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CORRELATED STATES OF A QUANTUM OSCILLATOR ACTED BY SHORT PULSES

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ABSTRACT

Correlated squeezed states for a quantum oscillator acted by very short in time pulses modeled by special dependence on time of frequency of oscillator in the form of sequence of three delta-kickings of frequency are constructed based on the method of quantum integrals of motion. Also the correlation coefficient and quantum variances of operators of coordinate and momenta are written in explicit form.

The aim of the paper is to discuss the squeezing phenomenon and correlations in the system of quantum parametric oscillator with special dependence on time of frequency of oscilllator. We consider the case when oscillator is acted by very short in time pulses. This dependence on time we will model by δ-kickings of frequency. In this paper we will consider the case of sequence of three δ-kickings of frequency. The cases of one and two δ-kickings of frequency were considered in [1]. Short pulses in the form of δ-kicking were discussed briefly in the case of two-mode squeezing [2] and for the chain of quantum oscillators [3].

Let us consider the quantum parametric oscillator which is acted by very short in time pulses. We are modeling this action by the special dependence on time of frequency of oscillator. We will use the model of δ-kickings of frequency.

Let the first kick be at initial moment of time $t_1=0$, the second one in the moment $t_2=\tau$, and the third one at $t_3=2\tau$.

The Hamiltonian of the system is of the form

$$\hat{H} = \hat{p}^2/2m + m\omega^2(t)\hat{q}^2/2 \qquad (1)$$

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where $\hat{q}$ is coordinate operator $\hat{p}$ is momentum operator, m is the mass and $\omega(t)$ is time-dependent frequency. We choose the following dependence on time of oscillator frequency

$$\omega^2(t)=\omega_0^2 - 2\sum_{n=1}^{3}\kappa\delta(t-t_n). \qquad (2)$$

The equations of motion corresponding to Hamiltonian (1) are of the form

$$\ddot{q} + (\omega_0^2 - 2\kappa\delta(t) - 2\kappa\delta(t-\tau) - 2\kappa\delta(t-2\tau))q = 0 \qquad (3)$$

Following the usual scheme [4] one can construct integral of motion for the Hamiltonian (1)

$$\hat{A}(t) = \frac{i}{2}\left\{\frac{\hat{p}\varepsilon}{(\hbar m\omega_0)^{1/2}} - \frac{\hat{q}\dot{\varepsilon}}{(\hbar\omega_0/m)^{1/2}}\right\} \qquad (4)$$

where function ε is the solution of equation of motion (3). If function ε satisfy additional condition

$$\dot{\varepsilon}\varepsilon^* - \dot{\varepsilon}^*\varepsilon = 2i\omega_0.$$

the integral of motion (4) and its hermitian conjugate satisfy boson commutation relations. The ground state of the system can be found from the condition

$$\hat{A}(t)\psi_0(q,t) = 0$$

and has the form

$$\psi_0(q,t) = \pi^{-1/4}\left[\frac{\hbar\varepsilon}{m\omega_0}\right]^{-1/2}\exp\left\{\frac{i\dot{\varepsilon}}{2\varepsilon}\frac{q^2}{(\hbar/m\omega)}\right\}. \qquad (5)$$

The coherent states of the system can be found as eigenfunctions of the integral of motion $\hat{A}(t)$

$$\hat{A}(t)\,\psi_\alpha(q,t) = \alpha\psi_\alpha(q,t),$$

where α is complex number and has the form

$$\psi_\alpha(q,t)=\psi_0(q,t)\exp\left\{-\frac{|\alpha|^2}{2} + \frac{\sqrt{2}\,\alpha q}{\varepsilon\sqrt{\hbar/m\omega_0}} - \frac{\alpha^2\varepsilon^*}{2\varepsilon}\right\}. \qquad (6)$$

One can see that ground and coherent states are Gaussian wavepackets with time-dependent coefficients in quadratic form under exponential function.

In order to write integral of motion in explicit form one has to solve equations (3) for the function ε in the case of sequence of three δ-kicks of frequencies (2). For the function $\varepsilon(t)$ one can write following solutions

$$
\begin{aligned}
\varepsilon_0(t) &= A_0 e^{i\omega_0 t} + B_0 e^{-i\omega_0 t}, \quad t<0, \\
\varepsilon_1(t) &= A_1 e^{i\omega_0 t} + B_1 e^{-i\omega_0 t}, \quad 0<t<t_2, \\
\varepsilon_2(t) &= A_2 e^{i\omega_0 t} + B_2 e^{-i\omega_0 t}, \quad t_2<t<t_3, \\
\varepsilon_3(t) &= A_3 e^{i\omega_0 t} + B_3 e^{-i\omega_0 t}, \quad t>t_3.
\end{aligned}
\tag{7}
$$

