N94-10591

SUPERSQUEEZED STATES FROM SQUEEZED STATES

Michael Martin Nieto[1]
Theoretical Division, Los Alamos National Laboratory
University of California
Los Alamos, New Mexico 87545, U.S.A.

Abstract

Using super-Baker-Campbell-Hausdorff relations on the elements of the supergroup OSP(2/2), we derive the supersqueeze operator and the supersqueezed states, which are the supersymmetric generalization of the squeezed states of the harmonic oscillator.

1 Introduction

The concept of supersymmetry became of wide interest to physicists because of attempts to obtain a grand unified theory of the fundamental interactions. In particular, such supersymmetric theories predict that there are fermion partners to fundamental bosons, and *vice versa*. However, searches for this fundamental supersymmetry have so far proven fruitless, and something of a, "Trust me, we'll find it at the next accelerator"-attitude has emerged.

On the other hand, phenomenological manifestations of supersymmetry have been found at low energies, e.g., in the contexts of nuclear physics [1], atomic physics [2], and WKB-theory [3]. The supersymmetry and atomic physics interests [2, 4] of Alan Kostelecký, Rod Truax, and myself, combined with our interest in coherent states [5, 6], led us to develop super-BCH relations [7, 8] as a precursor to deriving a complete supercoherent states formalism. With Alan's graduate student, Beata Fatyga [9], we gave supercoherent states for three distinct systems: (i) the super Heisenberg-Weyl algebra, which defines the supersymmetric harmonic oscillator; (ii) an electron in a constant magnetic field, which is a supersymmetric quantum-mechanical system with a Heisenberg-Weyl algebra plus another bosonic degree of freedom, and (iii) the electron-monopole system, which has an OSP(1/2) supersymmetry. (I also want to mention that Alan, Rod, and I have joined forces with Man'ko to obtain time-dependent supercoherent states [10].)

At the first International Workshop on Squeezed States [11], Alan reported on our supercoherent states [12]. In the question and answer session of Alan's talk, he was asked if we were trying to extend our results to the supersqueezed states of the harmonic oscillator. (Honest! That was not a set-up question.) Alan replied that we were, but that it was a harder problem. (If that response had come from me, instead of Alan, you might now suspect that it was a set-up answer.) Anyway, having committed ourselves, we hoped to do it before this Second International Workshop on Squeezed States. And we did–by the skins of our teeth. The last calculation (although not the last check) was finished on May 18.

[1]Email: mmn@pion.lanl.gov

In Sec. 2, I will give a quick review of coherent states and squeezed states. (See, also, Ref. [5]). Then, I go on to show how, given the superdisplacement operator for coherent states [9, 12], one can obtain supersqueezed states if one can first obtain the supersqueeze operator. This supersqueeze operator is derived in the following section. I conclude with a description of the supersqueezed states. Further details and results will appear elsewhere [13].

2 Coherent states and squeezed states

In the Schrödinger formalism, those states which minimize the $x - p$ uncertainty relation are

$$\psi(x) = [2\pi\sigma^2]^{-1/4} \exp\left[-\left(\frac{x - x_0}{2\sigma}\right)^2 + ip_0 x\right], \tag{1}$$

$$\sigma = \mathcal{S}\sigma_0 = \mathcal{S}/[2m\omega]^{1/2}. \tag{2}$$

When $\mathcal{S} = 1$, these Gaussians have the width of the ground state of the harmonic oscillator with natural frequency $\nu = \omega/(2\pi)$, and are the coherent states. When $\mathcal{S} \neq 1$, they are the "squeezed states" of the harmonic oscillator. Their uncertainty product evolves with time as

$$[\Delta x(t)]^2[\Delta p(t)]^2 = \frac{1}{4}\left[1 + \frac{1}{4}\left(\mathcal{S}^2 - \frac{1}{\mathcal{S}^2}\right)^2 \sin^2(2\omega t)\right]. \tag{3}$$

