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A Component Modes Projection and Assembly Model Reduction Methodology for Articulated, Multi-Flexible Body Structures

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Abstract

A two-stage model reduction methodology, combining the classical Component Mode Synthesis (CMS) method and the newly developed Enhanced Projection and Assembly (EP&A) method, is proposed in this research. The first stage of this methodology, called the COmponent Modes Projection and Assembly model REduction (COMPARE) method, involves the generation of CMS mode sets, such as the MacNeal-Rubin mode sets. These mode sets are then used to reduce the order of each component model in the Rayleigh-Ritz sense. The resultant component models are then combined to generate reduced-order system models at various system configurations. A composite mode set which retains important system modes at all system configurations is then selected from these reduced-order system models. In the second stage, the EP&A model reduction method is employed to reduce further the order of the system model generated in the first stage. The effectiveness of the COMPARE methodology has been successfully demonstrated on a high-order, finite-element model of the cruise-configured Galileo spacecraft.

1. Background and Motivation

Multibody dynamics simulation packages are gaining in popularity among dynamists for the simulation and analysis of systems of interconnected bodies (some or all of which are flexible). One such program, DISCOS,[1] was used in the development of control systems on board the Galileo spacecraft. The dual-spin Galileo spacecraft was modeled as a three-body system, consisting of a flexible spinning rotor, a flexible stator, and a rigid scan platform.

For complex systems such as the Galileo spacecraft, practical considerations (e.g., simulation time) impose limits on the number of modes that each flexible body can retain in a given simulation. Modal truncation procedures must be used to select and retain a limited number of "important" modes which capture the salient features of the component dynamics. The Enhanced Projection and Assembly (EP&A)[6,7] technique is one way of performing this task.

The EP&A method,[6,7] is a model reduction methodology for articulated, multi-flexible body systems. In this method, a composite mode set, consisting of "important" system modes from all system configurations of interest, and not just from one particular system configuration, is first selected. It is then augmented with static correction modes before being "projected" onto the component models to generate reduced-order component models. To generate the composite mode set, eigenvalue problems concerning the full-order system models, at all configurations of interest, must be solved repetitively. This is a drawback of the EP&A method because solving large eigenvalue problems can be costly. To overcome this difficulty, a two-stage model reduction methodology, combining the classical Component Mode Synthesis (CMS) method and the Enhanced Projection and Assembly method (EP&A), is proposed in this research.

The stages involved in the proposed technique, to be called the COmponent Modes Projection and Assembly model REduction (COMPARE) method, are illustrated in Fig. 1. First, CMS mode sets, such as the MacNeal-Rubin mode sets, are generated and used to reduce the order of each component model in the Rayleigh-Ritz sense. These component mode sets are then assembled using the interface compatibility conditions to generate reduced-order system models at various system configurations. The order of these reduced-order system models is typically smaller than that of the full-order system model.

In the second stage, the newly developed EP&A model reduction method is employed to reduce further the order of the system model generated in the first stage. As described above, the composite mode set is augmented with static correction modes before being projected on the CMS-generated component models to generate the final reduced-order system model. In this way, COMPARE retains the merits of both the CMS and EP&A methods, without their demerits. The effectiveness of COMPARE will be verified using a cruise-configured Galileo spacecraft model.

2. Component Mode Synthesis Method Revisited[2,8]

The Component Mode Synthesis (CMS) method is a Rayleigh-Ritz based approximation method that is commonly used to analyze linear, high-order structural dynamics problems. To use this method, the structure is first subdivided into a number of components (or substructures), and a Ritz transformation is employed to reduce the model orders of these substructures. Many component mode sets may be used to perform this reduction but the MacNeal-Rubin (M-R) and Craig-Bampton (C-B) mode sets were shown to have good convergence properties in the sense of CMS. Once reduced, the reduced-order component models are then coupled using the interface compatibility conditions to form the reduced-order system model.

Since the Craig-Bampton and MacNeal-Rubin mode sets will be used in the present research to reduce the orders of the component models, their constructions are first briefly reviewed. To this end, consider a multi-flexible body structure as depicted in Fig. 2. The undamped motion of each component of the structure can be described by a matrix differential equation

$$M_{nn}\ddot{x}_n + K_{nn}x_n = F_n, \tag{1}$$

where x_n is an $n \times 1$ displacement vector, and M_{nn} and K_{nn} are $n \times n$ mass and stiffness matrices of the component, respectively. Note that the matrix dimensions are indicated by the matrix subscripts. The $n \times 1$ force vector acting on the component is denoted by F_n. A similar equation can also be written for component B.

To generate the C-B or M-R mode set, the last equation is partitioned as follows :

$$\begin{bmatrix} M_{ii} & M_{ij} \\ M_{ji} & M_{jj} \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{x}_j \end{bmatrix} + \begin{bmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} F_i \\ 0 \end{bmatrix}, \tag{2}$$

where x_i and x_j represent the interface and interior coordinates, respectively.

2.1 Craig-Bampton Mode Set

The Craig-Bampton mode set is generated by augmenting a low-frequency subset of the fixed interface (I/F) normal modes with a set of static-shape functions termed constraint modes. The first k fixed I/F normal modes Φ_{jk}, and the ordered eigenvalue matrix Λ_{kk} are related by the following relation :

$$-M_{jj}\Phi_{jk}\Lambda_{kk} + K_{jj}\Phi_{jk} = 0. \tag{3}$$

There are ways to decide on the number of modes to be kept in Φ_{jk}. One way is to keep all modes whose frequencies are less than twice a characteristic frequency of the system (e.g., control bandwidth).[2] The above determined normal modes are then augmented with i constraint modes, where constraint modes are static-shape functions that result by imposing unit displacement on one coordinate of the i-set while holding the remaining coordinate in that set fixed. It can be shown that the interior displacement for these constraint modes is given by[8]

$$\Psi_{ji} = -K_{jj}^{-1} K_{ji} . \tag{4}$$

The C-B mode set is

$$\begin{bmatrix} O_{ik} & I_{ii} \\ \Phi_{jk} & \Psi_{ji} \end{bmatrix}$$

which is then used to reduce the full-order component model.

