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Flutter Analysis Using Transversality Theory

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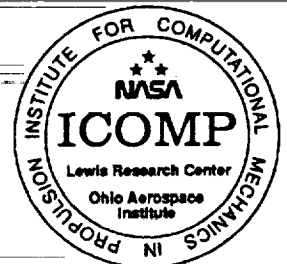
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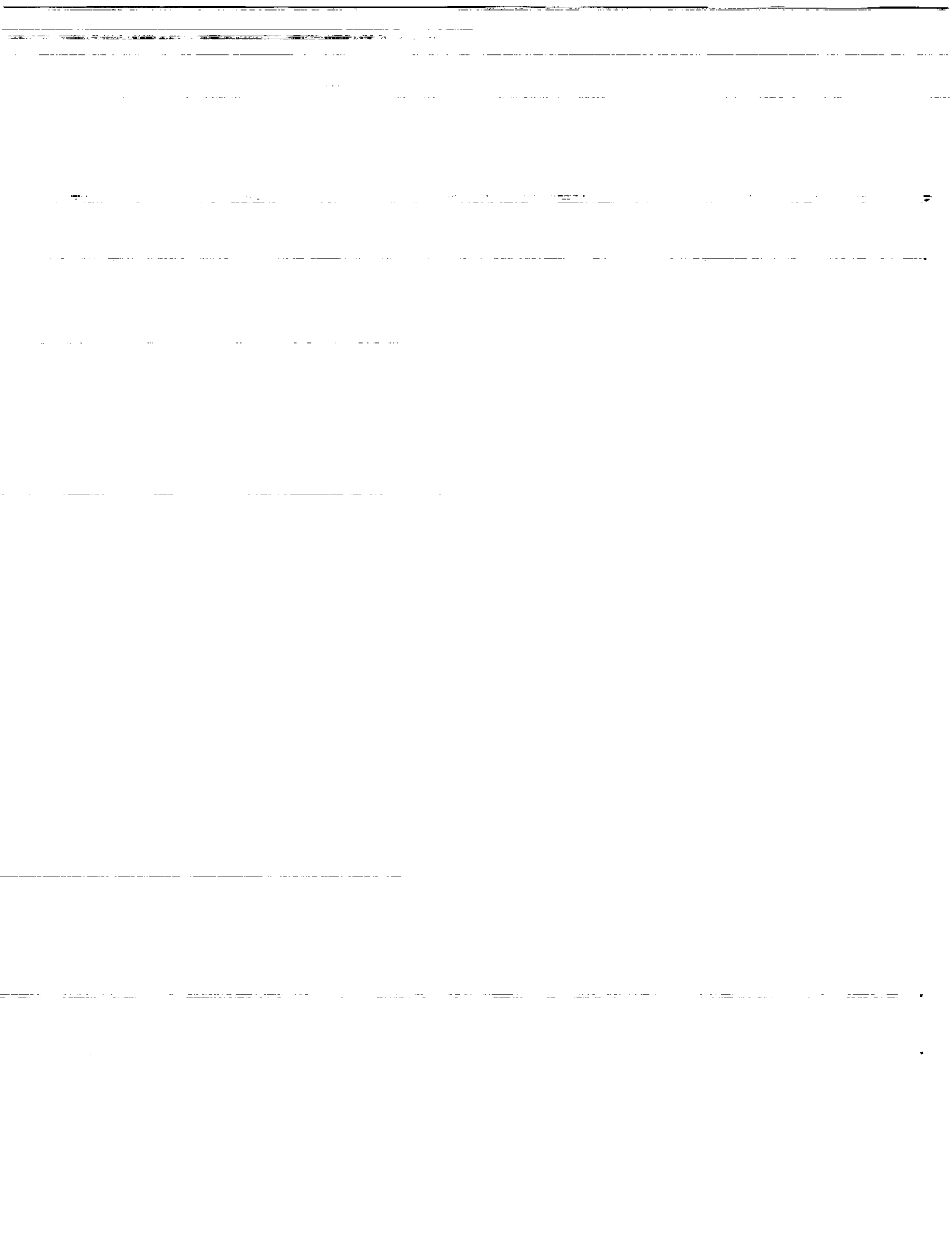
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FLUTTER ANALYSIS USING TRANSVERSALITY THEORY

