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## Why does preferential diffusion strongly affect premixed turbulent combustion?

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### 1. Introduction

Combustion of premixed reactants in a turbulent flow is a classical but unresolved problem. The key problem is to explain the following data: the maximal turbulent and laminar burning velocities  $u_t$  and  $u_L$  occur at different equivalence ratios  $\Phi$  (for a review of experimental data, see Kuznetsov & Sabel'nikov (1990), Chapter 6). Some examples of fuel behavior are:  $H_2$  displays a large shift of the maximum value of  $u_t$  towards the lean mixture,  $CH_4$  has a small lean shift,  $C_2H_6$  has no shift,  $C_3H_8$  has a moderate rich shift, and benzene has a large rich shift. The shift can be quite large. For example, the maximum of  $u_t$  occurs at  $\Phi = 1.0$  for  $H_2$  and at  $\Phi = 1.4$  for benzene, while, the maximum of  $u_L$  is at  $\Phi = 1.7$  for  $H_2$  and  $\Phi = 1$  for benzene. This shift is observed over a large range of Damköhler number, but is more pronounced at low Damköhler number. A theory should explain these data.

It can be seen that the fuels in the above-mentioned sequence are arranged according to the ratio of the molecular diffusivities of oxygen ( $D_o$ ) and fuel ( $D_f$ ). It can, therefore, be hypothesized that preferential diffusion strongly affects turbulent combustion in all regimes. The correlation between  $u_t$  and blow-off velocity, based on this assumption, is very good over a wide range of conditions (Kuznetsov & Sabel'nikov (1990), Chapter 6).

If the reaction zone were distributed, the influence of molecular diffusivity variation should be unimportant since only large-scale fluctuations should affect the reaction and their properties do not depend on Reynolds number. On the other hand, if the flame front were thin (which was verified by direct numerical simulation (Rutland *et al.*, 1989; Trouvé, 1991)), the Reynolds number (based on the flame front thickness) would be small and the influence of preferential diffusion could be significant.

It is known that the equivalence ratio varies along a curved flame if  $D_f \neq D_o$ . However, the mean flame radius of curvature is much larger than the laminar flame thickness  $\delta_L$ . Therefore, significant influence of preferential diffusion should occur only if the flame propagation speed varies with flame curvature. This conclusion agrees with Zel'dovich's long-standing idea about the important role of leading points of a flame (the points  $L_1, L_2, \dots$  in Fig. 1 which are deep inside the fresh mixture).

The main objective of this paper is to prove Zel'dovich's hypothesis. An equation for the mean flame surface area density (MFSAD) will be employed for this purpose.

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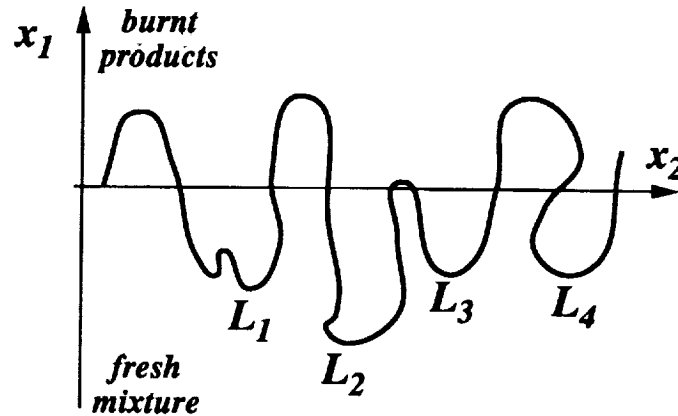


FIGURE 1. A sketch of turbulent flame front.  $L_1, L_2, \dots$  are leading points.

a popular treatment (for a review see Candel, *et al.*, 1990). An equation for MFSAD can be written

$$\left( \frac{\partial \Sigma_f}{\partial t} + \langle u_k \rangle \frac{\partial \Sigma_f}{\partial x_k} \right) = \frac{\partial}{\partial x_l} D_t \frac{\partial \Sigma_f}{\partial x_l} + H \Sigma_f - G \Sigma_f^2 \quad (1)$$

where  $D_t$  is a turbulent diffusivity,  $H$  and  $G$  are positive functions depending on turbulence properties, and  $\Sigma_f$  is MFSAD.

