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## Response of a supersonic boundary layer to a compression corner

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On the basis of direct numerical simulations of rapidly compressed turbulence, Zeman and Coleman (1991) have developed a model to represent rapid directional compression contribution to the pressure dilatation term in the turbulent kinetic energy equation. The model has been implemented in the CFD code for simulation of supersonic compression corner flow with an extended separated region. The computational results have shown a significant improvement with respect to the baseline solution given by the standard  $k - \epsilon$  turbulence model which does not contain any compressibility corrections.

### 1. Introduction

One of the critical problems in the field of compressible fluid dynamics is the response of turbulence to compressibility effects. Pioneering works in this area of research appeared in early 70's; the contribution of Wilcox & Alber (1972), Oh (1974), and Rubesin (1976) attempted to elucidate the modeling problem of the supersonic mixing layer by solving a transport equation for the turbulent kinetic energy. Later, Vandromme (1983) made an extension in the framework of a two-equation turbulence model by including compressibility effects in the dissipation equation. A detailed review of other extensions of incompressible models to high speed flows is in Vandromme (1991).

In recent years, progress has been made in understanding the compressibility effect on turbulence thanks principally to advances in direct numerical simulations (DNS) of 3D compressible turbulence (Feireisen *et al.* 1981, Blaisdell *et al.* 1991, Coleman & Mansour 1991, Lee 1991, Erlebacher *et al.* 1990). New theory models have been developed for the terms in the Reynolds stress equations, containing explicit compressibility effects: dilatation dissipation  $\epsilon_d \propto \nu \overline{(u_{j,j})^2}$  and the pressure-dilatation correlation  $\overline{p u_{j,j}}$  (Zeman, 1990, 1991a,b; Sarkar *et al.* 1991; Taulbee and VanOsdol 1991; Zeman and Coleman 1991; Durbin and Zeman 1992). In spite of the theoretical progress, the treatment of turbulence in supersonic flow codes is still inadequate. The principal reason for this is the lack of experiments to validate the variety of new modeling assumptions; furthermore, the new model are often fairly complex and difficult to implement in the compressible flow codes.

The present work is primarily concerned with the testing of novel modeling ideas which concern the effect of the so-called rapid compression (or volume deformation)

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on turbulence dynamics in a flow configuration of practical interest: a supersonic turbulent boundary layer (TBL) subjected to distortion through a compression corner. The qualifier *rapid* signifies that the rate of compression given by the mean flow divergence  $\nabla \cdot \mathbf{U}$  is rapid with respect to the large eddy turnover time scale  $\tau \propto k/\epsilon$  (see the following section for notation); i.e.,  $\nabla \cdot \mathbf{U} \tau \gg 1$ . The condition of rapid compression is satisfied when turbulence passes through a shock or sequence of shocks near the corner. An engineering example of turbulence compression is a flow within the combustion chamber of a piston engine. However, here the condition of rapid compression is not satisfied since in the piston engine  $\nabla \cdot \mathbf{U} \tau \approx 1$ . The combustion chamber flow problem has been addressed from a modeling point of view by various investigators, and their work has led mainly to modification of the (solenoidal) dissipation equation to account for the compression effect on the turbulence scales (see e.g. Reynolds, 1980, Morel & Mansour, 1982).

The process of the rapid compression of (homogeneous) turbulence has been simulated by a DNS method developed by Coleman and Mansour (1991). Here, the turbulence could be subjected to both spherical (isotropic) or one-dimensional (1D) compression with the initial value of  $\nabla \cdot \mathbf{U} \tau$  as high as 50 and initial  $M_t = 0.05 - 0.44$ . Thanks to these DNS results, Zeman (1991) and Zeman and Coleman (1991) were able to identify the directional rapid compression effect on the pressure dilatation term  $\overline{p u_{j,j}}$ : when turbulence was subjected to spherical compression  $\overline{p u_{j,j}}$  remained virtually zero (w.r.t.  $\epsilon$ ); however, during 1D rapid compression,  $\overline{p u_{j,j}}$  grew very large and negative, causing a significant drain on the turbulent kinetic energy  $k$ . Surprisingly, this process was more effective for initially low  $M_t$  ( $= 0.05$ ). By now, physical and theoretical understanding of this phenomena has been achieved through the rapid distortion theory (Durbin and Zeman, 1992), and a realistic model for the rapid compression contribution to  $\overline{p u_{j,j}}$  has been developed by Zeman and Coleman (1991).