2.2 MacNeal-Rubin Mode Set

In a manner similar to generating the Craig-Bampton mode set, the MacNeal-Rubin mode set is generated by augmenting a low-frequency subset of the free I/F normal modes with a set of static force response functions termed residual modes. The first k free I/F normal modes Φ_{nk}, and the ordered eigenvalue matrix Λ_{kk}, are related by the following relation :

$$-M_{nn}\Phi_{nk}\Lambda_{kk} + K_{nn}\Phi_{nk} = 0 . \tag{5}$$

The kept eigenvector matrix Φ_{nk}, which has been normalized with respect to the mass matrix, may be partitioned into its rigid-body and flexible parts: $[\Phi_{nr} \; \Phi_{nf}]$. Let Λ_{ff} be the eigenvalue matrix associated with the kept flexible modes. Then, the residual modes may be determined by

$$\Psi_{na} = (P_{nn}^T \, S_{nn} \, P_{nn} - \Phi_{nf} \, \Lambda_{ff}^{-1} \, \Phi_{nf}^T \,) \, F_{na} \, , \tag{6}$$

where $P_{nn} = I_{nn} - M_{nn} \, \Phi_{nr} \, \Phi_{nr}^T$, S_{nn} is the "pseudo" flexibility matrix of the component,[8] defined as follows. Let the set of all physical coordinates be divided into three subsets : r, a, and w. The r set may be any statically determinate constraint set which provides restraint against rigid-body motion. The a set consists of coordinates where unit forces are to be applied to define attachment modes (i.e., at the interface coordinates, and at coordinates where external forces are applied). Finally, the w set consists of the remaining coordinates in z. Using these definitions, the component stiffness matrix K_{nn} is partitioned as follows :

$$\begin{bmatrix} k_{rr} & k_{ra} & k_{rw} \\ k_{ar} & k_{aa} & k_{aw} \\ k_{wr} & k_{wa} & k_{ww} \end{bmatrix} .$$

The pseudo-flexibility matrix S_{nn} is now given by a matrix of the form[8]

$$\begin{bmatrix} 0 & 0 \quad 0 \\ 0 & \begin{bmatrix} k_{aa} & k_{aw} \\ k_{wa} & k_{ww} \end{bmatrix}^{-1} \\ 0 & \end{bmatrix} .$$

Finally, the matrix F_{na} is given by $[0_{ar},\ I_{aa},\ 0_{aw}]^T$, where the identity matrix I_{aa} is associated with the a interface coordinates. The M-R mode set is $[\Phi_{nk}\ \Psi_{na}]$, which is then used to generate a reduced-order model for the component.

3. A Component Modes Projection and Assembly Model Reduction (COMPARE) Methodology

Once CMS-based reduced-order component models are generated, they are assembled using the interface compatibility conditions to produce reduced-order system models at various system configurations of interest. Since the orders of these reduced-order system models are typically smaller than those of the full-order system models, we have accomplished the first of the two model reduction steps of the COMPARE methodology. However, note that these CMS-generated reduced-order system models were obtained without using knowledge of any system-level input-output information. This drawback is remedied in the second stage, in which the EP&A methodology[6,7] is used to further reduce the order of the models generated in the first stage. Since the EP&A methodology has been described elsewhere,[6,7] it will only be briefly reviewed here.

Consider a system with two flexible components. The undamped motion of component A, as described by either its M-R or C-B mode set, is given by

$$\begin{aligned} I_{pp}\ddot{\eta}_p^A + \Lambda_{pp}^A \eta_p^A &= G_{pa}^A u_a^A , \\ y_b^A &= H_{bp}^A \eta_p^A . \end{aligned} \tag{7}$$

Here, η_p^A and Λ_{pp}^A are the generalized coordinates and the diagonal stiffness matrix of component A, respectively. The dimension of η_p^A is p. The matrix G_{pa}^A is an control distribution matrix, and u_a^A is an $a \times 1$ control vector. Similarly, the matrix H_{bp}^A is an output distribution matrix, and y_b^A is an $b \times 1$ output vector. Similar equations can also be written for component B.

The system equations of motion at a particular articulation angle α may be constructed using these component equations, and enforcing displacement compatibility conditions at the component interface. To this end, let $P(\alpha) = [P_{pe}^{A^T}(\alpha), P_{qe}^{B^T}(\alpha)]^T$ be any full-rank matrix mapping a minimal system state η_e into

$$\begin{bmatrix} \eta_p^A \\ \eta_q^B \end{bmatrix} = \begin{bmatrix} P_{pe}^A(\alpha) \\ P_{qe}^B(\alpha) \end{bmatrix} [\eta_e] , \tag{8}$$

where η_e is an $e \times 1$ reduced-order system coordinate, and $e = p + q - i$ (i is the number of I/F constraint relations). For ease of notation, the dependencies of the matrices P_{pe}^A, P_{qe}^B, η_p^A, η_q^B, and η_e on α are dropped in the sequel. Substituting $\eta_p^A = P_{pe}^A \eta_e$ and $\eta_q^B = P_{qe}^B \eta_e$ into (7) and the corresponding equations for component B, pre-multiplying the resultant equations by $P_{pe}^{A^T}$ and $P_{qe}^{B^T}$, respectively, and summing the resultant equations gives

$$M_{ee}\ddot{\eta}_e + K_{ee}\eta_e = G_{ea}u_a , \tag{8a}$$

$$y_s = H_{se}\eta_e , \tag{8b}$$

where M_{ee}, K_{ee}, G_{ea}, and H_{se}, all functions of α, are given by

$$\begin{aligned} M_{ee} &= P_{pe}^{A^T} P_{pe}^A + P_{qe}^{B^T} P_{qe}^B , \\ K_{ee} &= P_{pe}^{A^T} \Lambda_{pp}^A P_{pe}^A + P_{qe}^{B^T} \Lambda_{qq}^B P_{qe}^B , \\ G_{ea} &= P_{pe}^{A^T} G_{pa}^A + P_{qe}^{B^T} G_{qa}^B , \\ H_{se} &= \begin{bmatrix} H_{bp}^A P_{pe}^A \\ H_{lq}^B P_{qe}^B \end{bmatrix} , \end{aligned}$$

where $y_s = [y_b^{A^T} \; y_l^{B^T}]^T$, and $s = b + l$. To arrive at the equation for G_{ea}, we have assumed that $u_a^A = u_a^B = u_a$. Otherwise, the term $G_{ea} u_a$ in (8a) should be replaced by $[P_{pe}^{A^T} G_{pa}^A, P_{qe}^{B^T} G_{qa}^B] [u_a^{A^T}, u_a^{B^T}]^T$. Since M_{ee} is symmetric and positive-definite while K_{ee} is symmetric and positive semi-definite, a transformation Φ_{ee} that diagonalizes M_{ee} and K_{ee} simultaneously can always be found. Let ϕ_e be the corresponding generalized coordinate, i.e., $\eta_e = \Phi_{ee} \phi_e$. Substituting this relation into (8a), and pre-multiplying the resultant equation by Φ_{ee}^T gives