The second objective of this paper is to suggest a different approach to the derivation of the equation for MFSAD. It is based on the pdf equation for the reaction progress variable  $C$  and the relation between the pdf and MFSAD (Kuznetsov & Sabel'nikov (1990)). As will be seen later, this treatment suggests an entirely different closure assumption.

## 2. Assumptions

We wish to prove the hypothesis about the crucial role of leading points for the case of equal diffusivities, so we consider the case in which the diffusivities of all species and heat are equal.

The main properties of turbulent flames can be correlated using non-dimensional numbers based on two characteristics of the laminar flame,  $u_L$  and  $\delta_L$  (Kuznetsov & Sabel'nikov (1990), Chapter 6). If this is the case, the detailed chemical source term is irrelevant, and it is important only to model  $u_L$  and  $\delta_L$  correctly. This reasoning is widely used (see Rutland, *et al.*, 1989). It can then be assumed that the source term in the equation for the reaction progress variable,  $C$ , depends only on  $C$  itself so that the governing equation is

$$\rho \frac{\partial C}{\partial t} + \rho u_k \frac{\partial C}{\partial x_k} = \nabla D \rho \nabla C + W(C) \quad (2)$$

We shall assume that the dependence of  $D$  or  $\rho$  on  $C$  is known.

If only these two quantities (i.e.,  $u_L$  and  $\delta_L$ ) are important, it is sufficient to study the high activation energy limit, i.e., it is assumed that

$$W = 0 \quad \text{if} \quad C \leq C_o. \quad (3)$$

$$\int_{C_o}^1 wdC = \text{const} \quad (4)$$

where  $C_o \rightarrow 1$ . It is clear from Eq. (4) that  $u_L$  remains constant as  $C_o \rightarrow 1$ .

If  $C_o \rightarrow 1$ , the chemical reaction zone is located on the surface on which  $C = 1^-$ . This definition is required since a large volume is occupied by fully burnt products in which  $C \equiv 1$ . If  $C_o$  is close to unity, the usual analysis of the solution within the reaction zone proves that

$$\frac{\partial C}{\partial n} = \frac{\rho_o u_L}{\rho_b D_b} \quad (5)$$

Here  $n$  is normal to the surface  $C = 1^-$ , and subscripts 0 and  $b$  denote fresh mixture and burnt products. It should be kept in mind that Eq. (5) is exact even when the flamelet model is not valid.

There may be some confusion if the limit  $C_o \rightarrow 1$  is studied. Indeed, Eq. (4) shows that  $W$  tends to  $\text{const} \delta(C-1)$ . At the same time, there is a term proportional to  $\delta(C-1)$  in the pdf (because there is a non-zero volume where  $C = 1$ ). Since there is a term  $\partial(WP)\partial C$  in the equation for the pdf, there can be some confusion. To avoid it, let us choose  $W(C)$  with ( $C_o \neq 1$ ) so that value  $C = 1$  occurs far from the flame. For example, we might let  $W = \text{const}$  if  $C_o < C < 1$ . For the plane, steady laminar flame, one can prove that  $C = 1$  at a finite distance from the flame. The equation for the pdf can be obtained, and only then will the limit  $C_o \rightarrow 1$  be investigated.

It will also be assumed that  $\delta_L$  is much smaller than the Kolmogorov scale. From the mathematical point of view, this means that  $\delta_L \rightarrow 0$  (or equivalently  $D \rightarrow 0$  since  $\delta_L \sim D/u_L$ ) while other quantities are kept constant.

### 3. Some geometrical considerations

According to the Zel'dovich model, reaction takes place only on a surface. Hence there are combustion products ( $C = 1$ ) on just one side of this surface, and the volume of products is not zero, i.e. the pdf of  $C$  contains a singular term proportional to  $\delta(C-1)$ , in agreement with the Bray-Moss (1977) model.

There is another singular term in the Bray-Moss model which is proportional to  $\delta(C)$  and represents pockets of fresh mixture. However, this term should be absent at any finite Reynolds number (Kuznetsov & Sabel'nikov (1990), Chapter 1) since heat spreads simultaneously throughout the whole volume of fluid due to molecular diffusion. This means that pockets of fresh mixture are diluted by the products. The degree of such a dilution is small at large Reynolds number, i.e. this term arises only at infinite Reynolds number. In this limiting case, the flame front should be treated as infinitely thin and the continuous term in the Bray-Moss model should be absent. It will be assumed in the present treatment that the Reynolds number

is finite and, hence, the second singular term is absent. This means that there is a minimal value  $C = C_{min}$  of the reaction progress variable (which depends on distance  $C_{min}$  from the flame).

