Probability density distribution of velocity differences at high Reynolds numbers

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1. Motivation and objectives

Recent understanding of fine-scale turbulence structure in high Reynolds number flows is mostly based on Kolmogorov's original (1941) and revised (1962) models. The main finding of these models is that intrinsic characteristics of fine-scale fluctuations are universal ones at high Reynolds numbers, i.e., the functional behavior of any small-scale parameter is the same in all flows if the Reynolds number is high enough. The only large-scale quantity that directly affects small-scale fluctuations is the energy flux through a cascade (see remarks by Kraichnan 1974). In dynamical equilibrium between large- and small-scale motions, this flux is equal to the mean rate of energy dissipation ε.

Kolmogorov obtained some general relations which are the foundation of almost all recent models. In particular, he found that for distances r within the inertial subrange, i.e., if $\eta \ll r \ll L$, the moments of velocity difference $\Delta u(r) = u(x) - u(x+r)$ can be described as (see Monin & Yaglom 1967)

$$< [(\Delta u(r)]^p >= C_p \varepsilon^{p/3} r^{p/3} \left(\frac{r}{L}\right)^{\zeta_p - p/3}. \quad (1)$$

Hereafter, x and u denote coordinate and velocity component in the mean flow direction, L and η denote the integral scale and Kolmogorov's viscous scale, C_p are constants, and ζ_p is some unknown function.

The main problem in a creation of the theory of fine-scale turbulence structure is clearly seen in Equation (1). Indeed, this equation was derived from general physical considerations and dimensional analysis. However, it is an incomplete result because the function ζ_p cannot be obtained by such a method, nor, at least at the moment, can it be found directly from the Navier-Stokes equations. The same is valid for the probability density distribution (pdd) $P(\Delta u)$ of Δu which is a more general function than ζ_p. As a result, various heuristic models to describe $P(\Delta u)$ have been proposed (for critical review and classification of the models see She 1991).

We believe that further progress in development of more adequate models of $P(\Delta u)$ is hindered by the lack of reliable experimental data. All known measurements were analyzed by Gagne, Hopfinger & Frisch (1988 hereinafter referred to as G,H&F). Two novel results were obtained there: (i) the functional behavior of the

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tails of the pdd can be represented by $P(\Delta u) \propto \exp(-b(r)|\Delta u/\sigma_{\Delta u}|)$; (ii) the logarithmic decrement $b(r)$ scales as $b(r) \propto r^{0.15}$ when separation r lies in the inertial subrange (symbol σ_φ denotes the rms value of any quantity φ).

The pdd of velocity difference is a very important characteristic for both the basic understanding of fully developed turbulence and engineering problems. Hence, it is important to test the findings (i) and (ii) in high Reynolds number laboratory shear flows.

2. Accomplishments

2.1. Apparatus and measurement techniques

Velocity time series taken in two different high Reynolds number laboratory shear flows were analyzed. The first one was obtained in the large wind tunnel of the Central Aerohydrodynamic Institute (Moscow). The mixing layer between a jet issuing from an elliptical nozzle ($14 \times 24\text{m}^2$) and ambient air was studied. The wind tunnel had an open 24m long working section. Measurements were performed on the line which continued the nozzle wall at a distance $x = 20$m downstream of the nozzle. The free jet velocity was equal to $U_0 = 11.8$m/s.

The second time series was obtained in the return channel of the same wind tunnel. The channel was 175m long and 22m wide. Its height rose linearly from 20m up to 32m. Measurements were done in the plane of symmetry from a tower 5m above floor level.

In both experiments, standard thermoanemometers were used. X-wire probes with perpendicular wires were operated at an overheat ratio of 1.8. Wires were made of platinum-plated tungsten with diameter of 2.5μm. Both the active length and the distance between the wires were 0.5mm. Signals from both wires were filtered to reduce noise level, digitized, and processed on a computer. The low-pass filter cut-off frequency was equal to $f_c = 1.7$kHz. The sampling frequency f_s and one-channel time series length N were equal to 8kHz and 2000000 respectively. These values were chosen to investigate the inertial subrange. To measure the energy dissipation rate ε, the values of f_c, f_s, and N were doubled.