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ABSTRACT

A new method of calculating flutter boundaries of undamped aeronautical structures is presented. The method is an application of the weak transversality theorem used in catastrophe theory. In the first instance, the flutter problem is cast in matrix form using a frequency domain method, leading to an eigenvalue matrix. The characteristic polynomial resulting from this matrix usually has a smooth dependence on the system's parameters. As these parameters change with operating conditions, certain critical values are reached at which flutter sets in. Our approach is to use the transversality theorem in locating such flutter boundaries using this criterion: *at a flutter boundary, the characteristic polynomial does not intersect the axis of the abscissa transversally*. Formulas for computing the flutter boundaries and flutter frequencies of structures with two degrees of freedom are presented, and extension to multi degree of freedom systems is indicated. The formulas have obvious applications in, for instance, problems of panel flutter at supersonic Mach numbers.

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1. INTRODUCTION

Flutter prevention is a very important consideration in the design and development of various engineering structures and components for aeronautics and space propulsion applications. The development of advanced propulsion engines and their components has been on-going at NASA Lewis Research Center. Part of the development effort is devoted to the aeroelastic behavior of rotating bladed structures. The work reported here is part of that development.

In this work, a computationally efficient method is developed for calculating the flutter boundaries of an engineering structure with two degrees of freedom, based on the typical section model. The typical section model of an airfoil is a simple but very effective model used to predict the aeroelastic behavior of structures such as fixed airplane wings (Bairstow [1]; Frazer & Duncan [2]; Theodorsen [3]), and the rotary wings of helicopters (Chopra & Johnson [4]). By an extension, the model has been used in the aeroelastic analysis of a cascade of turbomachine blades in various regimes of flow (Whitehead [5], [6]; Kaza & Kielb [7]; Dugundji & Bundas [8]; Bahkle *et al.* [9]). This methodology is used in a variety of mathematical models of engine components such as propfans, compressor fans, and turbine bladed-disk assemblies.

Flutter problems in aeroelasticity are characterized by nonlinearity and frequency dependence. This means that elements of the mass and stiffness matrices of an aeroelastic problem are normally frequency dependent. In flutter analysis, one usually solves a *flutter determinant*, which is essentially a characteristic equation, and one determines from the real parts of the complex roots of the equation the stability or otherwise of the system. In our new method, one is able to determine the onset of flutter without explicitly solving the flutter determinant or computing the roots of the characteristic equation. The method described in this paper is therefore very general, and may be applied to other "flutter" problems which do not necessarily originate from aeroelasticity, e.g. as in mechanical systems with follower forces. The method as currently developed in this paper, however, is limited to applications of undamped systems.

1.1. Qualitative Approach

The conventional approach in theoretical flutter analysis is basically *quantitative*, in which computationally intensive codes are developed for calculating flutter boundaries. However, the ultimate consideration in a flutter analysis is, essentially, a *qualitative* one: will flutter occur in the designed system under its normal operating conditions or not? The qualitative nature of the problem to be solved is, in some cases, masked by quantitative computational strategies.

An innovative aspect of our method is that it enables the solution of the qualitative flutter problem by means of a qualitative method of mathematical analysis well known in catastrophe theory or singularity theory. It is based on the concept of "structural stability" of mathematical objects such as matrices, smooth functions or differential equations; see, for instance, Poincaré [10, Lemma IV, p. LXI], Andronov & Pontryagin [11], Thom [12], and Arnol'd [13]–[14], among others.

1.2. Structural Stability and Flutter

The term "structural stability" as used in mathematical texts is quite different from what is normally understood to be structural stability in engineering. In order to avoid any confusion here, we shall use the term "dynamic stability" when engineering concepts are being discussed, while stability in the mathematical sense will be referred to as "structural stability", in those situations where the intended meaning is not obvious from the context.

