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ENHANCEMENT EFFECTS IN POLARIMETRIC RADAR RETURNS: PHASE DIFFERENCE STATISTICS

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ABSTRACT

In this paper, the probability density functions (pdfs) of the co- and cross-polarized phase differences are derived for backscatter from vegetation using the coherent and incoherent scattering theories. Unlike previous derivations, no assumptions or observations other than the applicability of the Central Limit Theorem (CLT), the low fractional volume of the medium, the reciprocity of the scatterers, and the azimuthal symmetry of the scatterer's orientation statistics, are employed. Everything else follows logically via the mathematics. The difference between the coherent theory and the incoherent theory is referred to as the backscatter enhancement effect. The influence of this enhancement effect on the phase difference pdfs is examined and found to be important under combined conditions of scatterer anisotropy and appropriate reflection coefficient values.

Keywords: Polarimetric, Backscatter, Vegetation, Phase Statistics.

INTRODUCTION

The phase difference statistics of polarimetric radar returns from vegetation have been a subject of growing interest in the remote sensing community. This is due to the possibility that a strong dependence between the biophysical parameters and the measured phase difference statistics of the backscattered electric field components exists. Eom and Boerner [1] first derived the single-look co-polarized phase difference pdf as a function of one parameter: the amplitude of the correlation coefficient of the two scattered field components. This was subsequently generalized by Touzi and Lopes [2] to include a second parameter: the phase of the correlation coefficient. A multi-look co-polarized phase difference pdf was later derived by Lopes et al. [3]. Both co- and cross-polarized phase difference pdfs have been derived by Sarabandi [4] in terms of the Mueller matrix elements. Again, the co-polarized pdf is shown to be completely specified by two parameters: the degree of correlation and the polarized phase difference, as named in [4].

In this study, the co- and cross-polarized phase difference pdfs for backscattering from vegetation are derived using the coherent and incoherent theories, and the two are compared. The vegetation is modeled as a layer containing a random distribution of uncorrelated discrete scatterers over a flat surface. By considering a large number of scatterers, the CLT is employed and a multivariate Gaussian distribution for the real and imaginary parts of the backscattered fields results. The analysis confirms certain assumptions in [4] that appear to be made from observations; namely, the independence of the co-polarized and cross-polarized scattered fields. By assuming reciprocity of the scatterers and azimuthal symmetry of the scatter's orientation statistics, we show the latter to be true for dipole scatterers and conjecture its correctness for larger scatterers. In addition, the two parameters which completely specify the co-polarized phase difference pdf are related to analytical expressions obtained via each theory.

Since the coherent (or Distorted Born Approximation (DBA)) theory is based on the addition of fields and the incoherent (or first-order Vector Transport (VT-1)) theory is based on the addition of powers, certain interference terms appear in the DBA theory which do not appear in the VT-1 theory. These terms are due to the coherent interaction of counter-propagating fields following a scatterer-ground path, and give rise to the backscatter enhancement effect. Consequently, the results considered will look at how the enhancement effect influences the phase difference pdfs for model parameters which simulate vegetation canopies. In addition, since the co-polarized phase difference pdf is completely specified by two parameters, these parameters will also be examined as the angle of incidence, for example, is varied.

PROBLEM FORMULATION

The co- and cross-polarized phase difference pdfs for the electromagnetic backscatter from vegetation are derived in this section. The vegetation is modeled as a layer of thickness d consisting of a sparse distribution of identical electrically-thin lossy scatterers. The scatterers are assumed to be uncorrelated and uniformly distributed azimuthally, but can be assigned arbitrary elevation statistics. The underlying ground is represented by a lossy dielectric half-space having a flat surface.

