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Inverse Dynamics: Simultaneous Trajectory Tracking and Vibration Reduction with Distributed Actuators

Santosh Devasia

Eduardo Bayo

Department of Mechanical Engineering
University of California
Santa Barbara, CA 93106

ABSTRACT

This paper addresses the problem of inverse dynamics for articulated flexible structures with both lumped and distributed actuators. This problem arises, for example, in the combined vibration minimization and trajectory control of space robots and structures. A new inverse dynamics scheme for computing the nominal lumped and distributed inputs for tracking a prescribed trajectory is given.

1. Introduction

Inverse dynamics is an important problem in the control of articulated flexible structures such as space stations and manipulators. A solution for the nonredundant lumped actuator case has been provided by Bayo et. al., [1] and Book, [2]. This method produces bounded inputs which move a reference point on the structure along a desired trajectory. The inputs are necessarily non-causal when the structure dynamics are nonminimum phase. Elastic deformation which may cause vibration of the structure is also determined by the trajectory; our goal is to minimize such vibrations. The viability of distributed actuators for the control of structural vibrations, [3] , [4] and [5] , has motivated their use here for trajectory tracking.

Trajectory tracking of the structure can be accomplished by the use of the joint actuators alone [6] and in this sense the distributed actuators are redundant. We introduce the concept of using the extra actuation available through the distributed actuators in the structure to not only satisfy the trajectory tracking constraint but also minimize the accompanying elastic displacements during the motion. To obtain these new feedforward inputs, the inverse dynamics method suggested in [1] is extended to cover cases of redundantly-actuated structures. This use of distributed actuators in feedforward for end effector trajectory control is contrasted with the use of only the joint actuators in feedforward in an example of a flexible two link truss structure with distributed piezo-electric actuators to verify the efficacy of the proposed method.

The remainder of the paper is organized in the following format. The modeling of flexible structures with joint and distributed actuators, the formulation of the problem and its solution are presented in Section 2. Section 3 deals with an application of the proposed method to the example of a two link flexible truss. The discussions and conclusions are presented in section 4.

2. Formulation

The solution to the general multi-link inverse dynamics problem involves studying an individual link in the chain, coupling the equations of the individual links, and then recursively converging to the desired actuator inputs and corresponding displacements. This approach is presented below, beginning with a single link.

2.1 Equation of motion of a single link

To simplify the equations, we present the equations for a link with a revolute joint. The flexible link depicted in figure 1 forms part of a multi-link system. The link is shown with a revolute joint, however the formulation remains identical for a link with translational joint. The elastic deflections in the structure are defined with respect to a nominal position characterized by a moving frame whose origin coincides with the location of the hub of the link. The nominal motion of this frame is prespecified by its angular velocity ω_h, angular acceleration α_h and the translational motion of its origin. The above definition of the elastic displacements with respect to this nominal frame permits the linearization of the problem from the outset. Incorporating the kinematic model followed by Naganathan and Soni [7] in a finite element model (FEM), the equations of motion for a single link at any time t can be written as [1]

$$M\ddot{z} + \left[C + C_c(\omega_h)\right]\dot{z} + \left[K + K_c(\alpha_h,\omega_h)\right]z = B_T T + B_p V_p + F. \tag{2.1}$$

Where z is an R^n vector of the finite element degrees of freedom. M and K belong to $R^{n\times n}$ and are the conventional finite element mass and stiffness matrices respectively; C_c and $K_c \in R^{n\times n}$ and are the time varying Coriolis and centrifugal stiffness matrices, respectively. The $R^{n\times n}$ matrix C represents the internal viscous damping of the material. T is the unknown joint actuation. $F \in R^n$ contains the reactions at the end of the link, and the known forces produced by the rotating frame effect. The distributed actuator inputs at time t are The equivalent nodal forces at the FEM degrees of freedom due to the distributed actuators are represented by V_p, a R^{np} vector, where np is the number of distributed actuator inputs. B_T and B_p are constant matrices of dimensions R^n and $R^{n\times np}$, respectively. The set of finite element equations (2.1) may be partitioned as follows

$$M\begin{bmatrix}\ddot{\theta}_h\\ \ddot{z}_i\\ \ddot{z}_t\end{bmatrix} + \left[C + C_c(\omega_h)\right]\begin{bmatrix}\dot{\theta}_h\\ \dot{z}_i\\ \dot{z}_t\end{bmatrix} + \left[K + K_c(\alpha_h,\omega_h)\right]\begin{bmatrix}\theta_h\\ z_i\\ z_t\end{bmatrix} =$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix}T + \begin{bmatrix}B_{p_h}\\ B_{p_i}\\ B_{p_t}\end{bmatrix}V_p + \begin{bmatrix}F_h\\ F_i\\ F_t\end{bmatrix} \tag{2.2}$$

where θ_h is the elastic rotation of the hub, z_t is the elastic deflection at the tip in the y direction, and the other $n-2$ finite element degrees of freedom are included in the vector z_i. The force vector, F, and the B_p and B_T matrices are also partitioned similarly.