So, one has four regions of changing the function $\varepsilon(t)$. At three points of time t_1, t_2, t_3 we have following conditions for functions ε_i

$$
\begin{aligned}
\varepsilon_i(t_i) &= \varepsilon_{i-1}(t_i), \\
\dot{\varepsilon}_i(t_i) - \dot{\varepsilon}_{i-1}(t_i) &= 2\kappa\, \varepsilon_{i-1}(t_i).
\end{aligned}
$$

From this conditions one can find the conditions for coefficients A_i and B_i. Taking in the initial moment of time the wave with $A_0=1$ and $B_0=0$ one has the solutions for ε-function after δ-kickings

$$\varepsilon_0 = e^{i\omega_0 t}, \quad t<0, \tag{8}$$

$$\varepsilon_1(t) = (1-i\kappa/\omega_0)\, e^{i\omega_0 t} + i\kappa/\omega_0 e^{-i\omega_0 t}, \quad 0<t<\tau, \tag{9}$$

$$
\begin{aligned}
\varepsilon_2(t) = {} & \left[(1-i\kappa/\omega_0)^2 + \frac{\kappa^2}{\omega_0^2} e^{-2i\omega_0 t}\right] e^{i\omega_0 t} + \Big[(i\kappa/\omega_0)(1+i\kappa/\omega_0) \\
& + (i\kappa/\omega_0)(1-i\kappa/\omega_0) e^{2i\omega_0 \tau}\Big] e^{-i\omega_0 t}, \quad \tau<t<2\tau,
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\varepsilon_3(t) = {} & \left[(1-i\kappa/\omega_0)(\chi^2 - 1)e^{-i\omega_0 \tau} - \chi e^{-2i\omega_0 \tau}\right] e^{i\omega_0 t} + \\
& + \frac{i\kappa}{\omega_0}(1 + \chi^2) e^{3i\omega_0 \tau}\, e^{-i\omega_0 t}, \quad t>2\tau .
\end{aligned}
\tag{11}
$$

where $\chi = 2\cos\omega_0\tau + \dfrac{2\kappa}{\omega_0}\sin\omega_0\tau$.

If before δ-kickings the system was in coherent states then after the sequence of δ-kickings of frequency the oscillator will be in correlated squeezed state determined by formulae (6) with function ε given by formulae (11). In order to have explicit expression for these states in another periods of time one has to put in formulae (6) the explicit expression for ε function in this period of time given by formulae (7).

The disperssion of coordinate after sequence of δ-kickings will be equal to

$$\sigma_q = \langle\psi_\alpha|\hat{q}^2|\psi_\alpha\rangle - \langle\psi_\alpha|\hat{q}|\psi_\alpha\rangle^2 = \frac{\hbar}{2m\omega_o}\Big\{1 + \frac{4\kappa^2}{\omega_o^2}(\chi^2-1)^2\sin^2\omega_o(t-2\tau) + \frac{2\kappa}{\omega_o}\Big[\chi^2-1\Big]^2\sin 2\omega_o(t-2\tau) + \frac{2\kappa^o}{\omega_o}\chi\Big[\chi^2-1\Big]\sin(2\omega_o t-5\omega_o\tau).$$

The correlation between coordinate and momenta in this state is not equal to zero and is of the form

$$\sigma_{qp} = \frac{1}{2}\langle\psi_\alpha|\hat{q}\hat{p}+\hat{p}\hat{q}|\psi_\alpha\rangle - \langle\psi_\alpha|\hat{q}|\psi_\alpha\rangle\langle\psi_\alpha|\hat{p}|\psi_\alpha\rangle == \frac{\hbar}{2}\Big\{\Big[1 + \frac{4\kappa^2}{\omega_o^2}(\chi^2-1)^2\sin^2\omega_o(t-2\tau) + \frac{2\kappa}{\omega_o}\Big[\chi^2-1\Big]^2\sin 2\omega_o(t-2\tau) + \frac{2\kappa}{\omega_o}\chi\Big[\chi^2-1\Big]\sin(2\omega_o t-5\omega_o\tau)\Big]\circ\Big[1 + \frac{4\kappa^2}{\omega_o^2}(\chi^2-1)^2\cos^2\omega_o(t-2\tau) - \frac{2\kappa}{\omega_o}\Big[\chi^2-1\Big]^2\sin 2\omega_o(t-2\tau) + \frac{2\kappa}{\omega_o}\chi\Big[\chi^2-1\Big]\sin(2\omega_o t-5\omega_o\tau)\Big] - 1\Big\}^{1/2}.$$

So one has statistical dependence of operators of coordinate and momentum after series of δ-kickings and in some periods of time the disperssion of coordinate is less then before δ-kickings. So we have two phenomena due to seria of short in time pulses acted on oscillator: squeezing phenomenon and phenomenon of statistical dependence of operators of coordinates and momenta.

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