The resulting backscattered field with polarization p, due to an incident field of polarization q, can be expressed in the radiation zone as

$$\mathcal{E}^s_{pq}(\mathbf{x},\mathbf{s}) = \frac{\exp(ik|\mathbf{x}|)}{|\mathbf{x}|}\, E_{pq}(\mathbf{s}) \ , \ p,q \in \{h,v\} \qquad (1)$$

where

$$E_{pq}(\underline{s}) = \sum_{n=1}^{N} e_{pq}(\underline{s}_n) = \sum_{n=1}^{N} (X^{(n)}_{pq} + Y^{(n)}_{pq}) = X_{pq} + iY_{pq} \qquad (2)$$

and h and v represent horizontal and vertical polarization, respectively. The variable, $e_{pq}(\underline{s}_n)$, in (2) represents the scattered field pattern for a single scatterer located at $\underline{s}_n$ in the layer containing N scatterers. Both $E_{pq}(\underline{s})$ and $e_{pq}(\underline{s}_n)$ are complex quantities.

As in [1]-[4], the Central Limit Theorem (CLT) is applied. The CLT states that the pdf of a sum of independent random variables (rvs) approaches a Gaussian as $N\to\infty$. Hence, if we assume that the $X^{(n)}_{pq}$'s, as well as the $Y^{(n)}_{pq}$'s, are independent rvs, then by the CLT, X_{pq} and Y_{pq} are each Gaussian rvs. Furthermore, since p and q can each assume two different states, a total of eight Gaussian rvs result.

Assigning $X_1 = X_{vv}$, $X_2 = Y_{vv}$, $X_3 = X_{hh}$, $X_4 = Y_{hh}$, $X_5 = X_{vh}$, $X_6 = Y_{vh}$, $X_7 = X_{hv}$ and $X_8 = Y_{hv}$, the joint Gaussian pdf can be written as

$$f_{\mathbf{X}}(x_1,\ldots,x_8) = \frac{1}{16\pi^4|\Lambda|^{1/2}}\exp\left(-\frac{1}{2}(\mathbf{x}^T\Lambda^{-1}\mathbf{x})\right) \qquad (3)$$

where Λ is the covariance matrix and is given by

$$\Lambda = (\langle X_i X_j\rangle) \quad , \quad i,j \in \{1,\ldots,8\} \qquad (4)$$

and $|\Lambda|$ denotes its determinant. Also, $\mathbf{x}^T$ denotes the transpose of the column vector $\mathbf{x} = (x_i)$, $i = 1,\ldots,8$. Using (2), it is observed that

$$\langle X_{pq}X_{p'q'}\rangle = \frac{1}{2}\,\mathcal{R}e(\langle E_{pq}E^*_{p'q'}\rangle + \langle E_{pq}E_{p'q'}\rangle) \qquad (5a)$$

$$\langle Y_{pq}Y_{p'q'}\rangle = \frac{1}{2}\,\mathcal{R}e(\langle E_{pq}E^*_{p'q'}\rangle - \langle E_{pq}E_{p'q'}\rangle) \qquad (5b)$$

$$\langle X_{pq}Y_{p'q'}\rangle = -\frac{1}{2}\,\mathcal{I}m(\langle E_{pq}E^*_{p'q'}\rangle - \langle E_{pq}E_{p'q'}\rangle) \qquad (5c)$$

$$\langle Y_{pq}X_{p'q'}\rangle = \frac{1}{2}\,\mathcal{I}m(\langle E_{pq}E^*_{p'q'}\rangle + \langle E_{pq}E_{p'q'}\rangle) \qquad (5d)$$

However, by the DBA theory (which is valid for small fractional volume, $\epsilon \ll 1$), we have the analytical results

$$\langle E_{pq}E^*_{p'q'}\rangle = (A/4\pi)\,\sigma^\circ_{pqp'q'} + o(\epsilon) \qquad (6)$$

and

$$\langle E_{pq}E_{p'q'}\rangle = o(\epsilon) \qquad (7)$$

with

$$\sigma^\circ_{pqp'q'} = \sigma^\circ_{pqp'q'}(d) + \sigma^\circ_{pqp'q'}(dri) + \sigma^\circ_{pqp'q'}(drc) \qquad (8)$$

