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THE UNCERTAINTY PRINCIPLE AND QUANTUM CHAOS

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Abstract

The conception of quantum chaos is described in some detail. The most striking feature of this novel phenomenon is in that all the properties of classical dynamical chaos persist here but, typically, on the finite and different time scales only. The ultimate origin of such a universal quantum stability is in the fundamental uncertainty principle which makes discrete the phase space and, hence, the spectrum of bounded quantum motion. Reformulation of the ergodic theory, as a part of the general theory of dynamical systems, is briefly discussed.

1 Introduction

The main purpose of this talk is to explain new physical ideas in the so-called *quantum chaos* which since recently attracts ever growing interest of many researchers [1–5, 10]. In appendix I also briefly discuss the concept of coherent/squeezed states in nonlinear, particularly, chaotic systems in a more close relation to the topic of this Workshop.

The recent breakthrough in understanding of the quantum chaos has been achieved, particularly, due to a new philosophy accepted, explicitly or more often implicitly, in most studies of quantum chaos. Namely, the whole physical problem of quantum dynamics was separated into two different parts:

- The *proper quantum dynamics* as described by a specific dynamical variable, the wavefunction $\psi(t)$, and by some deterministic equation, for example the Schrödinger equation. This part naturally belongs to the general theory of dynamical systems and is essentially mathematical; the problem is well-posed and this allows for extensive studies.
- The *quantum measurement* including the registration of the result and, hence, the collapse of the ψ function. This part still remains very vague to the extent that there is no common agreement even on the question whether this is a real physical problem or an ill-posed one so that the Copenhagen interpretation of (or convention in) quantum mechanics answers all the admissible questions. In any event, there exists as yet no dynamical description of the quantum measurement including the ψ collapse.

In this way one can single out a very difficult problem of the fundamental randomness in quantum mechanics which is related to the second part only, and which is foreign, in a sense, to the proper quantum system. On the other hand, there is a close relation of this separate problem to the quantum chaos itself (see Section 4 below and Ref.[4]).

The importance of quantum chaos is not only in that it represents a new unexplored field of nonintegrable quantum dynamics with many applications but also, and this is most interesting for the fundamental science, in reconciling the two seemingly different dynamical mechanisms for the statistical laws in physics.

Historically, the first mechanism was related to the *thermodynamic limit* $N \to \infty$ in which the completely integrable system becomes chaotic for typical (random) initial conditions (see, e.g.,Ref.[6]). A natural question – what happens for a large but finite number of freedoms N – has still no rigorous answer but the new phenomenon of quantum chaos, at least, presents an insight into this problem too. This mechanism, which is equally applicable in both classical and quantum mechanics, may be called the *traditional statistical mechanics* (TSM).

The second (new) mechanism is based upon the strong (exponential) local instability of motion characterized by positive Lyapunov's exponents $\Lambda > 0$ [6, 7]. It is not at all restricted to large N, and is possible, e.g., for $N > 1$ in a Hamiltonian system. However, this mechanism has been operative, until recently, in the classical mechanics only. This phenomenon is called *dynamical chaos* as it does not require any random parameters or any noise in the equations of motion. Notice that in a Hamiltonian (time–reversible) system the motion is unstable in both directions of time because for each positive Λ there is the equal negative one, and for almost all trajectories the instability depends on positive (in a given direction of time) exponents only. Hence, the dynamical chaos is also time–reversible, and no 'time arrow' exists or is required in the theory.

The quantum system bounded in phase space has a discrete energy (frequency) spectrum and is similar, in this respect, to the finite–N TSM. In both cases the motion is almost periodic. Moreover, such quantum systems are even completely integrable in the Hilbert space (see, e.g., Ref.[3]). Yet, the fundamental correspondence principle requires the transition to the classical mechanics, including the dynamical chaos, in the *classical limit* $q \to \infty$, where q is some (big) quasi-classical parameter, e.g., the quantum number n (the action variable, $\hbar = 1$). Again, a natural physical conjecture is that for finite but large q there must be some chaos similar to the finite–N TSM. Yet, in a chaotic quantum system the number of freedoms N does not need to be large as well as in the classical chaos. The quantum counterpart of N is q, both quantities determining the number of frequencies which control the motion. Thus, mathematically, the problem of quantum chaos is similar to that for the finite–N TSM.