2.2 Minimization Objective

The requirement is to accurately track the end effector of the link along the given nominal trajectory without overshoot and residual vibrations. Additionally we also seek to minimize the ensuing structural vibrations during this motion by minimizing $J(T,V_p)$, a measure of elastic deflections in the structure defined as follows

$$J(T,V_p) = \int_{-\infty}^{\infty} z(t)^T z(t) dt. \tag{2.3}$$

Mathematically the objective can be stated as

$$\min_{(T,V_p) \in \hat{T}} J(T,V_p). \tag{2.4}$$

Where $\hat{T}$ is the set of all pairs of stable joint torque and distributed actuator inputs that when used to actuate the system defined by equation (2.2) yields $z_t(t) = 0$ for all t.

2.3 Solution Methodology

An iterative scheme is described below for each link. Equation (2.2) can be rewritten as

$$M\ddot{z} + C\dot{z} + Kz = B_T T + B_p V_p + F - C_c(\omega_h)\,\dot{z} - K_c(\alpha_h,\omega_h)\,z \tag{2.5}$$

where the time dependent Coriolis and centrifugal terms are kept on the RHS of the equation. The iteration procedure starts with the absence of the last two terms involving C_c and K_c in the right hand side. Then, the system of equations can be transformed into independent sets of simultaneous complex equations by means of the Fourier transform. For each of the evaluation frequency ϖ, equation (2.5) becomes

$$\left[M + \frac{1}{i\varpi}C - \frac{1}{\varpi^2}K\right] \begin{bmatrix} \bar{\ddot{z}}_h \\ \bar{\ddot{z}}_i \\ \bar{\ddot{z}}_t \end{bmatrix} = \begin{bmatrix} \bar{T} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \bar{F}_h \\ \bar{F}_i \\ \bar{F}_t \end{bmatrix} + \begin{bmatrix} B_{p_h} \\ B_{p_i} \\ B_{p_t} \end{bmatrix} \overline{V_p} \tag{2.6}$$

where the bar stands for the Fourier transform, and F represents the known forcing terms. After the first iteration it will also include the updated contributions from the Coriolis and centrifugal terms appearing in the RHS of equation (2.5). For any $\varpi \neq 0$, the matrix

$$H = \left[M + \frac{1}{i\varpi}C - \frac{1}{\varpi^2}K\right] \tag{2.7}$$

is a complex, symmetric and invertible matrix. For $\varpi = 0$ the system undergoes a rigid body motion and $H = M$ which is the positive definite invertible mass matrix. Let $G = H^{-1}$. Then the above equation can be re-written as

$$\begin{bmatrix} \overline{\ddot{z}_h} \\ \overline{\ddot{z}_i} \\ \overline{\ddot{z}_t} \end{bmatrix} = \begin{bmatrix} G_{hh} & G_{hi} & G_{ht} \\ G_{ih} & G_{ii} & G_{it} \\ G_{th} & G_{ti} & G_{tt} \end{bmatrix} \left\{ \begin{bmatrix} \overline{T}(\varpi) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \overline{F}_h \\ \overline{F}_i \\ \overline{F}_t \end{bmatrix} + \begin{bmatrix} B_{p_h} \\ B_{p_i} \\ B_{p_t} \end{bmatrix} \overline{V_p} \right\} . \tag{2.8}$$

The condition that the tip should follow the nominal motion is equivalent to $\overline{\ddot{z}_t} = 0$ for all ϖ. This induces a relationship between the joint actuation and the distributed actuator inputs and is obtained from the last row of the previous equation.

$$\overline{T} = -\, G_{th}{}^{-1} \begin{bmatrix} G_{th} & G_{ti} & G_{tt} \end{bmatrix} (\overline{F} + B_p \overline{V_p}) . \tag{2.9}$$

Substituting this expression for the input hub torque in equation (2.8) and using the property that $\overline{\frac{d^2 z}{dt^2}} = -\varpi^2 \overline{z}$ yields

$$\overline{z} = -\frac{1}{\varpi^2} (A \overline{V_p} + B) . \tag{2.10}$$

Where

$$A = [-G_{th}{}^{-1}\, GB_T (G_{th}\ G_{ti}\ G_{tt}) + G]\, B_p \tag{2.11}$$

and

$$B = [-G_{th}{}^{-1}\, GB_T (G_{th}\ G_{ti}\ G_{tt}) + G]\, \overline{F} . \tag{2.12}$$