Hence, one has

$$p = \gamma\delta(C - 1) + F \quad (6)$$

where  $F$  is continuous if  $C_{min} < C < 1$  and  $\gamma$  is the probability of observing the value  $C = 1$ . Clearly,  $\gamma$  can be treated as a combustion efficiency if the flamelet model is valid.

To calculate the continuous term in Eq. (6), one can use the equation (Cant *et al.*, 1990)

$$\frac{d\langle \Sigma | C = const \rangle}{dv} = F \left\langle \frac{dC}{dn} \middle| C = const \right\rangle \quad (7)$$

where  $\Sigma$  is the area of surface on which  $C = const$  (so the left-hand side of Eq. (7) is the mean surface area density). Surfaces with different values of  $C$  have almost the same area since the thickness of the flame is small compared to the Kolmogorov scale. Hence,

$$\frac{d\langle \Sigma | C = const \rangle}{dv} = \Sigma_f(\bar{x}). \quad (8)$$

To calculate the conditionally averaged gradient of the reaction progress variable, let us choose some point on the surface  $C = 1^-$  and the frame moving with the velocity of this point. Then, at a distance much smaller than the Kolmogorov scale and much larger than  $\delta_L$ , one sees an almost plane, steady laminar flame, so that Eq. (2) reduces to

$$\rho_o u_L \frac{dC}{dn} = \frac{d}{dn} D\rho \frac{dC}{dn} + W$$

which is familiar from laminar flame theory. If  $C_o \rightarrow 1$ , the chemical source term can be neglected. Hence, after the integration one has

$$\rho_o u_L C = D\rho \frac{dC}{dn} \quad (9)$$

There are no random parameters, thus

$$\left\langle \frac{dC}{dn} \middle| C = const \right\rangle = \frac{\rho_o u_L}{D\rho} C \quad (10)$$

Combining Eqs. (7), (8), and (10), one has

$$F = \frac{D\rho}{\rho_o u_L C} \Sigma_f \quad (11)$$

It should be kept in mind that Eq. (11) is exact for  $C = 1^-$  since Eq. (5) is exact if  $C_o \rightarrow 1$ . Eq. (11) is approximate for  $C < 1$  since any influence of turbulence on

flame structure was neglected in Eq. (9). This influence is not necessarily small for small  $C$  since surfaces  $C = \text{const} \sim C_{\min}$  are located far from the flame where the structure of the scalar field is heavily affected by the turbulence. This conclusion agrees with direct numerical simulations (Rutland *et al.*, 1989) performed at large Damköhler number.

This is not a serious deficiency of Eq. (11) since the mean value of any function  $\varphi(C)$  can be easily calculated using identities

$$\langle \varphi \rangle = \langle \varphi(C) - \varphi(0) \rangle + \varphi(0) = \gamma\varphi(1) + (1-\gamma)\varphi(0) + \frac{\Sigma_f}{\rho_o u_L} \int_0^1 D\rho \frac{\varphi(C) - \varphi(0)}{C} dC$$

The integrand has no singular points, i.e. the domain  $C \sim 0$  where Eq. (11) is not valid does not play a significant role.

One can see that the pdf of reaction progress variable depends only on two functions of coordinates, the mean flame surface area density  $\Sigma_f$  and the combustion efficiency  $\gamma$ .

#### 4. Exact equations for $\Sigma_f$ and $\gamma$

It is natural to try to obtain equations for  $\Sigma_f$  and  $\gamma$  using an exact (but unclosed) equation for the pdf. Using methods developed by Kuznetsov & Sabel'nikov (1990), one can obtain two equivalent forms of the pdf equation:

$$\frac{\partial}{\partial x_k} [\rho v_k(\vec{x}, C)F + \rho v_k(\vec{x}, 1)\gamma\delta(C-1)] = -\frac{\partial^2}{\partial C^2} N(\vec{x}, C)\rho F - \frac{\partial}{\partial C} WF \quad (12)$$

