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Functional Approximation Using Artificial Neural Networks in Structural Mechanics

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SUMMARY

The artificial neural networks (ANN) methodology is an outgrowth of research in artificial intelligence. In this study the feed-forward network model that was proposed by Rumelhart, Hinton, and Williams was applied to the mapping of functions that are encountered in structural mechanics problems. Several different network configurations were chosen to train the available data for problems in materials characterization and structural analysis of plates and shells. By using the recall process the accuracy of these trained networks was assessed.

INTRODUCTION

The nonlinear stress analysis of complex structural systems by using finite element analysis (FEA) programs requires an accurate representation of the material behavior, which is usually available through experiments in tabular form. In the case of nonlinear material properties, including material behavior in an FEA program leads to large computation times. There is a need to develop new ways of material characterization that are suitable for the FEA, can capture the essence of material behavior, and are computationally efficient. The use of artificial neural networks (ANN) seems to be particularly appealing for this type of problem.

For a large class of structural systems the analysis results are available in the form of tables, charts, and equations. In designing these structures the values are often needed at intermediate points, and they are computed by using linear interpolation schemes. This process is error prone and time consuming whenever values at large numbers of intermediate points are needed. The application of the ANN methodology could be useful for solving this type of problem.

ARTIFICIAL NEURAL NETWORKS

The problems discussed in the introduction can be solved by developing efficient procedures for generalized multidimensional functional mapping. Ken-Ichi Funahasi (ref. 1) proved mathematically that any continuous mapping can be approximated by multilayer neural networks with at least one hidden layer. This work was further extended by Hornik et al. (ref. 2) to include other types of squashing functions. They also provided the mathematical proof (ref. 3) that these types of networks are capable of approximating the arbitrary functions, including their derivatives. Further refinement of this work can be found in reference 4. These mathematical proofs along with other work provide an excellent basis for using the multilayer feed-forward networks with a continuous squashing function for approximate functional mapping.

One of the popular ANN models of the multilayer feed-forward network is based on the studies of Rumelhart, Hinton, and Williams (ref. 5). It consists of an input, an output, and a minimum of one intermediate layer (fig. 1). The network training is accomplished by using the backpropagation algorithm as described in reference 5. It establishes the weights of the interconnections and the bias values for the processing elements. They are saved in a small file for use in the network recall process. This ANN model has been successfully used in pattern recognition tasks, such as text-to-speech synthesis (ref. 6), image processing and compression (ref. 7), and nonlinear signal processing (ref. 8).

The application of the ANN that is based on the backpropagation algorithm in computational structures technology (CST) is relatively new in origin. Rehak et al. (ref. 9) used ANN for simulating the dynamic behavior of structures. Troudet and Merrill (ref. 10) adopted a similar approach for estimating the fatigue life of structural components. Berke and Hajela (ref. 11) used ANN for structural analysis and shape optimization of trusses. The ANN approach has shown considerable promise in material properties characterization. Brown et al. (ref. 12) used it to model composite ply micromechanics. Ghaboussi et al. (ref. 13) have modeled the nonlinear behavior of concrete. McCauley (ref. 14) has explored the optical implementation of neural networks for engineering design.

The mathematical proofs for the convergence of an ANN that are based on the backpropagation algorithm do not provide guidelines for creating an appropriate network configuration or for network training. Presently, guidelines are provided by creating different network configurations and testing them numerically for accuracy and convergence characteristics. Extensive numerical experimentation is required before appropriate ANN models can be developed for a given problem. This approach has been tried in applying ANN in CST. In many cases a large number of processing units are used for intermediate layers, leading to an excessive amount of training time and a redundancy in the ANN configurations.

OBJECTIVE AND SCOPE OF STUDY

A main objective of this study was to obtain the smallest possible ANN configurations for CST problems. The problems were selected to reflect different types of functional approximations. The first two problems involved material property characterization. They were mainly chosen to develop a suitable form wherein trained networks could be added to a nonlinear FEA program without major modifications. This interfacing is needed to provide material data to an FEA program. The plate and shell problems were used to test the capability of the ANN method for multidimensional functional approximations. In both cases tubular data were used to train the ANN models and to test the accuracy of the trained networks' interpolation capability at the intermediate points. The details for these problems are provided in the next section.

PROBLEM DESCRIPTION

The first problem of material characterization maps the strain values to the known stress values. The following equation relates strains to stresses:

$$\begin{aligned} \sigma &= E_0\left(\varepsilon - 5\varepsilon^2\right) \qquad \text{for} \qquad \varepsilon \geq 0 \\ \sigma &= E_0\left(\varepsilon + 5\varepsilon^2\right) \qquad \text{for} \qquad \varepsilon < 0 \end{aligned} \tag{1}$$

The ANN model is given the strain values ε as input, and the stress values σ are obtained as output.

The second problem also falls into the category of material property characterization. The ANN models are given the strain values ε as input, and predictions are made for the stresses σ and the tangent modulus $d\sigma/d\varepsilon$ that are needed for the elastic-plastic stress analysis. This constitutes a mapping of one independent variable to two dependent variables. It allows the inclusion of the variable and its slope. The slope of the function is given as

$$\frac{d\sigma}{d\varepsilon} = E_0(1 - 10\varepsilon) \qquad \text{for} \qquad \varepsilon \geq 0 \tag{2}$$

The distribution of bending moment factors in a simply supported rectangular plate is given in tabular form in reference 15. The two input units of the neural network model are the aspect ratio b/a and the x coordinate of the plate (fig. 2). The y coordinates for all the points are zero. The two outputs from the ANN model are the factors for the bending moments M_x and M_y. This third problem was chosen to assess the modeling capability of ANN for a two-independent-variables-to-two-dependent-variables functional mapping.

The fourth problem is for an elliptical paraboloid shell from reference 16. In this case the input variables are x/a, y/b, and c_1/c_2, defining the location of the points at which the stress resultants are computed and the geometry of the shell, respectively, (fig. 3). The outputs for the ANN models are the coefficients for the stress resultants N_y, N_x, and N_{xy}. The problem allows us to investigate a more generalized functional mapping where the three input variables defining geometry are mapped to a space of the three stress resultant coefficients.