$$\begin{aligned} I_{ee}\ddot{\phi}_e + \Lambda_{ee}\phi_e &= \bar{G}_{ea}u_a , \\ y_s &= \bar{H}_{se}\phi_e , \end{aligned} \tag{9}$$

where $\Phi_{ee}^T M_{ee} \Phi_{ee} = I_{ee}$, $\bar{H}_{se} = H_{se}\Phi_{ee}$, $\bar{G}_{ea} = \Phi_{ee}^T G_{ea}$, and where $\Lambda_{ee} = \Phi_{ee}^T K_{ee} \Phi_{ee}$ contains the undamped, reduced-order system eigenvalues along its diagonal. Equation (9) represents the reduced-order system model obtained from the first stage of the COMPARE methodology.

The EP&A method is used in the second stage of COMPARE. With the EP&A method, only k of the system's e modes are kept while the remaining t $(= e - k)$ modes are removed. The kept mode set is a composite mode set, consisting of "important" system modes from all system configurations of interest, and not just from one particular configuration. With this understanding, we have

$$\eta_e = \Phi_{ee}\phi_e = [\,\Phi_{ek} \quad \Phi_{et}\,] \begin{bmatrix} \phi_k \\ \phi_t \end{bmatrix} \doteq \Phi_{ek}\phi_k \,, \tag{10}$$

where ϕ_k and ϕ_t are generalized coordinates associated with the "kept" and "truncated" modes, respectively, and Φ_{ek} and Φ_{et} the corresponding eigenvector matrices.

The composite mode set Φ_{ek} may now be projected onto the CMS-generated component models: $\eta_p^A \doteq P_{pe}^A \Phi_{ek} \phi_k^A = \Psi_{pk}^A \phi_k^A$. Here, ϕ_k^A denotes reduced sets of generalized coordinates of component A. The substitution of the last relation into (7) produces the "constrained" equations of motion for component A:

$$\Psi_{pk}^{A^T} \Psi_{pk}^A \, \ddot{\phi}_k^A + \Psi_{pk}^{A^T} \Lambda_{pp}^A \Psi_{pk}^A \, \phi_k^A = \Psi_{pk}^{A^T} G_{pa}^A \, u_a \,. \tag{11}$$

A second reduced-order system model can now be constructed using (11), a similar equation for component B, and the displacement compatibility conditions at the component I/F. The order of this new reduced-order model is smaller than that obtained from the first step (cf. (9)) due to the truncation of "t" modes in (10). In addition, it has been proven that the modes retained in the composite mode set Φ_{ek} are captured exactly by the resultant reduced-order system model (with a number of extraneous modes [3,4,5,6,7]). However, the static gain of the resultant reduced-order system model is not the same as that given by (9).[7]

Two different approaches were introduced in Lee and Tsuha[7] to preserve the static gain of the system described by (9) in the second reduced-order system model. The first approach involves augmenting the k "kept" modes of the system with an additional a modes so as to create a statically complete mode set. The enlarged mode set is then "projected" onto the components, and the reduced components are assembled as usual.[7] In the second approach, the k "kept" modes of the system are first projected onto the components. After the projections, the reduced-component mode sets are each augmented with static correction modes. The augmented mode sets are then used to reduce the components, and the resultant component models are assembled to generate the reduced-order system model. The details of these approaches are given below.

3.1 Component-level Augmentation Techniques[6,7]

In the component-level augmentation approach, the k "kept" modes are first projected onto the components. We first write

$$\begin{aligned} \eta_p^A &= \Psi_{pk}^A \, \Xi_{kk}^A \, \nu_k^A = \Upsilon_{pk}^A \, \nu_k^A = [\Upsilon_{pr}^A \; \Upsilon_{pf}^A] \, \nu_k^A \, , \\ \eta_q^B &= \Psi_{qk}^B \, \Xi_{kk}^B \, \nu_k^B = \Upsilon_{qk}^B \, \nu_k^B = [\Upsilon_{qr}^B \; \Upsilon_{qf}^B] \, \nu_k^B \, , \end{aligned} \tag{12}$$

where Ξ_{kk}^A is the eigenvector matrix associated with an eigenvalue problem of (11), and ν_k^A is the corresponding generalized coordinate. The transformation matrices Υ_{pk}^A and Υ_{qk}^B are defined in (12). Matrix partitions Υ_{pr}^A and Υ_{pf}^A denote eigenvectors associated with the rigid-body and flexible modes of the projected component A model. Similar partitions are made for the eigenvectors associated with the projected component B model.

Let Υ_{pr}^A, Υ_{pf}^A, etc. be normalized such that

$$\begin{aligned} \Upsilon_{pr}^{A^T} \Upsilon_{pr}^A &= I_{rr} \, , \qquad & \Upsilon_{pr}^{A^T} \Lambda_{pp}^A \Upsilon_{pr}^A &= 0_{rr} \, , \\ \Upsilon_{pf}^{A^T} \Upsilon_{pf}^A &= I_{ff} \, , \qquad & \Upsilon_{pf}^{A^T} \Lambda_{pp}^A \Upsilon_{pf}^A &= \bar{\Lambda}_{ff}^A \, , \end{aligned} \tag{13}$$

where $\bar{\Lambda}_{ff}^A$ is an eigenvalue matrix associated with the projected flexible modes of component A. Similar expressions can also be written for component B. Using the matrices defined in (12-13), residual modes,[8] described in Section 2.2, may once again be used to augment the projected mode sets of the components. From (6), the residual modes are given by

$$\Upsilon_{pa}^A = (\bar{P}_{pp}^{A^T} \, \bar{S}_{pp}^A \, \bar{P}_{pp}^A - \Upsilon_{pf}^A \, \bar{\Lambda}_{ff}^{A^{-1}} \, \Upsilon_{pf}^{A^T} \,) \, F_{pa}^A \, , \tag{14}$$

where

$$\bar{P}_{pp}^A = I_{pp}^A - \Upsilon_{pr}^A \, \Upsilon_{pr}^{A^T} \, , \tag{15}$$

and $\bar{S}_{pp}^A$ is the "pseudo" flexibility matrix of component A. If we assume that the diagonal stiffness matrix of component A (see Λ_{pp}^A in (7)) is ordered so that all the rigid-body modes (with zero frequency) are given first, then $\bar{S}_{pp}^A$ is given by

$$\begin{bmatrix} 0 & 0 \\ 0 & \Lambda_{\bar{p}\bar{p}}^{A^{-1}} \end{bmatrix} . \tag{16}$$

Here $\bar{p} = p - r$ is the total number of flexible modes in the CMS mode set of component A. In (14), the matrix F^A_{pa} is given by $[0_{ar},\ I_{aa},\ 0_{a\,p-r-a}]^T$. Similar expressions can also be written for component B.