Also, the term "flutter" is used in many texts in Applied Mechanics² to describe "dynamic instability", whether or not such instability is caused by aerodynamics. Thus, systems which are subject to follower forces may lose stability in a dynamic manner, and such a loss of stability is often called "flutter". However, some aeroelasticians reserve the term flutter for cases in which dynamic instability is caused by aerodynamic forces *only*, thereby excluding instability problems caused by follower forces. In this paper, 'flutter' is used to describe dynamic instability in general, and

²see, for instance: G. HERRMANN, *Dynamics and Stability of Mechanical Systems with Follower Forces*. NASA CR 1782, 1971.

this includes loss of stability by causes other than aerodynamics.

1.3. *Parametric Dependence*

The problem of flutter in aeroelasticity may be formulated as a problem of matrices depending on parameters, in which the parameters are derived from the geometry, flow and frequencies of the aeroelastic model. The parametric dependence is usually non-linear. The mathematical problem of matrices depending on parameters has been solved by Arnol'd [15]. The transversality theorem was used in [15] in arriving at the versal deformation theorem for matrices depending on parameters. In this paper, we draw motivation from Arnol'd's work, but do not apply the transversality theorem to *matrices* directly, as he did. Instead, we apply the transversality theorem to the *characteristic polynomials* of matrices depending on parameters. In this way, we obtain a computationally efficient method for calculating flutter boundaries.

The format of this paper is as follows. In §2, a brief review of pertinent definitions and concepts from matrix theory and algebraic geometry is presented. In §3, we show that, in an undamped vibrating system, the condition of a non-transversal intersection of the characteristic polynomial with the axis of the abscissa may be used to detect the onset of flutter. The material in §4 is a brief outline of aeroelastic problems, and is included here for continuity. Our main results are in §5, where various formulas for calculating flutter boundaries are presented. These formulas are applied to the computation of flutter boundaries in the remainder of the paper, §§6ff.

2. TRANSVERSALITY

The weak transversality theorem is one of the foundations of catastrophe theory, Thom & Levin [16]. It arises in the context of intersections of manifolds, a discussion of which has been given by, for instance, Abraham & Robbin [17]. The mathematical term "manifold" may be conceived as a smooth surface in n dimensions for engineering purposes. For example, a smooth curve is a 1-dimensional manifold, while a smooth surface is a 2-manifold. The significance of transversality in algebraic geometry has been outlined by Brieskorn & Knörrer [18] and Zeeman [19], among others. The weak

transversality theorem asserts that if two manifolds intersect in such a way that the intersection is not in general position, then an arbitrarily small perturbation will lead to its bifurcation, and place the resulting intersections in general position.

In Fig 1a, the two intersections between the horizontal line and the curve are in general position, and are called *transversal*. At each intersection, the local tangent to the curve is different from that to the line, and the set of local tangents spans the ambient space. On the other hand, the intersection shown in Fig 1b is *non-transversal*. The tangent to the curve at its only intersection with the line cannot be distinguished from the tangent to the line at that point. At this non-transversal intersection, the local tangents do not span the two-dimensional ambient space. In Fig 1c there are no real intersections between the curve and the line, but there is an imaginary or complex intersection. The imaginary or complex intersection in Fig 1c is just as transversal as the real intersections in Fig 1a. For a further discussion of these ideas see, for instance, Poston & Stewart [20].

3. NON-TRANSVERSALITY IMPLIES FLUTTER

Many flutter problems may be analyzed as vibrating systems with two or more degrees of freedom; see, for instance, Bisplinghoff & Ashley [21], Dowell *et al* [22], Dugundji & Bunas [8], and other references cited earlier. Often, the typical section model is used, in which there exists a coupling between two coordinates of vibration such as torsion and bending. In what follows, we consider such a coupled two degree-of-freedom system in order to illustrate how a non-transversal intersection of its characteristic polynomial with the axis of the abscissa indicates the onset of flutter.

3.1. Undamped System

If a coupled vibrating system with two degrees of freedom has no damping, then its characteristic equation may be written as a quadratic polynomial in λ ,

$$p(\lambda) = \lambda^2 + a\lambda + b = 0; \quad \lambda, a, b \in \mathbb{R}. \quad (1)$$

The eigenvalues $\lambda = \omega^2$ must be real and positive in order for the structure to have

elastic stability. A complex value of λ in a coupled undamped system implies flutter instability, while a real but negative value of λ implies divergence instability. Equation (1) may also be expressed in alternative form as

$$p(\omega) = \omega^4 + a\omega^2 + b = 0; \quad \omega, a, b \in \mathbb{R}, \quad (2)$$

where ω is the vibration frequency.

