LES versus DNS : a comparative study

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We have performed Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES) of forced isotropic turbulence at moderate Reynolds numbers. The subgrid scale model used in the LES is based on an eddy viscosity which adjusts instantaneously the energy spectrum of the LES to that of the DNS. The statistics of the large scales of the DNS (filtered DNS field or fDNS) are compared to that of the LES. We present results for the transfer spectra, the skewness and flatness factors of the velocity components, the PDF's of the angle between the vorticity and the eigenvectors of the rate of strain, and that between the vorticity and the vorticity stretching tensor. The above LES statistics are found to be in good agreement with those measured in the fDNS field. We further observe that in all the numerical measurements, the trend was for the LES field to be more gaussian than the fDNS field. Future research on this point is planned.

1. Introduction

Direct Numerical Simulation (DNS) of turbulent flows has become an indispensable tool in turbulence research. The importance of DNS was universally recognized when researchers started to obtain new qualitative results. While a DNS can reproduce basic turbulence constants or statistics determined previously from laboratory experiments, it is also able to provide statistical information difficult to obtain by experimental measurements. Among effects observed in the DNS prior to laboratory experiments are alignments of vorticity and velocity vectors (Pelz et al., 1985), alignments of vorticity vector and the eigenvectors of the rate of strain (Ashurst et al., 1987), and reduction of nonlinearity (Kraichnan & Panda, 1989). All of these effects are not present in gaussian fields. Some of the above observations can be qualitatively predicted in the framework of the DIA or the EDQNM closure approximations (see, for instance, Chen et al., 1987). Others, such as the spottiness of the vorticity field that was observed in the DNS, have not yet been demonstrated by closures. Nevertheless, the DNS (with all its advantages) is still limited to relatively low Reynolds numbers. Attainment of a high Reynolds number simulation requires use of a subgrid scale model to represent the effects of the unresolved small-scale turbulence on the explicitly simulated large-scale flow.

The most important assumption in this Large Eddy Simulation (LES) approach is that the subgrid scale model may be parameterized in terms of the resolved large-scales and a relatively small set of additional parameters. The basis for such

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an assumption is the experimental evidence supporting the 1941 phenomenology of Kolmogorov that the low-order statistics of the small scales are self-similar in a high Reynolds number turbulence. The task of modeling is to find a subgrid scale model which can represent the effects of a strongly nongaussian, intermittent small scale field of turbulence on a large scale field. For homogeneous and isotropic turbulence, Kraichnan (1976) introduced an effective eddy viscosity $\nu_e(k|k_m, t)$ acting at time t on scales of wavenumber k due to the effects of scales with wavenumbers greater than k_m. In this model, the eddy viscosity is derived from the turbulence energy equation. Clearly, one should construct $\nu_e(k|k_m, t)$ (from analytical theories or the DNS) in such a way that, at a minimum, the low order statistics of the flow field (*e.g.*, the energy spectrum) is preserved. The major role played by a well-chosen eddy-viscosity is to "adjust" the spectrum to the value it is supposed to have from analytical or DNS considerations. Strict eddy-viscosity models, however, suffer from a lack of phase information and an eddy-viscosity model combined with a random gaussian force has been shown to somewhat better reproduce the inertial range energy spectrum (Chasnov, 1991). However, it is not clear if such eddy-viscosity subgrid models or their refinements are capable of reproducing higher-order statistical moments of the large scales. Ideally, an LES field should be statistically the same as the large-scales of a fully-resolved DNS, not withstanding the inaccuracy in the representation of the small scales by a subgrid scale model.

2. The numerical experiment

Let us consider a DNS with resolution N^3. One can filter (in k-space) the field resulting from this simulation. Then we obtain an M^3 field ($M << N$). Simultaneously we will perform an LES with resolution M^3. We will use the same initial conditions and the same Reynolds number. Does the LES field remain the same as the filtered DNS field after a long time of evolution? To answer this question, we define a correlation coefficient for the filtered DNS and LES fields:

$$\eta = \frac{< \mathbf{u} \cdot \mathbf{u}' >}{< \mathbf{u}^2 >^{\frac{1}{2}} < \mathbf{u}'^2 >^{\frac{1}{2}}}. \tag{1}$$