The report is organized as follows: the turbulence modeling equations and the corresponding model expressions are described in the next section; Section 3 describes the main features of the numerical method which has been used for the solution of the Reynolds averaged Navier-Stokes (RANS) equations, including the new modeling ideas, when applied to the supersonic boundary layer submitted to a sudden compression along a 24-degree wedge (Section 4). In section 5, the results obtained for that specific test case are discussed.

## 2. Turbulence model

Considering a generic form of the classical two equation turbulence ( $k - \epsilon$ ) model, the transport equation can be written as:

$$\begin{aligned} \frac{D\bar{\rho}k}{Dt} - \text{Diffusion} &= \underbrace{-\bar{\rho} \widetilde{v_\alpha^n v_\beta^n} \frac{\partial \widetilde{v_\alpha^n}}{\partial x_\beta}}_{\text{Production}} - \underbrace{\bar{\rho} \left( \widetilde{\epsilon} + 2\nu \left( \frac{\partial \sqrt{k}}{\partial x_n} \right)^2 \right)}_{\text{Destruction}} - \underbrace{\overline{v_\alpha^n} \frac{\partial \bar{p}}{\partial x_\alpha} + \overline{p' \frac{\partial v_\alpha^n}{\partial x_\alpha}}}_{\text{Compressibility}} \\ \frac{D\bar{\rho}\epsilon}{Dt} - \text{Diffusion} &= \underbrace{-C_{\epsilon 1} \frac{\epsilon}{k} \bar{\rho} \widetilde{v_\alpha^n v_\beta^n} \frac{\partial \widetilde{v_\alpha^n}}{\partial x_\beta}}_{\text{Production}} - \underbrace{C_{\epsilon 2} \frac{\epsilon}{k} \bar{\rho} \widetilde{\epsilon} + 2\mu_t \frac{\mu}{\bar{\rho}} \left( \frac{\partial^2 \widetilde{v_\alpha^n}}{\partial x_n^2} \right)^2}_{\text{Destruction}} \end{aligned}$$

with all source terms in the RHS. The source terms include the low turbulent Reynolds number treatment proposed by Jones & Launder (1972). Subscripts *ta* and *no* stand here for tangential and normal components with respect to the solid wall.

The various options in order to account for compressibility effects concern either the  $\epsilon$  or the  $k$  equation. The first correction, which is supported by DNS results of 1D or spherical strain, has been suggested by Reynolds (1980) based on the behavior of decaying isotropic turbulence submitted to a mean strain. Thus the various contributions to the production of dissipation can be subjected to different constants. That yields that, in  $\epsilon$ -equation:

$$\text{Production} = C_{\epsilon 1} \frac{\epsilon}{k} 2\nu_t S_{\alpha\beta} - C'_{\epsilon 1} \frac{\epsilon}{k} \frac{2}{3} \nu_t \delta_{\alpha\beta} \frac{\partial \tilde{v}_\alpha}{\partial x_\beta} \frac{\partial \tilde{v}_\gamma}{\partial x_\gamma} - C''_{\epsilon 1} \frac{\epsilon}{k} \frac{2}{3} k \delta_{\alpha\beta} \frac{\partial \tilde{v}_\alpha}{\partial x_\beta}$$

in which the  $C_{\epsilon 1}$  constants take the following values (Reynolds, 1980):

$$C_{\epsilon 1} = 1.45; \quad C'_{\epsilon 1} = 1.45; \quad C''_{\epsilon 1} = 3.50$$

The compressibility contributions in the turbulent kinetic energy equation are detailed in the following. They are the dilatational contribution to the dissipation, a proper model for the density-velocity correlations, and the pressure-dilatation term.

### 2.1. Dilatation dissipation parameterization

The first idea is based on the assumption that, for sufficiently high turbulent Mach number values, shocklets exist, at least statistically, and can be responsible for an extra amount of dissipation induced by the bulk deformation in the flow. Zeman (1990) proposed a model for this extra dissipation based on the splitting between the solenoidal and dilatational parts of the strain tensor, which can be written as:

$$\epsilon_d = \epsilon_s F(M_t, K) \tag{1}$$

in which  $M_t = \sqrt{2k/a^2}$  is the rms (turbulent) Mach number,  $F(M_t, K)$  is an integral functional of the pdf  $p(m_t, K)$  of fluctuating Mach number  $m_t = \sqrt{u_j u_j}/a$ , and  $K = \overline{m_t^4}/(\overline{m_t^2})^2$  is the kurtosis of the  $m_t$ -distribution, which characterizes the departure from Gaussianity (intermittency) of  $m_t$ .