$$\frac{\partial}{\partial x_k} [\rho v_k(\vec{x}, C)F + \rho v_k(\vec{x}, 1)\gamma\delta(C-1)] = -\frac{\partial}{\partial C} F\Delta(\vec{x}, C) - \frac{\partial}{\partial C} WF \quad (13)$$

where  $v_k$  is a flow velocity averaged at the condition  $C = \text{const}$ ,  $N$  and  $\Delta$  are quantities  $D(\partial C/\partial x_k)^2$  and  $\nabla D\rho\nabla C$  averaged at the same condition. These equations are valid only for the chemistry model adopted in Section 2. The presence of a  $\delta$ -function on the left-hand side can lead to some confusion. To clarify it, let us note the  $F \equiv 0$  if  $C > 1$  and  $F \neq 0$  if  $C < 1$ . It can be guessed that another  $\delta$ -function will appear on the right-hand side on differentiation of the function  $F$ .

Quantities  $N$  and  $\Delta$  can be calculated to first approximation using the flamelet model. For example, using Eq. (9), one has

$$N = \frac{\rho_o^2 u_L^2}{\rho^2 D} C^2. \quad (14)$$

Note that this equation is exact if  $C = 1$  and  $C_o \rightarrow 1$  since Eq. (5) is exact. Similarly, one has

$$\Delta = \frac{\rho_o^2 u_L^2}{D\rho} C. \quad (15)$$

The flamelet model is valid if  $D \rightarrow 0$  and other quantities are kept constant. It is seen from Eq. (15) that  $\Delta \rightarrow \infty$  if  $D \rightarrow 0$ . Therefore, Eq. (13) reduces to

$$\frac{\partial}{\partial c} F \Delta = 0 \quad \text{if } C < C_o.$$

This equation is easily integrated. After the use of Eq. (15), one again has Eq. (11), which was obtained previously using purely geometrical considerations.

Thus Eq. (11) is the first term of the asymptotic expansion of the solution of the pdf equation. The difficulty is that to calculate MFSAD (which appears as a constant of integration), other terms must be investigated, i.e. small, previously neglected terms in Eq. (14) or (15) should be taken into account. This means that the small influence of turbulence on the inner structure of the flame front, flame stretch, and flame curvature should be included. Clearly, in the general case, it will also include effects of preferential diffusion.

There is a simpler approach based on integral relations which can be deduced from the exact Eqs. (12) and (13) without use of the approximate relations (14) and (15). In performing the calculations, one should keep in mind that the pdf is defined for all values of  $C$  ( $-\infty < C < \infty$ ) and

$$F \equiv 0 \quad \text{if } C < 0 \quad \text{or} \quad C > 1. \quad (16)$$

To obtain the first equation, let us integrate Eq. (13) from  $C_o$  to  $C = \infty$ . Using Eqs. (3) and (16) and assuming that  $C_o \rightarrow 0$ , one has

$$\frac{\partial}{\partial x_i} \rho_b v_i(\vec{x}, 1) \gamma = F(\vec{x}, 1^-) \Delta(\vec{x}, 1).$$

Small corrections to Eq. (15) do not play a significant role here. Hence, using Eqs. (11) and (15), one has

$$\frac{\partial}{\partial x_i} \rho_b v_i(\vec{x}, 1) \gamma = \rho_o u_L \Sigma_f. \quad (17)$$

To obtain the second equation, one should multiply Eq. (12) by  $C$  and integrate from  $C = -\infty$  to  $C = C_o$ . Using Eqs. (3) and (16), one has

$$\frac{\partial}{\partial x_i} \int_0^{C_o} \rho v_i F dC = [N F \rho - C \frac{\partial}{\partial C} N F \rho]_{C=C_o}.$$

Assuming  $C_o \rightarrow 1$  and using Eq. (14), which is exact for  $C = 1$ , one has

$$\frac{\partial}{\partial x_i} \int_0^1 \rho v_i C F dC = \left[ \frac{\rho_o^2 u_L^2}{D_b \rho_b} F - \frac{\partial N \rho F}{\partial C} \right]_{C=1}. \quad (18)$$

Note that the second term on the right-hand side of Eq. (18) can not be calculated exactly using Eqs. (11) and (14). To calculate it, let us compare Eqs. (12) and (13). One has

$$\frac{\partial^2}{\partial C^2} N \rho F = \frac{\partial}{\partial C} F \Delta$$