In the (displacement) operator formalism, the coherent states are given by

$$D(\alpha)|0\rangle = \exp[\alpha a^\dagger - \alpha^* a]|0\rangle = \exp\left[-\frac{1}{2}|\alpha|^2\right]\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle \equiv |\alpha\rangle, \tag{4}$$

where $|n\rangle$ are the number states. The displacement operator,$D(\alpha)$, is the unitary exponentiation of the elements of the factor algebra, spanned by a and $a^\dagger$:

$$D(\alpha) = \exp[\alpha a^\dagger - \alpha^* a] = \exp\left[-\frac{1}{2}|\alpha|^2\right]\exp[\alpha a^\dagger]\exp[-\alpha^* a], \tag{5}$$

where the last equality comes from using a BCH relation. With the identifications $Re(\alpha) = [m\omega/2]^{1/2}x_0$ and $Im(\alpha) = p_0/[2m\omega]^{1/2}$, these are the same as the minimum-uncertainty coherent states, up to an irrelevant phase factor.

Obtaining the squeezed states from the displacement operator coherent states is more complicated than from the minimum-uncertainty coherent states. One starts with the "unitary squeeze operator"

$$\begin{aligned} S(z) &= \exp\left[z\frac{a^\dagger a^\dagger}{2} - z^*\frac{aa}{2}\right] && (6)\\ &\equiv \exp\left[G_+\frac{a^\dagger a^\dagger}{2}\right]\exp\left[G_0\frac{\left(a^\dagger a + \frac{1}{2}\right)}{2}\right]\exp\left[G_-\frac{aa}{2}\right] && (7)\\ &= \exp\left[e^{i\phi}(\tanh r)\frac{a^\dagger a^\dagger}{2}\right]\left(\frac{1}{\cosh r}\right)^{(\frac{1}{2}+a^\dagger a)}\exp\left[-e^{-i\phi}(\tanh r)\frac{aa}{2}\right], && (8)\end{aligned}$$

where $z \equiv re^{i\phi}$ and Eq. (8) is obtained from a BCH relation. A normal-ordered form for the second term in Eq. (7) is

$$\left(\frac{1}{\cosh r}\right)^{(\frac{1}{2}+a^\dagger a)} = \left(\frac{1}{\cosh r}\right)^{\frac{1}{2}} \left[\sum_{n=0}^{\infty} \frac{(\operatorname{sech} r - 1)^n}{n!}(a^\dagger)^n(a)^n\right]. \tag{9}$$

Note that $S(z)$ by itself can be considered to be the displacement operator for the group SU(1,1) defined by

$$K_+ = \frac{1}{2}a^\dagger a^\dagger, \qquad K_- = \frac{1}{2}aa, \qquad K_0 = \frac{1}{2}(a^\dagger a + \frac{1}{2}). \tag{10}$$

The squeezed states equivalent to the ψ of Eqs. (1-2) are obtained by operating on the ground state by

$$T(\alpha, z)|0\rangle = D(\alpha)S(z)|0\rangle \equiv |(\alpha, z)\rangle, \tag{11}$$

$$z \equiv re^{i\phi}, \qquad r = \ln \mathcal{S}. \tag{12}$$

[ϕ is a phase which defines the starting time, $t_0 = (\phi/2\omega)$, and $\mathcal{S}$ is the wave-function squeeze of Eq. (2).]

Although the operator method appears, at first sight, to be more complicated, it has a distinct advantage when one wants to consider supersymmetry. The operator method has a direct supersymmetric generalization. The mathematics is clear, and so one does not have to solve the problem of how to include the fermionic sector in the wave-functon formalism. That answer will come out in the end.

