NASA-CR-194199

(NASA-CR-194199) EXPLOITING STRUCTURE: INTRODUCTION AND MOTIVATION Semiannual Status Report, 1 Mar. - 31 Aug. 1993 (Texas Univ.) 10 p

N94-13828

Unclas

G3/61 0185668

0 CIT.

185...5

1993 SEMIANNUAL STATUS REPORT FOR
NASA Grant NAG 5-2210

Exploiting Structure: Introduction and Motivation

Zhong Ling Xu
Department of Mathematics
University of Texas at Brownsville
80 Fort Brown
Tel: (210) 544-8901, Fax: 544-8988
E-mail: BINET%"ZHONG2UTB" SMTP%"ZHONG@UTB.EDU"

Submitted to
NASA Goddard Space Flight Center
Greenbelt, MD 20771

NASA Technical Officer: Mr. Gary E. Mosier

Report Period: March 1 - August 31, 1993

September 3, 1993 - Submitted

Contents

1. Summary

This semiannual report briefly summarizes research activities performed during the period of June 26, 1993 through August 31, 1993. We developed the Robust Stability of Systems where transfer function or Characteristic polynomial are multilinear affine functions of parameters of interest in two directions, Algorithmic and Theoretical. In the algorithmic direction, a new approach that reduces the computational burden of checking the robust stability of the system with multilinear uncertainty is found. This technique is called "Stability by linear process." In fact, the "Stability by linear process" we describe here gives an algorithm. But we still have something else to be done. In analysis, we obtained a robustness criterion for the family of polynomials with coefficients of multilinear affine function in the coefficient space and obtained the result for the robust stability of diamond families of polynomials with complex coefficients also. We obtained the limited results for SPR design and we provide a framework for solving ACS. Finally, copies of the outline of our results are provided in the appendix. Also, there is administration issue in the appendix.

2.1 Parametric Robust Stability.

Let $p(s,Q) = \Sigma^{n}_{i=0} a_i(q) S^i$, $q \in Q \subset R^n$

and $a_i(q)$ are multilinear affine functions. As we know

$$\sigma[p(s,Q)] \subset C^{\circ}_{-} \Leftrightarrow p(s,Q) \in H \Leftrightarrow 0 \notin p(j\omega, Q)$$

Hurwitz and Routh criterion

Zero exclusion principle

In this section, we develop

$$p(s,Q) \in H \Leftrightarrow 0 \notin p(j\omega, Q)$$

by algorithm, and

$$\sigma\ [p(s,Q)] \subset C \Leftrightarrow p(s,Q) \in H$$

by theoretical analysis.

2.1.1 Stability by linear process.

Our problem is to find a quick way to determine if the origin 0 in the complex plane C within the image of hyperrectangle $Q \subset R^n$ under the mapping

$$p(j\omega,q) = f(q) + g(q)j \text{ for fixed } \omega$$

Where f(q) and g(q) are multilinear affine functions for $\forall\, q \in Q$.

We completely utilize linearity of multilinear affine function to study the line segments parallel to axis in Q mapping to the line segments in C. In accordance with our investigation, these line segments in C can be classified two kinds, one is so-called "perpendicular" and the other is so-called "non-perpendicular."

Now we define that a line segment l′ in C is said to be a perpendicular if there is a point A′on l′ such that the line through O and A′, OA′ perpendicular to l′. If not, then this line segment l′ is called non-perpendicular. (see figure 1)