In the context of unpredictability studies using closure theories (Leith & Kraichnan, 1972), it was demonstrated that two turbulent fields which are identical in the large-scales but differ in the small scales at high Reynolds numbers will become decorrelated after a time on the order of a large-eddy turnover time. The implication is that an LES can not hope to follow a single realization of a turbulent flow. Although this may have some practical importance to problems such as weather prediction, most engineering applications only require an LES to obtain the correct statistics of the large scales. The more important question we will therefore address is: are the filtered DNS field and the LES field still statistically the same after a sufficiently long time evolution after which the fields themselves are completely different? We will check that commonly accepted effects associated with the non-gaussian nature of turbulence fields, such as the above mentioned alignments, are observed in an LES and are quantitatively similar to those measured in the filtered DNS field.
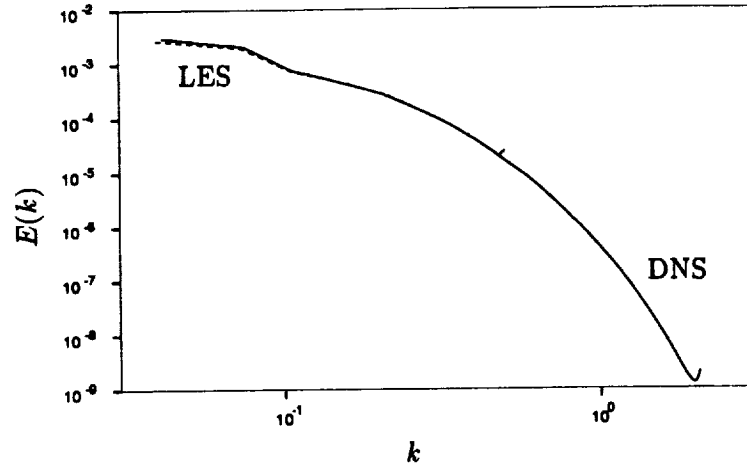
FIGURE 1.   Final energy spectrum of the DNS and the LES.

Toward this end, we performed simulations of forced isotropic, homogeneous tur-
bulence with $Re \approx 70$ and a resolution of 128^3. A fully-developed field was used
as the initial conditions for the DNS and LES comparison. Starting with this field,
we have time-evolved a 128^3 DNS and a 32^3 LES. Our subgrid scale model for the
LES consists of adjusting the shell-averaged spectrum of the LES to the value ob-
tained from the DNS by a simple rescaling of the Fourier amplitudes within each
shell without phase modification. Instead of the 128^3 DNS field, we therefore have
a 32^3 LES field plus the energy spectrum of the truncated DNS (averaged in shells
of unit thickness), which consists of 15 real numbers. Thus the LES preserves the
instantaneous spectrum of the DNS, which is better than all existing subgrid scale
models. Although such an LES is not realistic in practice since one needs to perform
a fully-resolved DNS concurrently, the failure of this LES could very well imply the
failing of the approach itself.

3. Results

In figure 1, we present the final energy spectrum for the DNS and the LES. The
plot demonstrates that all the scales of the DNS are fully-resolved. The DNS has
a maximum Kolmogorov wavenumber of 2 while the LES and the fDNS have a
maximum Kolmogorov wavenumber of 0.5.

In figure 2, we present the time-evolution of the correlation coefficient η (eq. (1)).
This plot demonstrates the impossibility of an LES to follow a particular realization
of the DNS field. Indeed, after approximately two large-eddy turnover times, the
filtered DNS field and the LES field are completely decorrelated. Clearly, this result
should not discourage us since even two DNS fields having slightly different initial
conditions will diverge exponentially with time. Our real goal is to check whether
the LES field has the same statistics as the filtered DNS field.

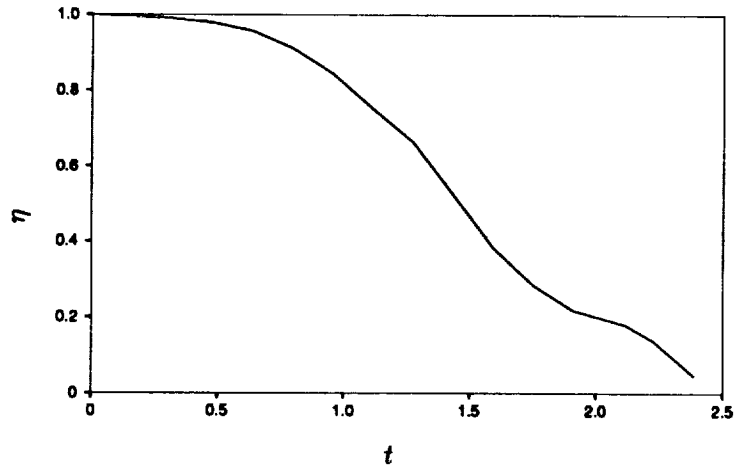Some statistics of turbulent fields (*e.g.*, the PDF of vorticity and dissipation) are