For the purpose of numerical computation, the function  $F$  is approximated as

$$F(M_t, K) = C_d \left( 1 - \exp\left\{-\left(\frac{M_t - M_{t0}}{\sigma_M}\right)^2\right\}\right) \tag{2}$$

$$F(M_t) = 0, \quad \text{if } M_t \leq M_{t0}$$

where the quantities  $C_d$ ,  $M_{t0}$ , and  $\sigma_M$  are functions of  $K$ . For values of  $M_t$  lower than the threshold of 0.2, the function  $F(M_t, K)$  is set to zero, which is consistent with the results of DNS (Lee *et al.* 1991).

In adiabatic turbulent boundary layers,  $M_t$  appears to be below this threshold level for  $M_e \leq 5$ . However, this is not so in hypersonic TBL's with wall cooling or in the vicinity of a separation bubble in compression corner flow.

## 2.2. Pressure-dilatation correlation $\overline{p u_{j,j}} \equiv \overline{p\theta}$

In TBL flows, two important contributions to  $\overline{p\theta}$  have been identified and modeled: the density-gradient and rapid-compression contributions.

Relying on the balance of the transport equation for the pressure fluctuations and assuming that the gradient flux law is valid for the transport of density fluctuations in the framework of a thin layer approximation, Zeman (1993) suggested the following model for the mean density gradient contribution:

$$(\overline{p\theta})_\rho = f_\rho(M_t) \tau k a^2 \left( \frac{\partial \bar{p}}{\partial x_2} \frac{1}{\bar{p}} \right)^2 \quad (3)$$

Here,  $\tau = k/\epsilon_s$  is a (vortical) turbulent time scale, and the function  $f_\rho \rightarrow M_t^2$  as  $M_t \rightarrow 0$ . The above contribution is positive and reflects the process of conversion of the potential (pressure) to kinetic energies. Although this contribution is indispensable in the mode equations for preservation of the proper (Van-Driest) scaling in the constant stress layer, it has not been used in the present validation tests.

A model for the rapid compression contribution to  $\overline{p u_{j,j}}$  has been first proposed by Zeman (1991b) and Zeman and Coleman (1991) in the form:

$$(\overline{p\theta})_R = -c_{d1} \bar{\rho} \frac{(\overline{p^2})^{1/2}}{\bar{p} M_t^2} k \tau \{ (S_{ij}^*)^2 + c_{d2} b_{ik} S_{kj}^* S_{ij}^* \} \quad (4)$$

The model reflects the sensitivity to the directionality of compression strain and is bilinear in the (trace-free) strain rate tensor  $S_{ij}^* = \frac{1}{2}(U_{i,j} + U_{j,i} - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{U})$ ;  $b_{ij}$  is the anisotropy tensor associated with the Reynolds stresses, and  $\overline{p^2}$  is the fluctuation pressure variance. The above expression yields results agreeing with the DNS data for both 1D and 3D rapid compression; however, it requires an addi-

tional equation for  $\overline{p^2}$  (for details see Zeman and Coleman (1991)). In the present  $k$ - $\epsilon$  model, because  $b_{ij}$  and  $\overline{p^2}$  are not accessible at this level of closure, we set  $\frac{(\overline{p^2})^{1/2}}{\bar{p} M_t^2} \approx 1$  and used a simpler version

$$(\overline{p\theta})_R = -c_{d1} \bar{\rho} k \tau (S_{ij}^*)^2. \quad (5)$$

As discussed in the previous section, the rapid compression contribution to  $\overline{p\theta}$  is expected to be very important in shock/turbulence interactions.

## 3. Numerical method

The numerical method is a predictor-corrector scheme developed initially by MacCormack. To the basic explicit version, various improvements have been added in order to make the code more accurate and robust as well as more efficient. These improvements, which have been described in MacCormack (1985) and Vandromme (1991), are mainly:

- Finite volume discretization
- Flux vector splitting

- Implicit approximation
- Line Gauss-Seidel relaxation
- Energy coupling with the turbulence
- Implicit treatment of the source terms

Performance of this code is remarkable. Although it allows the treatment of complex geometries, large values for the integration time step can be used (which would correspond to a CFL number above  $10^6$  for the Euler equations). Furthermore, special treatment of the source terms of the turbulence equations in the implicit part eliminates the stiffness of the equations during the transient preceding the convergence to steady state.