Next we determine $\overline{V_p}$. Using Parseval's theorem, minimizing $J(T, V_p)$ in equation (2.4) is equivalent to minimizing $\|\overline{z}\|_2^2$ at each ϖ. This is a standard least squares approximation problem [8] and results in the following solution for the distributed actuator inputs,

$$\overline{V_p} = -U \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & 0 \end{bmatrix} V^* B \tag{2.13}$$

where Σ, U and V define the standard singular value decomposition of A as follows

$$V^* A U = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} . \tag{2.14}$$

Where the conjugate transpose matrix operator is denoted by $*$. In addition if A has rank np, which is the number of distributed actuator inputs, then the least squares approximation yields

$$\overline{V_p} = -(A^* A)^{-1} A^* B . \tag{2.15}$$

A sufficient and necessary condition for A to have rank np is given next.

Lemma

$$\text{rank } [A] = np \quad \text{if and only if} \quad \text{rank } [B_T \mid B_p] = np+1 \tag{2.16}$$

Proof

$$\text{Rank } [B_T] = 1 \Rightarrow \text{rank } \begin{bmatrix} GB_T(G_{th}\ G_{ii}\ G_{tt})] = 1 \\ \Rightarrow \text{rank } \left[\hat{A} = [-G_{th}^{-1}\ GB_T(G_{th}\ G_{ti}\ G_{tt}) + G\,]\right] \end{bmatrix} \geq n-1.$$

Since $B_T = [1\ 0\ 0]^*$, it is easy to see that the null space of $\hat{A}$ is the span of $[1\ 0\ 0]^*$. Hence rank $\hat{A}$ is $n-1$. Noting that $A = \hat{A}\ B_p$, the lemma follows easily. □

The above lemma requires that all the columns of the input matrices B_T and B_p be independent. This is computationally more efficient than checking the rank of A for each ϖ. Next, the corresponding joint torque component, $\bar{T}$ is then evaluated from equation (2.9). The inverse Fourier transforms for the feedforward inputs completes the first iteration and results in torques, T^1 and distributed inputs V_p^1. Then the forward dynamic analysis is carried out to compute K_c and C_c. F in the RHS of equation (2.5) is updated and the process is repeated to find the new input torques and voltages. The process is stopped at the n^{th} iteration if $\|T^n - T^{n-1}\|_2 + \|V_p^{\,n} - V_p^{\,n-1}\|_2 < \varepsilon$, where ε is some small positive constant. It may be noted that for slow motions the terms involving K_c and C_c are small relative to the other terms in equation (2.1) and the iterations converge in a few steps [1].

2.4 The Algorithms for the Multi-Link Cases

In the previous sub-section the procedure to evaluate the joint actuations of a single link was presented. This can be recursively extended for multi-link flexible manipulators. Algorithms are presented below for both open and closed chain multi-link mechanisms. These are similar to those proposed by Bayo et. al. [1].

Multi-Link Open Chain Case

1. Define the nominal motion (Inverse Kinematics of rigid manipulator).
2. For each link j, starting from the last one in the chain:
   a) Compute torque (or force) T^j and distributed actuator inputs P_v^j imposing $z_i^j = 0$ (Section 2).
   b) Compute the link reaction forces R^j from equilibrium.
3. Use equation (2.1) to compute the elastic displacement and joint angles.
4. Compute the inputs for the next link, $j-1$.

Multi-Link Closed Chain Case

1. Define the nominal motion (Inverse Kinematics of rigid robot).
2. Define an independent set of joint forces and reactions equal in number to the degrees of freedom of the robot.
3. For each link j, starting from the last one in the chain:
   a) Compute torque (or force) T^j and distributed actuator inputs P_v^j imposing $z_i^j = 0$ (Section 2).

b) Compute the link reaction forces R^j from equilibrium.

4. Use equation (2.1) to compute the elastic displacements and joint angles
5. Use elastic deflections to correct the nominal motion of each link.
6. Repeat steps 3 to 5 until convergence in the forces/torques is obtained.

This concludes the methodology. In the next section we present an application to a two-link flexible manipulator.