The standard configurations of a feed-forward network that includes an input layer, an output layer, and an intermediate layer were utilized for this study. A typical network configuration is shown in figure 1. The computer program NETS (ref. 17) was used for all the network training and recall. In the program the backpropagation algorithm was implemented at the NASA Johnson Space Center. The number of processing units in the intermediate layer was established by arbitrary selection, and then the accuracy of the trained network model was assessed.

RESULTS AND DISCUSSION

Materials Characterization

For stress-strain curve modeling, the following ANN configurations were chosen:

(1) Case 1, 1-5-1.13
(2) Case 2, 1-10-1.13
(3) Case 3, 1-15-1.13
(4) Case 4, 1-5-1.19
(5) Case 5, 1-10-1.19

The first number denotes the number of input units. The second number represents the number of hidden units, and it varies from 5 to 15. The third number (1) is the number of output units. The number after the period is the total number of input-output pairs that were used for network training. These pairs were obtained from equation (1). All the training data were scaled between 0 and 1 because of the restriction that is placed by the backpropagation algorithm which is implemented in NETS. The networks were trained with a maximum error not exceeding 1.8 percent and a root-mean-square (rms) error less than 1 percent. After the training the files containing weights and biases were saved for each network to use in assessing the accuracy of all the neural network models.

The input strain values used for training were augmented by additional strain values from equation (2) to propagate the data. The predicted stress values from the neural networks were plotted along with the actual values obtained from equation (2). Figure 4(a) shows good prediction capability for cases 1 to 3, with case 3 being closest to the actual stress-strain curve. Cases 4 and 5 (fig. 4(b)) were in good agreement with the known results. Cases 3 and 4 (fig. 4(c)) were very close to the chosen stress-strain curve. It is difficult to select the best case from these plots. Therefore, for a closer look at the accuracy of the results, the error in neural network interpolation versus strain is plotted in figure 5. The error was within ±3 percent when the strains used for training were also used for predicting stresses. For other strain values these errors could be significant, especially at the two end points of the stress-strain curve, where strain values were nearly ±0.2. The other location where errors were significant was near the strain value of zero. Note that at these strain levels the actual stress is approaching zero. Any small variation in the neural network prediction causes a large relative error because in calculating the error the difference between the actual and predicted stress is divided by a stress value that is small in magnitude. This division artificially magnifies the magnitude of the error. Therefore, the ANN predictions, although very accurate, could be in error at a few points, and careful checking is necessary before selecting an appropriate ANN configuration for material characterization.

Several network configurations were tried for the second problem, where the strains ε were used as input to predict the stresses σ and the tangent modulus $d\sigma/d\varepsilon$ given by equations (1) and (2). The two networks with the most accurate results were

(1) Case I, 1-20-2.11 (26 000 training cycles)
(2) Case II, 1-20-2.21 (4000 training cycles)

Both networks have identical configurations with 1 input unit, 2 output units, and 20 hidden units. They only differ in the number of training pairs used. For case I, 11 of the 21 input-output pairs were used and for case II all 21 input-output pairs were used. For both cases all the 21 pairs were used for propagation, resulting in rote memorization for the second network model. The maximum allowed errors in training were 0.2 and 8 percent for cases I and II, respectively. Figure 6 contains the plot for the exact curve from equation (1) and the predicted stress values from cases I and II. The relative errors in stresses are shown in figure 7. The errors for case I were within ±1.5 percent for all the points except at two points where they were nearly 15 percent. The errors for case II were within ±11 percent, making it less accurate than the case I ANN model. Figure 8 shows the plot of tangent modulus versus strain. The relative errors are plotted in figure 9. A trend similar to the stress prediction can be observed here. The inaccuracy of the case II ANN model in predicting results can be attributed to the maximum error that was allowed for training the network. However, a low maximum error leads to a large number of training cycles, which may not be feasible for some problems.

Plate Problem

For the plate problem two input units were used to supply the values of x and b/a. The two output units were for the bending moment factors M_x and M_y, as defined in reference 15. Three different values were chosen for the number of hidden units. A set of 45 input-output pairs was used for training. A different set of 25 pairs was used for obtaining the bending moments at intermediate points. Table I shows the number of cycles and the maximum and root mean square (rms) errors obtained in training the ANN models that were used for the plate problems. This table shows that the 2-15-2 network model had the smallest maximum error.

For the plate problem it was difficult to plot the predicted bending moment factors with the exact solution. Therefore, an absolute relative error distribution in predictions by different ANN models using the training data set are shown in figure 10(a) as a bar chart. These predictions can be considered as a rote memorization because the same data were used for interpolation purposes that were used for training. The results were extremely accurate for all the cases. Approximately 90 percent of the predicted values had errors that were below 3 percent.

Figure 10(b) shows the same quantities as discussed before. However, in this case a different set of data points was used for predicting the bending moment factors for the plate problem than was used for training. This could be termed "generalization" by the network. In this case 84 percent of the predicted values had errors that were below 3 percent, showing very good generalization capability for all the constructed ANN models. Overall, for the plate problem the ANN approach gave extremely good results. For a closer look at the predicted and exact values of the bending moment factors for the plate problem, see table II.

Shell Problem

For the elliptical paraboloid shell problem three input units were used for x/a, y/b, and c_1/c_2, defining the location of the points at which the stress resultants are computed and the geometry of the shell. The three output units were used for the three coefficients for the stress resultants as defined in reference 16. Three ANN configurations were tried with 6, 10, and 15 hidden units, respectively. The network configuration with 6 hidden units had a very low rate of convergence and was discarded. The network with 10 hidden units has a maximum error of 0.03 and an rms error of 0.008 with 4504 cycles. The network with 15 hidden units was allowed to run for 22 439 cycles with a maximum error of 0.039 and an rms error of 0.005, which was less than that for the second configuration. However, note that for all these configurations most of the error reduction was accomplished in the first few thousand cycles and after that the convergence rate was very low. For training purposes 100 input-output pairs were used. For interpolation at intermediate points a separate set of 25 pairs was used that included a value of infinity for the coefficient for N_{xy} at five points.