Here, σ° is the total polarimetric scattering coefficient and is defined in the regular manner as (see, for example, Borgeaud, et. al.[5])

$$\sigma^\circ_{pqp'q'} = \lim_{\substack{|x|\to\infty \\ A\to\infty}} \frac{4\pi|x|^2}{A}\langle \mathcal{E}^s_{pq}\mathcal{E}^{s*}_{p'q'}\rangle \quad , \; p,q,p',q' \in \{h,v\} \qquad (9)$$

and $\sigma^\circ(d)$, $\sigma^\circ(dri)$, and $\sigma^\circ(drc)$ represent, respectively, the direct, direct-reflected incoherent, and direct-reflected coherent contributions to the total scattering coefficient. The three contributions are shown in Figure 1. It should be mentioned that (6) becomes of order ϵ (i.e. $o(\epsilon)$) for very thin slabs and consequently a more detailed analysis, retaining terms of order ϵ, is needed for this special case.

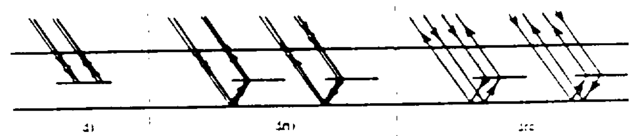

Figure 1. The three different contributions to the total scattering coefficient σ°

To zeroth order in ϵ, then, (5) with (6) and (7) becomes

$$\langle X_{pq}X_{p'q'}\rangle = \langle Y_{pq}Y_{p'q'}\rangle = \frac{A}{8\pi}\,\mathcal{R}e(\sigma^\circ_{pqp'q'}) \qquad (10a)$$

$$\langle X_{pq}Y_{p'q'}\rangle = -\langle Y_{pq}X_{p'q'}\rangle = \frac{-A}{8\pi}\,\mathcal{I}m(\sigma^\circ_{pqp'q'}) \qquad (10b)$$

Explicit analytical expressions have been derived for the different contributions to σ° for all polarization combinations, and can be found scattered in several papers (see Chauhan et. al.[6], for instance). Below, for the purpose of demonstrating the independence of the co- and cross-polarized scattered fields, we give the analytical expression for the direct contribution to σ°_{pppq} ($p\neq q$), i.e.

$$\sigma^\circ_{pppq}(d) = 4\pi\hat{\rho}\,\sigma_{pppq}(d)\left[\frac{1-e^{i\epsilon[2\Delta K^-_p - \Delta K^{-*}_p - \Delta K^{-*}_q]d}}{-i[2\Delta K^-_p - \Delta K^{-*}_p - \Delta K^{-*}_q]}\right] \qquad (11)$$

where

$$\sigma_{pppq}(d) = \overline{f_{pp}(-\hat{i}^-,\hat{i}^-)f^*_{pq}(-\hat{i}^-,\hat{i}^-)} \qquad (12a)$$

$$\Delta K^-_\nu = \frac{2\pi\hat{\rho}}{k_o\cos\theta_i}\,\overline{f_{\nu\nu}(\hat{i}^-,\hat{i}^-)} \quad , \; \nu \in \{p,q\} \qquad (12b)$$

Here $f_{pq}(-\hat{i}^-,\hat{i}^-)$ represents the scattering amplitude of a single scatterer with incident radiation of polarization q in a direction $\hat{i}^-$, and scattered radiation of polarization p in a direction $-\hat{i}^-$ (i.e. the backscatter direction); the overbar denotes the average with respect to orientation; the variable $\hat{\rho}$ represents the density of scatterers; and d represents the vegetation layer thickness.