3. Example

A twolink truss experiment under development at UCSB is shown in figure 2. The trusses are made of lexan and have lumped masses (net 2 Kg for each link) distributed along their lengths. The first and the second links are tip loaded with 3.5 and 1 Kg respectively. Equivalent beam properties of the trusses used in the FEM model for simulations are Youngs modulus = 7 e^9 GPa, Link length = 1.2 m, density = 1500 Kg/m^3, cross sectional area = 4.378 e^{-5} m^2 and cross sectional area moment of inertia = 4.7244 e^{-9} m^4. Of the 10 spans in each link, two are piezo-electrically actuated. They are located at the second and ninth spans as shown in the figure 2. The piezo-electric stack actuators in those spans have the following properties. Cross sectional area, A_{cs} = 7.3 e^{-6} m^2, piezo strain to voltage constant, d_{sv} = .731 e^{-6} V^{-1}, Youngs modulus, E_p = 73 e^9 Gpa and distance of the actuator from the neutral axis of the truss, d_t = 1.27 e^{-2} m. Following the standard Bernoulli-Euler modeling for an applied voltage V_{input}, the piezo-electric actuation can be considered as two concentrated moments M acting at the two ends of the actuator [9] and [10]. Where M is given by

$$M = (d_{sv}\ N_p E_p A_{cs} d_t) V_{input} \tag{3.1}$$

and $N_p = 4$ is the number of piezos in each span. For the truss considered above $M = \ = 0.0198 V_{input}$. The desired trajectory is a rest to rest motion of the structure with initial conditions given by $\theta_1 = \theta_2 = 0$ and final conditions $\theta_1 = 11.25^o$ and $\theta_2 = -22.5^o$. $\theta's$ are the absolute angles of the links with respect to a frame fixed on the ground and are shown in figure 2. The nominal motion of the tip for each link are the trajectories followed by the tips of the links if the structure were rigid and followed the nominal angular motions shown in figure 3. Using the procedure in section 2.4 for open-chain mechanisms, open loop simulations were performed (1) using only the joint actuation for feedforward and (2) using the distributed piezo-electric actuators along with joint actuators in feedforward and the results are presented below.

Plots of the input piezo voltages and joint torques are presented in figures 4 and 5 respectively. To illustrate the viability of the proposed method figures 6 and 7 show the transverse structural midpoint deflections of the two links during the motion with and without the distributed actuators. Similar plots for the elastic hub rotations are shown in figures 8 and 9.

Thus the piezo-electric actuators show a significant reduction in the structural vibrations and demonstrate the viability of the proposed method. The consequent reduction in the induced strains in the structure allows the use of lighter elements and smaller actuators, especially in space structures where the loads are mainly inertial.

4. Conclusion

Typically distributed actuators like the piezo-electric ones cannot garner enough actuation to cause large motions in the structure. However they could be very effective in controlling the small structural deformations in the structure. Their use in the feedforward to aid the joint actuators for trajectory tracking is a novel idea developed in this paper. The method proposed was shown to be extremely efficient in removing structural vibrations from structures as seen in the example. Thus these feedforward actuations, obtained through the proposed inverse dynamics, augmented with joint angle feedback based closed loop controllers seem promising in the slewing control of flexible manipulators. This encouraging result motivates further work on distributed actuators in the control of flexible structures.

ACKNOWLEDGEMENT

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References

1. E. Bayo, M.A. Serna, P. Papadopoulos, and J. Stubbe, "Inverse Dynamics and Kinematics of Multi-Link Elastic Robots. An Iterative Frequency Domain Approach," *The International J. of Robotics Research*, vol. 8, No 6, Dec 1989.
2. D. Kwon. and W.J. Book, "An inverse dynamic method yielding flexible manipulator state trajectories," *Proc. of ACC*, pp. 186-193, 1990.
3. Fanson, J. L. and Garba, J. A., "Experimental Studies of Active members in Control of Large Space Structures," *Proc. AIAA 29th SDM Conf.*, pp. 9-17, 1988.
4. L. Meirovitch and H. Baruh, "Control of Self-Adjoint Distributed-Parameter Systems," *AIAA*, vol. 5, No 1, pp. 60-66, 1980.
5. M. J. Balas, "Active Control of Flexible Systems," *Journal of Optimization Theory and Applications*, vol. 25, No 3, pp. 415-436, 1978.
6. B. Paden, D. Chen, R. Ledesma, and E. Bayo, "Exponentially Stable Tracking Control for Multi-Joint Flexible-Link Manipulators," *ASME J. of Dynamic Systems, Measurement and Control*, vol. accepted for publication.
7. G. Naganathan and A.H. Soni, "Coupling Effects of Kinematics and Flexibility in Manipulators," *International Journal of Robotics Research*, vol. 6, No 1, pp. 75-85, 1987.
8. G.W. Stewart, *Introduction to Matrix Computations*, pp. 319-325, Academic Press, Inc., 1973.
9. E.F. Crawley and E.H. Anderson, "Detailed Models of Piezoceramic Actuation of Beams," *J. of Intelligent Material Systems and Structures*, vol. 1, pp. 4-24, 1990.
10. S. Devasia, "Modeling of Piezo Electric Actuators," *M.S. Thesis*, UCSB, 1990.