Some researchers believe that the only way out of the above apparent contradiction is the failure of the correspondence principle [37]. If it were so the quantum chaos would be, indeed, a great discovery. 'Unfortunately', there exists a less radical (but also interesting and important) resolution of this difficulty to be discussed below.

The main difficulty here is in that the both problems suggest some chaos in the discrete spectrum which is completely contrary to the existing theory of dynamical systems and to the ergodic theory where such dynamics corresponds to the opposite limit of regular motion.

The ultimate origin of the quantum integrability is discreteness of the phase space (but not, as yet, of the space–time!) or, in the modern mathematical language, the noncommutative geometry of the former. This is the very basis of the whole quantum physics directly related to the fundamental uncertainty principle which implies a finite size of an elementary phase-space cell: $\Delta x \cdot \Delta p \gtrsim \hbar$ (per freedom).

As an illustration I will make use of the simple model described classically by the *standard map* (SM) [7, 8]:

$$\overline{n} = n + k \cdot \sin\theta; \qquad \overline{\theta} = \theta + T \cdot \overline{n} \tag{1}$$

with action-angle variables n, θ, and perturbation parameters k, T. The quantized standard map (QSM) is given by [9, 10]

$$\overline{\psi} = \exp(-ik \cdot \cos\hat{\theta}) \cdot \exp\left(-i\frac{T}{2}\hat{n}^2\right)\psi, \tag{2}$$

where momentum operator $\hat{n} = -i\partial/\partial\theta$. To provide the complete boundedness of the motion consider SM on a torus of circumference (in n)

$$L = \frac{2\pi m}{T} \tag{3}$$

with integer m to avoid discontinuities. The quasi-classical transition corresponds to quantum parameters $k \to \infty$, $T \to 0$, $L \to \infty$ while classical parameters $K = kT = \text{const}$, and $m = LT/2\pi = \text{const}$ remain unchanged.

The QSM models the *energy shell* of a conservative system which is the quantum counterpart of the classical energy surface.

In the studies of dynamical systems, both classical and quantal, most problems unreachable for rigorous mathematical analysis are treated "numerically" using computer as a universal model With all obvious drawbacks and limitations such "numerical experiments" have very important advantage as compared to the laboratory experiments, namely, they provide the complete information about the system under study. In quantum mechanics this advantage becomes crucial as in laboratory one cannot observe (measure) the quantum system without a radical change of its dynamics.

2 The definition of quantum chaos

The common definition of the classical chaos in physical literature is the *strongly unstable motion*, that is one with positive Lyapunov's exponents $\Lambda > 0$. The Alekseev – Brudno theorem then implies that almost all trajectories of such a motion are unpredictable, or random (see Ref.[11]). A similar definition of quantum chaos, which still has adherents among both mathematicians as well as a few physicists, fails because, for the bounded systems, the set of such motions is empty due to the discreteness of the phase space and, hence, of the spectrum.

The common definition of quantum chaos is *quantum dynamics of classically chaotic systems* whatever it could happen to be. Logically, this is most simple and clear definition. Yet, in my opinion, it is completely inadequate (and even somewhat helpless) from the physical viewpoint just because such a chaos may turn out to be a perfectly regular motion as, for example, in case of the *perturbative localization* [12]. In QSM the latter corresponds to $k \lesssim 1$ when all quantum transitions are suppressed independent of classical parameter K which controls the chaos.

I would like to define the quantum chaos in such a way to include some essential part of the classical chaos. The best definition I have managed to imagine so far reads:

the quantum chaos is finite-time (transient) dynamical chaos in discrete spectrum

In other words this new phenomenon reveals an intrinsic complexity and richness of the motion with discrete spectrum which has been considered since long ago as the most simple and regular. This is certainly in contradiction with the existing ergodic theory. In what follows I will try to explain a new approach to the ergodic theory which is necessary to describe the peculiar phenomenon of quantum chaos.