The set of RANS equations, completed with a two equation turbulence model, takes the following form:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = H$$

in which the vectors  $U$ ,  $F$ ,  $G$  and  $H$  are defined as:

$$U = \begin{bmatrix} \bar{\rho} \\ \bar{\rho}\bar{u} \\ \bar{\rho}\bar{v} \\ \bar{\rho}\bar{E} \\ \bar{\rho}k \\ \bar{\rho}\epsilon \end{bmatrix} \quad F = \begin{bmatrix} \bar{\rho}\bar{u}^2 + \bar{p} + \frac{2}{3}\bar{\rho}k + \sigma_{xx} \\ \bar{\rho}\bar{u}\bar{v} + \tau_{xy} \\ \bar{\rho}\bar{u}\bar{E} + (\bar{p} + \frac{2}{3}\bar{\rho}k + \sigma_{xx})\bar{u} + \tau_{xy}\bar{v} - Q_x \\ \bar{\rho}\bar{u}k - (\mu + \frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial x} \\ \bar{\rho}\bar{u}\epsilon - (\mu + \frac{\mu_t}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial x} \end{bmatrix}$$

$$G = \begin{bmatrix} \bar{\rho}\bar{v} \\ \bar{\rho}\bar{u}\bar{v} + \tau_{yx} \\ \bar{\rho}\bar{v}^2 + \bar{p} + \frac{2}{3}\bar{\rho}k + \sigma_{yy} \\ \bar{\rho}\bar{v}\bar{E} + (\bar{p} + \frac{2}{3}\bar{\rho}k + \sigma_{yy})\bar{v} + \tau_{yx}\bar{u} - Q_y \\ \bar{\rho}\bar{v}k - (\mu + \frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial y} \\ \bar{\rho}\bar{v}\epsilon - (\mu + \frac{\mu_t}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial y} \end{bmatrix} \quad H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ H_k \\ H_\epsilon \end{bmatrix}$$

Define the jacobian matrices as:

$$A = \frac{\partial F}{\partial U}; \quad B = \frac{\partial G}{\partial U}; \quad C = \frac{\partial H}{\partial U}$$

This implicit approximation can be solved either by a factorate approximation method or by a relaxation scheme:

$$(I + \Delta t \frac{\partial A_\bullet}{\partial x} + \Delta t \frac{\partial B_\bullet}{\partial y} - \Delta t \cdot C) \delta U^{(n+1)} = \Delta U^{(n)}$$

$$\delta U^{(n+1)} = \Delta t \frac{\partial U^{(n+1)}}{\partial t}; \quad \Delta U^{(n+1)} = \Delta t \frac{\partial U^{(n)}}{\partial t}$$

in which the source terms can be imbedded on the diagonal terms or a separate factorization can be performed for the sources. Inversion of the implicit source

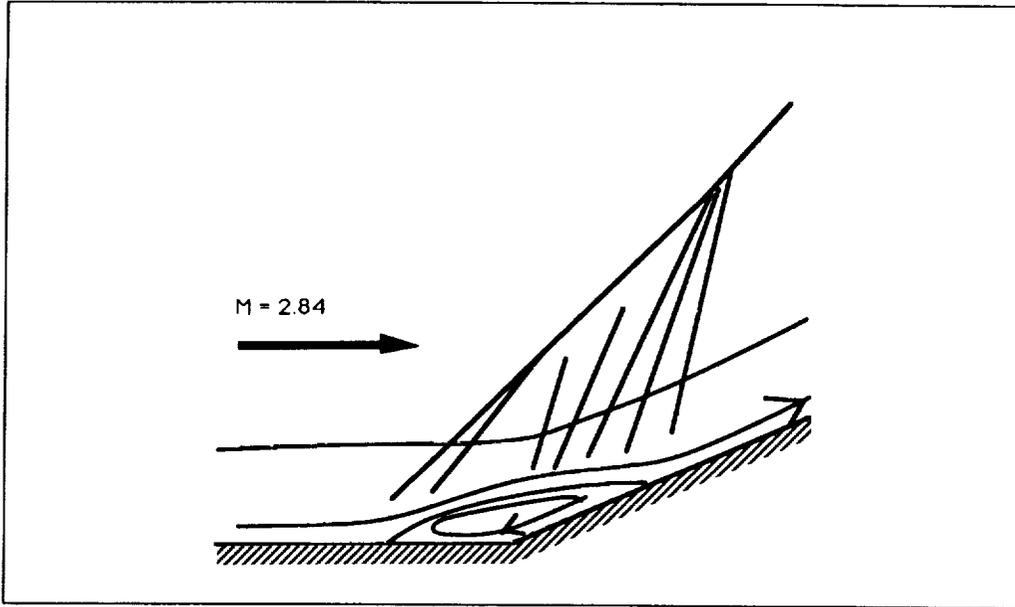


FIGURE 1. Flow Sketch.