3 How to obtain supersqueezed states

Recently, we used the operator method to find supercoherent states [9]. Among the examples in this study, the supercoherent states of the harmonic oscillator were obtained. From the super Heisenberg-Weyl algebra defined by

$$[a, a^\dagger] = I, \quad \{b, b^\dagger\} = I, \tag{13}$$

the superdisplacement operator was obtained:

$$\begin{aligned} \mathbf{D}(A,\theta) &= \exp[Aa^\dagger - \overline{A}a + \theta b^\dagger + \overline{\theta}b] &(14)\\ &= \left(\exp[-\frac{1}{2}|A|^2]\exp[Aa^\dagger]\exp[-\overline{A}a]\right)\left(\exp[-\frac{1}{2}\overline{\theta}\theta]\exp[\theta b^\dagger]\exp[\overline{\theta}b]\right). &(15)\end{aligned}$$

θ and $\overline{\theta}$ are odd Grassmann numbers. They are nilpotent and they satisfy anticommutation relations among themselves and with the fermion operators b and $b^\dagger$. A and $\overline{A}$ are complex, even, Grassmann numbers. Explicit calculation yields

$$\mathbf{D}(A,\theta)|0,0\rangle = [1 - (1/2)\overline{\theta}\theta]|A,0\rangle + \theta|A,1\rangle. \tag{16}$$

The two labels of $|0,0\rangle$ in Eq. (16) represent the even (bosonic) and odd (fermionic) spaces. The bosonic space contains an ordinary coherent state $|A\rangle$ and the fermionic space has zero or one fermions. (See Ref. [9] for further details.)

From the above it is clear that the supersymmetric generalization of the SU(1,1) squeeze operator of Eqs. (6-8) is what is needed to obtain the supersqueeze operator and, hence, the supersqueezed states. The group involved is the supergroup OSP(2/2). In addition to the su(1,1) algebra elements of Eq. (10), it has five more:

$$\begin{aligned} M_0 &= \frac{1}{2}(b^\dagger b - \frac{1}{2}), \\ Q_1 &= \frac{1}{2}a^\dagger b^\dagger, \qquad Q_2 = \frac{1}{2}ab, \qquad Q_3 = \frac{1}{2}a^\dagger b, \qquad Q_4 = \frac{1}{2}ab^\dagger. \end{aligned} \tag{17}$$

4 The supersqueeze operator

To obtain the supersqueeze operator as a product, one solves the t-dependent equation

$$\begin{aligned} \mathbf{S}(Z,\theta_j,t) &= \exp[t(ZK_+ - \overline{Z}K_- + \theta_1 Q_1 + \overline{\theta_1}Q_2 + \overline{\theta_2}Q_3 + \theta_2 Q_4)] \\ &= e^{\gamma_+ K_+} e^{\gamma_0 K_0} e^{\gamma_- K_-} e^{\beta_1 Q_1} e^{\mu M_0} e^{\beta_4 Q_4} e^{\beta_3 Q_3} e^{\beta_2 Q_2} \\ &\equiv \mathbf{S}_1(\mu,\gamma_i,\beta_k,t). \end{aligned} \tag{18}$$

By construction, μ, the γ_i, and the β_k's are functions of t. Thus, taking the derivative of Eq. (18) with respect to t and then multiplying on the right by $\mathbf{S}^{-1}$ yields

$$\left[\frac{d}{dt}\mathbf{S}\right]\mathbf{S}^{-1} = \left[\frac{d}{dt}\mathbf{S}_1\right]\mathbf{S}_1{}^{-1}. \tag{19}$$

This can explicitly be written as ("dot" signifies $\frac{d}{dt}$)