3 The time scales of quantum dynamics

Already the first numerical experiments with QSM revealed the quantum diffusion in n close to the classical one under conditions $K \gtrsim 1$ (classical stability border) and $k \gtrsim 1$ (quantum stability border) [9]. Futher studies confirmed this conclusion and showed that the former followed the latter in all details but on a *finite time interval* only [10, 13]. This observation was the clue to understanding the dynamical mechanism of the diffusion, which is apparently an aperiodic process, in discrete spectrum. Indeed, the fundamental uncertainty principle implies that the discreteness of the spectrum is not resolved on a sufficiently short time interval. Whence, the estimate for the ***diffusion (relaxation) time scale*** :

$$t_R \sim \varrho_0 \leq \varrho. \tag{4}$$

Here ϱ is the density of (quasi)energy levels, and ϱ_0 is the same for the *operative eigenstates* which are actually present in the initial quantum state $\psi(0)$ and, thus, do actually control the dynamics. In QSM the quasi–energies are determined mod $2\pi/T$ and, surprisingly, $\varrho = LT/2\pi = m$ is a classical parameter (3). As to ϱ_0, it depends on the dynamics and is given by the estimate [10, 13]:

$$\frac{\varrho_0}{T} \sim \frac{t_R}{T} \equiv \tau_R \sim D \equiv \frac{\langle(\Delta n)^2\rangle}{\tau} \leq \frac{m}{T} \tag{5}$$

Here τ is discrete map's time (the number of iterations), and D is the classical diffusion rate. This remarkable expression relates an essentially quantum characteristic (τ_R) to the classical one (D). The latter inequality in Eq.(5) follows from that in Eq.(4), and it is explained by the boundedness of QSM on a torus. In the quasi–classical region $\tau_R \sim k^2 \to \infty$ (see Eq.(1)) in accordance with the correspondence principle.

Besides relatively long time scale (5) there is another one given by the estimate [14, 10]

$$t_r \sim \frac{\ln q}{\Lambda} \to \frac{\ln k}{\ln(K/2)} \tag{6}$$

where q is some (large) quasi–classical parameter, and where the latter expression holds for QSM. It may be termed the *random time scale* since here the quantum motion of a narrow wave packet is as random as classical trajectories according to the Ehrenfest theorem. This was well confirmed in a number of numerical experiments [15]. The physical meaning of scale t_r is in fast spreading of a wave packet due to the strong local instability of classical motion.

Even though the random time scale t_r is very short it grows indefinitely in the quasi-classical region (q, $k \to \infty$), again in agreement with the correspondence principle.

Big ratio t_R/t_r imlies another peculiarity of quantum diffusion: it is dynamically stable as was demonstrated in striking numerical experiments [16] with time reversal.

Thus, the quantum chaos possesses all the finite-time properties of the 'true' (classical-like) chaos on the corresponding time scales in spite of the discrete spectrum. To put it another way, the phenomenon of quantum chaos demonstrates that the limiting case of the regular motion in the general theory of dynamical systems, which appears to be fairly simple and transparent, reveals, in the quantum chaos, its internal complexity and richness to the extent of approaching its opposite, the 'true' classical chaos, or deterministic randomness.

I think that the conception of characteristic time scales of quantum dynamics is a satisfactory resolution of the apparent contradiction between the correspondence principle and the quantum transient (finite-time) pseudochaos. Some physicists, however, feel that such an explanation is, at least, ambiguous because it includes the two limits which do not commute:

$$\lim_{|t|\to\infty}\ \lim_{q\to\infty} \neq \lim_{q\to\infty}\ \lim_{|t|\to\infty}$$

While the first order leads to the classical chaos, the second one results in an essentially quantum behavior with no chaos at all. To relax these doubts I notice that in physics one does not need any limits at all, and can describe, principally, anything quantum-mechanically. If, nevertheless, we would like to make use of the much simpler classical mechanics (for practical purposes) the only one limit ($q \to \infty$) is quite sufficient as the physical time is certainly finite. At last, even if it would be helpful for some reason to formally take the limit $|t| \to \infty$ this should be *conditional* that is one for a fixed ratio $|t|/t_R(q)$, for example. The limit $|t| \to \infty$ is related to the existing ergodic theory which is asymptotic in t. Meanwhile the new phenomenon of the quantum chaos requires the modification of the theory to a finite time which is a difficult mathematical problem still to be solved. The main difficulty is in that even the distinction between the two opposite limits in the ergodic theory - discrete and continuous spectra - is asymptotic only.

In any event, since quantum mechanics is commonly accepted as the universal theory, the phenomenon of the 'true' (classical-like) dynamical chaos strictly speaking does not exist in nature. Nevertheless, it is very important in the theory as the limiting pattern to compare with the real quantum chaos. On the other hand, the practical importance of statistical laws even for a finite time interval is in that they provide a relatively simple description of the *essential* behavior for a very complicated dynamics.