operator is based on the knowledge of the analytical or the numerical form of the jacobian matrices. The standard solution procedure is to run line Gauss-Seidel in the streamwise direction (usually a backward-forward sweep), whereas the cross-wise lines are solved in a direct mode with a classical block-tridiagonal algorithm. When using the relaxation scheme, after application of the flux vector splitting, the resulting implicit approximation has one of the following form:

$$a_{i,j}\delta U_{i-1,j}^n + b_{i,j}\delta U_{i,j}^{n+1} + c_{i,j}\delta U_{i+1,j}^{n+1} + d_{i,j}\delta U_{i,j-1}^{n+1} + e_{i,j}\delta U_{i,j+1}^{n+1} = f_{i,j}\delta U_{i,j}^n + H_{i,j}^n$$

in a backward sweep, or

$$a_{i,j}\delta U_{i-1,j}^{n+1} + b_{i,j}\delta U_{i,j}^{n+1} + c_{i,j}\delta U_{i+1,j}^n + d_{i,j}\delta U_{i,j-1}^{n+1} + e_{i,j}\delta U_{i,j+1}^{n+1} = f_{i,j}\delta U_{i,j}^n + H_{i,j}^n$$

in a forward sweep.  $H_{i,j}^n$  is the explicit source terms, and the  $b_{i,j}$  coefficient contribute also to the implicit source treatment.

Independently, the sign of the sources is used to discriminate between stable and unstable implicit approximations (Vandromme, 1991). For stable cases, unlimited time step values can be used; nevertheless, experience shows that the turbulence equations (especially the dissipation equation) never do reach a "machine-zero" type of convergence.

#### 4. Flow description

The main flow features are described with the sketch in Figure 1. A supersonic equilibrium boundary layer experiences a sudden deviation of 24 degrees. That deviation causes an oblique shock wave which induces a strong flow separation in

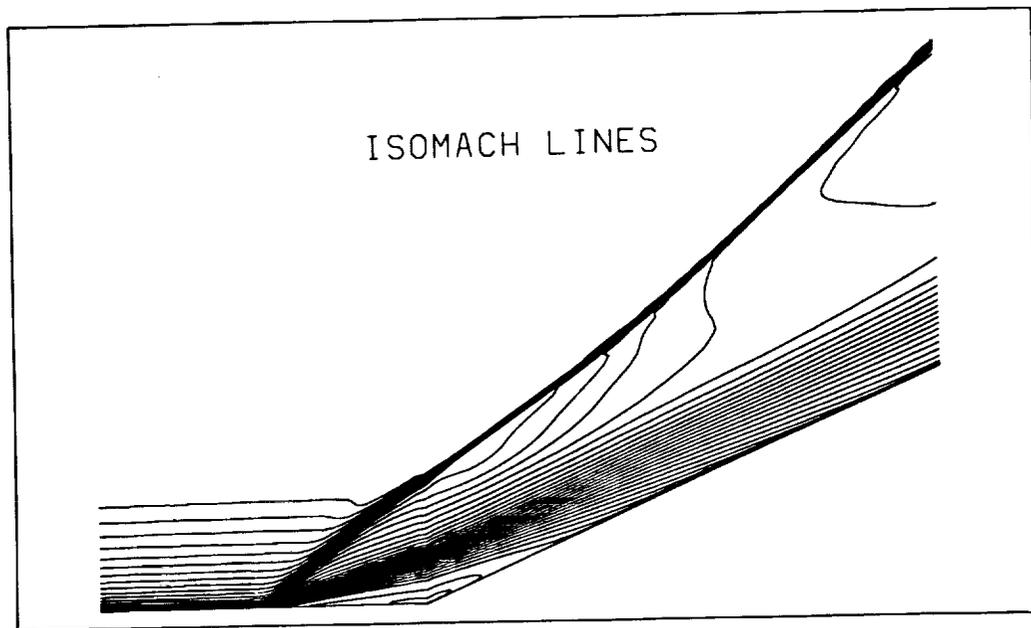


FIGURE 2. Isomach lines (basic solution).