Description of the experiments and analysis of measurement errors (temporal and spatial resolution, statistical convergence, use of Taylor's hypothesis, nonlinearity of the hot-wire response, etc.) can be found in Karyakin, Kuznetsov & Praskovsky (1991).

The main flow characteristics at the measurement points are listed in Table 1 (abbreviations RC and ML stand for return channel and mixing layer). The longitudinal velocity component u was processed at each point. Taylor's hypothesis was used to convert the temporal into the spatial coordinate. The mean energy dissipation rate was estimated using the local-isotropy relation: $\varepsilon = 15\nu < (\partial u/\partial x)^2 >$. The integral length scale L, Taylor's microscale λ, Kolmogorov's scale η, and Reynolds number R_λ were estimated by standard formulas defined by

$$L = \frac{<u>}{\sigma_u^2} \int_0^\infty < [u(t+\tau) - <u>][u(t) - <u>] > d\tau,$$
$$\lambda = \frac{\sigma_u}{\sigma_{\partial u/\partial x}}, \qquad \eta = (\nu^3/\varepsilon)^{1/4}, \qquad R_\lambda = \frac{\sigma_u \lambda}{\nu}, \tag{2}$$

where t is time and τ is a time delay.

TABLE 1.

Apparatus	RC	ML
$< u >, m/s$	10.8	7.87
$\sigma_u, m/s$	1.03	1.67
$\varepsilon, m^2/s^3$	0.11	1.9
λ, mm	46	18
$R_\lambda \cdot 10^{-3}$	3.2	2.0
L, m	4.8	1.3
η, mm	0.41	0.21
$(L/\eta) \cdot 10^{-3}$	12	6.2

Table 1. Main turbulence characteristics at the measurement points.

It is seen from Table 1 that $L/\eta > 6000$. This indicates that in both flows under consideration, fairly large inertial subrange regions should exist.

2.2. Results

It is necessary to determine the inertial subrange bounds for each measurement point since Kolmogorov's definition $\eta \ll r \ll L$ is not exact. It was assumed that the distance r belongs to the inertial subrange if $20\eta \leq r \leq L/5$. The structure functions $< [\Delta u(r)]^p >$ for $p = 2, 3, 4, 6$ are presented in Figure 1, where the bounds are indicated by vertical arrows. The chosen bounds appear plausible: "the two-thirds law" and Kolmogorov's exact equation

$$< [\Delta u(r)]^3 > = -\frac{4}{5}\varepsilon r \tag{3}$$

are in agreement with experimental data within these bounds. (The absolute values of the third-order structure functions are plotted.)

Typical pdd of velocity differences in the two measurement points are presented in Figure 2. Results for three widely different values of r/η within the inertial subrange are presented for every point. All curves in Figure 2 are truncated just before the first point at which $P(\Delta u) = 0$ (no samples in the "bin" of width $\pm 0.2\sigma_{\Delta u}$). Thus, implausible values of $P(\Delta u)$ caused by insufficient samples are ignored. It is seen in Figure 2 that, for sufficiently high amplitudes $|\Delta u/\sigma_{\Delta u}| \geq h$, the obtained pdd may be approximated by the expression proposed by G,H&F