Once again it was difficult to plot the predicted results versus the exact results; therefore an error distribution was computed for the predicted values when the training set and the intermediate points were used for propagation. Only the network model with configuration 3-15-3 was used because it had the smallest rms error. The results are plotted in figure 11. The error distribution shows that the predicted results were most accurate for the coefficients for N_y and least accurate for the coefficients for N_{xy}. It also shows that the prediction accuracy for the training set was extremely high (i.e., 96 percent of the predicted values had errors that were below 3 percent for the coefficients for N_y). The interpolation accuracy for the shell problem was low relative to that for the plate problem. This could be attributed to the small magnitudes of these coefficients. However, in the case of the coefficients for N_{xy}, at five points the actual magnitude was infinity. The ANN model cannot be trained for this value. For a closer look at the magnitudes of all three coefficients of the stress resultants at 125 points, which included the training and intermediate data sets, see table III. It can be observed that the actual numbers are much closer than shown by the error distributions on the plots.

CONCLUSIONS

For all the problems the artificial neural network (ANN) approach led to very small files containing the weights and biases that were used for reconstructing the original functions. It captured all the essential characteristics of these functions, leading to a significant amount of data compression. Also, the trained networks in their present forms for the material characterization could easily be incorporated with minimal modifications into an existing finite element program.

The ANN approach for functional approximation offers a viable alternative to other methods that are used for similar purposes. It is capable of mapping multidimensional functions as shown by the different solutions to the problems. All the ANN models that were trained in this study were considerably smaller than the networks reported in other studies. The results show that ANN approximations are very good for associative recall with rote memorization. They can also extract the general trend from the data. However, caution must be exercised in using this type of interpolation, as can be seen from the shell example.

RECOMMENDATIONS AND SUGGESTIONS FOR FUTURE WORK

It is difficult to establish guidelines for configuring an appropriate artificial neural network (ANN) for different problems. Similarly, it is not possible to predict a priori the number of cycles needed for training an accurate ANN. Therefore, there is a strong need to establish some of these guidelines either by mathematical proofs or by an extensive numerical experimentation. The backpropagation algorithm has a tendency to move toward a lower convergence rate in the training process. This problem can be partially alleviated by changing the learning rate and the momentum term in the learning equation. It is suggested to try other ANN methods, such as a counterpropagation network, to investigate the convergence rate during training and to achieve more accurate results.

REFERENCES

1. Funahasi, K.-I.: On the Approximate Realization of Continuous Mapping by Neural Network. Neural Networks, vol. 2, no. 3, 1989, pp. 183–192.

2. Hornik, K.; Stinchcombe, M.; and White, H.: Multilayer Feedforward Networks Are Universal Approximators. Neural Networks, vol. 2, no. 4, 1989, pp. 359–366.

3. Hornik, K.; Stinchcombe, M.; and White, H.: Universal Approximation of an Unknown Mapping and Its Derivatives Using Multilayer Feedforward Networks. Neural Networks, vol. 3, no. 4, 1990, pp. 551–560.

4. Hornik, K.: Approximation Capabilities of Multilayer Feedforward Networks. Neural Networks, vol. 4, 1991, pp. 251–257.

5. Rumelhart, D.E.; Hinton, G.E.; and Williams, R.J.: Learning Internal Representation by Error Propagation. Parallel Distributed Processing: Explorations in the Microstructure of Cognition, D.E. Rumelhart and J.L. McClelland, eds., M.I.T. Press, Cambridge, MA, 1986, pp. 318–362.

6. Sejnowski, T.J.; and Rosenberg, C.R.: NeTtalk: A Parallel Network That Learns To Read Aloud. Neurocomputing: Foundations of Research, J.A. Aderson and E. Rosenfield, eds., MIT Press, Cambridge, MA, 1988, pp. 662–672.

7. Cottrell, G.W.; Munro, P.; and Zisper, D.: Learning Internal Representations from Gray-Scale Images: An Example of Extensional Programming. Proceedings of Ninth Annual Conference of the Cognitive Science Society, Lawrence Erlbaum Associates, Hillsdale, NJ, 1987, pp. 461–463.

8. Lapedes, A.; and Farber, R.: Nonlinear Signal Processing Using Neural Networks: Prediction and System Modelling. Report LA–UR–87–2662, Los Alamos National Laboratory, NM, 1987.

9. Rehak, D.E.; Thewalt, C.; and Doo, L.: Neural Network Approaches in Structural Mechanics Computations. Presented at the Structures Congress '89: Computer Utilization in Structural Engineering, San Francisco, CA, May 1–5, 1989.

10. Troudet, T.; and Merrill, W.: A Real Time Neural Network Estimator of Fatigue Life. NASA TM–103117, 1990.

11. Berke, L.; and Hajela, P.: Applications of Artificial Neural Networks in Structural Mechanics. Lecture Series on Shape and Layout Optimization of Structural Systems. Proceedings of the Conference, International Center for Mechanical Science, Italy, July 16-20, 1990, Springer-Verlag, New York, 1992.

12. Brown, D.A.; Murthy, P.L.N.; and Berke, L.: Applications of Artificial Neural Networks to Composite Ply Micromechanics. NASA TM–104365, 1991.

13. Ghaboussi, J.; Garrett, J.H., Jr.; and Wu, X.: Knowledge-Based Modeling of Material Behavior With Neural Networks. J. Eng. Mech., vol. 117, no. 1, Jan. 1991, pp. 132–153.

14. McCauley, A.D.: Optical Neural Network for Engineering Design. IEEE 1988 National Aerospace and Electronics Conference, NAECON 1988, Vol. 4, IEEE, New York, 1988, pp. 1302–1306.

15. Timoshenko, S.P.; and Woinowsky-Krieger, S.: Theory of Plates and Shells, McGraw-Hill, New York, 1959.

16. Billington, D.P.: Thin Shell Concrete Structures. Second ed., McGraw-Hill, New York, 1982.

17. Baffes, P.T.: NETS User's Guide. Software Technology Branch, NASA Lyndon B. Johnson Space Center, Houston, TX, Sept. 1989.