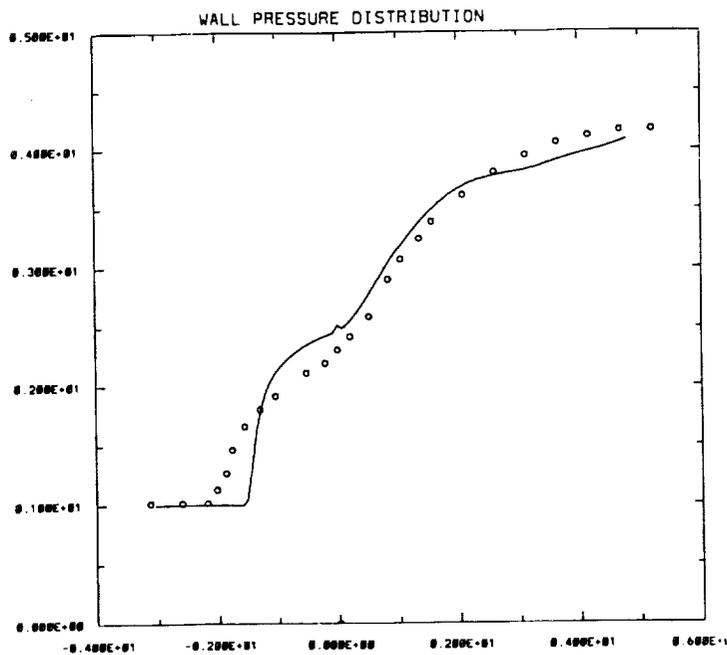


FIGURE 3. Wall pressure. (solid line = basic solution, symbols = experiments)

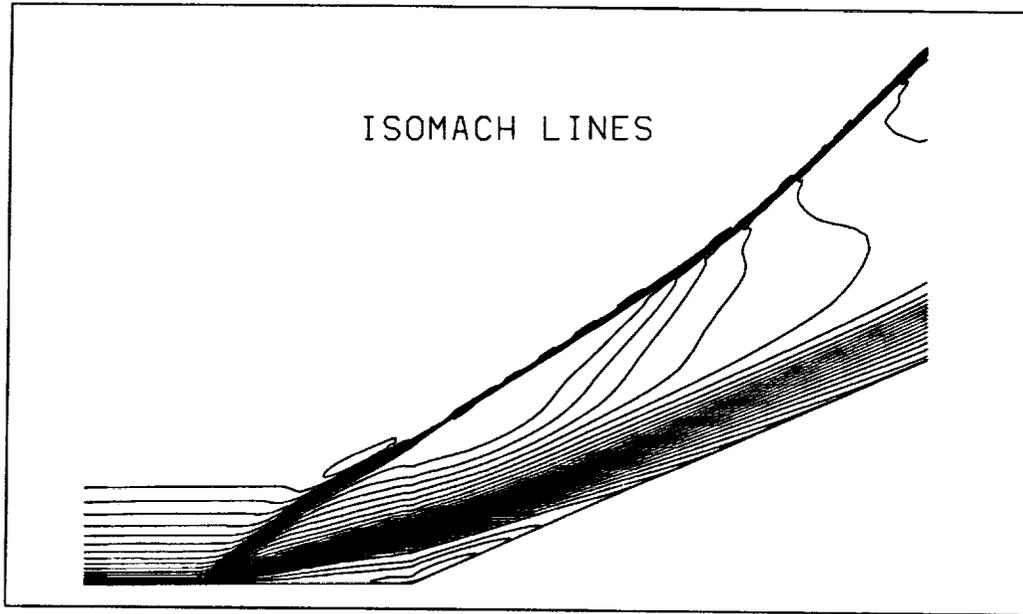


FIGURE 4. Isomach lines (with compressibility correction).