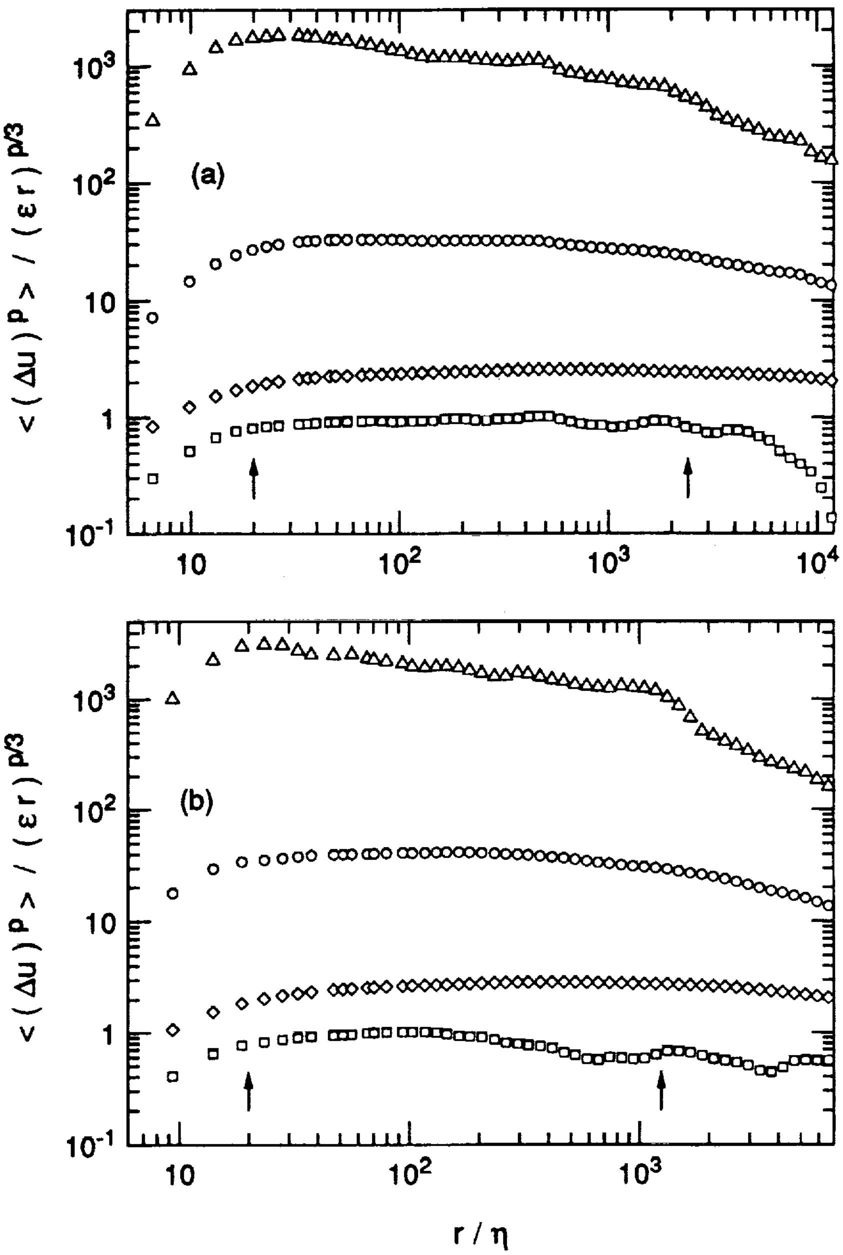

FIGURE 1. Higher-order velocity structure functions (absolute values for $p = 3$). (a), return channel; (b), mixing layer; $\diamond$, $p = 2$; $\square$, 3; $\circ$, 4; $\triangle$, 6. Vertical arrows correspond to the inertial-range bounds.