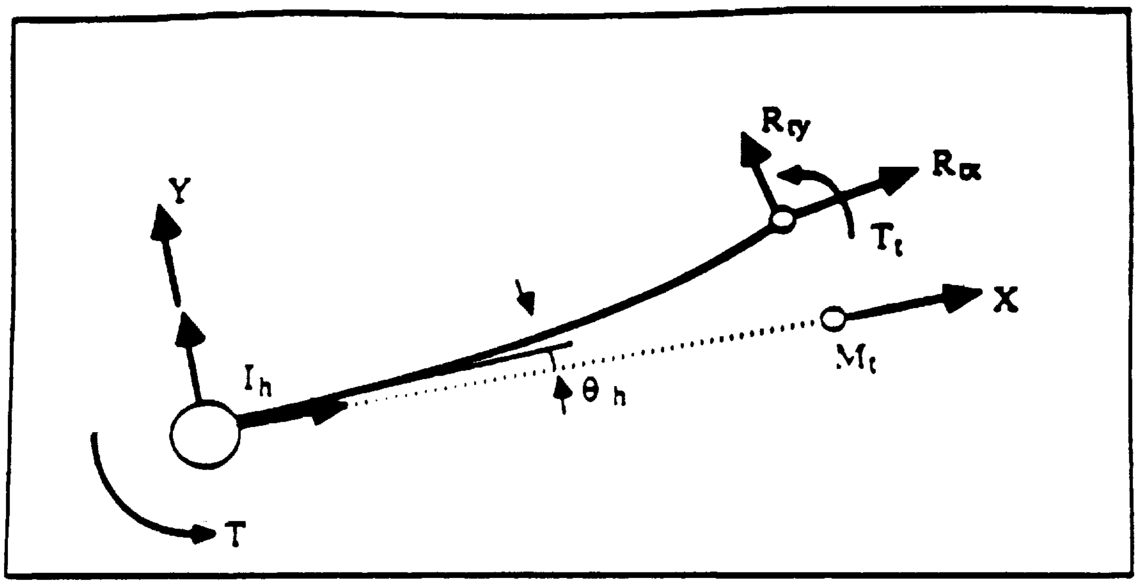

Figure 1. A Single Flexible Link

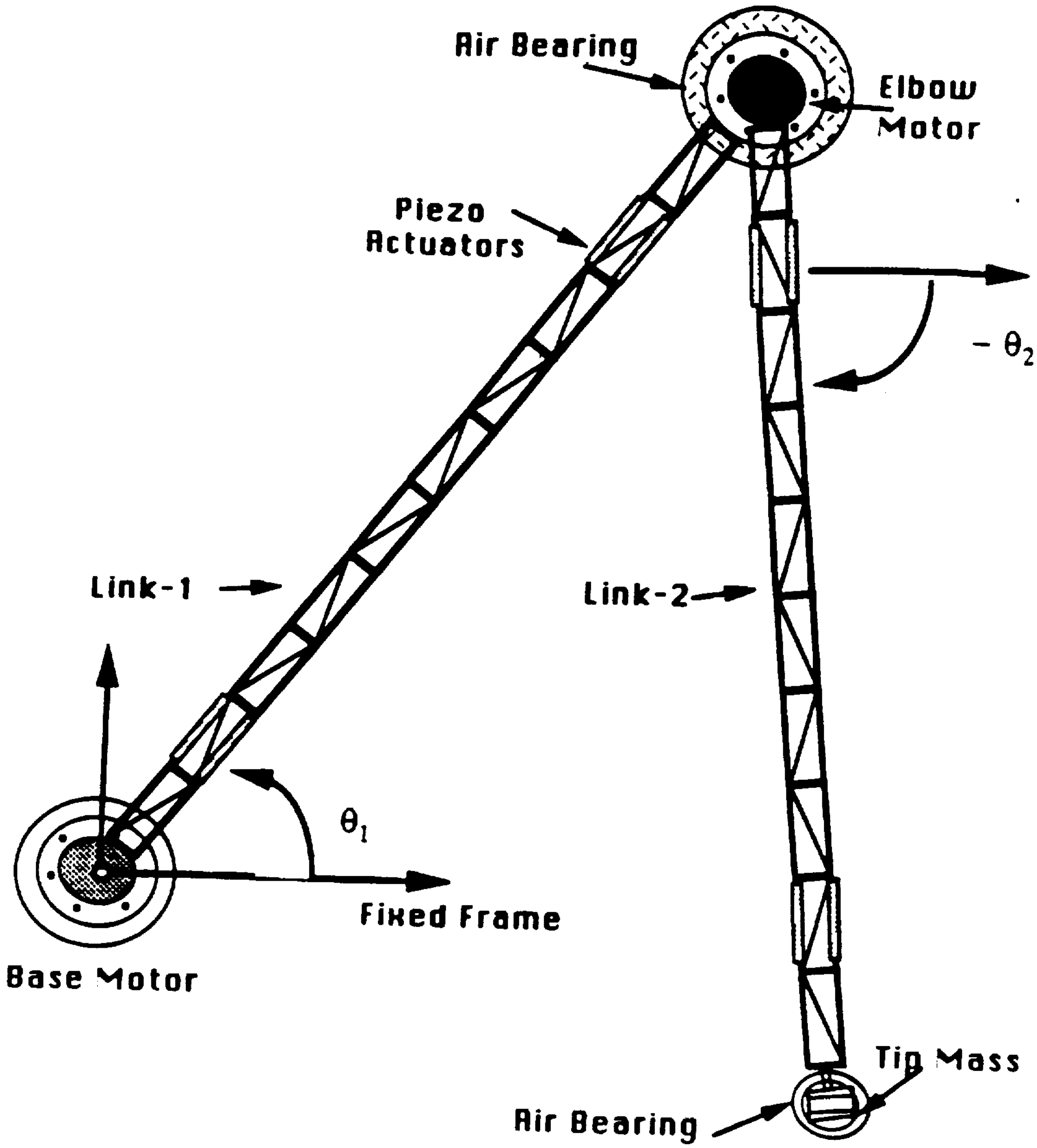

Figure 2. The Two Link Truss Structure

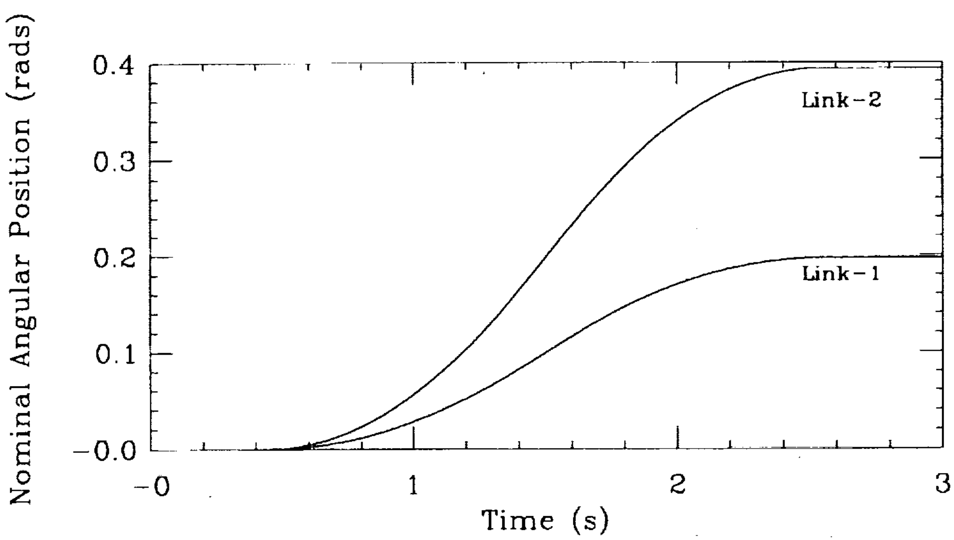