FIGURE 2.   Time-evolution of the correlation coefficient $\eta(t)$, defined in eq. (1), between the LES and the fDNS fields. Time t is in units of a large-eddy turnover time.
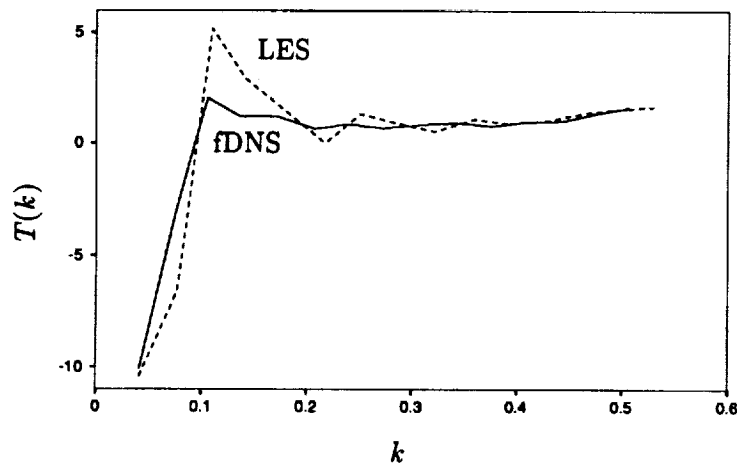

FIGURE 3.   Energy transfer spectrum

close to those for gaussian fields (Shtilman *et al.*, 1992) while some statistics (*e.g.*, helicity fluctuations) require statistical averaging over realizations for accuracy. We exclude results related to these quantities in the present study.   In figure 3, we present a comparison between the energy transfer spectrum for the LES and the fDNS. We note that the spectrum preservation in the LES does not necessarily imply that the fDNS and the LES have the same transfer spectrum in the large-scales. The energy equation for isotropic turbulence is

$$\frac{\partial E(k)}{\partial t} = T(k) - 2\nu k^2 E(k),$$

Statistic	DNS	fDNS	LES	Gaussian
S_3	-0.54	-0.44	-0.37	0
S_4	4.62	3.74	3.69	3
S_5	-8.52	-5.01	-4.18	0

Table 1. Derivative skewness, flatness and hyperflatness from the DNS, the filtered DNS, and the LES. The values for a gaussian field are shown for comparison.

where the transfer term $T(k)$ is defined as the shell integral of

$$T(\mathbf{k}) = \text{Re}[\lambda(\mathbf{k}) \cdot \mathbf{v}^*(\mathbf{k})],$$

where $\lambda(\mathbf{k})$ is the Fourier transform of the Lamb vector, $\lambda(\mathbf{x}) = (\mathbf{v} \times \omega)(\mathbf{x})$ and the asterisk denotes complex conjugate. Clearly T(k) depends on the absolute value of $\mathbf{v}(\mathbf{k})$ and on its phase. While the mean-square Fourier amplitude in a shell is adjusted to its DNS value, the individual Fourier phases are determined by solution of the Navier-Stokes equations. Nevertheless, from figure 3 we learn that the transfer term of LES has values close to those of fDNS.

In Table 1, we present high-order derivative statistics of the flow fields – the skewness S_3, flatness S_4 and hyperflatness S_5, where

$$S_n = \frac{1}{3} \sum_{i=1,3} < \left(\frac{\partial u_i}{\partial x_i}\right)^n > / < \left(\frac{\partial u_i}{\partial x_i}\right)^2 >^{\frac{n}{2}}$$

It is seen from Table 1 that the fDNS and the LES values are both more gaussian than those values obtained from the full DNS, as one would expect for high-order statistics. We also note that the LES values are more gaussian than their fDNS counterparts, although we do not yet know if this is only a statistical fluctuation or if it is a shortcoming of LES. Additional numerical experiments which directly address this question are planned for the future.

In figure 4, the PDF of the cosine of the angle between ω and the eigenvectors of the rate of strain S_{ij} is presented. The results for the LES and the fDNS are seen to be in good agreement. These PDF's are flat for a gaussian field and most authors relate this alignment to the tube-like nature of the vorticity field. A detailed examination of this plot demonstrates again that the LES field has a tendency to be more gaussian than the fDNS field.