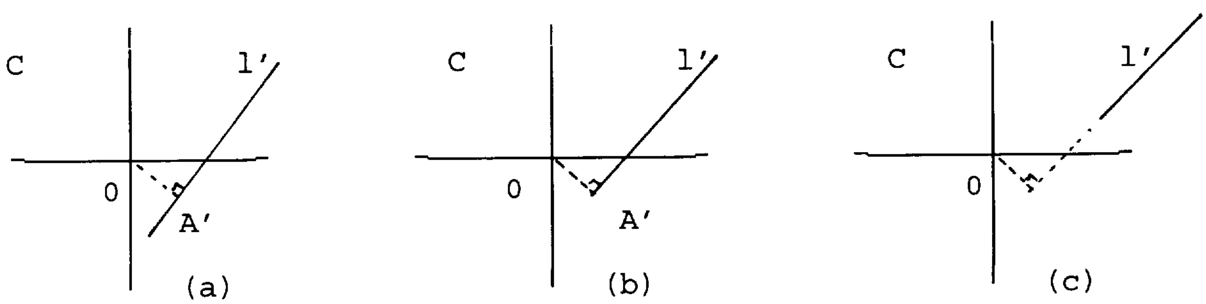

Figure 1

In Figure 1 (a) and (b) are the case of perpendicular; (c) is the case of non-perpendicular.

It is evident that (1) if all l′ in C are perpendicular then $0 \in p(j\omega, Q)$, (2) if all l′ in C are non-perpendicular then $0 \notin p(j\omega, Q)$. Certainly, if we check out all l′ in c whether perpendicular or not we would suffer from the heavy computational burden. Thus, our main goal is to develop a computational feasible algorithm for it. The algorithm is indicated in the appendix and our conjectures appear in the appendix also.

2.1.2 Family of polynomials with the coefficients of multilinear affine function.

For the family of characteristic polynomials

$$p(s,q) = \Sigma^{n}_{i=0}\ a_i(q)s^i \qquad \forall\ q \in Q \subset R^n$$

Where $a_i(q)$ is multilinear affine function for each i, and its zeros

$$\sigma[p(s,Q)] = \{z \in C : p(z,Q) = 0\}$$

Our problem is finding necessary and sufficient conditions for

$$\sigma[p(s,Q)] \subset C^{\circ}_{-}\ ,$$

That is, $\qquad p(s, Q) \in$ Hurwitz.

As we know, the image of Q in the coefficient space,

$$a(Q) = \{a(q) : a(q) = (a_0(q),\ a_1(q),\ \cdots,\ a_n(q)),\ q \in Q\} \subset R^{n+1}$$

is not necessarily convex. The stability of a whole family of polynomials can not be inferenced from the stability of just the set's vertices, edges or boundaries. But consider the box

$$A^* \subset R^{n+1},$$

$$A^* = [A^-_0,\ A^+_0] \times [A^-_1,\ A^+_1] \times \cdots \times [A^-_n,\ A^+_n], \text{ and}$$

$$A^-_i = \min_{q \in Q}[a_i(q)],\ A^+_i = \max_{q \in Q}[a_i(q)]$$

representing polynomials with independently varying coefficients in the intervals defined by the minimum and maximum values of the multilinear affine coefficient functions, $a_i(q)$, defined on Q.

For new box A^*, we have a family of polynomials

$$p(s,A^*) = \{\ p(s,A)\ :\ p(s,A) = \Sigma^{n}_{i=0}\ A_i s^i, A = (A_0, A_1, \cdots, A_n), A \subset A^*\}$$

and it becomes interval polynomials. Then we have

$$p(s,Q) \subset p(s,A^*)$$

While Kharitonov's Theorem guarantees us that the interval polynomials in A^* are Hurwitz if and only if Kharitonov's polynomials $\{K_{11}(\cdot),\ K_{12}(\cdot),\ K_{21}(\cdot),\ K_{22}(\cdot)\} \subset A^*$ are Hurwitz.

It is evident that

$$p(s,A^*) \in H \Rightarrow p(s,Q) \in H$$

Therefore we have the following theorem:

Theorem 1.

$p(s,Q) \in H$ if and only if Kharitonov's polynomials $\{K_{11}(\cdot),\ K_{12}(\cdot),\ K_{21}(\cdot),\ K_{22}(\cdot)\} \subset A^*$ are Hurwitz.

where $A^* = [A^-_0,\ A^+_0] \times \cdots \times [A^-_n,\ A^+_n]$

and $A^-_i = \min_{q\in Q} a_i(q)$; $A^+_i = \max_{q\in Q} a_i(q)$. $i = 0,1,\ldots,n$.