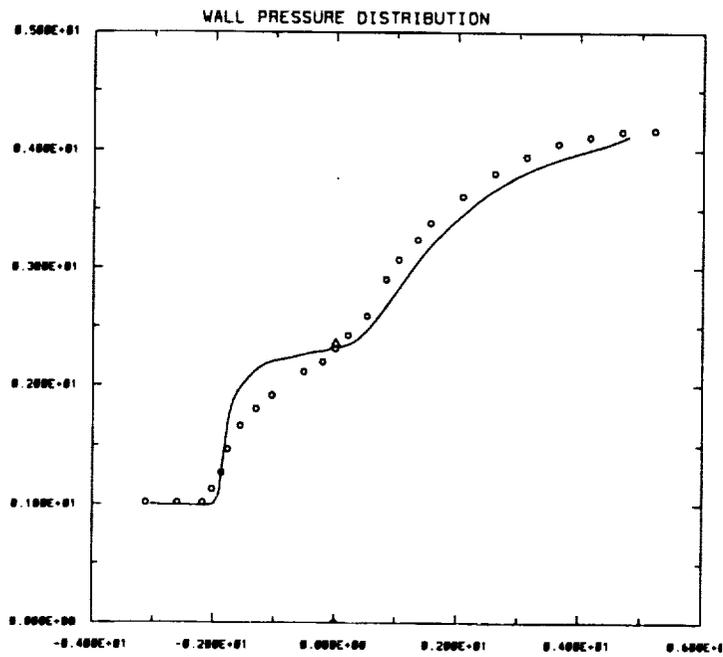


FIGURE 5. Wall pressure (solid line = with compressibility correction, symbols = experiments).

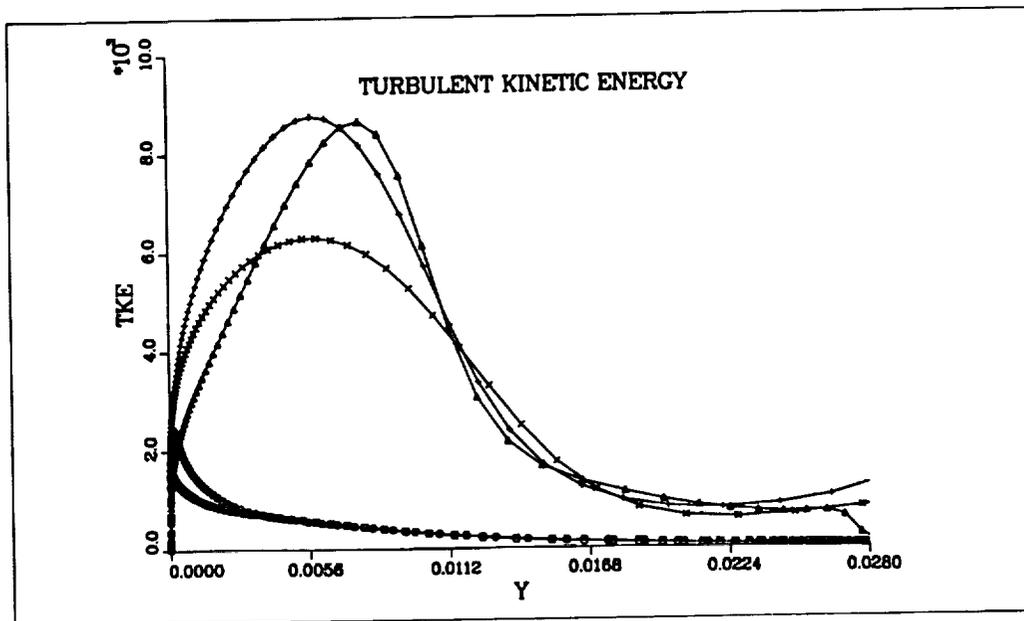


FIGURE 6. TKE profiles (basic solution).