The projected mode sets Υ^A_{pk} and Υ^B_{qk} can now be supplemented with the residual modes

$$
\begin{aligned}
\eta^A_p &= [\Upsilon^A_{pk}\ \Upsilon^A_{pa}] \begin{bmatrix} \nu^A_k \\ \nu^A_a \end{bmatrix} = \Upsilon^A_{pv}\, \nu^A_v\,, \\
\eta^B_q &= [\Upsilon^B_{qk}\ \Upsilon^B_{qa}] \begin{bmatrix} \nu^B_k \\ \nu^B_a \end{bmatrix} = \Upsilon^B_{qv}\, \nu^B_v\,,
\end{aligned}
\tag{17}
$$

where ν^A_a and ν^B_a are generalized coordinates associated with the residual modes. Using the Ritz transformations suggested in (17), the new component-projected equations of motion are

$$
\Upsilon^{A^T}_{pv}\Upsilon^A_{pv}\ \ddot{\nu}^A_v + \Upsilon^{A^T}_{pv}\Lambda^A_{pp}\Upsilon^A_{pv}\ \nu^A_v = \Upsilon^{A^T}_{pv} G^A_{pa} u_a\,, \tag{18}
$$

$$
\Upsilon^{B^T}_{qv}\Upsilon^B_{qv}\ \ddot{\nu}^B_v + \Upsilon^{B^T}_{qv}\Lambda^B_{qq}\Upsilon^B_{qv}\ \nu^B_v = \Upsilon^{B^T}_{qv} G^B_{qa} u_a\,. \tag{19}
$$

Using (18-19), the reduced-order system equations of motion at a particular system configuration α can now be formed by enforcing displacement compatibility at the interface

$$
C^A_{ip}(\alpha)\eta^A_p + C^B_{iq}(\alpha)\eta^B_q = 0\,, \tag{20}
$$

where C^A_{ip} and C^B_{iq} are matrices that establish the constraint relations between the generalized coordinates of CMS generated component models, and i is the number of constraint relations. Using (17), (20) becomes

$$
[C^A_{ip}\Upsilon^A_{pv}\ \ C^B_{iq}\Upsilon^B_{qv}] \begin{bmatrix} \nu^A_v \\ \nu^B_v \end{bmatrix} = [D_{ic}] \begin{bmatrix} \nu^A_v \\ \nu^B_v \end{bmatrix} \doteq 0\,, \tag{21}
$$

where $[D_{ic}]$ is defined in (21), and $c = 2v$. To construct the reduced-order system model, we partition the "compatibility" matrix $[D_{ic}]$ using the Singular Value Decomposition (SVD) technique

$$
[D_{ic}] = [C^A_{ip}\Upsilon^A_{pv},\ C^B_{iq}\Upsilon^B_{qv}\,], = [U_{ii}]\,[\Sigma_{ii}\,, O_{id}\,] \begin{bmatrix} V^T_{ci} \\ V^T_{cd} \end{bmatrix}, \tag{22}
$$

where $d = c - i$, and $[O_{id}]$ is an $i \times d$ null matrix. We can now write[3,6,7]

$$
\begin{bmatrix} \nu^A_v \\ \nu^B_v \end{bmatrix} = [P_{cd}]\,\nu_d = \begin{bmatrix} P^A_{vd} \\ P^B_{vd} \end{bmatrix} \nu_d\,, \tag{23}
$$

where $[P_{cd}] = [V_{cd}]$ (cf. (22)) is a full column rank mapping matrix, and ν_d denotes a minimum set of generalized coordinates of a statically complete reduced-order system. Substituting $\nu_v^A = [P_{vd}^A]\,\nu_d$ and $\nu_v^B = [P_{vd}^B]\,\nu_d$ into (18) and (19), pre-multiplying the resultant equations by $P_{vd}^{A^T}$ and $P_{vd}^{B^T}$, respectively, and summing the resultant equations give

$$M_{dd}\ddot{\nu}_d + K_{dd}\nu_d = G_{da}u_a\,, \tag{24}$$

$$y_s = H_{sd}\nu_d\,, \tag{25}$$

where M_{dd}, K_{dd}, G_{da}, and H_{sd}, all functions of α, are given by

$$M_{dd} = P_{vd}^{A^T}\Upsilon_{pv}^{A^T}\Upsilon_{pv}^{A}P_{vd}^{A} + P_{vd}^{B^T}\Upsilon_{qv}^{B^T}\Upsilon_{qv}^{B}P_{vd}^{B}\,, \tag{26}$$

$$K_{dd} = P_{vd}^{A^T}\Upsilon_{pv}^{A^T}\Lambda_{pp}^{A}\Upsilon_{pv}^{A}P_{vd}^{A} + P_{vd}^{B^T}\Upsilon_{qv}^{B^T}\Lambda_{qq}^{B}\Upsilon_{qv}^{B}P_{vd}^{B}\,, \tag{27}$$

$$G_{da} = P_{vd}^{A^T}\Upsilon_{pv}^{A^T}G_{pa}^{A} + P_{vd}^{B^T}\Upsilon_{qv}^{B^T}G_{qa}^{B}\,, \tag{28}$$

$$H_{sd} = \begin{bmatrix} H_{bp}^{A}\Upsilon_{pv}^{A}P_{vd}^{A} \\ H_{lq}^{B}\Upsilon_{qv}^{B}P_{vd}^{B} \end{bmatrix}. \tag{29}$$