TABLE I.—NEURAL NETWORK CONFIGURATIONS WITH CORRESPONDING MAXIMUM AND RMS ERRORS AND NUMBER OF CYCLES FOR PLATE PROBLEM

ANN configuration	Maximum error	rms error	Number of cycles
2-6-2	0.0431	0.0145	6 000
2-10-2	.0263	.0092	30 000
2-15-2	.0180	.0060	23 000

TABLE II.—NUMERICAL FACTORS FOR BENDING MOMENTS OF SIMPLY SUPPORTED RECTANGULAR PLATE UNDER UNIFORM PRESSURE FOR ANN CONFIGURATION 2-15-2 AND EXACT SOLUTION

(a) Bending moment M_x; interpolation at training set

b/a	Solution	$x = 0.1a$	$x = 0.2a$	$x = 0.3a$	$x = 0.4a$	$x = 0.5a$
		M_x at $y = 0$				
1.0	ANN	0.0227	0.0343	0.0421	0.0462	0.0472
	Exact	.0209	.0343	.0424	.0466	.0479
1.2	ANN	.0253	.0424	.0536	.0597	.0607
	Exact	.0256	.0432	.0545	.0607	.0627
1.4	ANN	.0288	.0504	.0644	.0720	.0738
	Exact	.0297	.0509	.0649	.0730	.0755
1.6	ANN	.0325	.0571	.0734	.0822	.0846
	Exact	.0330	.0572	.0736	.0831	.0862
1.8	ANN	.0356	.0623	.0804	.0905	.0933
	Exact	.0357	.0623	.0806	.0913	.0948
2.0	ANN	.0381	.0662	.0858	.0972	.1004
	Exact	.0378	.0663	.0861	.0978	.1017
2.5	ANN	.0416	.0723	.0943	.1080	.1126
	Exact	.0413	.0729	.0952	.1085	.1129
3.0	ANN	.0431	.0754	.0988	.1137	.1187
	Exact	.0431	.0763	.1000	.1142	.1189
4.0	ANN	.0443	.0791	.1037	.1186	.1224
	Exact	.0445	.0791	.1038	.1185	.1235

(b) Bending moment M_y; interpolation at training set

b/a	Solution	$x = 0.1a$	$x = 0.2a$	$x = 0.3a$	$x = 0.4a$	$x = 0.5a$
		M_y at $y = 0$				
1.0	ANN	0.0170	0.0295	0.0398	0.0460	0.0479
	Exact	.0168	.0303	.0400	.0459	.0479
1.2	ANN	.0174	.0315	.0421	.0477	.0495
	Exact	.0174	.0315	.0417	.0480	.0501
1.4	ANN	.0174	.0316	.0419	.0475	.0496
	Exact	.0175	.0315	.0418	.0481	.0502
1.6	ANN	.0171	.0309	.0409	.0465	.0492
	Exact	.0171	.0309	.0411	.0472	.0492
1.8	ANN	.0167	.0300	.03966	.0453	.0483
	Exact	.0167	.0301	.0399	.0459	.0479
2.0	ANN	.0162	.0290	.0384	.0439	.0469
	Exact	.0162	.0292	.0387	.0444	.0464
2.5	ANN	.0153	.0270	.0356	.0409	.0430
	Exact	.0152	.0272	.0359	.0412	.0430
3.0	ANN	.0146	.0258	.0337	.0389	.0403
	Exact	.0145	.0258	.0340	.0390	.0406
4.0	ANN	.0140	.0246	.0322	.0371	.0381
	Exact	.0138	.0246	.0322	.0369	.0384

TABLE II.—Concluded.

(c) Bending moment M_x; interpolation at intermediate points

b/a	Solution	$x = 0.1a$	$x = 0.2a$	$x = 0.3a$	$x = 0.4a$	$x = 0.5a$
		M_x at $y = 0$				
1.1	ANN	0.0239	0.0383	0.0478	0.0529	0.0538
	Exact	.0234	.0389	.0486	.0541	.0554
1.3	ANN	.0270	.0465	.0592	.0661	.0675
	Exact	.0277	.0472	.0599	.0671	.0694
1.5	ANN	.0306	.0539	.0691	.0774	.0795
	Exact	.0314	.0544	.0695	.0783	.0812
1.7	ANN	.0341	.0598	.0771	.0866	.0892
	Exact	.0344	.0599	.0773	.0874	.0908
1.9	ANN	.0370	.0644	.0833	.0940	.0970
	Exact	.0368	.0644	.0835	.0948	.0985

(d) Bending moment M_y; interpolation at intermediate points

b/a	Solution	$x = 0.1a$	$x = 0.2a$	$x = 0.3a$	$x = 0.4a$	$x = 0.5a$
		M_y at $y = 0$				
1.1	ANN	0.0173	0.0309	0.0415	0.0473	0.0490
	Exact	.0172	.0311	.0412	.0475	.0493
1.3	ANN	.0175	.0317	.0422	.0477	.0496
	Exact	.0175	.0316	.0417	.0482	.0503
1.5	ANN	.0173	.0313	.0415	.0471	.0495
	Exact	.0173	.0312	.0415	.0478	.0498
1.7	ANN	.0169	.0305	.0403	.0459	.0488
	Exact	.0169	.0306	.0405	.0466	.0486
1.9	ANN	.0164	.0295	.0390	.0446	.0476
	Exact	.0165	.0297	.0393	.0451	.0471

TABLE III.—COEFFICIENTS FOR COMPUTING STRESS RESULTANTS IN ELLIPTIC PARABOLOID SHELL FROM EXACT SOLUTION AND ANN PREDICTION

(a) $c_1/c_2 = 1.0$

y/b	Solution	$x = 0.00a$			$x = 0.25a$			$x = 0.50a$			$x = 0.75a$			$x = 1.00a$		
		N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}
0	ANN	0.255	0.244	0.001	0.258	0.241	0.004	0.319	0.180	0.009	0.402	0.097	0.005	0.499	0	0.001
	Exact	.250	.250	0	.267	.233	0	.318	.182	0	.399	.101	0	.500	0	0
.25	ANN	.232	.267	.002	.238	.261	.025	.298	.201	.061	.384	.115	.070	.499	0	.105
	Exact	.233	.267	0	.25	.250	.029	.301	.199	.068	.389	.111	.096	.500	0	.108
.50	ANN	.122	.277	.003	.198	.301	.062	.302	.197	.166	.356	.143	.205	.499	0	.243
	Exact	.182	.318	0	.199	.301	.068	.250	.250	.140	.350	.150	.210	.500	0	.243
.75	ANN	.097	.402	0	.124	.375	.079	.145	.354	.212	.248	.251	.347	.499	0	.458
	Exact	.101	.399	0	.111	.389	.096	.150	.350	.210	.250	.250	.356	.500	0	.465
1.00	ANN	0	.499	0	0	.499	.098	0	.499	.234	.001	.498	.473	.493	.006	.508
	Exact	0	.500	0	0	.500	.108	0	.500	.244	0	.500	.465	0	0	inf.