Another quantity we consider is the statistics of the angle between the vorticity and the vorticity stretching vector

$$W_j = \omega_i S_{ij}.$$

It was shown in laboratory experiments (Dracos *et al.*, 1991) and numerical experiments (Shtilman *et al.*, 1992) that W_j has a strong tendency to be aligned with
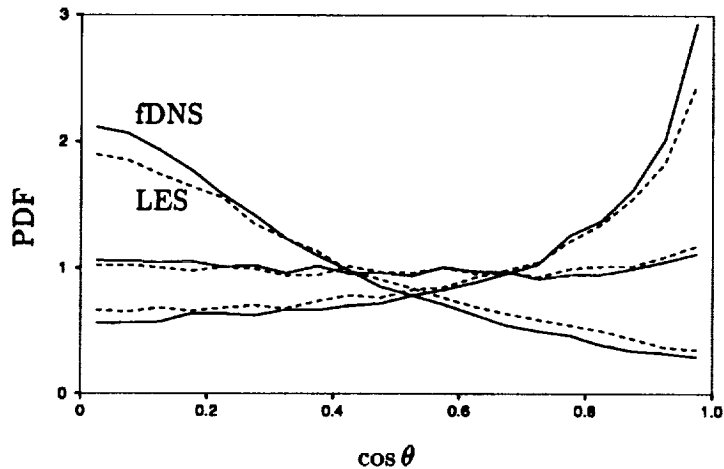
FIGURE 4.　PDF of cosine of the angle between ω and the eigenvectors of the rate of strain tensor S_{ij}.
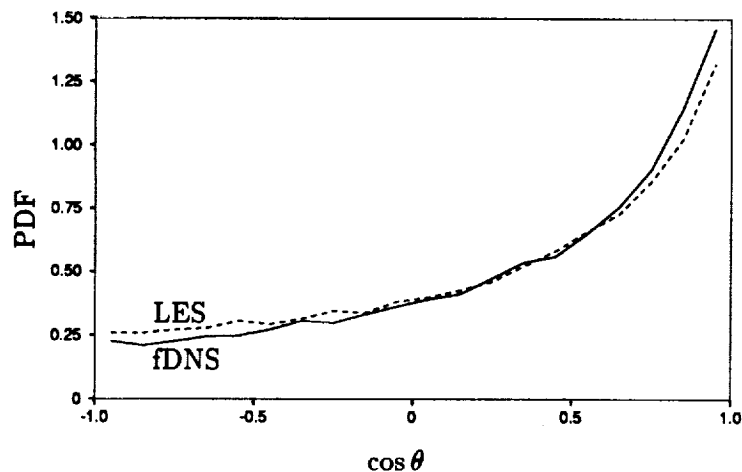

FIGURE 5.　PDF of the cosine of the angle between ω and the vorticity stretching vector $W_j = \omega_i S_{ij}$. .

vorticity. This alignment reflects the total positive production of enstrophy. In figure 5, we present the PDF of the cosine of the angle between **W** and ω. While the LES and fDNS curves are in reasonable agreement, we again note the tendency of the LES curve to be more gaussian than the fDNS curve.

4. Conclusions

We have compared the statistics of the large scales of the DNS field with the LES field for forced isotropic turbulence at moderate Reynolds numbers. The subgrid

scale model used is based on an eddy viscosity which adjusts the instantaneous shell-averaged energy spectrum of the LES to that of the DNS at each time-step. After a couple of large-eddy turnover times, the LES field is uncorrelated with the fDNS field. Several statistical parameters of the large scales of the fDNS were compared to that of the LES. Among them were the transfer spectrum, skewness and flatness factors of velocity components, and PDF's of the angle between vorticity and eigenvectors of the rate of strain and that between the vorticity and vorticity stretching tensor. The overall agreement between the LES and the fDNS statistics was quite good, although we did observe a tendency for the LES field to be slightly more Gaussian than the fDNS field. Nevertheless, the preliminary results presented here point to the promising future of LES.

REFERENCES

ASHURST, W. T., KERSTEIN, A. R., KERR, R. M., & GIBSON, C. H. 1987 Alignment of vorticity and scalar gradient with strain rate in simulated Navier-Stokes turbulence. *Phys. Fluids.* **30**, 2343.

CHASNOV, J. R. 1991 Simulation of the Kolmogorov inertial subrange using an improved subgrid model. *Phys. Fluids A.* **3**, 188-200.

CHEN, H., HERRING, J. R., KERR, R. M., & KRAICHNAN, R. H. 1989 Nongaussian statistics in isotropic turbulence. *Phys. Fluids A.* **1**, 1844-1854.

KRAICHNAN, R. H. 1976 Eddy viscosity in two and three dimensions. *J. Atmos. Sci.* **33**, 1521-1536.

KRAICHNAN, R. H. & PANDA, R. 1988 Depression of nonlinearity in decaying isotropic turbulence. *Phys. Fluids.* **31**, 2395.