Since M_{dd} is symmetric and positive-definite while K_{dd} is symmetric and positive semi-definite, a transformation Φ_{dd} that diagonalizes M_{dd} and K_{dd} simultaneously can always be found. Let Φ_{dd} be normalized with respect to the mass matrix, and let χ_d be the corresponding generalized coordinate, i.e.,

$$\nu_d = \Phi_{dd}\,\chi_d\,. \tag{30}$$

Substituting (30) into (24-25) and pre-multiplying the resultant equation by Φ_{dd}^T give

$$I_{dd}\ddot{\chi}_d + \Lambda_{dd}\chi_d = \bar{G}_{da}u_a\,, \tag{31}$$

$$y_s = \bar{H}_{sd}\eta_d\,, \tag{32}$$

where

$$\bar{H}_{sd} = H_{sd}\Phi_{dd}\,, \tag{33}$$

$$\bar{G}_{da} = \Phi_{dd}^T G_{da}\,, \tag{34}$$

$$\Lambda_{dd} = \Phi_{dd}^T K_{dd}\Phi_{dd}\,. \tag{35}$$

Here Λ_{dd} is a diagonal matrix with the undamped, reduced-order system squared frequencies along its diagonal. It was proven in Lee and Tsuha[7] that Φ_{ek} are captured exactly in Φ_{dd}

despite the augmentations of the projected component models with residual modes. The matrix Λ_{dd} also contains a number of extraneous modes.[6]

3.2 System Level Augmentation

The equation (9) may be decomposed into its kept and truncated parts:

$$\begin{aligned} I_{kk}\ddot{\phi}_k + \Lambda_{kk}\phi_k &= \Phi_{ek}^T G_{ea} u_a , \\ I_{tt}\ddot{\phi}_t + \Lambda_{tt}\phi_t &= \Phi_{et}^T G_{ea} u_a . \end{aligned} \tag{36}$$

The mode set Φ_{et}, which is truncated, will now be replaced by a smaller but statically equivalent mode set Φ_{ea}. To find Φ_{ea}, consider the following Ritz transformation[6]

$$\begin{aligned} \phi_t &= R_{ta}\phi_a , \\ \text{where } R_{ta} &= \Lambda_{tt}^{-1} \Phi_{et}^T G_{ea} . \end{aligned} \tag{37}$$

Here ϕ_a is the generalized coordinate associated with the augmented mode set Φ_{ea}. It can be shown that the static gain due to the mode set $[\Phi_{ek}\ \Phi_{ea}] = [\Phi_{ek}\ \Phi_{et}R_{ta}]$ is identical to that of the original system (cf. (9)).[6] Hence, $[\Phi_{ek}\ \Phi_{et}R_{ta}]$ is a statically complete mode set.

Several observations regarding the augmented mode set $[\Phi_{ek}\ \Phi_{et}\, R_{ta}]$ are in order. First, note that the augmented mode set contains two parts. The first part contains a selected number of eigenvectors which satisfy the eigenvalue problem defined by M_{ee} and K_{ee}. The second part is formed using the residual flexibility matrix and the distribution matrix of the external load. Hence, $\Phi_{et}\, R_{ta}$, unlike Φ_{ek}, is a function of the external load distribution.

Next, we observe that $\Phi_{et}\, R_{ta}$ is identical to the residual mode defined in Section 2.2. Also, note that this mode set is mass and stiffness orthogonal to the retained mode set Φ_{ek}. Hence, the two parts of the augmented mode set are linearly independent. This statically complete mode set is now ready for projections onto the various flexible component models to generate reduced-order component models. The reduced-order component models may then be assembled to generate the reduced-order system model using the procedures outlined in Section 3.1.

4. Applying COMPARE on A High-order Finite-element Galileo Model

The effectiveness of the proposed COMPARE methodology will now be demonstrated using a high-order finite-element model of the cruise-configured Galileo spacecraft. The three-body topology of the dual-spin Galileo spacecraft is illustrated in Fig. 3.[7] The rotor is the largest and most flexible component represented, with 243 dof. The smaller and more rigid stator is represented with 57 dof. Lastly, the scan platform is the smallest body idealized as rigid, with 6 dof.

For the purpose of controller design, a low-order system model, accurate at all configurations of interest and over a frequency range of interest (0-10 Hz) is needed. To this end, we apply the MacNeal-Rubin version of the COMPARE methodology on the Galileo model. The first stage of COMPARE requires the generation of M-R mode sets for all the flexible components. Following standard procedures, free interface normal modes of both the rotor and stator are first determined, and then truncated at twice the frequency of interest (20 Hz). Next, these truncated normal mode sets are each augmented with residual modes to generate the needed M-R mode sets for both the stator and rotor.

Next, the MacNeal-Rubin mode sets and the interface compatibility conditions are used to construct system models at all system configurations of interest, and determine from them important system-level modes at all clock angles of interest. The selected composite mode set has 8 rigid-body and 21 flexible modes.

The next step is to augment the composite mode set with one or more static-correction modes. For the Galileo example, we augment the composite mode set with two residual modes, one for an input torque about the Z-axis on the rotor side of the rotor/stator interface, and a second equal and opposite torque on the stator side of the interface. The enlarged mode set is then projected onto the flexible components. The resultant reduced-order models of the rotor and stator have 29 (with 6 rigid-body) and 21 (with 8 rigid-body) modes, respectively. The assembled reduced-order model has 44 (with 8 rigid-body) modes. The natural frequencies of these reduced-order component models and of the system model at a clock angle of 300 degrees are tabulated in Table 1. All system flexible modes retained in the composite mode set have been captured <u>exactly</u> in the reduced-order system model.

Comparisons of Bode plots of full-order and reduced order models, at clock angles of 60, 180, and 300 degrees, are given in Figs. 4, 5, and 6 respectively. Actuation was done at the Spin Bearing Assembly (SBA) located at the rotor-stator interface (along the Z-axis, cf. Fig. 3), and sensing was done by a gyroscope located on the scan platform. From these comparisons, we observe that the frequency responses of the full-order models, at all configurations of interest, have been closely captured by their reduced-order counterparts, over the frequency range of interest (0-10 Hz). Results obtained at other clock angles are similar to those depicted in Figs. 4-6.

5. Concluding Remarks

A two-stage model reduction methodology, called COMPARE, is proposed in this research. The first stage of this methodology involves the generation of CMS mode sets for the flexible components. The resultant component models are then combined to generate reduced-order system models at various system configurations. A composite mode set which retains important system modes at all system configurations is then determined from these reduced order system models. In the second stage, the EP&A model reduction method is employed to reduce further the order of the system model generated in the first stage.