If the coefficients a and b in (1) or (2) are real, then the graph of $p(\lambda)$ or $p(\omega)$ in \mathbb{R}^2 is a *real algebraic curve*, Brieskorn & Knörrer [18]. Consider, for now, equation (1). The zero level set of this graph comes from the intersections of the polynomial with the axis of the abscissa, and are the eigenvalues of the coupled vibrating system. It follows from a corollary of the fundamental theorem of algebra, that there are at most two roots of (1), counting multiplicities. If the magnitudes of the roots are distinct, then the roots must be real; if the magnitudes are equal, the roots are either real and degenerate, or are complex conjugates.

The coefficients a and b have parametric dependence on system variables, such as the air speed in an aeroelastic system or the magnitude of the force in a system with follower force. As these system variables change with operating conditions, a and b also vary, and the graph of (1) becomes a family of curves in the plane. There are exactly three *qualitatively* different types of intersections with the axis of the abscissa, with regard to the number and nature of the roots in this family. All three are illustrated in Figs. 1a to 1c.

In Fig 1a, there are two distinct real roots; two real but coincident roots in Fig 1b; and no real roots at all in Fig 1c. The only case where transversal intersections do not occur is Fig 1b. We shall now show how the loss of transversality, as in Fig 1b, marks the flutter boundary in a coupled two degree-of-freedom system without damping.

Coupled vibrating systems with two degrees of freedom having the graph in Fig 1a *cannot* flutter because the eigenvalues $\lambda = \omega^2$, being the two roots of the polynomial in (1), are always real and distinct. Coupled two degree-of-freedom vibrating systems having the graph in Fig 1c *must flutter*. The system flutters because the eigenvalues

$\lambda_{1,2}$, which should always be real and positive if flutter is to be avoided, have now become complex. Intermediate between the two cases is that of Fig. 1b, i.e. a non transversal intersection. The following points may now be made.

From the mathematical point of view, the intersection of Fig. 1b is not non-transversal, is not in general position, and is structurally unstable. If flutter occurs at any time in an initially stable system (1) as its parameters a and b are varied, then the graph of the characteristic polynomial must have changed from that of Fig. 1a to that of Fig 1c. There is *only one route* for passing from Fig. 1a to Fig 1c, and that is through Fig. 1b. Therefore, the case of Fig. 1b constitutes a *flutter boundary*.

From what has been said above, we come to the following result:

the flutter boundaries of a coupled two degree-of-freedom system without damping may be obtained simply by inspecting its characteristic polynomial, and noting the critical parameters at which a non-transversal intersection with the axis of the abscissa, such as in Fig. 1b, occurs.

Although we reached the above result by considering the transversal intersection of a real curve with a real axis of the abscissa, similarly useful results could be obtained by considerations of the transversality of complex algebraic curves intersecting with a complex axis of the abscissa, using the appropriate singularity theory for complex polynomial germs; see, for instance, Milnor [23] or Arnol'd *et al* [24].

3.2. Computational Aspects and Degeneracies

Computationally, the loss of transversality of the characteristic polynomial with the axis of the abscissa is indicated by the occurrence of *degenerate eigenvalues*. Degenerate eigenvalues, like all degenerate mathematical objects, are not in “general position”. Therefore, they are structurally unstable, and should not normally be encountered in realistic models in engineering analysis. If they are encountered in the mathematical model of a physical system, it is only because one has made a theoretical *assumption* which is not *qualitatively* valid in the actual physical problem. For example, one might have assumed perfect symmetry when, in fact, there is a

small but non-vanishing amount of imperfection, leading to a coupling between, say, two modes of vibration. Although the imperfection may be *quantitatively* small, the dynamic behavior of the coupled systems could be dramatically different from that predicted by ignoring the small imperfection altogether.