(b) $c_1/c_2 = 0.8$

y/b	Solution	$x = 0.00a$			$x = 0.25a$			$x = 0.50a$			$x = 0.75a$			$x = 1.00a$		
		N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}
0	ANN	0.289	0.210	0	0.301	0.198	0.003	0.343	0.156	0.009	0.415	0.084	0.006	0.499	0	0.001
	Exact	.289	.211	0	.304	.196	0	.347	.153	0	.416	.084	0	.500	0	0
.25	ANN	.270	.229	.002	.267	.232	.025	.329	.170	.061	.395	.104	.075	.499	0	.100
	Exact	.270	.230	0	.285	.215	.034	.331	.169	.065	.406	.094	.091	.500	0	.101
.50	ANN	.145	.354	.003	.230	.269	.069	.317	.182	.178	.370	.129	.200	.499	0	.226
	Exact	.213	.287	0	.228	.272	.069	.277	.223	.139	.369	.131	.201	.500	0	.229
.75	ANN	.116	.383	0	.139	.360	.092	.167	.332	.217	.272	.227	.340	.499	0	.438
	Exact	.119	.381	0	.130	.370	.100	.169	.331	.215	.270	.230	.353	.500	0	.443
1.00	ANN	.001	.498	.003	0	.499	.114	0	.499	.248	.001	.498	.485	.484	.015	.508
	Exact	0	.500	0	0	.500	.114	0	.500	.255	0	.500	.480	0	0	inf.

TABLE III.—Continued.

(c) $c_1/c_2 = 0.6$

y/b	Solution	$x = 0.00a$			$x = 0.25a$			$x = 0.50a$			$x = 0.75a$			$x = 1.00a$		
		N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}
0	ANN	0.333	0.166	0	0.351	0.148	0.002	0.375	0.124	0.009	0.434	0.065	0.006	0.499	0	0.001
	Exact	.336	.164	0	.348	.152	0	.383	.117	0	.436	.064	0	.500	0	0
.25	ANN	.312	.187	.002	.305	.194	.023	.364	.135	.056	.411	.088	.077	.498	.001	.089
	Exact	.316	.184	0	.329	.171	.031	.367	.133	.060	.426	.074	.081	.500	0	.089
.50	ANN	.177	.322	.004	.267	.232	.073	.335	.163	.183	.389	.110	.185	.499	0	.205
	Exact	.252	.248	0	.267	.233	.067	.312	.188	.132	.392	.108	.185	.500	0	.208
.75	ANN	.141	.358	0	.157	.342	.100	.193	.306	.218	.298	.201	.329	.499	0	.408
	Exact	.143	.357	0	.155	.345	.103	.197	.304	.216	.296	.204	.342	.500	0	.413
1.00	ANN	.001	.498	.001	0	.499	.125	0	.499	.263	.001	.498	.492	.460	.039	.508
	Exact	0	.500	0	0	.500	.120	0	.500	.265	0	.500	.494	0	0	inf.

(d) $c_1/c_2 = 0.4$

y/b	Solution	$x = 0.00a$			$x = 0.25a$			$x = 0.50a$			$x = 0.75a$			$x = 1.00a$		
		N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}
0	ANN	0.395	0.104	0	0.414	0.085	0.001	0.425	0.074	0.007	0.463	0.036	0.004	0.498	0.001	0.001
	Exact	.395	.105	0	.403	.097	0	.425	.075	0	.459	.041	0	.500	0	0
.25	ANN	.368	.131	.001	.363	.136	.017	.411	.088	.044	.439	.060	.065	.497	.002	.065
	Exact	.374	.126	0	.383	.117	.026	.410	.090	.049	.451	.049	.065	.500	0	.070
.50	ANN	.227	.272	.003	.318	.181	.066	.369	.130	.168	.419	.080	.154	.498	.001	.167
	Exact	.307	.193	0	.319	.181	.060	.357	.143	.115	.419	.081	.156	.500	0	.173
.75	ANN	.179	.320	.001	.185	.314	.097	.232	.267	.209	.334	.165	.306	.498	.001	.358
	Exact	.180	.320	0	.192	.308	.101	.235	.265	.208	.331	.169	.316	.500	0	.363
1.00	ANN	.001	.498	.001	0	.499	.127	0	.499	.275	.001	.498	.495	.404	.010	.507
	Exact	0	.500	0	0	.500	.125	0	.500	.274	0	.500	.506	0	0	inf.

TABLE III.—Concluded.

(e) $c_1/c_2 = 0.2$

y/b	Solution	$x = 0.00a$			$x = 0.25a$			$x = 0.50a$			$x = 0.75a$			$x = 1.00a$		
		N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}	N_y	N_x	N_{xy}
0	ANN	0.459	0.040	0	0.469	0.030	0	0.471	0.028	0.003	0.486	0.013	0.002	0.499	0	0
	Exact	.462	.038	0	.465	.035	0	.473	.027	0	.485	.015	0	.500	0	0
.25	ANN	.437	.062	.001	.439	.060	.008	.461	.038	.024	.472	.026	.040	.497	.002	.034
	Exact	.446	.054	0	.451	.049	.014	.462	.038	.027	.480	.020	.034	.500	0	.038
.50	ANN	.320	.179	.002	.396	.103	.044	.427	.072	.118	.460	.039	.098	.498	.001	.105
	Exact	.388	.112	0	.396	.104	.040	.414	.086	.074	.456	.044	.098	.500	0	.108
.75	ANN	.252	.247	.001	.248	.251	.075	.309	.190	.172	.396	.103	.249	.497	.002	.261
	Exact	.248	.252	0	.261	.239	.088	.303	.197	.174	.383	.117	.246	.500	0	.262
1.00	ANN	.002	.497	.001	.001	.498	.110	.001	.498	.271	.002	.497	.494	.333	.165	.506
	Exact	0	.500	0	0	.500	.128	0	.500	.280	0	.500	.510	0	0	inf.