4 The quantum steady state

As a result of quantum diffusion and relaxation some steady state is formed whose nature depends on the *ergodicity parameter*

$$\lambda = \frac{l_s}{L} \approx \frac{D}{L}. \tag{7}$$

where l_s is the so-called *localization length* (see Eq.(10) below). If $\lambda \gg 1$ the quantum steady state is close (at average) to the classical statistical equilibrium which is described by ergodic phase density $g_{cl}(n)$ =const (for SM on a torus) where n is a continuous variable. In quantum mechanics n is integer, and the quantum phase density $g_q(n,\tau)$ in the steady state fluctuates [17, 5], the ergodicity meaning

$$g_q(n) = \overline{|\psi_s(n,\tau)|^2} = \frac{1}{L} \tag{8}$$

where the bar denotes time averaging.

According to numerical experiments the ergodicity does not depend on the initial state which implies that all eigenfunctions $\phi_m(n)$ are also ergodic, at average, with Gaussian fluctuations [17, 5] and the dispersion

$$\langle|\phi_m(n)|^2\rangle = \frac{1}{L}. \tag{9}$$

This is always the case sufficiently far in the quasi-classical region as $\lambda \sim k^2/L \sim Kk/m \to \infty$ with $k \to \infty$ ($K = kT$ and $m = LT/2\pi$ remain constant) in accordance with old Shnirelman's theorem [18].

An interesting unsolved problem is the microstructure of ergodic eigenfunctions, particularly, the so-called 'scars' [29] which reveal the set of classical (unstable) periodic trajectories (see Ref.[30] for the theory of scars).

Finite fluctuations (9) show that a single chaotic quantum system in a pure state described by $\psi_s(n,\tau)$ represents, in a sense, a finite statistical ensemble of $M \sim L$ "particles". Moreover, Eq.(9) shows that all the basis states in a chaotic quantum system are statistically independent as if the system were in a mixed state and not in a pure one as it actually is. This means that the quantum chaos provides the *dynamical* loss of quantum coherence which is of principal importance in many problems, for example, in the theory of quantum measurement. The fluctuations result, particularly, in partial recurrences toward the initial state but the recurrence time is much longer as compared to the relaxation time scale τ_R and sharply depends on the recurrence domain.

If $\lambda \ll 1$ the quantum steady state is qualitatively different from the classical one. Namely, it is localized in n within the region of size l_s around the initial state if the size of the latter $l_0 \ll l_s$. Numerical experiments show that the phase space density, or the *quantum statistical measure* , is approximately exponential [10, 13]

$$g_s(n) \approx \frac{1}{l_s}\exp\left(-\frac{2|n|}{l_s}\right); \qquad l_s \approx D \tag{10}$$

for initial $g(n,0) = \delta(n)$. The quantum ensemble is now characterized by $M \sim l_s \sim k^2$ "particles".

The relaxation to this steady state is called *diffusion localization* , and it is described approximately by the diffusion equation [19, 28]

$$\frac{\partial g}{\partial \tau'} = \frac{1}{2}\frac{\partial}{\partial n}D\frac{\partial g}{\partial n} + \frac{\partial g}{\partial n}\cdot\frac{n}{|n|} \tag{11}$$

for initial $g(n,0) = \delta(n)$ where new time

$$\tau' = \tau_R \ln\left(1 + \frac{\tau}{\tau_R}\right) \tag{12}$$

accounts for the discrete motion spectrum [20]. The last term in Eq.(11) describes the "backscattering" of ψ wave propagating in n which eventually results in the diffusion localization. The fitting parameter $\tau_R \approx 2D$ was derived from the best numerical data available (see Ref.[21] where a different theory of diffusion localization was also developed).