Fig.3: nominal angular positions

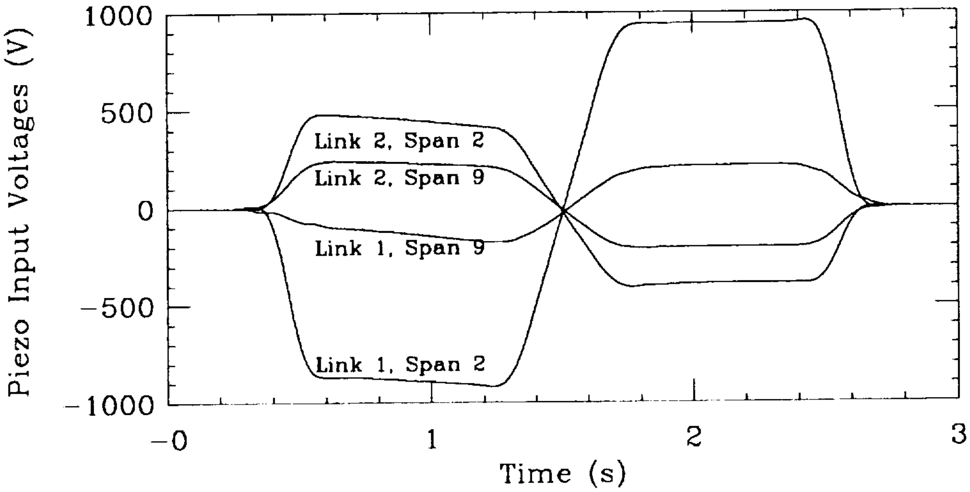

Fig.4: input piezo voltages

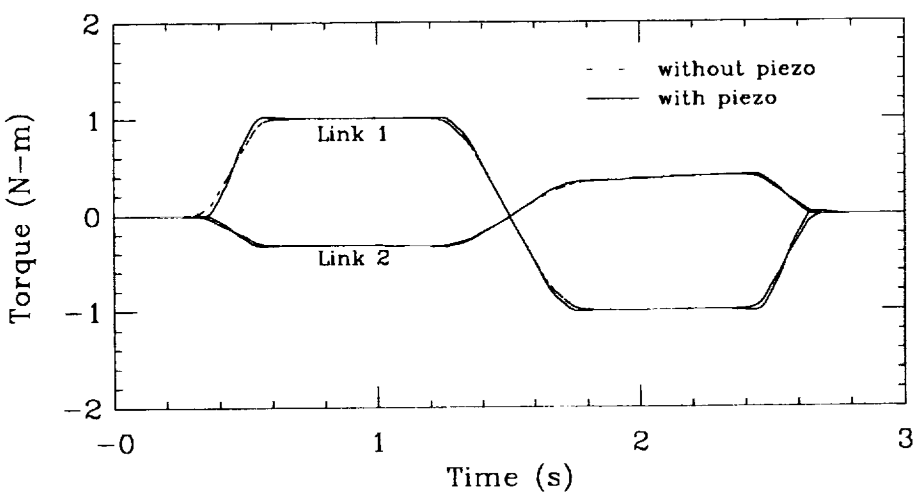

Fig.5: inverse dynamics torques