$$\begin{aligned} [ZK_+ - \overline{Z}K_- &+ \theta_1 Q_1 + \overline{\theta}_1 Q_2 + \overline{\theta}_2 Q_3 + \theta_2 Q_4] \\ &= \dot{\gamma}_+ K_+ \\ &\quad + [e^{\gamma_+ K_+}]\dot{\gamma}_0 K_0 [e^{-\gamma_+ K_+}] \\ &\quad + [e^{\gamma_+ K_+} e^{\gamma_0 K_0}]\dot{\gamma}_- K_- [e^{-\gamma_0 K_0} e^{-\gamma_+ K_+}] \\ &\quad + S_B \dot{\beta}_1 Q_1 S_B^{-1} \\ &\quad + S_B [e^{\beta_1 Q_1}]\dot{\mu} M_0 [e^{-\beta_1 Q_1}] S_B^{-1} \\ &\quad + S_B [e^{\beta_1 Q_1} e^{\mu M_0}]\dot{\beta}_4 Q_4 [e^{-\mu M_0} e^{-\beta_1 Q_1}] S_B^{-1} \\ &\quad + S_B [e^{\beta_1 Q_1} e^{\mu M_0} e^{\beta_4 Q_4}]\dot{\beta}_3 Q_3 [e^{-\beta_4 Q_4} e^{-\mu M_0} e^{-\beta_1 Q_1}] S_B^{-1} \\ &\quad + S_B [e^{\beta_1 Q_1} e^{\mu M_0} e^{\beta_4 Q_4} e^{\beta_3 Q_3}]\dot{\beta}_2 Q_2 [e^{-\beta_3 Q_3} e^{-\beta_4 Q_4} e^{-\mu M_0} e^{-\beta_1 Q_1}] S_B^{-1}, \end{aligned} \tag{20}$$

where

$$S_B = [e^{\gamma_+ K_+} e^{\gamma_0 K_0} e^{\gamma_- K_-}]. \tag{21}$$

Note that S_B is the form of the ordinary squeeze operator defined in Eq. (7).

All the terms on the right hand side of Eq. (20) can be written in nonexponential form by using super-BCH formulas and the graded commutation relations. When this is done, there are really eight equations, one for each of the factors multiplying the eight elements of the algebra osp(2/2); i.e., an equation for each of the factors multiplying K_+, K_0, etc. With some algebra, each of the eight equations can be changed to a set of equations having only one time-differential in each.

These eight equations can actually be solved as twenty separate, coupled, differential equations, of simpler form. This is because the four even group parameters $\{\mu, \gamma_+, \gamma_0, \gamma_-\}$ can each be written as having three terms, containing products of zero, two, or four of the θ_j, respectively, and the four odd group parameters $\{\beta_k\}$ can be written as having two terms, containing products of one or three of the θ_j, respectively. (We wll use a presubscript to denote this; e.g., $\beta_1 = ({}_1\beta_1) + ({}_3\beta_1)$.) One takes the eight equations and expands all of the expressions in powers of the θ_j. The order-zero, -two, and -four pieces of the even equations are separated and, similarly, the order-one and -three pieces of the odd equations are separated. One places the lower-order solutions into the higher-order equations. (Note that the boundary conditions needed are that the solutions must all be zero when $t = 0$. Then the supersqueeze operator will be obtained when we set $t = 1$.)

One can do this in a well-defined manner. In particular, the solutions shown below were obtained by finding, in order: $({}_0\mu)$, $({}_0\gamma_+)$, $({}_0\gamma_0)$, $({}_0\gamma_-)$, $({}_1\beta_1)$, $({}_1\beta_2)$, $({}_1\beta_3)$, $({}_1\beta_4)$, $({}_2\mu)$, $({}_2\gamma_+)$, $({}_2\gamma_0)$, $({}_2\gamma_-)$, $({}_3\beta_1)$, $({}_3\beta_2)$, $({}_3\beta_3)$, $({}_3\beta_4)$, $({}_4\mu)$, $({}_4\gamma_+)$, $({}_4\gamma_0)$ and $({}_4\gamma_-)$.