The merit of the COMPARE methodology is that system models (at various system configurations) assembled using CMS-generated component models are smaller in size than the full-order system models. Hence, COMPARE alleviates the need to solve large-order eigenvalue problems repetitively. The need to generate the components' M-R or C-B mode sets is not a disadvantage because efficient software exists for their construction (see, e.g., Tsuha[9]). The effectiveness of COMPARE, using the M-R version of COMPARE, has been successfully demonstrated on a high-order, finite-element model of the cruise-configured Galileo spacecraft.

6. References

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Table 1 Frequencies of Reduced-order Stator, Rotor, and System (at a Clock Angle of 300 degrees) Flexible Modes

Flexible Mode	ω_{stator} (Hz)	ω_{rotor} (Hz)	ω_{system} (Hz)
1	7.105	0.143	0.127
2	9.129	0.866	0.864†
3	10.561	1.237	1.236†
4	14.560	1.483	1.479†
5	43.172	1.728	1.707†
6	50.070	2.286	1.734†
7	63.530	2.809	2.072†
8	80.867	3.647	2.351†
9	86.989	3.996	2.815†
10	96.605	5.207	3.707†
11	166.34	5.337	4.167
12	240.18	5.994	5.231†
13	254.52	6.410	5.433
14		9.503	5.467†
15		10.291	6.150
16		10.553	6.436
17		13.536	7.056†
18		15.613	8.153†
19		29.188	9.606†
20		41.181	9.613
21		58.524	10.294
22		69.003	10.420†
23		77.722	10.555†
24			13.534†
25			13.997
26			15.860
27			16.800†
28			20.591
29			21.280†
30			28.462
31			34.316
32			41.214
33			48.011
34			58.318
35			71.770†
36			82.129†

† Exactly captured system flexible modes.

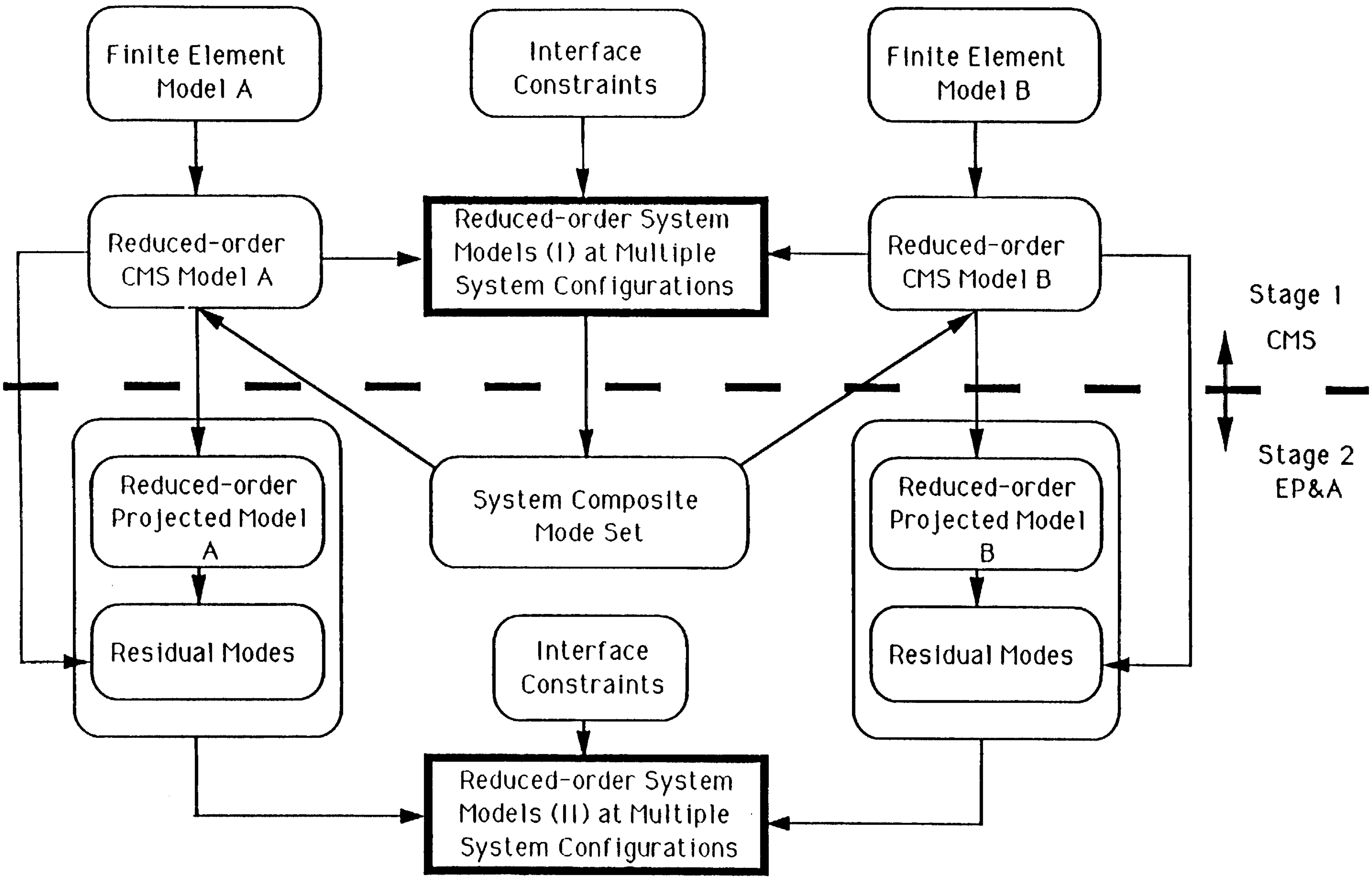
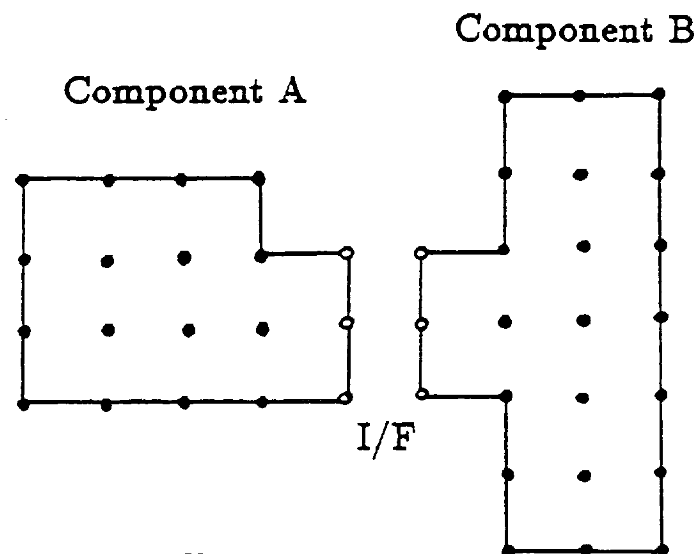