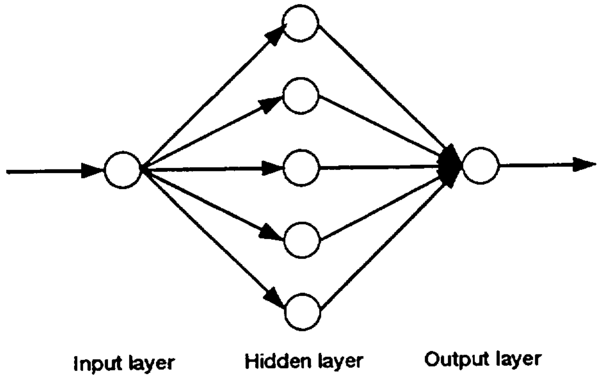


Figure 1.— Configuration of a neural network.

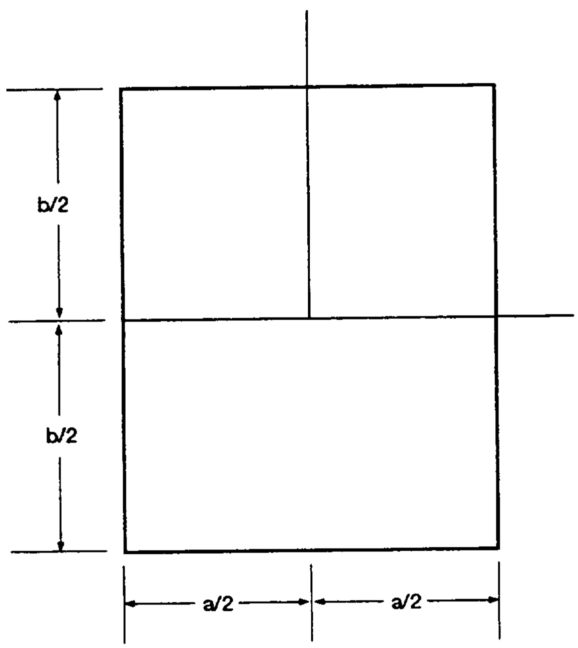


Figure 2.— Simply supported rectangular plate.

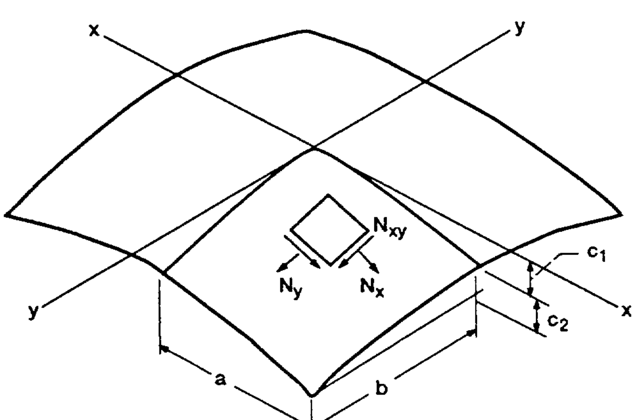
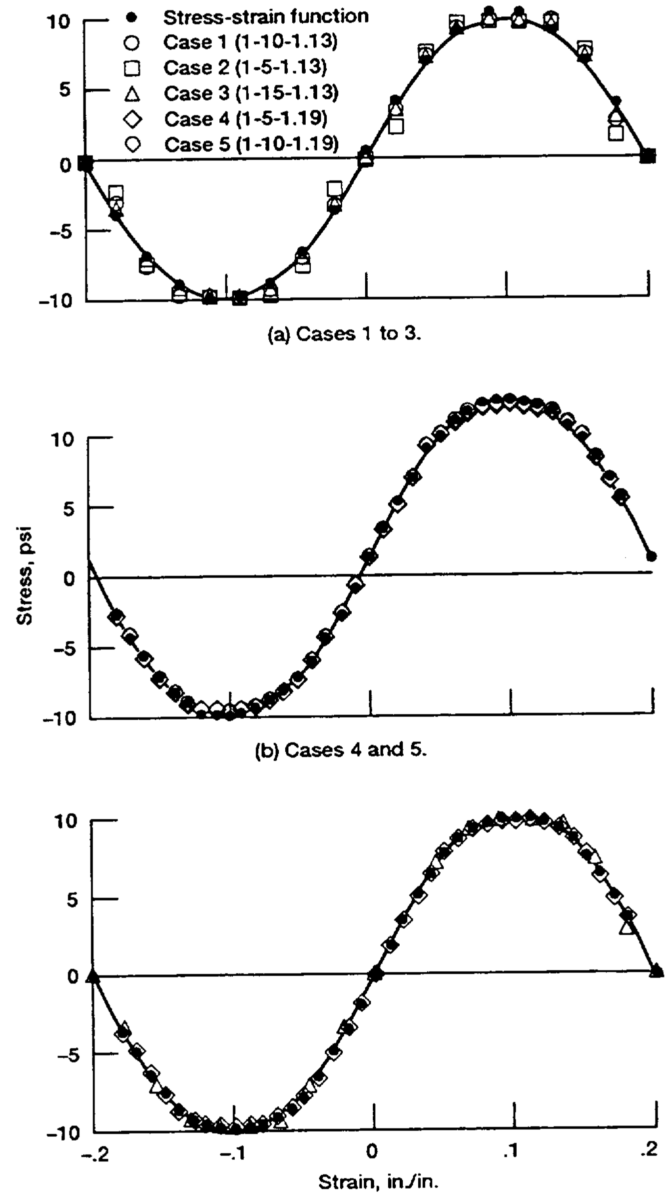


Figure 3.— Elliptic paraboloid shell geometry and stress resultants.



(a) Cases 1 to 3.

(b) Cases 4 and 5.

(c) Cases 3 and 4.

Figure 4.—Neural network predictions for cases 1 to 5.