There are also other kinds of ambiguities associated with the eigenvalue degeneracy. For example, to which form of (3) below does the system eigenmatrix corresponding to Fig. 1*b* reduce under a similarity transformation: a diagonal matrix \mathbf{D}_2 of order 2, or a jordan matrix \mathbf{J}_2 of order 2?

$$\mathbf{D}_2 = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}, \quad \mathbf{J}_2 = \begin{bmatrix} \lambda & 0 \\ 1 & \lambda \end{bmatrix}. \quad (3)$$

Computationally, in order to resolve whether or not a coupled two degree-of-freedom will flutter, one has to calculate the eigenvalues in the first instance. If degenerate eigenvalues are encountered, then it means that the characteristic polynomial is not transversal to the abscissa axis. We then must inspect the corresponding *eigenvectors* or, equivalently, the eigenvalue matrix at the point where transversality is lost. If the eigenvectors are linearly dependent or, equivalently, the eigenmatrix is not diagonalizable, then flutter *must* occur.

3.3. A Note on Divergence

The graph of (1) loses transversality with the axis of its abscissa in only one way, as in Fig 1*b*. In contrast, the graph of (2) intersects the axis of its abscissa non-transversally in two ways as in Fig 2*b* or Fig 2*c*. Now, (1) is a quadratic in λ , whereas (2) is a biquadratic in ω , and both describe the same system. The loss of transversality depicted in Fig. 2*b* is that which signifies a flutter condition.

4. EQUATIONS OF MOTION

Many problems of static and dynamic instability encountered in engineering are analogous to the two instability phenomena known as “flutter”, and “divergence”. Problems of the flutter type are characterized by the fact that the equivalent “stiffness

matrix" of the undamped system is no longer symmetric, i.e. Maxwell's reciprocal theorem is not obeyed by such systems. Consequently, such systems are not governed by a potential, and are often called *non-conservative systems*; see, for instance, Bolotin [25]. Before the onset of flutter, it is admissible to assume that motion is harmonic, with small amplitudes in the neighborhood of equilibrium. This is the essence of linear stability analysis, and the onset of flutter is often correctly predicted by linear stability analysis. Nonlinear analysis becomes important for post-flutter prediction.

The powerful techniques of catastrophe theory may, at first glance, seem to be inapplicable to the solution of the physical problems outlined above since, in the first instance, such problems are *linear* or linearizable and, secondly, they are not gradient dynamic systems, or systems governed by potentials. However, if we use matrix techniques such as the receptance method, we may apply catastrophe theoretic ideas to gain insight into the stability of such systems, simply by studying the transversality of the *characteristic polynomial* of the system's matrix to the axis of the abscissa.

The technical term "receptance" as proposed by Duncan, Biot, Johnson & Bishop [26] relates to a concept initially called mechanical admittance; see, for instance, Duncan [27], or Bisplinghoff & Ashley [21, p204]. It is a powerful technique that enables one to make a frequency domain analysis of a complex engineering structure. A detailed account of this technique has been provided by Bishop & Johnson [28]. Similar ideas are also used in the static analysis of engineering structures, where receptances are called "displacement influence coefficients".

The basic concept of receptance is to relate generalized forces to generalized displacements in a multi degree of freedom system vibrating at a frequency ω using matrix methods. If \mathbf{f} and \mathbf{x} represent the generalized force vector and generalized displacement vector respectively, then the relationship between the two may be expressed as

$$\mathbf{f} = \mathbf{D}(\omega)\mathbf{x}, \quad \mathbf{x} = \mathbf{A}(\omega)\mathbf{f}, \quad \mathbf{AD} = \mathbf{DA} = \mathbf{I} \in \mathbb{C}^{n \times n}, \quad \mathbf{x}, \mathbf{f} \in \mathbb{C}^n. \quad (4)$$

where $\mathbf{A}(\omega)$ is the receptance matrix and its inverse, $\mathbf{D}(\omega)$, is the "dynamic stiffness

matrix" in the frequency domain. If \mathbf{f} is due to aerodynamic forces, then $\mathbf{D}(\omega)$ may be termed the "aerodynamic stiffness matrix".

The flutter problem, being essentially a problem of mechanical vibration analysis, may be treated by the method of receptance. This means that the equation of motion of an aeroelastic system undergoing small displacements in the neighborhood of equilibrium may be written in the standard notation of mechanical vibration as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}_s\mathbf{x} = \mathbf{f}, \quad (5)$$

where \mathbf{K}_s is the static stiffness matrix.