In the solutions we will use the suggestive notation

$$r \equiv [Z\overline{Z}]^{1/2}, \qquad\qquad e^{i\phi} \equiv [Z/\overline{Z}]^{1/2}, \tag{22}$$

where r and $e^{i\phi}$ are now understood to represent Grassmann-valued quantities. Then, one can make the replacements

$$Z \rightarrow re^{i\phi}, \qquad\qquad \overline{Z} \rightarrow re^{-i\phi}. \tag{23}$$

Some care is needed because the quantity $e^{i\phi}$ is strictly defined only for $|\overline{Z}| \neq 0$ and $\overline{z} \neq 0$, where $\overline{z}$ is the body of $\overline{Z}$. However, the solutions given below are not affected by this. Even so, the physical meaning of Grassmann numbers remains an open question [14].

We also define

$$c \equiv \cosh y, \qquad s \equiv \sinh y, \qquad y \equiv rt, \tag{24}$$

$$\Phi \equiv \overline{\theta_2}\theta_2\overline{\theta_1}\theta_1 = \overline{\theta_2}\overline{\theta_1}\theta_1\theta_2. \tag{25}$$

With this, the complete solutions to the group parameters are:

$$\begin{aligned}
\mu &= 0 \\
&+\frac{1}{2r^2}\{[\overline{\theta}_1\theta_1 - \overline{\theta}_2\theta_2](c-1) + [\overline{\theta}_2\theta_1 e^{-i\phi} - \overline{\theta}_1\theta_2 e^{i\phi}](s-y)\} \\
&+\frac{\Phi}{r^4}\left[c - 1 - \frac{1}{2}sy\right],
\end{aligned} \tag{26}$$

$$\begin{aligned}
\gamma_+ &= \left[e^{i\phi}\frac{s}{c}\right] \\
&-\frac{e^{i\phi}}{4r^2c^2}[\overline{\theta}_1\theta_1(sc-y) + e^{i\phi}\overline{\theta}_1\theta_2(c-1)^2 \\
&\qquad + e^{-i\phi}\overline{\theta}_2\theta_1 s^2 + \overline{\theta}_2\theta_2(sc+y-2s)] \\
&+\frac{\Phi e^{i\phi}}{8r^4c^3}\left[(2y+sy^2-s) + c(\frac{11}{8}y - 2s) + \left(-\frac{5}{8}sc^2 + \frac{1}{4}sc^4\right)\right],
\end{aligned} \tag{27}$$

$$\gamma_0 = [-2\ln c]$$

$$
\begin{aligned}
&+\frac{1}{2r^2}[\overline{\theta}_1\theta_1(\frac{-ys}{c}+c-1)+e^{i\phi}\overline{\theta}_1\theta_2(\frac{s}{c}-s)\\
&\qquad+e^{-i\phi}\overline{\theta}_2\theta_1(-\frac{s}{c}+s)+\overline{\theta}_2\theta_2(\frac{2+ys}{c}-c-1)]\\
&+\frac{\Phi}{8r^4c^2}[(y^2-1-2ys)-c(\frac{11}{4}ys+4)\\
&\qquad+c^2(2\ln c+8c-3-4ys-\frac{1}{4}s^2)],
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\gamma_- &= \left[-e^{-i\phi}\frac{s}{c}\right]\\
&\quad+(\frac{e^{-i\phi}}{4r^2c^2})[\overline{\theta}_1\theta_1(sc-y)-e^{i\phi}\overline{\theta}_1\theta_2 s^2\\
&\qquad-e^{-i\phi}\overline{\theta}_2\theta_1(c-1)^2+\overline{\theta}_2\theta_2(sc+y-2s)]\\
&\quad-\frac{\Phi e^{-i\phi}}{8r^4c^3}\left[(2y+sy^2-s)+c(\frac{11}{8}y-2s)+\left(sc^2(\frac{15}{8}+2\ln c)-\frac{9}{4}c^3y\right)\right],
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
\beta_1 &= \frac{1}{r}[s\theta_1+(c-1)e^{i\phi}\theta_2]\\
&\quad+\frac{1}{4r^3}[\overline{\theta}_2\theta_1\theta_2(y-2cs+yc)+e^{i\phi}\overline{\theta}_1\theta_1\theta_2(2c(1-c)+ys)],
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\beta_2 &= \frac{1}{r}[s\overline{\theta}_1+(c-1)e^{-i\phi}\overline{\theta}_2]\\
&\quad+\frac{1}{4r^3}[\overline{\theta}_2\theta_1\theta_2(yc-s+\frac{1}{2}(sc-y))+\overline{\theta}_2\overline{\theta}_1\theta_1e^{-i\phi}(ys-3(c-1)-\frac{1}{2}s^2)],
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
\beta_3 &= \frac{1}{r}[(c-1)e^{i\phi}\overline{\theta}_1+s\overline{\theta}_2]\\
&\quad+\frac{1}{4r^3}[e^{i\phi}\overline{\theta}_2\overline{\theta}_1\theta_1 2(ys-2(c-1))+\overline{\theta}_2\overline{\theta}_1\theta_1 2(yc-s)],
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
\beta_4 &= \frac{1}{r}[(c-1)e^{-i\phi}\theta_1+s\theta_2]\\
&\quad+\frac{1}{4r^3}[e^{-i\phi}\overline{\theta}_2\theta_1\theta_2(-4c^2+4c+2ys)+\overline{\theta}_1\theta_1\theta_2(-4sc+2s+2yc)].
\end{aligned}
\tag{33}
$$