Fig. 1 A 2-stage Component Modes Projection and Assembly Model Reduction Methodology (COMPARE)



o x_i, Interface Coordinates

• x_j, Interior Coordinates

Fig. 2 Coordinate Definitions of A 2-Component System

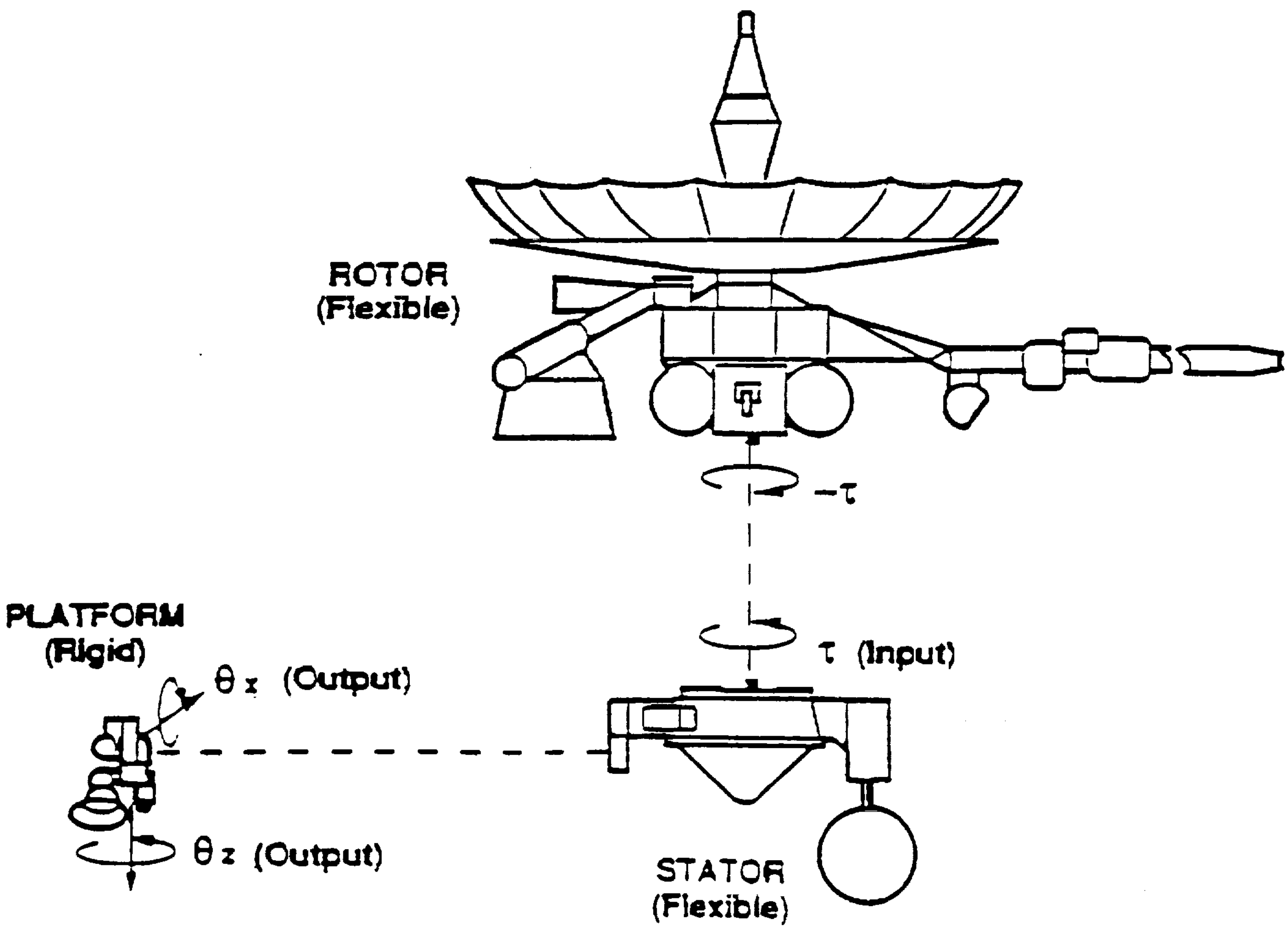


Fig. 3 Galileo Spacecraft Cruise Model[2,6,7]