Integrating, one has

$$\frac{\partial}{\partial C} N \rho F = F \Delta. \quad (19)$$

The term  $F \Delta$  can not be calculated exactly from Eqs. (11) and (15). However, the use of Eq. (19) has some advantages since  $\Delta$  is related to a mean velocity of chemical reaction zone which has a clear physical meaning. Since the reaction zone is located where  $C = 1^-$ , its velocity in a laboratory frame  $u_k^{ch}$  is given by

$$u_k^{ch} = -\frac{\partial C}{\partial t} \frac{\partial C}{\partial x_k} / \left( \frac{\partial C}{\partial x_i} \right)^2 \quad \text{if } C = C_o. \quad (20)$$

Using Eqs. (2) and (20), one has

$$u_k^{ch} = \frac{\Delta}{\rho_b} \frac{\partial C}{\partial x_k} / \left( \frac{\partial C}{\partial x_i} \right)^2 - u_k \quad \text{if } C = C_o. \quad (21)$$

Evidently, Eq. (21) is valid if  $C_o \rightarrow 1$ . So the normal component of chemical reaction zone velocity relative to the local velocity of burnt products is

$$\omega = (u_k^{ch} - u_k) \frac{\partial C}{\partial x_k} / \sqrt{\left( \frac{\partial C}{\partial x_k} \right)^2} = -\frac{\Delta}{\rho_b} / \sqrt{\left( \frac{\partial C}{\partial x_i} \right)^2}. \quad (22)$$

It should be kept in mind that  $\omega$  is negative (if the normal to the flame is directed toward the fresh mixture). To avoid confusion, it is convenient to change the sign in Eq. (22) so that  $\omega$  is positive. Using Eq. (15), one has

$$\omega = \frac{D_b}{\rho_o u_L} \Delta. \quad (23)$$

This equation is valid on the surface  $C = 1^-$  for instantaneous values of  $\omega$  and  $\Delta$ . At the same time, one needs to calculate the conditionally averaged value of  $\Delta$ . This means that in the homogeneous case, quantity  $\Delta$  should be averaged over a volume between surfaces  $C = 1^-$  and  $C = 1 - dC$ . Generally speaking, the averages over this volume and over the surface  $C = 1^-$  are different due to the random fluctuation of the distance between the surfaces  $C = 1^-$  and  $C = 1 - dC$ . However, in the studied case, such fluctuations are absent since this distance is

$$dn = \frac{dC}{(\partial C / \partial u)}$$

where  $\partial C / \partial u$  is constant according to Eq. (5). Hence, Eq. (22) can be averaged over surface  $C = 1^-$  to calculate the conditionally averaged value of  $\Delta$ .

Using Eqs. (18), (19), and (23), one has

$$\frac{\partial}{\partial x_i} \int_o^1 \rho v_i C F dC = \frac{\rho_o u_L}{D_b \rho_b} (\rho_o u_L - \rho_b \omega) F|_{C=1}.$$

The approximate Eq. (11) can now be used to obtain

$$\frac{\partial}{\partial x_i} \int_0^1 D \rho^2 v_i dC \Sigma_f = \rho_o^2 u_L (\Theta u_L - \omega) \Sigma_f \quad (24)$$

where  $\Theta = \rho_o / \rho_b$ .

This equation is exact if the flamelet model is used. It is seen that two quantities should be modeled: the conditionally averaged gas velocity and the difference  $\Theta u_L - \omega$ . It is clear that  $\Theta u_L$  is the velocity of a plane steady laminar flame relative to the burnt products. Hence,  $\omega$  would equal to  $\Theta u_L$  if the properties of a flame front in a turbulent flow were exactly the same as those of a laminar flame. Of course, this is not true. In fact, a small difference between  $\Theta u_L$  and  $\omega$  is caused by the flame curvature and strain, which can not be neglected since the left-hand side of Eq. (23) is also small (note that  $D \rightarrow 0$ ).