It can be shown, at least for dipole scatterers (but probably also for larger scatterers), that $\sigma_{pppq}(d) = 0$ ($p \neq q$). In a similar manner, $\sigma_{pppq}(dri) = \sigma_{pppq}(drc) = 0$. In addition, by reciprocity, $\sigma_{pppq} = \sigma_{pqpp} = \sigma_{qppp} =$

0 $(p \neq q)$ also holds. Thus, the polarimetric matrix P which results from the DBA theory reduces, for backscatter, to

$$P = \begin{pmatrix} \sigma^\circ_{hhhh} & \sigma^\circ_{hhhv} & \sigma^\circ_{hhvh} & \sigma^\circ_{hhvv} \\ \sigma^\circ_{hvhh} & \sigma^\circ_{hvhv} & \sigma^\circ_{hvvh} & \sigma^\circ_{hvvv} \\ \sigma^\circ_{vhhh} & \sigma^\circ_{vhhv} & \sigma^\circ_{vhvh} & \sigma^\circ_{vhvv} \\ \sigma^\circ_{vvhh} & \sigma^\circ_{vvhv} & \sigma^\circ_{vvvh} & \sigma^\circ_{vvvv} \end{pmatrix} \rightarrow \begin{pmatrix} \sigma^\circ_{hhhh} & 0 & 0 & \sigma^{\circ *}_{vvhh} \\ 0 & \sigma^\circ_{vhvh} & \sigma^\circ_{vhvh} & 0 \\ 0 & \sigma^\circ_{vhvh} & \sigma^\circ_{vhvh} & 0 \\ \sigma^\circ_{vvhh} & 0 & 0 & \sigma^\circ_{vvvv} \end{pmatrix} \quad (13)$$

Making use of (10) and (13), the covariance matrix of (4) becomes

$$\Lambda = \begin{pmatrix} \Lambda_{co} & 0 \\ 0 & \Lambda_{cross} \end{pmatrix} \quad (14)$$

where

$$\Lambda_{co} = \frac{A}{8\pi} \begin{pmatrix} \sigma^\circ_{vvvv} & 0 & \mathcal{R}e(\sigma^\circ_{vvhh}) & -\mathcal{I}m(\sigma^\circ_{vvhh}) \\ 0 & \sigma^\circ_{vvvv} & \mathcal{I}m(\sigma^\circ_{vvhh}) & \mathcal{R}e(\sigma^\circ_{vvhh}) \\ \mathcal{R}e(\sigma^\circ_{vvhh}) & \mathcal{I}m(\sigma^\circ_{vvhh}) & \sigma^\circ_{hhhh} & 0 \\ -\mathcal{I}m(\sigma^\circ_{vvhh}) & \mathcal{R}e(\sigma^\circ_{vvhh}) & 0 & \sigma^\circ_{hhhh} \end{pmatrix} \quad (14a)$$

and

$$\Lambda_{cross} = \frac{A}{8\pi} \begin{pmatrix} \sigma^\circ_{vhvh} & 0 & \sigma^\circ_{vhvh} & 0 \\ 0 & \sigma^\circ_{vhvh} & 0 & \sigma^\circ_{vhvh} \\ \sigma^\circ_{vhvh} & 0 & \sigma^\circ_{vhvh} & 0 \\ 0 & \sigma^\circ_{vhvh} & 0 & \sigma^\circ_{vhvh} \end{pmatrix} \quad (14b)$$

and the joint Gaussian pdf can now be expressed as the product

$$f_{\mathbf{X}}(x_1,\ldots,x_8) = f_{\mathbf{X}_{co}}(x_1,\ldots,x_4) \cdot f_{\mathbf{X}_{cross}}(x_5,\ldots,x_8) \quad (15)$$

where $f_{\mathbf{X}_{co}}$ and $f_{\mathbf{X}_{cross}}$ are the joint Gaussian pdfs for the co-polarized and cross-polarized rvs, respectively. It is evident from (15) that the two sets of rvs are independent.