Fig. 4 Bode Plot Comparison of Full- and Reduced-order Models

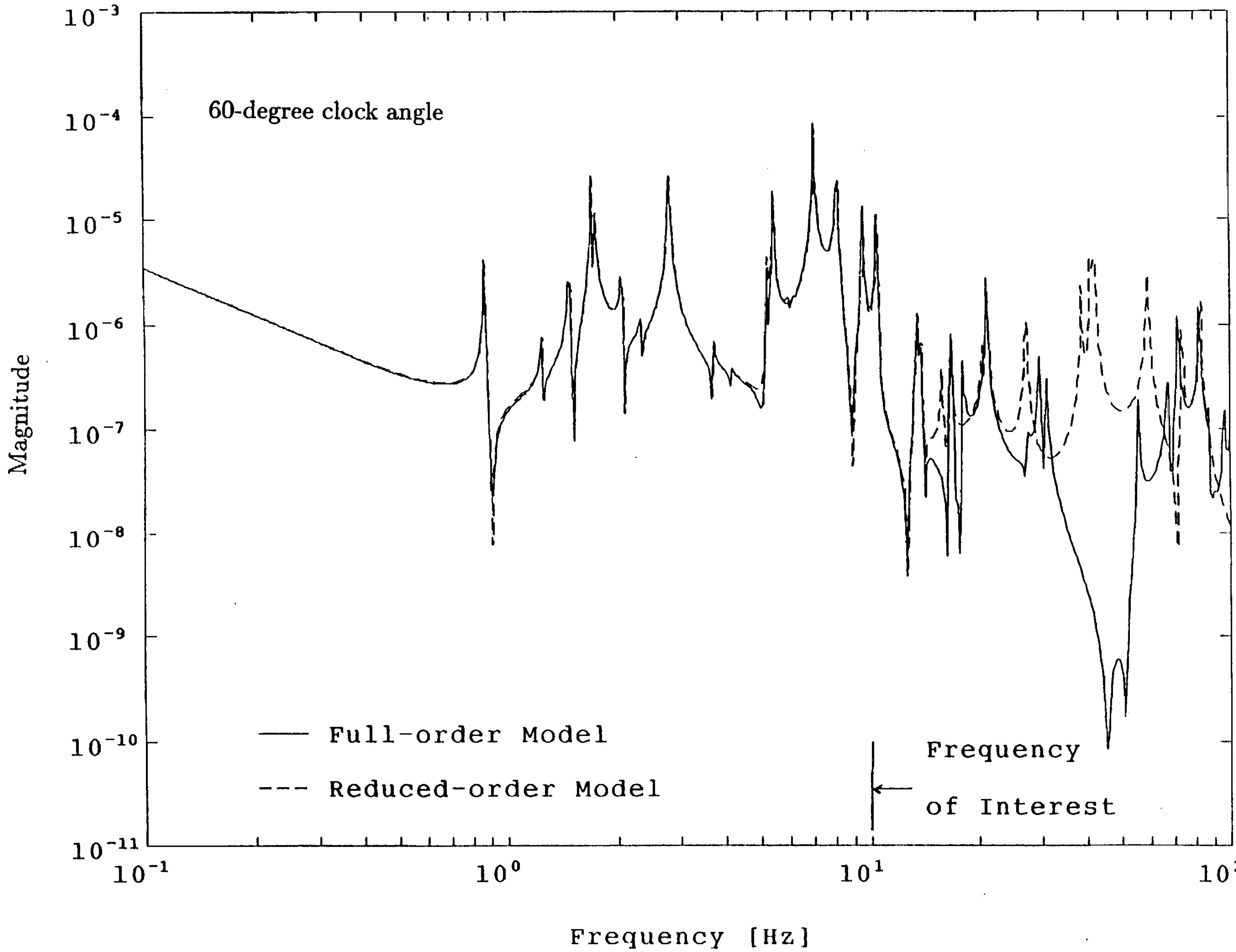


Fig. 5 Bode Plot Comparison of Full- and Reduced-order Models

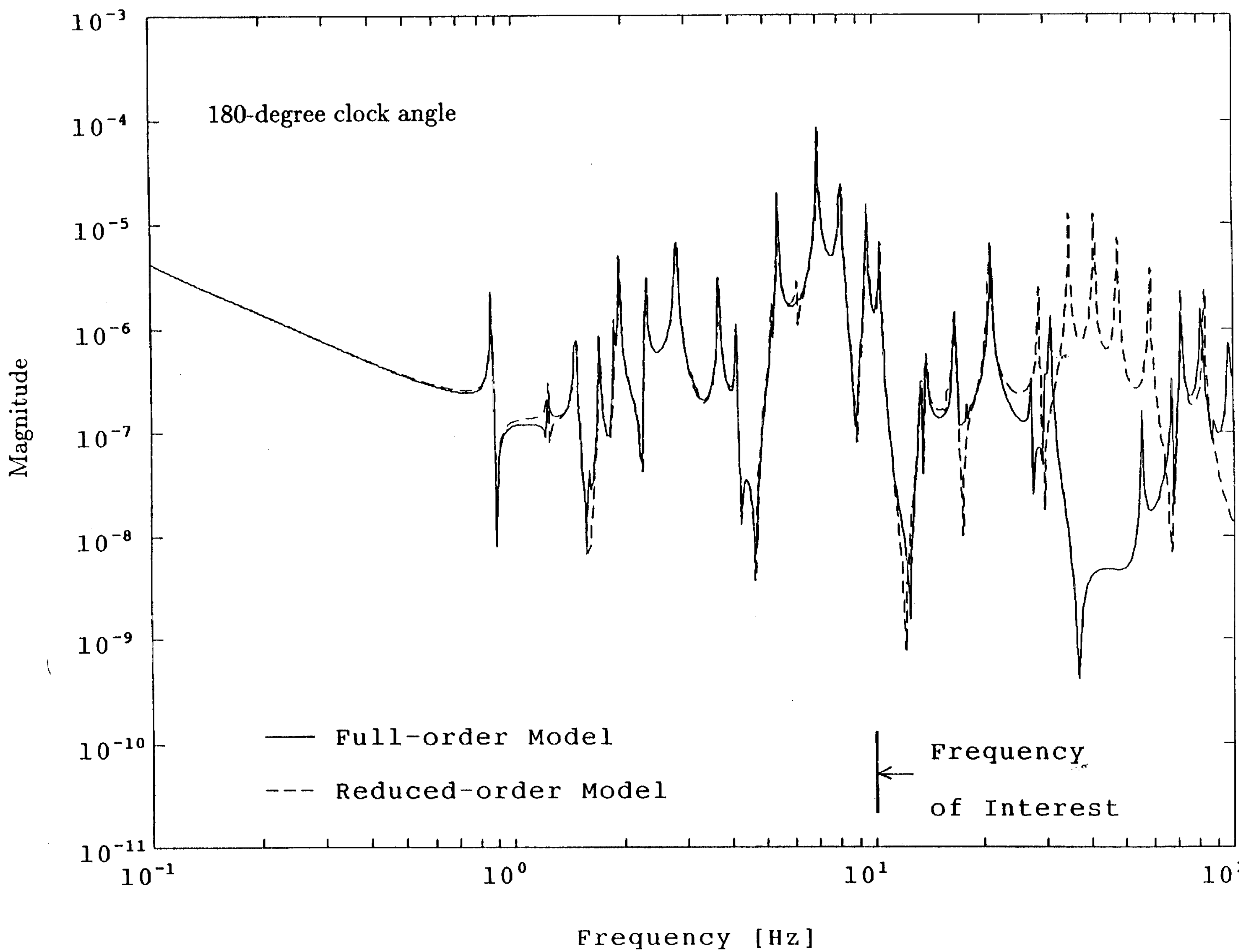


Fig. 6 Bode Plot Comparison of Full- and Reduced-order Models