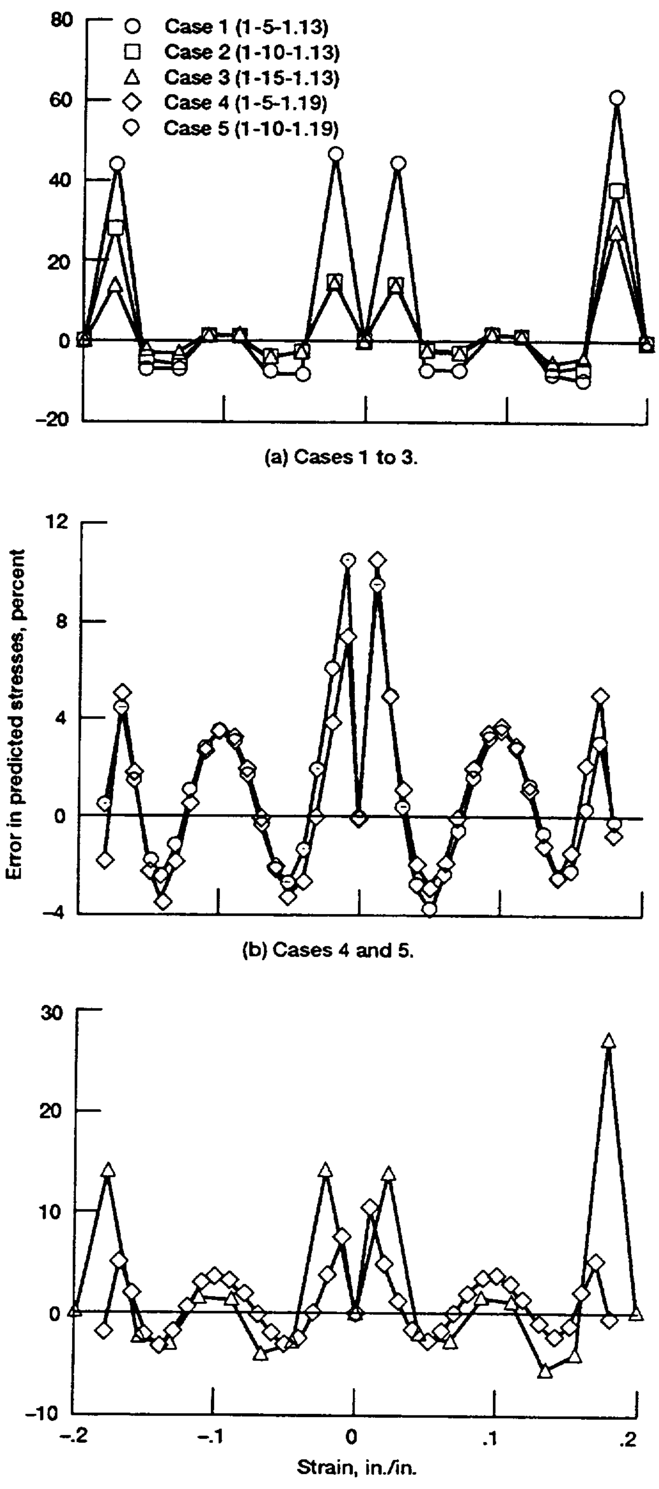


Figure 5.—Errors in neural network interpolation for cases 1 to 5.

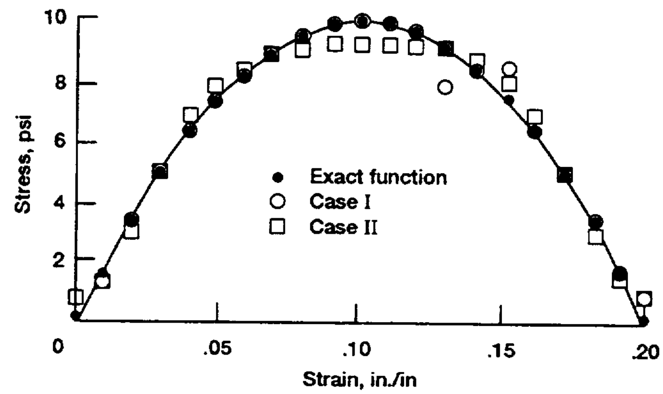


Figure 6.— Neural network stress predictions for cases I and II.

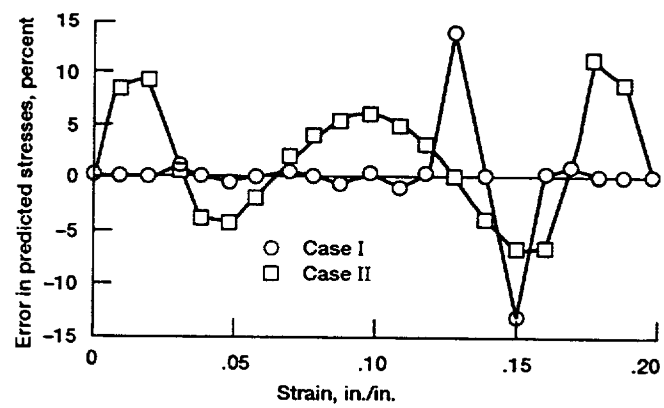


Figure 7.— Errors in neural network stress interpolation for cases I and II.

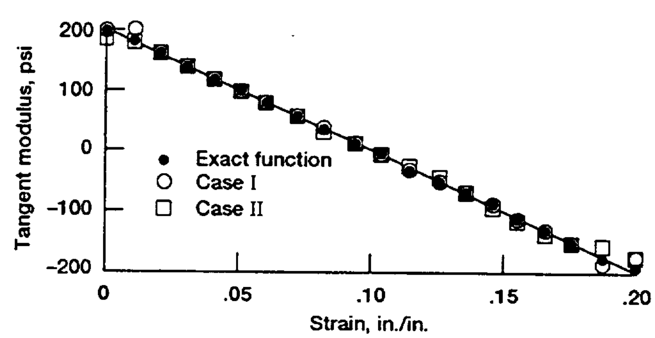


Figure 8.— Neural network tangent modulus predictions for cases I and II.

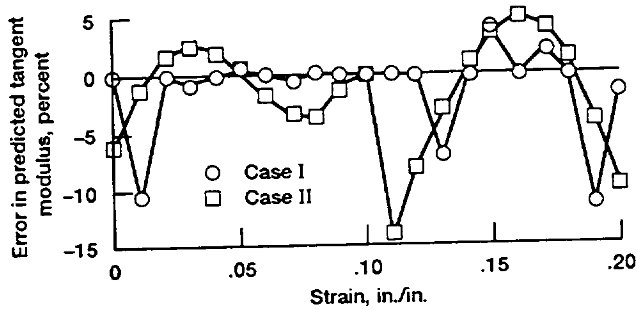


Figure 9.— Error in neural network tangent modulus interpolation for cases I and II.

