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Analysis of Interstellar Fragmentation Structure Based On IRAS Images
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The goal of this project was to develop new tools for the analysis of the structure of densely sampled maps of interstellar star-forming regions. A particular emphasis was on the recongition and characterization of nested hierarchical structure and fractal irregularity, and their relation to the level of star formation activity. The panoramic IRAS images provided data with the required range in spatial scale, greater than a factor of 100, and in column density, greater than a factor of 50.

In order to construct a densely sampled column density map of a cloud complex which is both self-gravitating and not (yet?) stirred up much by star formation, a column density image of the Taurus region has been constructed from IRAS data. The primary drawback to using the IRAS data for this purpose is that it contains no velocity information, and the possible importance of projection effects must be kept in mind. Destriped 60 μm and 100 μm IRAS images of a 9° x 9° area (1° = 2.4 pc at a distance of 140 pc) centered on $\alpha(1950) = 4^h30^m00^s$, $\delta(1950) = 26°00'00"$ in the core of the Taurus complex were obtained from IPAC. Each image contained 540^2 pixels of size 1 arcmin (=0.04 pc). The effective resolution is estimated at 2-3 arcmin. Subtraction of galactic emission was performed by fitting two-dimensional polynomials to a number of low-intensity spots on each image. For each pixel the dust temperature was taken as the color temperature derived from the observed 60 μm/100 μm flux ratio, assuming a λ^{-n} wavelength dependence of the far infrared emissivity. The 100 μm optical depth could then be derived from the Planck function using the observed 100 μm intensity. Assuming that the warm dust fraction is a constant, as suggested by other work and by the extensive comparisons mentioned below, the 100 μm optical depth is proportional to the total column density of gas. It was found that the resulting *relative* column density map (the absolute scale of the column densities is irrelevant for the present discussion) was virtually independent of the choice of emissivity law for n = 1 and 2, and also was not sensitive to different choices of background subtractions, except for the smallest optical depths.

The estimated noise at 100 μm is about 0.2 to 1 MJy/sr. For comparison, after subtraction of background (5-9 MJy/sr), the 100 μm intensities were >5 MJy/sr over most of the 100 μm image (>100 MJy/sr in the brightest spots), and were only as low as 1-3 MJy/sr in the darkest "holes." At 60 μm the relative background subtraction is larger, but the noise estimate is only about half as large as at 100 μm. It therefore appears that only the very smallest column densities may be affected by noise.

The resulting range of 100 μm optical depth τ_{100} is 1 x 10^{-5} to 4.4 x 10^{-3} (in the core of the L1495 cloud) for a λ^{-1} emissivity law. Comparison with available studies of extinction and ^{13}CO gives $A_V \approx 2000 \tau_{100}$ for this emissivity law, so the column density range corresponds to $A_V \approx 0.02$-0.05 mag (roughly at the noise level) to $A_V \approx 10$ mag. This range includes both conditions in which the gas is mostly atomic *and* in which it is mostly molecular, eliminating the artificial separation between (HI, reddening) and (CO, extinction) studies.

The validity of the derived column density structure was checked by comparison of various higher-column density ($0.5 \lesssim A_V \lesssim 5$) subregions of the map with gray scale representations of extinction maps for the dark clouds Heiles Cloud 2 (L1534 region), L1495, L1506, L1529, and L1539, and with ^{13}CO maps for Heiles Cloud 2, B216-217-218, and B18(=L1529 region). The agreement between these maps is for the most part very good, and in fact the pixel-to-pixel noise level appears significantly smaller in the IRAS structure, especially compared to the extinction maps. One disagreement appeared to occur in the core of the L1495 cloud, but it turns out that the ^{13}CO and extinction are saturated there; the $C^{18}O$ map of this region is in good agreement with the τ_{100} structure, showing that the IRAS data can be used to probe column densities as large as $A_V \approx 10$ mag, even when there is no internal heat source. Much of the lower-column density structure can be seen by careful inspection of POSS plates. These comparisons, along with independent comparisons of τ_{100} with ^{13}CO column densities and A_V by others demonstrates the ability of IRAS to probe the relative column density structure over a range of at least a factor of 100 in column density. The only major exceptions occur around the