Setting $t=1$ yields the general supersqueeze group parameters.

5 The supersqueezed states

Then, using the above group parameters, the graded commutation relations among the generators, and the properties of Grassmann algebra, the supersqueezed states can be found to be

$$
\begin{aligned}
\mathbf{T}(A,\theta,Z,\theta_i)|0,0\rangle &= \mathbf{D}(A,\theta)\mathbf{S}(Z,\theta_i)|0,0\rangle = |A,\theta;Z,\theta_i\rangle\\
&= \hat{\mu}\Gamma_- h_1(a^\dagger)\left[(1-\frac{1}{2}\overline{\theta}\theta)\,|(A,Z),0\rangle+\theta\,|(A,Z),1\rangle\right]\\
&\quad+\hat{\mu}\Gamma_+\frac{\beta_1}{2}h_2(a^+)\left[\overline{\theta}\,|(A,Z),0\rangle+(1+\frac{1}{2}\overline{\theta}\theta)\,|(A,Z),1\rangle\right],
\end{aligned}
\tag{34}
$$

where

$$\hat{\mu} = 1 - \frac{1}{4}[({}_2\mu) + ({}_4\mu)] + \frac{1}{32}({}_2\mu)^2, \tag{35}$$

$$\Gamma_\pm = 1 + \frac{(2 \pm 1)}{4}[({}_2\gamma_0) + ({}_4\gamma_0)] + \frac{(2 \pm 1)^2}{32}({}_2\gamma_0)^2, \tag{36}$$

$$h_1(a^\dagger) = 1 + \frac{1}{2}[({}_2\gamma_+) + ({}_4\gamma_+)](a^\dagger - \overline{A})^2 + \frac{1}{8}({}_2\gamma_+)^2(a^\dagger - \overline{A})^4, \tag{37}$$

$$h_2(a^\dagger) = \frac{(a^\dagger - \overline{A})}{c}\left[1 + \frac{1}{2}({}_2\gamma_+)(a^\dagger - \overline{A})^2\right]. \tag{38}$$

As with the supercoherent states, we find that the supersqueezed states are a linear combination of squeezed states in the bosonic sector with zero or one fermion in the odd sector. What is different, however, is that the squeezed states are multiplied by a linear combination of boson raising operators up to order four.