2.1.3 Diamond families of polynomials with complex coefficients.

As we know, in some cases the stability of a whole family of polynomials can be inferred from the stability of just the set's vertices, edges or boundaries. Here, we consider the case of the diamond family of polynomials. We have then a result as following:

Theorem 1. Let the complex diamond polynomial family be

$$W(s) = \{p(s) : p(s) = \Sigma^n_{i=0}(a_i + jb_i)s^i$$

$$\Sigma^n_{i=0}(|a_i-\hat{a}_i| + |b_i - \hat{b}_i|) \le r\} \subset P^n$$

Where $P^n = \{p(s) : p(s) = \Sigma^n_{i=0}(a_i+jb_i)s^i,\ a_n + jb_n \neq 0\}$

and $\hat{a}_i$, $\hat{b}_i$ are the coefficients of the nominal polynomial

$$p_0(s) = \Sigma^n_{i=0} (\hat{a}_i + j\hat{b}_i)s^i$$

Then $W(s) \in H$ if and only if $p_1(s),\ p_2(s),\ \cdots,\ p_8(s) \in H$

Where
$$\begin{aligned}
p_1(s) &= p_0(s) + r\\
p_2(s) &= p_0(s) - r\\
p_3(s) &= p_0(s) + jr\\
p_4(s) &= p_0(s) - jr\\
p_5(s) &= p_0(s) + rs^n\\
p_6(s) &= p_0(s) - rn^n\\
p_7(s) &= p_0(s) + jrs^n\\
p_8(s) &= p_0(s) - jrs^n
\end{aligned}$$

Theorem 2.

Let $W'(s) = \{p(s) : p(s) = \Sigma^n_{i=0} (a_i + jb_i)s^i,\ a_i \ge \hat{a}_i,\ b_i \ge \hat{b}_i,$

$i = 0,1,\cdots,n$ and $\Sigma^n_{i=0}[a_i-\hat{a}_i) + (b_i-\hat{b}_i)] \le r\} \subset P^n$

$W'(s) \in H$ if and only if 4 vertices are Hurwitz and 4 line segments are Hurwitz.

Details please see Appendix.

2.2 Strictly Positive Real Functions (SPR) for Robust Design.

We have known that the sets

$$SPR\{n_i(s)\} = \{d(s) : Re\{n_i(j\omega)/d(j\omega)\}>0 \text{ and } n_i(s) \in H \text{ for } i = 1,2\}$$

are convex cones, and the convex combination of $n_1(s)$ and $n_2(s)$ keeping Hurwitz is only a necessary condition for existence of d(s) such that both $n_1(s)/d(s)$ and $n_2(s)/d(s)$ are SPR. Unfortunately, we could not know if it is also a sufficient condition.

For this, we need to describe the set SPR{n(s)} in some way so that to prove $SPR\{n_1(s)\} \cap SPR\{n_2(s)\} \neq \phi$. Even though we have had 5 definitions for SPR{n(s)}, they would not work for our problem. Recently, we find the following new ways to define SPR{n(s)}.

Definition 1. $SPR\{n(s)\}=\{d(s):(a_0+j\alpha b_0,\cdots,a_n+j\alpha b_n)\in H, \forall\alpha\in R\}$

where $n(s)=\Sigma^n_{i=0}a_i s^i$ $\quad d(s) = \Sigma^n_{i=0}b_i s^i$

Now, we may then answer our question

$$SPR\{n_1(s)\}\cap SPR\{n_2(s)\}\neq\phi \text{ for } \lambda n_1(s)+(1-\lambda)n_2(s) \in H.$$

$\lambda\in[0,1]$.