### 5. Turbulent diffusion in a flame

Consider now the quantity  $v_k$ . It is convenient to begin with the constant density case. Since the flame front is very thin, there is no appreciable velocity difference across it, i.e.  $v_k$  does not depend on  $C$ . In a variable density case, the variation of gas velocity across the flame front can be calculated from the continuity equation. Since the flame front is almost planar, in a frame moving with the front

$$u_k = \frac{\rho}{\rho_o} u_L n_k$$

where  $n_k$  is a unit vector normal to a flame front. Hence, in the laboratory frame,

$$u_k = u_k(\vec{x}, 1^-) + \frac{\rho - \rho_b}{\rho_o} u_L n_k$$

where  $u_k(\vec{x}, 1^-)$  is the gas velocity on the surface  $C = 1^-$ . After averaging at the condition  $C = 1^-$ , one has

$$v_k = v_k(\vec{x}, 1^-) + \frac{\rho - \rho_b}{\rho_o} u_L \langle n_k | C = 1^- \rangle. \quad (25)$$

Consider now the case  $u' \gg u_L$  where  $u'$  is the rms flow velocity. Then the second term on the right-hand side of Eq. (25) can be neglected. Also, the flame moves with almost the same velocity as the gas. Hence, the theory of turbulent diffusion can be applied. This means that the quantity  $v_k$  should be treated as differential operator

$$v_k(\vec{x}, 1^-) = \langle u_k \rangle - D_t \frac{\partial}{\partial x_k} \quad (26)$$

where  $D_t$  is a turbulent diffusivity. Using Eqs. (24) and (26), one has

$$\frac{\partial \langle u_k \rangle \Sigma_f}{\partial x_k} = \frac{\partial}{\partial x_k} D \frac{\partial \Sigma_f}{\partial x_k} + \frac{\rho_o^2 (\Theta u_L - \omega) u_L \Sigma_f}{\int_0^1 D \rho^2 dC} \quad (27)$$



It is worth noting that the validity of Eq. (27) is questionable when  $u' \sim u_L$  because the surface  $C = 1^-$  is moving rapidly relative to the gas. At the same time, the surface on which a non-reactive scalar is constant moves with the Kolmogorov velocity relative to the gas. This is easily seen from Eq. (22) which is also valid for a non-reacting scalar if  $u_k^{ch}$  is the velocity of an isoscalar surface. Obviously, the quantities  $\Delta$  and  $(\partial C / \partial x_i)^2$  have Kolmogorov scaling. Hence, the left-hand side of Eq. (22) also has Kolmogorov scaling, i.e.  $\omega \ll u'$ .

Therefore, it could be expected that if  $u' \sim u_L$ , the quantity  $D_t$  in Eq. (27) is not the turbulent diffusivity of non-reactive scalar. Even counter-gradient diffusion can be expected.

The same approach can be used to close Eq. (17), i.e.

$$\frac{\partial}{\partial x_k} \langle u_k \rangle \gamma = \frac{\partial}{\partial x_k} D_t \frac{\partial \gamma}{\partial x_k} + \Theta u_L \Sigma_f. \quad (28)$$

### 6. Effects of flame curvature

As was mentioned earlier, one can not neglect the difference between  $\Theta u_L$  and  $\omega$ . It is easily seen that if one assumes  $\Theta u_L - \omega = \text{const} - \text{const} \Sigma_f$ , then Eq. (27) reduces to Eq. (1). However, we do not intend to develop a model for this difference since, as will be seen later, we can reach important conclusions by analyzing only the sign of difference  $\Theta u_L - \omega$  in various parts of a flame.

We shall consider only the effects of flame curvature. Obviously, flame curvature in low and high mean temperature regions have opposite predominant signs. This becomes clear from examination of Fig. 1 where a flame homogeneous in  $x_2$  direction is shown. In a low mean temperature region ( $x_2 < 0$ ), the flame front  $x_1(x_2)$  has more minima than maxima. In a high mean temperature region ( $x_2 > 0$ ), the flame front has more maxima than minima. One more example is the Gaussian curve  $x_1(x_2)$ . In this case, one has

$$\left\langle \frac{d^2 x_1}{dx_2^2} \middle| x_1 = \text{const} \right\rangle = - \left\langle \left( \frac{dx_1}{dx_2} \right)^2 \right\rangle \frac{x_1}{\langle x_1^2 \rangle},$$

i.e. the flame curvature is positive in a low mean temperature region and is negative in a high mean temperature region.