5 Concluding remarks

In conclusion I would like to briefly mention a few important results for unbounded quantum motion. In SM it corresponds to $L \to \infty$. First, there is an interesting analogy between dynamical localization in momentum space and the celebrated Anderson localization in disordered solids which is a statistical theory. The analogy was discovered in Ref.[22] and essentially developed in Ref.[23]. It is based upon (and restricted by) the equations for eigenfunctions. The most striking (and less known) difference between the two problems is in the absence of any diffusion regime in $1D$ solids [24]. This is because the energy level density of the operative eigenfunctions in solids

$$\rho_0 \sim \frac{l dp}{dE} \sim \frac{l}{u} \sim t_R \tag{13}$$

which is the relaxation time scale, is always of the order of the time interval for a free spreading of the initial wave packet at a characteristic velocity u. In other words, the localization length l is of the order of the electron scattering free path. On the contrary, in momentum space, for instance in the standard map, each scattering (one map's iteration) couples $\sim k$ unperturbed states, so that $\sim k^2 \gg 1$ scatterings are required to reach the localization $l \sim k^2$. Another (qualitative) explanation of this surprising difference is in that the density of quasienergy levels for driven systems is always much higher as compared to that of energy levels. The same is true for a conservative 2D system as compared with 1D motion in solids. Thus, the Anderson localization is the spreading, rather than diffusion, localization.

Another similarity between the two problems is in that the Bloch extended states in periodic potential correspond to a pecular quantum resonance in QSM for rational $T/4\pi$ [9, 10].

An interesting open question is the dynamics for irrational-Liouville's (transcendental) $T/4\pi$. As was proved in Ref.[25] the motion can be unbounded in this case unlike that for a typical irrational value. The latter is the result of numerical experiments, no rigorous proof of localization for $k \gg 1$ has been found as yet. In Ref.[28] the conjecture is put forward, supported by some semiqualitative considerations, that depending on a particular Liouville's number the broad range of motions is possible, from purely resonant one ($|n| \sim \tau$) down to complete localization ($|n| \lesssim l$).

If the quantum motion is not only unbounded but the growth of unbounded variables is exponential, the "true" chaos (not restricted to a finite time scale) can occur. A few exotic examples together with considerations from different viewpoints can be found in Refs.[10, 26]. One particular model is $3D$ linear oscillator with phase-dependent frequencies described by the Hamiltonian

$$\hat{H} = \frac{1}{2} \sum_{k=1}^{3} \left(\omega_k(\theta_1, \theta_2, \theta_3) \hat{n}_k + \hat{n}_k \omega_k(\theta_1, \theta_2, \theta_3) \right); \qquad \hat{n}_k = -i \frac{\partial}{\partial \theta_k}$$

However, such chaos does not seem to be a typical quantum dynamics.

The final remark is that the quantum chaos, as defined in Section 2, comprises not only quantum systems but also any linear, particularly classical, waves [27]. So, it is essentially the *linear wave chaos* . Moreover, a similar mechanism works also in completely integrable nonlinear systems like Toda lattice, for example [31]. From mathematical point of view all these new ideas require some *perestroika* in the existing ergodic theory. Perhaps, better to say that a new ergodic theory is wanted which, instead of benefiting from the asymptotic approximation ($|t| \to \infty$ or $N \to \infty$), could analyze the finite-time statistical properties of dynamical systems. In my opinion, this is the most important conclusion from the first attempts to comprehend the quantum chaos.

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Appendix: universal squeezing of coherent states

The coherent states have been introduced and are widely used as the special most narrow wave packets for the linear harmonic oscillator. In this and only this case the packets do not spread, and it allows, particularly, for the unambiguous distinction between the coherent and squeezed states which attract recently much attention [32]. The generalization of both onto nonlinear oscillations remains unclear as many different attempts do attest [33]. The main obstacle here is a universal phenomenon of the stretching/squeezing for any narrow wave packet in nonlinear dynamics. Even in a completely integrable system the linear (in time) local instability of motion always occurs just as a result of nonlinearity which makes the frequencies $\omega(n)$ dependent on initial conditions. In quantum mechanics it corresponds to unequal energy level spacings. As a result the squeezing parameter

$$s(t) \equiv \frac{d_{max}}{d_{min}} \sim n\,(\Delta\theta)^2 \sim (\Delta n)^2 t^2 \sim (\nu_1 \omega t)^2 \frac{(\Delta n)^2}{n}$$

permanently grows with time. Here $d_{max} \sim \sqrt{n}\Delta\theta \sim (\nu_1\omega t)\Delta n/\sqrt{n}$ and $d_{min} \sim 1/d_{max}$ are the maximal and minimal dimensions, respectively, of an initially 'round' (coherent) wave packet ($\Delta n/\sqrt{n} \sim \sqrt{n}\Delta\theta \sim 1$) on the action-angle phase plane in polar coordinates $\sqrt{n}$, θ; $\nu_1 = (n/\omega)|d\omega/dn|$ is dimensionless nonlinearity, and the minimum-uncertainty relation [34] $d_{max} \cdot d_{min} \sim 1$ used is the quantum counterpart of the classical phase-space area conservation. The former is not exact [35]