In fact, consider

$SPR\{\lambda n_1(s)+(1-\lambda)n_2(s)\}=\{d(s):[\lambda n_1(s)+(1-\lambda)n_2(s)]+j\alpha d(s)\in H \ \forall\alpha\in R\}$ by our definition 1

Then, we have that $\lambda n_1(s)+(1-\lambda)n_2(s)+\lambda j\alpha d(s)+(1-\lambda)j\alpha d(s) \in H$

$$\therefore \lambda n_1(s)+(1-\lambda)j\alpha d(s)+(1-\lambda)n_2(s)+\lambda j\alpha d(s) \in H$$

$$\therefore \lambda n_1(s)+(1-\lambda)j\alpha d(s) \in H \text{ and } (1-\lambda)n_2(s)+\lambda j\alpha d(s) \in H$$

Therefore there exists $d(s) \in SPR\{n_1(s)\}\cap SPR\{n_2(s)\}$

That is, there exists d(s) such that $n_1(s)/d(s)$ and $n_2(s)/d(s)$ are SPR. We will develop further results.

2.3 ACS.

After reviewing the XTE Subsystem Analysis Results Report, we have realized that ACS is a flexible body linear system with time-varying, uncertain parameter and with the uncertain disturbance of same movements. Therefore, ACS would be in the form

$$X(t)=[A(t)+\Delta A(r(t)]X(t)+[B(t)+\Delta B(s(t))]U(t)+D(t)W(t) \qquad (1)$$

where $\Delta A(r(t))$ and $\Delta B(s(t))$ are yielded by the parameters and the control U(t). W(t) is the disturbance of some moments. Thus, our framework for analyzing ACS would be the following:

1. Construct state equations (1) in time domain for ACS. That is, we should look for the conditions for A, ΔA; B, ΔB; D, W such that the control U(t) stabilizes ACS. For the system (1), we usually adopt the Quadratic cost functional

$$J_{min} = \int^{\infty}_{0}[Y(t)^{T}y(t) + U(t)^{T}RU(t)]dt$$

where R is a positive definite
to find the solution of its corresponding Riccati's equation to obtain the robustness conditions for this system.

2. Determine Stability Margin for ACS.

That is, to find the best U(t) such that the system ACS maintains asymptotic stability in the region of the uncertain parameters and the uncertain disturbances. Therefore we need to define a radius $\rho(p,u)$ and find

$$\mathrm{Max}_{u\epsilon U} \ \mathrm{Min}_{p\epsilon\pi} \ \{\rho(p,u)\}$$

Where u is a control, p is a disturbation

Thus, H_{∞} norm would be considered.

3. Adjust and revise u(t) according to the reality of ACS.

References

1. H. Lin, C.V. Hollot, A.C. Bartlett, "Stability of families of polynomials: geometric considerations in coefficient space", Int. Journal of Control, 1987, Vol. 45 No.2 649-660

2. R.J. Minnichelli, J.J. Anagnost, and C.A. DeSoer, "An Elementary Proof of Kharitonov's Stability Theorem With Extensions", IEEE Transactions on Automatic Control, Vol 34, No. 9, Sept. 1989

3. H. Chapellat, S.P. Bhattacharyya, " A Generalization of Kharitonov's Theorem: Robust Stability of Interval Plants", IEEE Transactions on Automatic Control, Vol 34, No. 3, March 1989

4. S. Bialas, "A Necessary and Sufficient Condition for the Stability of Convex Combinations of Stable Polynomials or Matrices", Bulletin of the Polish Academy of Sciences, Vol. 33 No. 9-10, 1985

5. L. Huang, C.V. Hollot, and Z.L. Xu, "Robust Analysis Of Strictly Positive Real Function Set", Published

6. S.P. Bhattacharyya, "On Robust Nonlinear Stability of Interval Control Systems", IEEE Transactions On Automatic Control, Vol. 36, No. 1, January 1991

7. B.D.O. Anderson, S.Dasgupta, P. Khargonekar, F.J. Kraus, and M. Mansour, IEEE Transactions On Circuits And Systems, Vol. 37, No. 7, July 1990

Appendix