It was shown long ago (Markstein, 1964) that  $\Theta u_L > \omega$  if the curvature is negative and  $\Theta u_L < \omega$  if the curvature is positive. So  $\Theta u_L < \omega$  in a high mean temperature part of a flame and  $\Theta u_L > \omega$  in a low mean temperature part of a flame. For the plane turbulent flame shown in Fig. 1, one has

$$\Theta u_L - \omega > 0 \quad \text{if} \quad x_1 \rightarrow -\infty \quad (28)$$

$$\Theta u_L - \omega < 0 \quad \text{if} \quad x_1 \rightarrow +\infty. \quad (29)$$

### 7. Plane turbulent flame

Consider now a steady plane turbulent flame such as the one shown in Fig. 1. One has

$$\begin{aligned} \langle u_1 \rangle &= u_t \quad \text{if } x_1 \rightarrow -\infty \\ \langle u_1 \rangle &= \Theta u_t \quad \text{if } x_1 \rightarrow +\infty. \end{aligned}$$

Let us analyze solutions of Eq. (27) in the regions  $x_1 \rightarrow +\infty$  and  $x_1 \rightarrow -\infty$ , assuming that all coefficients in this equation tend to constants in these limits.

One has

$$\Sigma_f = \text{const} \exp \left\{ \frac{x_1}{2D_t} \left( \Theta u_t \pm \sqrt{\Theta^2 u_t^2 - w} \right) \right\} \quad \text{if } x_1 \rightarrow +\infty$$

where

$$w = \frac{4D_t \rho_o^2 (\Theta u_L - \omega) u_L}{\int_o^1 D \rho^2 dC}.$$

One can see from Eq. (29) that  $w < 0$ ; hence, one solution increases without bound and the other tends to zero as  $x_1 \rightarrow \infty$ .

One has also

$$\Sigma_f = \text{const} \exp \left\{ \frac{x_1}{2D_2} \left( u_t \pm \sqrt{u_t^2 - w} \right) \right\}.$$

One can see from Eq. (28) that  $w > 0$ . So there are two cases. The first one is  $u_t < \sqrt{w}$ , in which case each solution changes sign and is meaningless. The second case is  $u_t \geq \sqrt{w}$ , and both solutions tend to zero as  $x_1 \rightarrow -\infty$ . Therefore, the spectrum of eigenvalues is continuous. Such spectra are often met in the combustion theory and other areas (Barenblatt, *et al.*, 1985). It was proven that the physical meaningful velocity is the minimal value of flame velocity (the other values are due to the initial disturbances). Hence, one has

$$u_t = \sqrt{\frac{4D u_L \rho_o^2 (\Theta u_L - \omega)}{\int_o^1 D \rho^2 dC}}.$$

Since  $u_t$  depends only on value  $w$  deep in the fresh mixture, this proves Zel'dovich's assumption about the crucial role of leading points.

As was said above, the difference  $\Theta u_L - \omega$  depends on flame curvature. In the general case, the value of this difference depends on effects caused by the preferential diffusion. This means that  $\Theta u_L - \omega$  and  $u_L$  should be calculated at an equivalence ratio different from the overall equivalence ratio. This is the cause of a shift between maximal values of  $u_t$  and  $u_L$ .

The sign of this shift can be established by studying simple configuration (say, a spherical flame). It was found that  $\Phi_f < \Phi$  if  $D_t > D_o$  and  $\Phi_f > \Phi$  if  $D_f < D_o$  (Kuznetsov & Sabel'nikov, 1990). Here,  $\Phi_f$  is the equivalence ratio in the reaction zone and  $\Phi$  is the overall equivalence ratio. Therefore, a lean shift should be observed in  $H_2$ -air flame and a rich shift should be observed in  $C_3H_8$ -air flame. This conclusion is fully consistent with experimental data (see Introduction).

## 8. Conclusion

This investigation led to three important conclusions:

- i) The influence of turbulence on the inner structure of a flame front is quite important even when the laminar flame thickness is much smaller than the Kolmogorov scale.
- ii) Turbulent flame propagation depends on conditions within the low temperature part of a flame. Preferential diffusion plays an important role here because flame curvature in this part of the flame has a predominant sign.
- iii) To develop a correct equation for the mean flame surface area density, one should properly model the difference between laminar flame velocity and the actual velocity of a flame front.

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