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SQUEEZED COLOUR STATES IN GLUON JET

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Abstract

The possibility of formation of squeezed states of gluon fields in quantum chromodynamics due to nonlinear nonperturbative selfinteraction during jet evolution in the process of e^+e^- annihilation into hadrons, which are analogous to the quantum foton squeezed states in quantum electrodynamics is demonstrated and the squeezing parameters are calculated.

1 Evolution equation for gluon field

The gluon part of the quantum chromodynamics Hamiltonian has the form [1]

$$H_g = H^0 + H_{int} = \int\{\tfrac{1}{2}(\vec{E}_a\vec{E}_a + \vec{B}_a\vec{B}_a) - \vec{E}_a g C_{abc}\vec{A}_b\vec{A}_c + \\ \tfrac{1}{2}g\vec{B}_a C_{abc}\vec{A}_b \times \vec{A}_c + \tfrac{1}{2}g^2(C_{abc}\vec{A}_b\vec{A}_c^0)^2 + \tfrac{1}{2}g^2(\tfrac{1}{2}C_{abc}\vec{A}_b \times \vec{A}_c)^2\}d^3x \tag{1}$$

where $\vec{E}_a = -\vec{\nabla}A_a^0 - \partial_t\vec{A}_a, \vec{B}_a = \vec{\nabla} \times \vec{A}_a, \vec{A}_a$–vector potential of gluon field, C_{abc}–structure constants of the SU(3) , $a, b, c, = 1, \dots, 8$ are colour indices; i, j, k, l–indices of 3-vectors.

The field of gluons appears in the form of gluon jet or cascade, which is produced by the quark with large transfered momentum. Due to the cubic and quadratic nonlinearities in (1) bremstrahlung gluons divide and at the end of perturbative cascade we have a jet of gluons with approximately equal energies and momenta [2].

At the end of cascade multiplicity distribution of gluons is close to negative binomial distribution [3, 4] which can be considered as a set of Poisson (coherent) distributions.

The importance of nonperturbative hadronisation stage is connected with confinement, subpoisson multiplicity distributions at this stage [5, 6], connection with intermittency [7], pairing of partons during colour loosing, nonlinearities of (1) hint on the possibilities of squeesed gluon states.

Let us take for simplicity that all gluons in jet have equal energies and momenta. Choose such the system of coordinates that has axe x_1 coinciding with the direction of gluon momentum. Then in the momentum representation the operator of gluon selfinteraction takes the form

$$V = \frac{1}{2}g^2 C_{abc}C_{adb}\left[\left(2 - \frac{m_g^2}{k_0^2}\right)A^{bcdf}_{1212} + \left(2 - \frac{m_g^2}{k_0^2}\right)A^{bcdf}_{1313} + A^{bcdf}_{2323}\right], \tag{2}$$

$$A^{bcdf}_{ijij} = A^{+b}_{i}A^{+c}_{j}A^{d}_{i}A^{f}_{j} + A^{+b}_{i}A^{c}_{j}A^{+d}_{i}A^{f}_{j} + A^{+b}_{i}A^{c}_{j}A^{d}_{i}A^{+f}_{j} + (\text{c.c.}), \tag{3}$$

where $A^{b}_{i}(A^{+b}_{i})$ are annihilation (production) operators of gluons with b-colour and i-vector component, k_0 and m_0 are gluon energy and mass: $k_0^2 - \vec{k}^2 = m_g^2$.

Evolution equation for gluon operator with indices a and k

$$i\partial_t A^{a}_{k} = [A^{a}_{k}, H] \tag{4}$$

then takes the form

$$i\partial_t A^{a}_{k} = f_1 A^{a}_{k} + f_2 A^{+a}_{k} + f \tag{5}$$

The function f_1, f_2, f do not contain explicitly A^{a}_{k} and A^{+a}_{k}, $f_1 = f_1^{*+}$ [9].