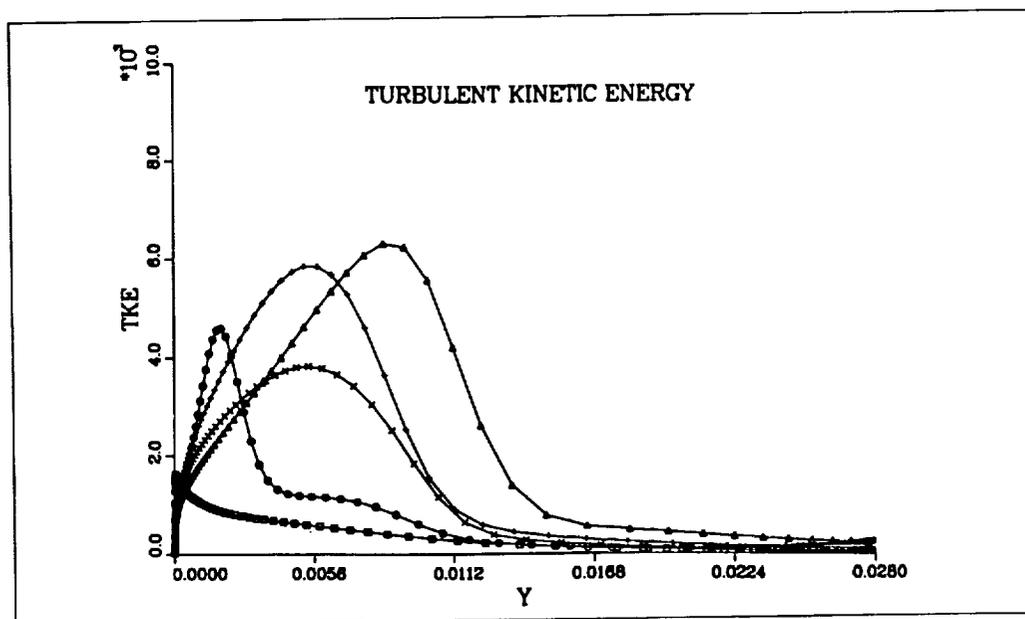


FIGURE 7. TKE profiles (with compressibility correction).

the wedge region. The separation point is at the foot of the shock, upstream of the wedge. The slight curvature of the streamlines above the separation bubble induces a weak expansion fan. Then, at the reattachment, the Mach lines focus again on the main shock and change its slope in the inviscid region. The flow conditions are as follows:

$$\begin{array}{llll} M_\infty = 2.84 & C_f = 11.5 \cdot 10^{-4} & P_t = 6.9 \cdot 10^5 \text{ Pa} & T_t = 270 \text{ K} \\ \delta = 2.6 \cdot 10^{-2} \text{ m} & \delta^* = 6.4 \cdot 10^{-3} \text{ m} & \theta = 1.3 \cdot 10^{-3} \text{ m} & Re = 1.78 \cdot 10^6 \end{array}$$

Experiments have been conducted at Princeton Gas Dynamics Laboratory (Settles *et al.* 1976, 1979). Similar flows have been studied with different ramp angles. i.e.  $8^\circ$ ,  $16^\circ$ , and  $20^\circ$ . In this work, only the  $24^\circ$  has been considered because of the importance of the separated region related to the turbulence field.

A striking feature of this type of separated flow is the strong dependence of the separated region on the incoming turbulence within the boundary layer and on the changes occurring across the shock wave. In order to validate the changes due to the proposed compressibility corrections, all calculations have been made first with a basic model (which is the classical Jones-Launder  $k - \epsilon$  model), and then the code was rerun with the compressibility corrections in (1) and (5) added to the basic model.

## 5. Results

The following results have been obtained during the course of the summer program. The basic solution is shown in Figures 2, 3, and 6. Figure 2 shows the distribution of isomach lines in the interaction region, and Figure 3 shows the wall pressure distribution compared to the experimental values (symbols). Figure 6 depicts profiles of turbulent kinetic energy (TKE) at successive streamwise locations beginning with an unperturbed boundary layer just upstream of the separation zone. Figures 4, 5, and 7 show the effect of inclusion of the compressibility contributions in (1), (2), and (5) in the code.

Comparing Figures 4 and 5 with the corresponding basic solution in Figures 2 and 3, it is evident that the compressibility corrections visibly improve the prediction of the extent of the separation zone. Comparing Figures 6 and 7, we observe that the compressibility corrections cause a marked reduction of overall TKE levels.

## 6. Conclusions

- The principal purpose of this project was to test the effect of compressibility corrections on computations of compression corner flow. These corrections consisted of i) a dilatation dissipation model representing additional dissipation of TKE due to eddy shocklets, and ii) a model for rapid compression contribution to the pressure dilation term.
- The suggested models have been implemented in a compressible R.A.N.S. solver, and computations with and without the compressibility corrections have been performed.

- A significant improvement has been gained in the prediction of a supersonic boundary layer interaction with an extended separation zone. The principal contributor of the improvement was the rapid compression term which becomes a large TKE sink in the vicinity of the shock regions. As expected, the dilatation dissipation effect was insignificant for this test case with the free stream Mach number  $M_\infty < 3$ . Dilatation dissipation (due to shocklets) does become significant in hypersonic boundary layers with wall cooling (Zeman 1993).
- Overall, the new compressibility corrections are expected to play a much more dominant role in higher Mach number flows, for which further validation work is desired.

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