For harmonic vibrations at the circular frequency ω one may write

$$\mathbf{D}(\omega)\mathbf{x} = \mathbf{f}, \quad \mathbf{D}(\omega) = (\mathbf{K}_s - \omega^2\mathbf{M}) + i\omega\mathbf{C}. \quad (6)$$

where \mathbf{D} is the dynamic stiffness matrix of the multi degree of freedom vibrating system. For sinusoidal motion of an airfoil in an air stream, the forcing vector \mathbf{f} in (5) may be written in matrix form

$$\mathbf{f} = \omega^2\mathbf{L}(\omega)\mathbf{x}, \quad (7)$$

where $\mathbf{L}(\omega)$ is an "aerodynamic stiffness matrix". It may be noted that \mathbf{L} generally has a smooth, nonlinear dependence on the vibration frequency ω .

From (5) and (7), one gets the equation of motion, when $\mathbf{C} = 0$ as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \omega^2\mathbf{L}(\omega)\mathbf{x}. \quad (8)$$

Under harmonic vibrations at small amplitudes, $\ddot{\mathbf{x}} = -\omega^2\mathbf{x}$, and the above becomes

$$\mathbf{K}\mathbf{x} = \omega^2 [\mathbf{M} + \mathbf{L}(\omega)] \mathbf{x}, \quad (9)$$

which may be written in the *eigenvalue problem form* as

$$\mathbf{A}(\lambda)\mathbf{u} = \lambda\mathbf{u}, \quad \mathbf{A}(\lambda) = \mathbf{K}^{-1} [\mathbf{M} + \mathbf{L}(\lambda)]. \quad (10)$$

The stability of \mathbf{A} in the above may be investigated by using the techniques published by Arnol'd [15] on matrices depending on parameters. However, our approach here is to map \mathbf{A} from the space of matrices to the space of polynomials, and treat it there as a problem of smooth functions depending on parameters. In this way we apply the transversality theorem in a more efficient way to suit the problem under analysis.

5. DETERMINATION OF FLUTTER FREQUENCIES

Equation (10) is a nonlinear eigenvalue problem, which is traditionally solved for damping and frequency, each requiring iteration, in a computationally intensive procedure, in order to determine the flutter boundaries. In this section, we shall outline a new and computationally more efficient procedure for finding the flutter boundaries, based on applications of the weak transversality theorem of catastrophe theory. This method, in the general sense, requires only frequency iteration.

First, we consider the special case where we fix λ in (10) at some nominal value λ_0 to get

$$\mathbf{A}(\lambda_0)\mathbf{u} = \lambda\mathbf{I}\mathbf{u}. \quad (11)$$

which may be written as

$$[\mathbf{A}(\lambda_0) - \lambda\mathbf{I}]\mathbf{u} = 0. \quad (12)$$

from which one obtains a “flutter determinant”,

$$|\mathbf{A}(\lambda_0) - \lambda\mathbf{I}| = 0. \quad (13)$$

Expanding the above determinant yields the characteristic polynomial, $p(\lambda)$. The characteristic polynomial is obtained from the eigenmatrix, $\mathbf{A}(\lambda_0)$ in (12), which may

be written as

$$\mathbf{A}(\lambda_0) = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}, \quad a_{ij} \in \mathbb{C}. \quad (14)$$

Since $\mathbf{A}(\lambda_0)$ is not a symmetric matrix, it may be decomposed into its symmetric and skew-symmetric parts,

$$\mathbf{A}(\lambda_0) = \begin{bmatrix} a_{11} & \frac{1}{2}(a_{12} + a_{21}) \\ \frac{1}{2}(a_{12} + a_{21}) & a_{22} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2}(a_{12} - a_{21}) \\ \frac{1}{2}(a_{12} - a_{21}) & 0 \end{bmatrix}, \quad (15)$$

in which all the matrix elements a_{ij} are functions of λ_0 .