locations of embedded IRAS point sources, where the column densities come out very small. This effect is due to temperature gradients along the lines of sight to the point sources, which cause an overestimate of the appropriate mean temperature and an underestimate of the optical depth. These stellar heating regions can be easily recognized as small dark circular disks in the column density image. Although the effect is minor for Taurus, it should be much more serious in regions with massive star formation.

The main conclusion to be drawn from the resulting image is that, when viewed with large dynamic range in spatial scale and column density, one sees complex, irregular, interconnected structure on all scales, with filaments, chains, tendrils, and cirrus-like structure present. This structure does not resemble the ideas of quasi-static evolution of virialized "clouds" or "clumps" popular in current models, but instead suggests a more dynamically active organizational process. In fact the irregularity and continuity of structure makes it difficult to clearly identify any separate entities which correspond to discrete "clouds," although of course regions with various density contrasts and forms can be operationally distinguished.

While the visual impression of a densely sampled map of a star-forming region can be quite informative, it is of obvious interest to develop quantitative descriptors of structure which can be used to directly compare the observed structure with future numerical hydrodynamic simulations of large spatial dynamic range. In the past, most empirical studies have concentrated on estimating total or average properties for an entire region and cataloguing and searching for correlations between the properties of operationally defined clouds within the mapped region, but not on characterizing the spatial structure itself.

Two-point second order spatial statistics such as the power spectrum, correlation function, and structure function have been evaluated for some regions, but they involve such a severe compression and smearing of the spatial *relational* information, and are so affected by structures whose sizes are a significant fraction of the image size, that they cannot provide an adequate characterization of the complex column density structures being discussed here (Houlahan and Scalo 1989, reprint attached). Even very high-order structure functions, which are used to characterize intermittency in incompressible turbulence, smear out most of the relational information in the data.

One of the characteristic features of complex systems is hierarchical structure, which is apparent in comparisons of maps of interstellar structures at different resolutions and has figured prominently in many older theoretical discussions of fragmentation. The recognition and description of a hierarchical spatial structure is a problem which has apparently not been discussed in the literature. For interstellar structures which can only be viewed as two-dimensional projections, the difficulties are magnified by the fact that projection will make a random three-dimensional distribution of density enhancements with a variety of scales appear somewhat hierarchical, while even a strictly hierarchical three-dimensional structure will appear more randomized due to the effects of projection.

With these considerations in mind, a new method of image analysis, called "structure tree analysis" was designed to recognize and characterize complex structure, especially hierarchical structures, in a manner well-suited for comparison of observations with theory. In addition, the technique automatically produces a catalogue of operationally defined clouds and their properties, and can be used to calculate the fractal dimension of boundary irregularities and estimate the topological genus.

Briefly, the procedure consists of successively thresholding the image at increasing grey levels (e.g., column densities) and identifying "clouds" as areas of connected pixels at each grey level, retaining information on the lineage of each cloud to larger "parent" clouds which were identified at smaller grey level thresholds. If the image is viewed as analogous to a mountain range, with height corresponding to column density, then the clouds are those parts of the plane, at a given height (grey level), that intersect the mountain range. A "path" is a sequence of clouds that preserves connectivity (or lineage). Paths can be illustrated by plotting the position of the centroid of each cloud in the sequence against the corresponding intensities. A "structure tree" is the set of all paths found in a given image.

Because the structure tree is basically a "skeleton image" or "primal sketch" of the observed structure, there is little loss of spatial relationship information in its construction; we have simply reduced the task of describing the complete structure to the more tractable problem of describing a tree. The usefulness of this method for the problem of identifying and characterizing hierarchical structure can be seen by noting that a randomly distributed collection of clouds with a range of scales will produce a tree with a very large branching ratio, while a strictly self-similar hierarchical arrangement yields a "fractal" tree.