LEITH, C. E., & KRAICHNAN, R. H. 1972 Predictability of turbulent flows. *J. Atmos. Sci.* **29**, 1041-1058.

PELZ, R. B., YAKHOT, V., ORSZAG, S. A., SHTILMAN L. & LEVICH, E. 1985 Velocity-vorticity patterns in turbulent flow. *Phys. Rev. Let.* **54**, 2505-2508.

SHTILMAN, L., SPECTOR, M., & TSINOBER, A. 1993 On some kinematic versus dynamic properties of homogeneous turbulence. *J. Fluid Mech.* in press.

TSINOBER, A., KIT, E., & DRACOS, T. 1990 Some experimental results on velocity-velocity gradients measurements in turbulent grid flows . *Topological Fluid Mechanics, Proc. of the IUTAM Symp.* edited by H. K. Moffatt and A. Tsinober. Cambridge University Press.

II. Turbulence physics group

Numerical simulation of turbulence has proven to be a powerful tool in studying the physics of turbulence. There are three papers in this group, each illustrating how numerical simulations are being used for this purpose. Lopez and Bulbeck analyzed an existing database to investigate vortex breakdown in a mixing layer. Orlandi, Homsy, and Azaiez reported preliminary results from modeling the effect of viscoelasticity on flow structures. The last paper by Reuss and Cheng was an attempt to develop new experimental techniques for characterizing vortices in a complex flow by exploring different approaches in a much simpler flow situation. Some highlights as well as critiques of these reports are given below:

Lopez and Bulbeck studied vortex breakdown in a time-developing plane mixing layer by analyzing the database obtained by Moser and Rogers. Vortex break-down in large-scale flows has been observed frequently, from which much of our knowledge of vortex breakdown is derived. There exists some evidence that such breakdown may also occur in smaller scales over a wide range of flows and that vortex breakdown may play a role in characterizing a length scale for vortical structures in turbulent flows. The objective of this paper was to investigate whether vortex breakdown occurs in the rib vortices in the plane mixing layer, where a previous study indicated a rapid change in the local topology. If vortex breakdown were found here, they postulated that it would also exist in other turbulent flows. Using the criteria developed by Brown and Lopez for breakdown of an isolated vortex, i.e., the sudden acceleration of the axial flow and the helix angle of the velocity vector being larger than that of the vorticity vector, they found evidence that the rib vortex downstream of the mid-braid plane began vortex breakdown. There was no evidence, however, of sudden core expansion or intense mixing, phenomena nominally associated with large-scale vortex breakdown flows. There were some discussions during the final presentation of the Summer Program as to whether what they observed here in the temporally developing mixing layer could be regarded as a true vortex breakdown.

Orlandi *et al.* performed numerical simulations of a two-dimensional mixing layer and the interaction of vortex dipole with a wall in order to investigate the effect of viscoelastic fluids on flow structures. Three different viscoelastic models were used to account for the viscoelasticity. For some models, however, they could not obtain a converged solution. In the case of mixing layer, they found that the viscoelasticity enhanced the formation of small scales, which produced intense gradients in the braid region of the mixing layer. These intense gradients led to a faster and more intense roll-up of the layer. This is contrary to the linear stability analysis by Azaiez and Homsy, who showed that viscoelasticity reduced the instability of the flow. The second part of the paper concerned with the effect of viscoelasticity on vortex dipole impinging on a wall, a model of streamwise vortices in a turbulent

boundary layer. They considered both free-slip and no-slip walls but found that the effect of the viscoelasticity for both cases was small. The results presented in this paper appear to be preliminary and they should be interpreted as such. As the authors pointed out, further numerical studies as well as experimental verifications are deemed necessary to validate the present result.

Reuss and Cheng explored different methods for characterizing vortex structures by examining a turbulent flow field obtained from a simulation of turbulent channel. The senior author has been conducting experiments to investigate vortex structures that influence flame wrinkling in reciprocating internal combustion engine, and the objective of this project was to develop an experimentally suitable technique for identifying the turbulence properties associated with these structures. They applied two-dimensional spatial filtering to the instantaneous flow field to separate different scales present in the flow field. As expected, they were able to identify vortical structures which were not apparent from the unfiltered field, but the results were highly dependent on the filter size used. They proceeded to use a conditional-averaging procedure in which the detection was based on the local peak vorticity. They presented results obtained from this procedure as representative of the coherent parts of the flow field. It should be pointed, however, that these results might also depend on the threshold value used for the detection and, to a lesser extent, on how the alignment for the averaging process was conducted. I might add that in the past other investigators have used an iterative procedure using a correlation technique to minimize this problem.

<div align="right">John Kim</div>