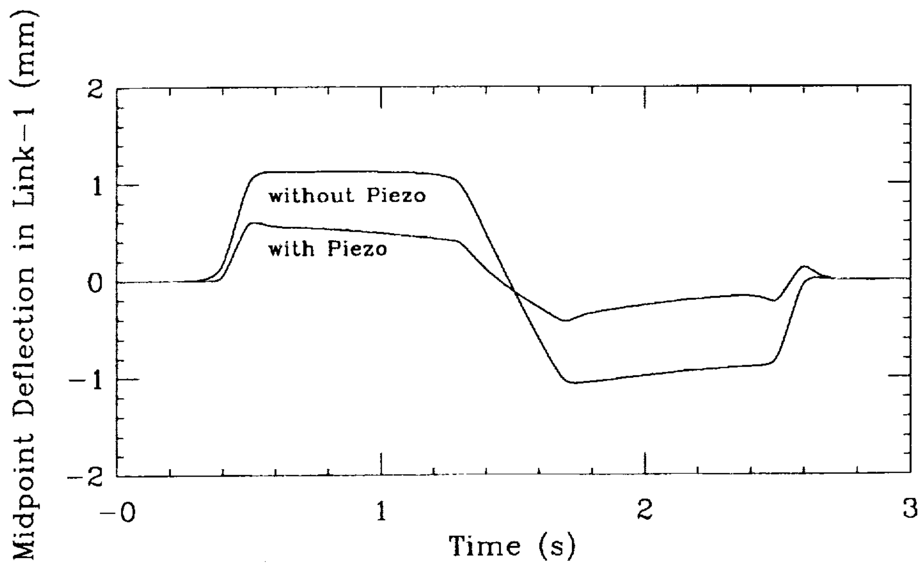

Fig.6: transverse deflection at midpoint of link 1

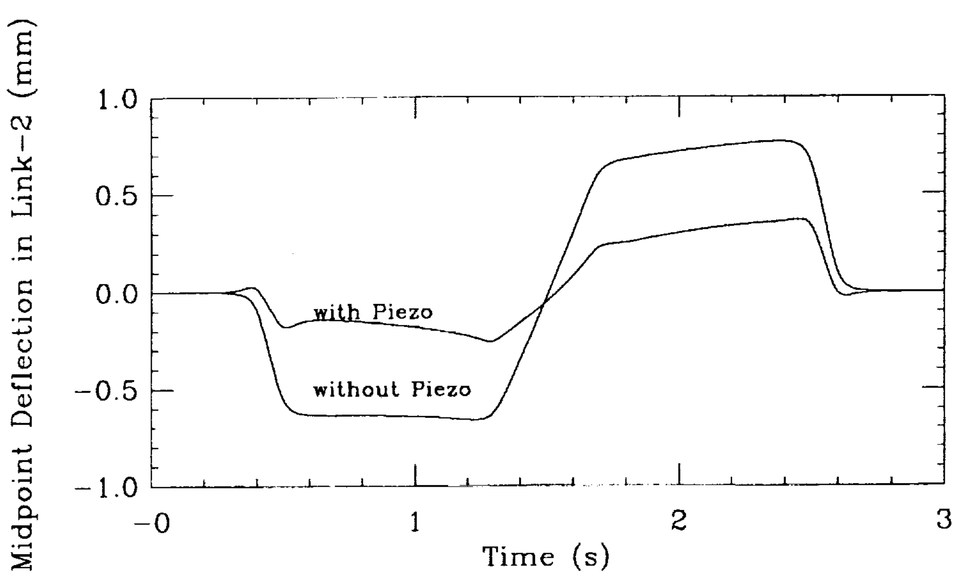

Fig.7: transverse deflection at midpoint of link 2