In the limits $A \to 0$ and $Z \to 0$, the supersqueezed states reduce to the "fermisqueezed states"

$$\begin{aligned} \mathbf{D}(0,\theta)\mathbf{S}(0,\theta_i)|0,0\rangle = & \left[1 - \frac{1}{2}\left(\frac{\overline{\theta}_1\theta_1}{4}\right) - \frac{1}{12}\left(\frac{\Phi}{16}\right)\right]\left[\left(1 - \frac{1}{2}\overline{\theta}\theta\right)|0,0\rangle + \theta|0,1\rangle\right] \\ + & \left[\frac{\theta_1}{2} - \frac{1}{3}\left(\frac{\overline{\theta}_2\theta_1\theta_2}{8}\right)\right]\left[\left(1 + \frac{1}{2}\overline{\theta}\theta\right)|1,1\rangle + \overline{\theta}|1,0\rangle\right] \\ + & \left[-\frac{1}{\sqrt{2}}\left(\frac{\overline{\theta}_2\theta_1}{4}\right)\right]\left[\left(1 - \frac{1}{2}\overline{\theta}\theta\right)|2,0\rangle + \theta|2,1\rangle\right]. \end{aligned} \tag{39}$$

6 Acknowledgments

Of course, I wish to thank and acknowledge my colleagues, Alan Kostelecký and Rod Truax, with whom this work was done [13].

References

[1] F. Iachello, Phys. Rev. Lett. **44**, 772 (1981); Nucl. Phys. **A370**, 284 (1981).

[2] V. A. Kostelecký and M. M. Nieto, Phys. Rev. Lett. **53**, 2285 (1984); V. A. Kostelecký, M. M. Nieto, and D. R. Truax, Phys. Rev. D **32**, 2627 (1985), V. A. Kostelecký, M. M. Nieto, and D. R. Truax, Phys. Rev. A **38**, 4413 (1988), and references therein.

[3] A. Comptet, A. D. Bandrauk, and D. K. Campbell, Phys. Lett. B **150**, 159 (1985).

[4] V. A. Kostelecký and D. K. Campbell, Physica **15D**, 3 (1985).

[5] M. M. Nieto, in: *Frontiers of Nonequilibrium Statistical Physics*, eds. G. T. Moore and M. O. Scully (Plenum, New York, 1986) p. 287; in: J. R. Klauder and B.-S. Skagerstam, *Coherent States – Applications in Physics and Mathematical Physics* (World Scientific, Singapore, 1985) p. 429; in: *Festschrift in Honor of John Klauder's 60th Birthday*, eds. G. G. Emch, G. C. Hegerfeldt, and L. Streit (to be published), and references therein.

[6] S. Gee and D. R. Truax, Phys. Rev. A **29**, 1627 (1984).

[7] D. R. Truax, V. A. Kostelecký, and M. M. Nieto, J. Math. Phys. **27**, 354 (1986); V. A. Kostelecký, M. M. Nieto, and D. R. Truax, J. Math. Phys. **27**, 1419 (1986).

[8] V. A. Kostelecký and D. R. Truax, J. Math. Phys. **28**, 2480 (1987); B. W. Fatyga, V. A. Kostelecký, and D. R. Truax, J. Math. Phys. **30** 291 (1989).

[9] B. W. Fatyga, V. A. Kostelecký, M. M. Nieto, and D. R. Truax, Phys. Rev. D **43**, 1403 (1991).

[10] A. Kostelecký, V. I. Man'ko, M. M. Nieto, and D. R. Truax, Phys. Rev. D. (to be submitted).

[11] D. Han, Y. S. Kim, and W. W. Zachary, eds., *Workshop on Squeezed States and Uncertainty Relations*, NASA Conference Publication 3135 (NASA, Washington, D. C., 1992).

[12] B. W. Fatyga, V. A. Kostelecký, M. M. Nieto, and D. R. Truax, in Ref. [11], p. 261.

[13] A. Kostelecký, M. M. Nieto, and D. R. Truax, Phys. Rev. D. (submitted).

[14] M. M. Nieto, in: *Santa Fe Workshop: Foundations of Quantum Mechanics*, eds. T. D. Black, M. M. Nieto, M. O. Scully, R. M. Sinclair, and H. S. Pilloff (World Scientific, Singapore, 1992) p. 95.