The cross-polarized covariance matrix given by (14b) is reducible to a 2x2 diagonal matrix with σ°_{vhvh} as the non-zero elements. This implies that

$$f_{\mathbf{X}_{vh}}(x_5,x_6) = f_{\mathbf{X}_{hv}}(x_7,x_8) = f_{X_{vh}}(x_5) \cdot f_{Y_{vh}}(x_6) \quad (16)$$

where X_{vh} and Y_{vh} are independent rvs with equal variance.

Making the polar coordinate transformations

$$X_{vh} = \rho_{vh}\cos\phi_{vh} \text{ and } Y_{vh} = \rho_{vh}\sin\phi_{vh} \quad (17a)$$

$$X_{pp} = \rho_{pp}\cos\phi_{pp} \text{ and } Y_{pp} = \rho_{pp}\sin\phi_{pp}, \quad p \in (h,v) \quad (17b)$$

it is found that ρ_{vh} is Rayleigh distributed and ϕ_{vh} is uniformly distributed, the two being independent. The conditional phase difference $(\phi_{vh} - \phi_{vv})$ pdf is thus always uniform. The resulting co-polarized pdf, after integrating out ρ_{vv} and ρ_{hh}, is a function of only the difference $\phi = \phi_{hh} - \phi_{vv}$ and can be written as [4]

$$f_\Phi(\phi) = \frac{1-\alpha^2}{2\pi[1-\gamma^2]}\left\{1+\frac{\gamma}{[1-\gamma^2]^{1/2}}\left[\pi/2+\tan^{-1}\frac{\gamma}{[1-\gamma^2]^{1/2}}\right]\right\} \quad (18)$$

where

$$\gamma = \alpha\cos(\phi - \beta) \quad (19)$$

and

$$\alpha = \frac{|\sigma^\circ_{vvhh}|}{[\sigma^\circ_{vvvv}\sigma^\circ_{hhhh}]^{1/2}} \quad (20a)$$

$$\beta = \tan^{-1}\left[\frac{-\mathcal{I}m(\sigma^\circ_{vvhh})}{\mathcal{R}e(\sigma^\circ_{vvhh})}\right]. \quad (20b)$$

The parameters α and β completely specify the pdf (18) and depend on the results obtained via either theory. The parameter α $(0 \le \alpha \le 1)$ denotes the degree of correlation between the co-polarized return signals and is related to the width of the pdf. The parameter β $(-\pi \le \beta \le \pi)$, on the other hand, has been called the polarized phase difference [4] and is related to the positioning of the center of the pdf (18).

RESULTS

The co-polarized phase difference pdf given by (18) is examined in this section for both the coherent (DBA) theory and the incoherent (VT-1) theory. Recall that the difference between both theories stems from the coherent direct-reflected contribution, $\sigma^\circ(drc)$, caused by counter-propagating waves following a scatterer-ground path. It is this contribution which gives rise to the backscatter enhancement effect.

The co-polarized phase difference pdf, via each theory, is shown in Figure 2 for a vegetation-like canopy consisting of a distribution of elliptical discs. The model parameters are given in Table 1. Note from Figure 2, that there is a marked difference between the α parameters obtained using the coherent theory (i.e. α_c) and the incoherent theory (i.e. α_i).

As mentioned earlier, α (given by (20a)) is one of two parameters needed to fully specify the pdf, and is related to the width of the pdf. It is interesting to note, via (20a), that $\alpha_c = \alpha_i$ when either the direct contributions of all the σ°s are much greater than the respective direct-reflected contributions (i.e. $|\sigma^\circ(d)| >> |\sigma^\circ(dr)|$), or vice versa (i.e. $|\sigma^\circ(dr)| >> |\sigma^\circ(d)|$). Thus, in order for $\alpha_i \neq \alpha_c$, a significant direct-reflected contribution must be necessary for at least one of the σ°s in the denominator of (20a).