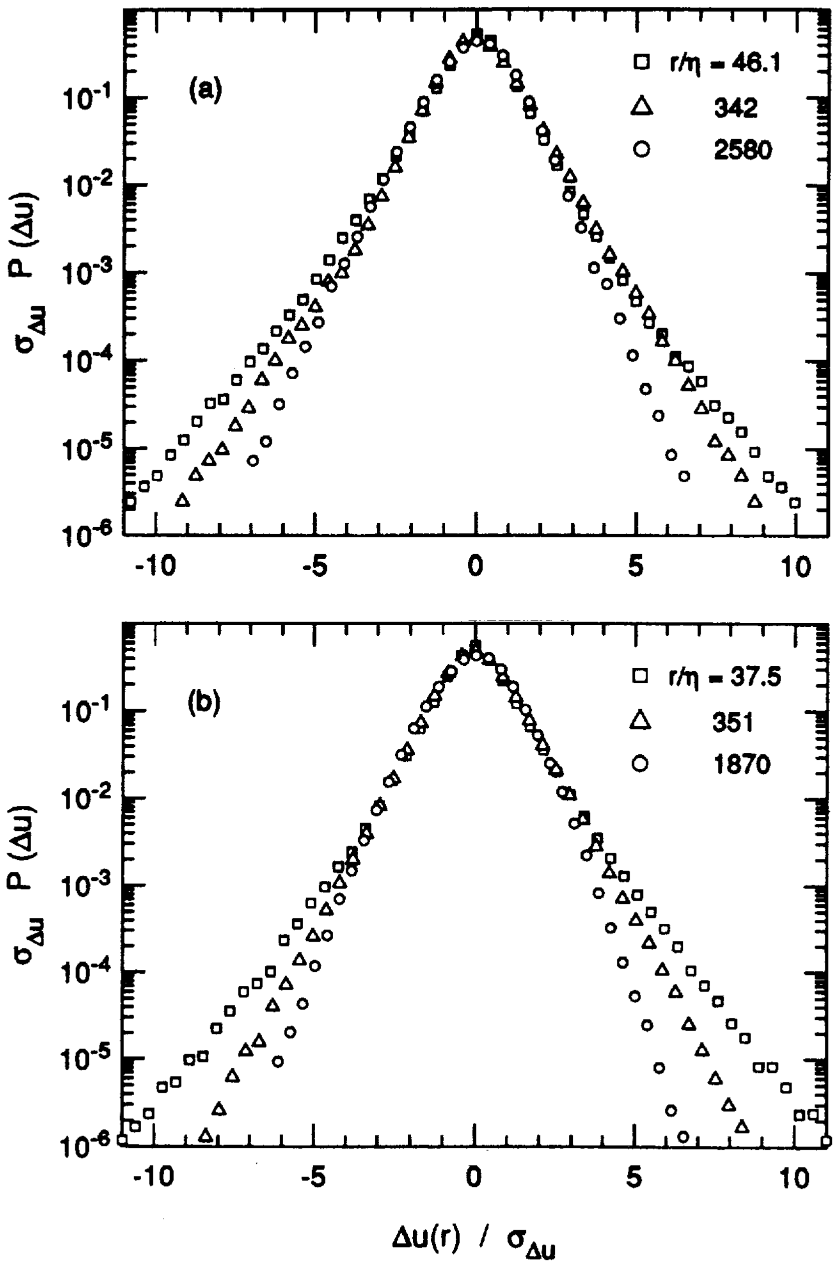

FIGURE 2. The pdd of velocity differences for distances r from the inertial range. (a), return channel; (b), mixing layer.

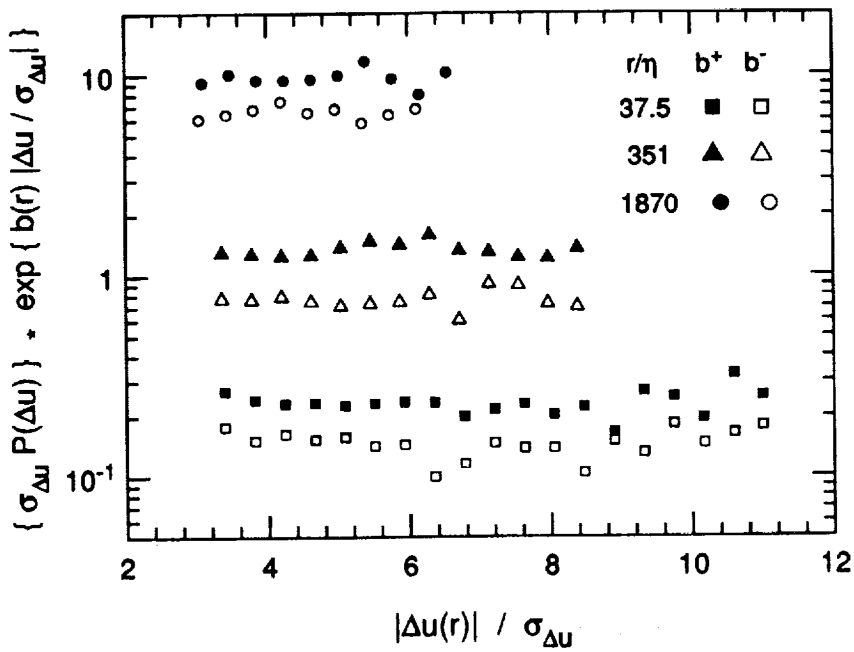

FIGURE 3. The tails of pdd premultiplied by exponentials with appropriate decrements in the mixing layer.