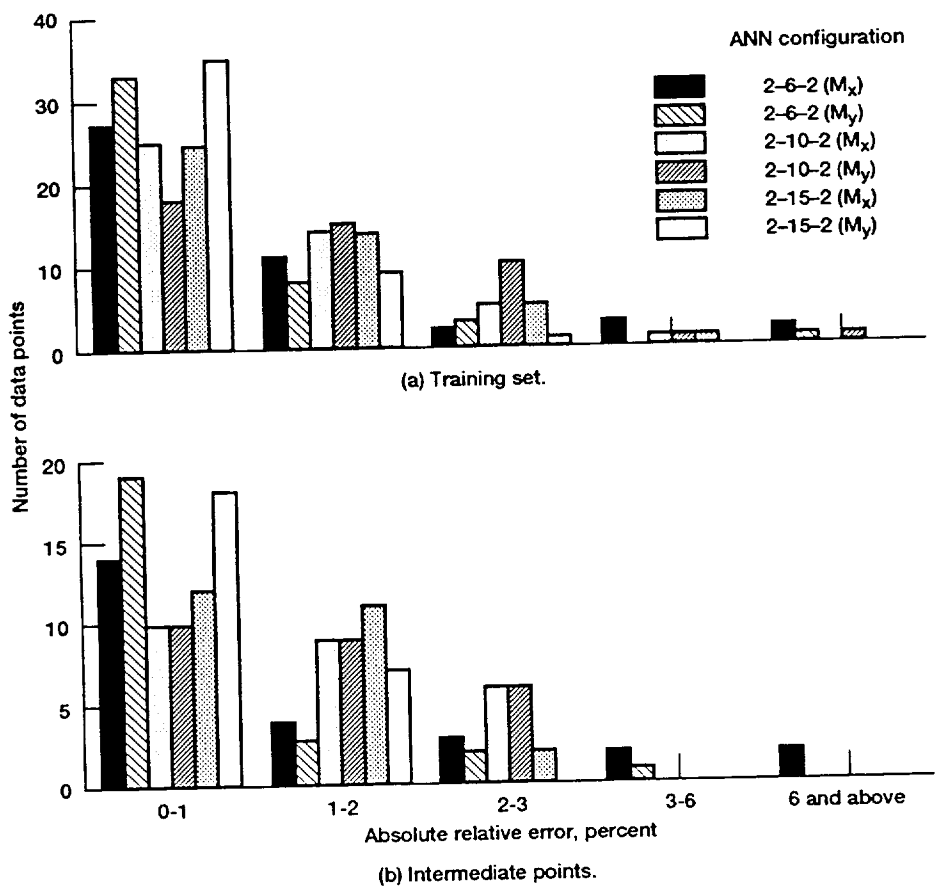
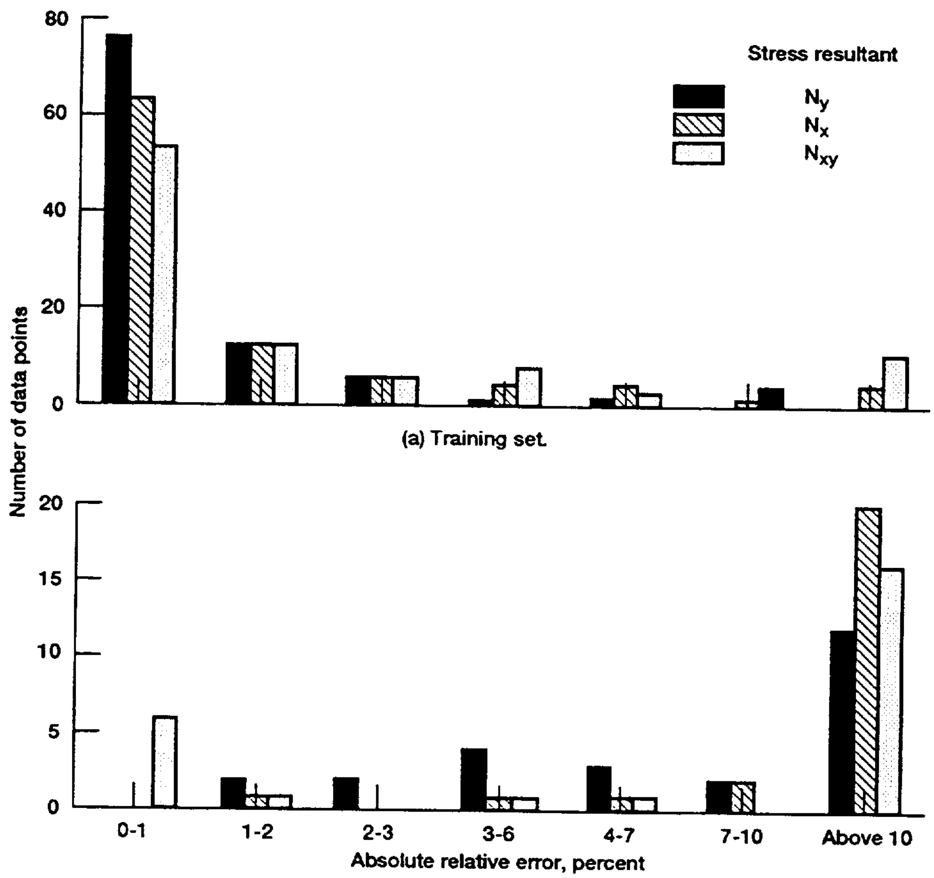


Figure 10.—Error distribution for plate problem.



(b) Intermediate points.

Figure 11.—Error distribution for elliptical paraboloid shell problem for ANN model 3-15-3.

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13. ABSTRACT *(Maximum 200 words)*

The artificial neural networks (ANN) methodology is an outgrowth of research in artificial intelligence. In this study the feed-forward network model that was proposed by Rumelhart, Hinton, and Williams was applied to the mapping of functions that are encountered in structural mechanics problems. Several different network configurations were chosen to train the available data for problems in materials characterization and structural analysis of plates and shells. By using the recall process the accuracy of these trained networks was assessed.

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