In order to actually use the structure trees, it is necessary to find descriptors which are sensitive to the various aspects of the tree structure. These aspects can be classified into two categories depending on whether or not they are invariant to "rubber-sheet" distortions applied to the tree. For example, branching nodes (branching sites) remain invariant whether the tree is stretched or expanded in either intensity or space—or any other transformations that preserve lineage. On the other hand, the density contrast and scale reduction encountered in going from one level to the next (which are measures of how a hierarchical system distributes its mass among its children and its levels) are not invariant to rubber-sheet distortions.

Therefore it is to be expected that branching nodes should play a key role in any tree descriptor designed to be sensitive to features such as the number of levels and the degree of fragmentation present in any hierarchical structure. For example, the average number of branching sites encountered in following a path from tip down to the root, and the progeny ratio, which is the ratio of progenies for successive branching nodes (a node's progeny is the total number of branching nodes on the branch between it and the tree tips), were descriptors designed to respond to invariants like fragmentation and lineage. Examples of quantities that were to measure non-invariants like the scale reduction and density contrast were the average density contrast and separation between the branches and their parent branching nodes. Descriptors of irregularity, like the dispersion in children separations at each branching node, were also used.

There is of course no guarantee that all of the descriptors from the invariant class are independent of those that are non-invariant, or that an individual descriptor may be able to itself to say whether an image is generally hierarchical or random. For these reasons, the descriptors were applied to an ensemble of trees obtained for 300 hierarchical and randomized projected simulated images with a variety of assigned parameters like image type (hierarchical/random), the total number of levels, the scale reduction factor per level, and the branching factor, and regressed against the known values of the underlying parameters. The resulting linear combinations for each parameter were found to be able to estimate that parameter's value for any of the individual models in the ensemble. The success with the simulated structures led us to attempt an application to the Taurus 100 μm column density map. There is no assurance, of course, that the Taurus structure is reasonably close enough to one of the types of simulated structures which were constructed assuming spherical independent clouds so that the derived parameters are meaningful. For this reason we feel that the major importance of the structure trees will be comparisons of observations with future numerical hydrodynamic calculations and quantitatively comparing the structures of regions with different levels of star formation activity.

The preliminary results are as follows. The hierarchical indicator have values intermediate between the hierarchical and randomized simulations, suggesting that Taurus has a column density structure which is a mixture of both components, or that Taurus cannot be represented by the systems of nested or random clouds which comprised the simulations. Assuming that the former is true, the parameters of the hierarchical component were estimated using the linear regressions of statistics which reliably estimated the parameters of the simulations. This yielded an average branching ratio, or number of children per parent, denoted η, of about 10, an average change in scale or "shrinkage factor," per level of hierarchy, denoted θ, of about 0.16, and an average volume density contrast D between child and parent clouds of about 9. The average number of levels of

hierarchical structure per path through the tree (from root to every terminal tree tip) was 2; this estimate does not include the "root," and refers to structure with linear scale between about 5-7 pc (sizes of largest clouds above the root) and about 0.2 pc (size of smallest parents). A physically significant quantity is the average number of hierarchical levels per unit of spatial dynamical range, which is $2/(6/0.2) \approx 0.07$.

As an illustration of how this tree analysis can yield physically interesting quantities, I will adopt the values of η, θ and D given above and assume that the structure is self-similar. The mean volume filling factor of children in parent clouds is then $\epsilon = \eta\theta^3 \approx 0.04$. Interpreting the hierarchy in terms of fragmentation, the mass efficiency of the fragmentation process per level of hierarchy must be $f = \eta\theta^3 D \approx 0.4$. While only a preliminary result, it should be clear that this type of estimate provides a direct constraint on numerical hydrodynamic models for cloud fragmentation, and that an application of this approach, including measures of irregularity, to regions with different levels of star formation could provide important evolutionary information. We are currently beginning such a study.