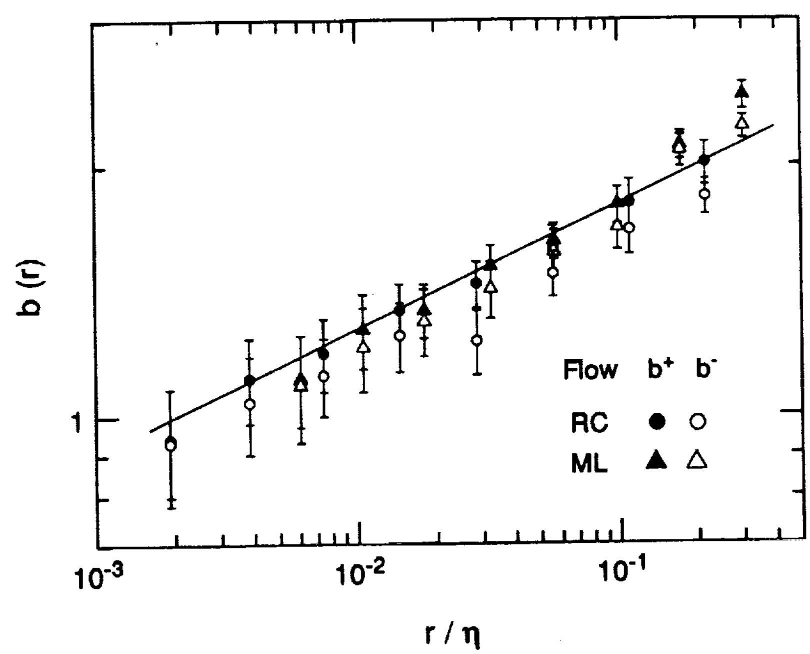

FIGURE 4. Scaling of the logarithmic decrements b^+ and b^-. Solid line corresponds to equation (5) with $\beta = 0.15$.

$$P(\Delta u) \propto \exp(-b(r)|\Delta u/\sigma_{\Delta u}|). \qquad (4)$$

The threshold level h was chosen to be $h = 3$ for all r at all points. Then the values of logarithmic decrements $b(r)$ were estimated for both positive (b^+) and negative (b^-) tails of the pdd by the least-squares method. As an example, the tails of the pdd in the mixing layer premultiplied by exponentials with the decrements so obtained are presented in Figure 3. The scatter of experimental data with respect to the horizontal looks random, i.e. no systematic trends are seen. Thus equation (4) seems to be valid. The rms deviation of the data from the horizontal in Figure 3 (and in plots of $\sigma_{\Delta u} P(\Delta u) \cdot \exp\{b(r)|\Delta u/\sigma_{\Delta u}|\}$ for other r and points) was treated as a measurement error for decrements b^+ and b^-. Such errors are supposed to include both deviations from assumed behavior, equation (4), and statistical scatter. The measured values of these logarithmic decrements with appropriate error bars are presented in Figure 4. In this figure, the solid line corresponds to the power-law scaling

$$b(r) \propto r^{\beta} \qquad (5)$$

with scaling exponent $\beta = 0.15$, proposed by G,H&F. It has to be noted that G,H&F verified equation (5) with $\beta = 0.15$ in four different flows: in an atmospheric boundary layer, $R_\lambda = 3000$ (Van Atta & Park 1971); in the return channel of the Modane wind tunnel, $R_\lambda = 2720$ (Gagne 1987); in an axisymmetric jet, $R_\lambda = 852$ and 536, and in a rectangular duct, $R_\lambda = 515$ (Anselmet *et al.* 1984). It is seen in Figure 4 that our experimental results are in agreement with those analyzed by G,H&F. Thus it can be assumed that equations (4) and (5) with $\beta \approx 0.15$ describe some universal behavior in high Reynolds number flows.

It has to be noted that no dependence of $b(r)$ on Reynolds number is seen in Figure 4. This result is opposite to that obtained by G,H&F.

3. Future plans

It is important to investigate two other relevant questions: (1) Within the inertial subrange, is there any "measurable" dependence of $b(r)$ on Reynolds number? and (2) For the viscous subrange, how should equations (4) and (5) be changed?

We believe new experiments in a wide range of Reynolds numbers are desirable to clarify these questions.

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