2 Squeezing of the gluon field in jet

Let us solve the equation (5) for small time $\Delta t << 1/E, E = \sqrt{f_1^2 - |f_2|^2}$. Then the solution of (5), is written in the matrix form

$$\partial_t \begin{pmatrix} A^{a}_{k}(t) \\ A^{+a}_{k}(t) \end{pmatrix} = \frac{1}{i}\begin{pmatrix} f_1 & f_2 \\ -f_2^{+} & f_1 \end{pmatrix}\begin{pmatrix} A^{a}_{k}(t) \\ A^{+a}_{k}(t) \end{pmatrix} + \frac{1}{i}\begin{pmatrix} f \\ f^{+} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} A^{a}_{k}(t) \\ A^{+a}_{k}(t) \end{pmatrix} = \exp\left[-i\int_0^{\Delta t}\begin{pmatrix} f_1 & f_2 \\ -f_2^{+} & f_1 \end{pmatrix} dt'\right] \times$$
$$\left\{\begin{pmatrix} A^{a}_{k}(0) \\ A^{+a}_{k}(0) \end{pmatrix} + \frac{1}{i}\int_0^{\Delta t}\exp\left[i\begin{pmatrix} f_1 & f_2 \\ -f_2^{+} & f_1 \end{pmatrix} dt''\right]\begin{pmatrix} f \\ -f^{+} \end{pmatrix} dt'\right\} \tag{7}$$

Let us take at some moment $t_0 = 0$ the conditions: $f = 0, \Im f_2 = 0$, and that f_1 and f_2 vary slowly. Then the solution takes the form

$$A^{a}_{k}(t) = A^{a}_{k}(0) - iA^{a}_{k}(0) f_1 \Delta t - iA^{+a}_{k}(0) f_2 \Delta t. \tag{8}$$

This expression coincides with the expression for ideal squeesed state [8]

$$A^{a}_{k} = A^{A}_{k}\operatorname{ch}r + e^{2i\theta}A^{+a}_{k}\operatorname{sh}r \tag{9}$$

$$\operatorname{ch}r = 1 - if_1 t, \qquad \operatorname{sh}r = f_2, \qquad e^{2i\theta} = -i \tag{10}$$

where r and θ are squeesing parameters. Thus the selfsqueesing is possible for the gluon field with fixed colour and Lorents component.

In quantum optics such states are named as pure quantum stated and operators $x_1 = (A - A^{+})/2$ and $x_2 = (a = A - A^{+})/2$ can have average fluctuations smaller then 1/4.

3 Evolution of gluon multiplicity distribution in jet

Take vector of state $|n_1, n_2, \ldots, n_a>$ where n_a- the number of gluons with definite indices i and a. The operator of full gluon number $\hat{N}$ acts on the vector as

$$\hat{N}|n_1, n_2, \ldots, n_a> = (n_1 + n_2 + \ldots + n_a)|n_1, n_2, \ldots, n_a> \tag{11}$$

It is clear that any part of H_{int} acts on the vector as

$$|New>= A_i^{+a} A_j^{+b} A_k^c A_l^d |n_1, n_2, \ldots, n_\alpha >= |n_1, n_2, \ldots, n_{\xi+1}, n_{\beta+2}, \ldots, n_{\sigma-1}, n_{\gamma-1}, \ldots, n_\alpha > \quad (12)$$

It does not change the number of particle

$$\hat{N}|New>= (n_1 + n_2 + \ldots + n_\alpha)|New> \quad (13)$$

and then

$$[\hat{N}, H] = 0. \quad (14)$$

Thus the total number of gluon is jet under made conditions ($\vec{k} = \overrightarrow{const}, k_0 = \text{const}$) does not change with the time and it is not difficult to see that gluon multiplicity distribution does not change with the time.

It can be also shown that the value squeesing shr for every mode is limited [9]. Foton multiplicity of squeesing states distributions have been used earlier for phenomenological description of some properties of hadron multiplicity distribution [10, 11].

Here we obtain for model gluon jet that the squeesed states of colour gluon field can appear due selfinteraction and nonperturbative mechanism of gluon selfinteraction and can be particularly important at nonperturbative stage of jet evolution.

Due to nonperturbativeness, pairing of gluon and subpoisson multiplicity distributions squeesing states can be responsible partly for hadronisation of colour partons (confinment) and intermittency (fractal dimension) phenomenon in multiparticle processes.

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