$$\frac{dW}{dt} \approx \frac{1}{24}\frac{d^2\omega}{dn^2}\frac{\partial^3 W}{\partial\theta^3} \sim W\,\omega\,\nu_2\,\frac{(\Delta n)^3}{n^2}$$

where $W(n, \theta, t)$ is the Wigner function, and $\nu_2 = (n^2/\omega)d^2\omega/dn^2$. This estimate determines the *inflation time scale* t_{if} when the phase-space area A, occupied by a quantum state, substantially increases ($\Delta A \sim A$):

$$\nu_2\,\omega\,t_{if} \sim \frac{n^2}{(\Delta n)^3} \to \sqrt{n} \quad (s_0 \sim 1)$$

The latter estimate holds for the coherent initial state ($s_0 = (\Delta n)^2/n = 1$).

It is instructive to compare t_{if} with the two other characteristic time scales of the packet dynamics, namely

- *squeezing time scale* ($\Delta s \sim 1$): $\nu_1 \omega t_{sq} \sim \sqrt{n}/\Delta n \to 1 \quad (s_0 \sim 1)$
- *stretching time scale* ($\Delta\theta \sim 1$): $\nu_1 \omega t_{st} \sim n/\Delta n \to \sqrt{n} \quad (s_0 \sim 1)$

In quasiclassics ($n \gg 1$) $t_{sq} \ll t_{st} \sim t_{if}$ ($s_0 \sim 1$). If $\Delta n \ll \sqrt{n}$ (initial squeezing parameter $s_0 \gg 1$) the discreteness (quantization) of action n comes into play and destroys the wave packet. Apparently, it happens when $\Delta n_c \sim 1$ at the packet center, or $\Delta\theta \sim 1$. Hence, beyond the stretching time scale t_{st} a single packet does no longer exist. In a sense, t_{st} is the packet life time.

The ultimate origin of the packet inflation is in that the uncertainty relations are generally inequality. An attempt [36] to reformulate them as the equality, using the universal relation

$$\int W^2 \, dp \, dq = \frac{1}{2\pi}$$

for any pure state, is very restrictive as W may be negative. Particularly, this is just the case during inflation when W oscillates around zero.

Recently another version of 'phase-space density' (also called Husimi distribution)

$$S(p, q, t) = \frac{1}{2\pi} | < \alpha | \psi > |^2$$

became very popular. Even though this function has a clear physical meaning as the expansion in the basis of the coherent states at points $\alpha = (q + ip)/\sqrt{2}$ and, moreover, is never negative it may substantially distort the picture of quantum evolution owing to the inherent restriction of resolution in both p and q separately. Particularly, for a classically unstable and, hence, chaotic motion the squeezing of a narrow wave packet is almost completely hidden, the stretching only showing up [15].

In the latter case the squeezing (as well as stretching) is most fast ($s(t) \sim \exp(2\Lambda t)$ where the instability rate Λ is the Lyapunov exponent), and it explains a very short random time scale (6). This scale essentially depends on the initial wave packet, estimate (6) corresponding to the special, least squeezing, packet with $\Delta n \sim (\Delta\theta)^{-1} \sim \sqrt{k}$. This is also a sort of coherent state but very unusual one which depends not on the action n but on perturbation parameter k ($\Delta n/\sqrt{k} \sim \sqrt{k}\Delta\theta \sim 1$). The squeezing due to the local instability is terminated at time (6) by the distruction of the packet which disintegrates into many scattered pieces [15] when $\Delta\theta \sim \Delta n \sim 1$ as explained above. However, if the packet resides on a classical (unstable) periodic trajectory of period $P \lesssim t_r$ the squeezing is restricted, due to periodicity, by the time $P/2$, and a quasistationary structure may exist. This phenomenon manifests itself in the so-called 'scars' on the chaotic eigenfunctions [29, 30]. The set of such almost 'frozen' packets may form a natural coherent basis for chaotic quantum systems [19].

In conclusion I would like to emphasize again that even though the distinction between coherent and squeezed states remaines, as yet, ambiguous the squeezing itself is generic.

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