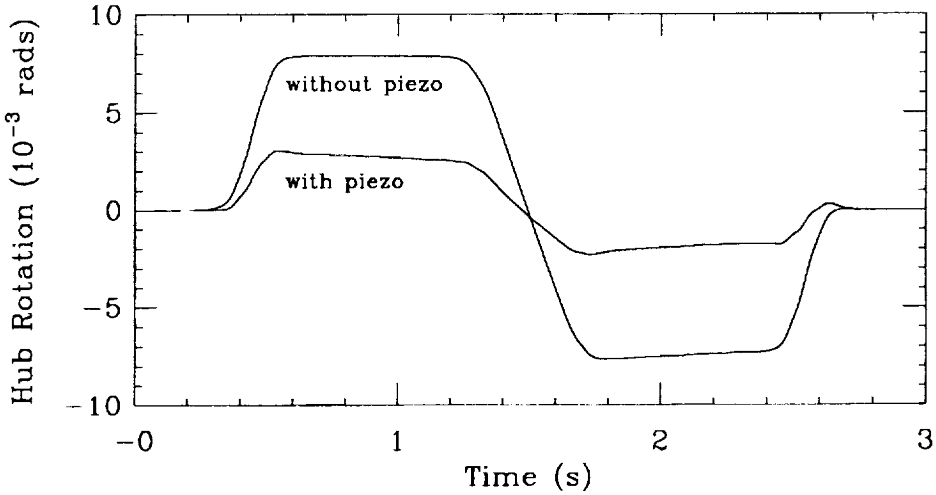

Fig.8: elastic hub rotation of link 1

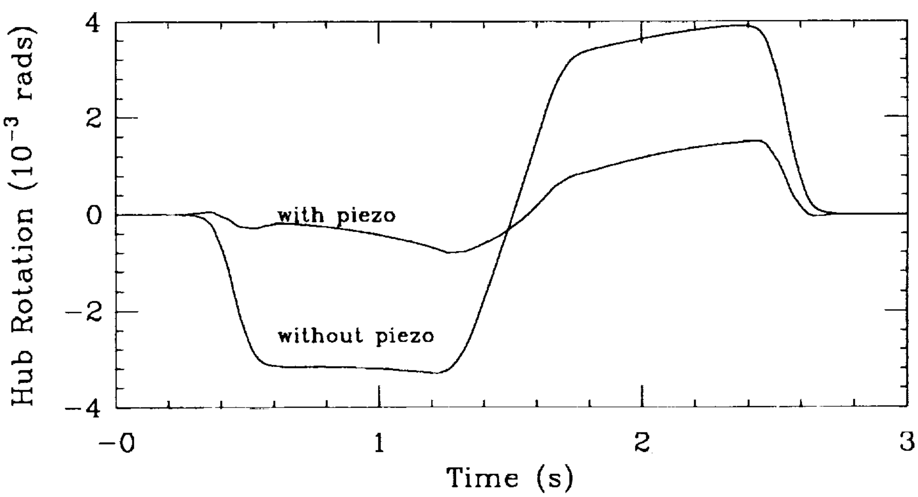

Fig.9: elastic hub rotation of link 2