For the particular situation presented in Figure 2 with the parameters listed in Table 1, the elliptical disc scatterers are very long and thin, and are distributed with a near-horizontal elevation angle pdf. In addition, the dielectric constant of the ground is high. This means that the scattering which takes place due to the discs has a strong preference for horizontal polarization, and this, combined with a relatively high horizontally polarized reflection coefficient, results in a high direct-reflected contribution for σ°_{hhhh} (relative to the direct contribution $\sigma^{\circ}_{hhhh}(d)$ and the total contribution σ°_{vvvv}), and consequently a substantial difference between α_i and α_c. Figure 3 shows the values of α_i and α_c as the angle of the incident wave impinging on the canopy varies. Note that the difference $\alpha_i-\alpha_c$ is about 0.2 for θ_i=55°. This is twice the difference for θ_i=45° (the situation of Figure 2).

In conclusion, it can be said that the pdf obtained from the coherent theory may be very different from the pdf obtained from the incoherent theory; or, more simply, that the backscatter enhancement effect may have an important affect on the resulting phase statistics.

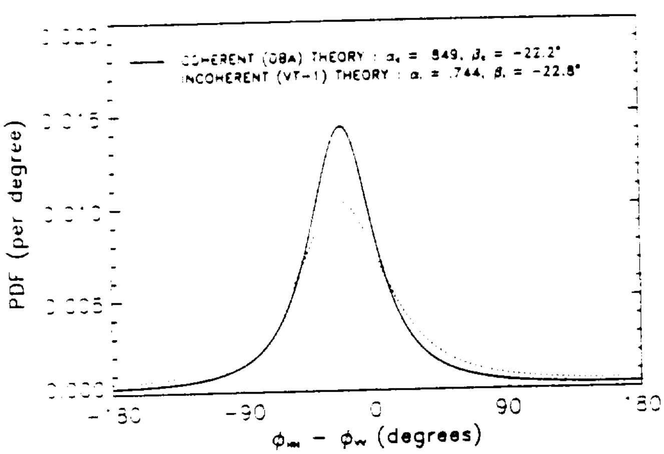

Figure 2. The co-polarized phase difference pdf for backscattering from vegetation characterized by the parameters in Table 1: DBA and VT-1 theories.

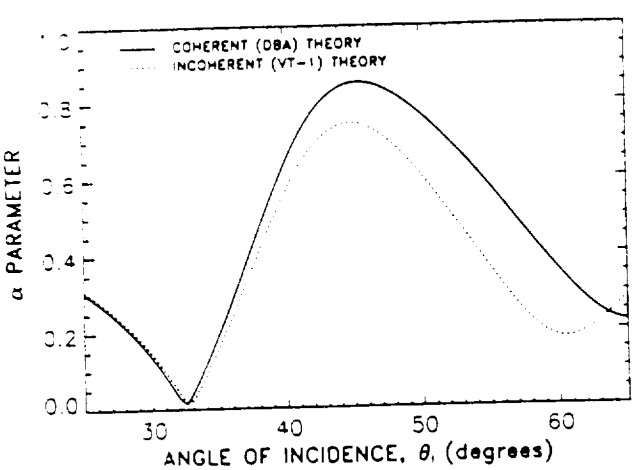

Figure 3. The variation of the α parameter of the co-polarized phase difference pdf vs. angle of incidence: DBA and VT-1 theories.

Vegetation Model Parameters	Value
General parameters:	
Frequency, f	1.5 GHz
Angle of incidence, θ_i	45°
Vegetation layer thickness, d	1 m
Dielectric constant of ground, ϵ_g	23.0+i14.1
Scatterer parameters:	
Elliptical discs with	
semi-major axis, a	10 cm
semi-minor axis, b	1 cm
thickness, t	0.2 mm
dielectric constant, ϵ_r	17.0+i5.55
density, $\hat{\rho}$	4000/m³
elevation pdf, p(θ)	$\begin{cases} \|4 \sin 8\theta\|, & \frac{3\pi}{8} \le \theta \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$

Table 1. Model parameters used to generate Figures 1 and 2.

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