrm{Re}\,\tilde{\mu}(\omega)\,\hbar\omega\,\{<a^+(\omega)\, a(\omega) + \frac{1}{2}>)\cos\omega(t - t') \\ &\qquad + \mathrm{Re} <a(\omega)\, a(\omega)> \cos\omega\,(t + t') \\ &\qquad + \mathrm{Im} <a(\omega)\, a(\omega)> \sin\omega\,(t + t')\},\end{aligned} \tag{6}$$

where the second equality follows from the use of the expession for Re $\tilde{\mu}(\omega)$ given by Eq. (4.16) of Ref. 1.

In the particular case of the bath being in a thermal state, at temperature T, the last two terms on the right-side of Eq.(6) are zero and Eq.(6) reduces to Eq.(5). In the case of a squeezed bath, all of the terms in Eq.(6) are non-zero and detailed expressions for the various quantities may be found, for example, in the work of Gardiner et al. [3].

As in the case of a thermal bath, the result for the symmetric position autocorrelation viz. $1/2 <x(t)\, x(t') + x(t')\, x(t)>$ is given by the right-side of Eq.(6) except that the integrand has an additional factor $|\alpha(\omega)|^2$, where $\alpha(\omega)$ is the generalized susceptibility. Such a relation is, in essence, a generalization of the fluctuation-dissipation theorem to the case of a non-thermal bath.

In conclusion, the results of Refs. 1 and 2, supplemented by Eq.(6) of the present paper, provide a general framework for discussing the problem of a quantum particle in a heat-bath whose modes are squeezed.

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