A striking feature of the contour map of the IRAS column density image of Taurus is the irregularity and convolution of the contours at all scales. An obvious question is whether this irregularity is self-similar, or "fractal."

The fractal dimension of a collection of two-dimensional objects can be determined by plotting log (perimeter) as a function of log (area). (Note that the fractal dimension here refers to the irregularity of the column density contours, not the hierarchical internal structure; the fractal dimension of the hierarchical structure is not very informative.) For planar objects with smooth shapes, $P \propto A^{1/2}$. But for objects with irregularity at all scales, $P \propto A^{D/2}$, where D, the fractal dimension ($1 \leq D \leq 2$), characterizes the irregularity of the boundary. As the boundaries get more and more complex, they eventually fill the plane, and $P \propto A$, while $D \to 2$. As part of its compression of an image, the structure tree automatically computes and stores the perimeter and area of every cloud (set of connected pixels) at every grey level, where the perimeter is the number of non-cloud pixels bordering a cloud. The Taurus complex is of course a 3-dimensional object, so we are only estimating the dimension of its 2-dimensional projection. It is known that 2-dimensional slices of a 3-dimensional fractal with D>2 are fractals with fractal dimension D-1. However little is known about the relation for projected fractals, although it is sometimes assumed that the same relation holds and we have tentatively done the same.

The logP-logA relation for all the clouds in the IRAS τ_{100} map is fit well by a single power law of slope 0.7, corresponding to a 2-dimensional fractal dimension of 1.4, over a factor of about 50 in size (taken as $A^{1/2}$). A similar result has been found independently by other workers for cirrus clouds, Lynds dark clouds, ^{12}CO emission, and HI emission.

The result that $D \approx 1.4$ over such a large range of size and column density has some interesting and important implications for both theoretical and empirical studies of cloud evolution and star formation.

First, the similarity of the dimensions measured over such a large range of column densities indicates that there is no fundamental difference between the dominant physics controlling the shapes of molecular and atomic clouds. This suggests that the separation of HI and H_2 clouds into distinct conceptual categories is largely artificial, and illustrates how theoretical speculations may have mistakenly generalized an operational distinction based on detection method to a distinction in spatial distribution, origin, and evolution.

Secondly, the dimension $D \approx 1.4$ is the same as that found for the turbulent-nonturbulent interface of incompressible fluids (using 2-dimensional slices or one-dimensional cuts), essentially independent of the type of flow (e.g. boundary layer, jet, wake). This is also very close to the fractal dimension of the boundaries of terrestrial clouds and rain areas and of hail clouds covering nearly four orders of magnitude in size. Thus, while the result does not yet provide any constraint on the specific types of physical processes at work, it does at least suggest a possible connection with turbulence. The connection must be generic, since interstellar cloud "turbulence" is expected to be

considerably different than ordinary incompressible turbulence (compressibility, shock dissipation, presence of magnetic fields, presence of stellar energy sources). The major feature in common would seem to be the nonlinear advection term in the momentum equation, whose power to generate, amplify and distort fluctuations is ignored in all discussions based on linear stability analyses or the virial theorem, and is largely suppressed by existing numerical simulations because of lack of spatial dynamic range.

Fractal contour structure raises several other theoretical questions. For example, what does it mean for some of these irregular structures to appear approximately "virialized?" Is it reasonable to imagine any sort of quasi-hydrostatic equilibrium for such clouds? Shouldn't such irregularity imply a highly disordered magnetic field?

To summarize, the analysis of the Taurus region carried out to date shows that IRAS data can be used to construct reliable panoramic column density maps of regions of low-mass star formation, and that the resulting structure appear qualitatively different from standard concepts of interstellar clouds. This complex structure requires new methods of analysis, and two of them, structure trees and fractal dimension, have already yielded interesting results.

A more detailed account of this work appears in the review paper "Perception of Interstellar Structure: Facing Complexity," which is attached.