In the general case, elements of the eigenmatrix \mathbf{A} are frequency dependent. However, there are special cases in which the elements are independent of frequency. In what follows, we consider the less general case where a_{ij} are either constants or independent of λ . By using the substitutions

$$\begin{aligned} a_0 &= \frac{1}{2}(a_{11} + a_{22}), & b_0 &= \frac{1}{2}(a_{12} + a_{21}), \\ c_0 &= \frac{1}{2}(a_{12} - a_{21}), & d_0 &= \frac{1}{2}(a_{11} - a_{22}), \end{aligned} \quad (16)$$

one obtains from (15)

$$\mathbf{A} = \begin{bmatrix} a_0 - d_0 & b_0 \\ b_0 & a_0 + d_0 \end{bmatrix} + \begin{bmatrix} 0 & -c_0 \\ c_0 & 0 \end{bmatrix}. \quad (17)$$

The characteristic polynomial of (17) is

$$p(\lambda) = \lambda^2 - 2a_0\lambda + (a_0^2 - b_0^2 + c_0^2 - d_0^2) = 0; \quad a_0, b_0, d_0 \in \mathbb{R}. \quad (18)$$

from which one obtains the following discriminant of p , as

$$\Delta = b_0^2 + d_0^2 - c_0^2. \quad (19)$$

When p is not a quadratic, its discriminant may be computed by means of Sylvester's eliminant; see, for instance, Afolabi [29].

The condition for a non-transversal intersection of (18) with the axis of the abscissa is equivalent to the vanishing of the discriminant of the polynomial. The vanishing of the discriminants of polynomials is very significant in catastrophe theory, Zeeman [19], where the projection of discriminant surfaces to the parameter space is called the *bifurcation set*. The geometry of discriminant surfaces of algebraic varieties in a more general context is discussed in the work of Brieskorn & Knörrer [18]. In the specific case of our two degree-of-freedom typical section model the non-transversality condition, of the vanishing of the discriminant of the characteristic polynomial, is also the same as eigenvalue degeneracy.

If we calculate the eigenvalues and corresponding eigenvectors of (17), we get

$$\lambda_1 = a_0 - \sqrt{\Delta}, \lambda_2 = a_0 + \sqrt{\Delta}, \mathbf{u}_1 = \begin{Bmatrix} 1 \\ -d_0 + \sqrt{\Delta} \end{Bmatrix}, \mathbf{u}_2 = \begin{Bmatrix} 1 \\ -d_0 - \sqrt{\Delta} \end{Bmatrix}. \quad (20)$$

When the discriminant vanishes, $\Delta = 0$ in (20), and one obtains the following degenerate eigenvalues, the corresponding *eigenvectors* of which *are also degenerate*:

$$\lambda_1 = \lambda_2 = a_0, \quad \mathbf{u}_1 = \mathbf{u}_2 = \begin{Bmatrix} 1 \\ -d_0 \end{Bmatrix}. \quad (21)$$

Thus, at the non-transversal condition signified by the vanishing of the discriminant, the eigenvalues are degenerate. It is precisely this kind of eigenvalue degeneracy, usually noted in undamped models of coupled bending-torsion vibrations, that gives rise to the well known terms, *coupled mode flutter* and *coalescence flutter*.

It is now pertinent to make the following remarks.

1. A *flutter boundary* corresponds to the parameter values where a simultaneous degeneracy of the eigenvalues and eigenvectors occurs.
2. The *degeneracy of eigenvectors necessarily implies flutter*, because the existence of degenerate eigenvectors at a flutter boundary implies that the system's eigenmatrix cannot be diagonalized at that condition; it is only reducible to a jordan matrix.

A summary of the foregoing is this. The flutter boundaries obtained from the transversality criterion, as determined by the vanishing of the discriminant, also corresponds to the conditions of simultaneous degeneracy of the eigenvalues and their corresponding eigenvectors.

Two types of flutter information may be deduced from the characteristic polynomial of an aeroelastic system. In the first place, one tests if flutter will occur at all. If flutter is to occur, the discriminant of the polynomial of the undamped system must vanish. The discriminant of the undamped system has real coefficients. If the occurrence of flutter has been determined, the second thing is to compute the flutter frequencies. The flutter boundaries are obtained simply by setting the discriminant to zero when solving for the roots of the characteristic polynomial. The formula for computing the discriminant of a quadratic equation is very well known in engineering, but not so for a polynomial of arbitrary order. A general algorithm for computing the discriminant of a polynomial of arbitrary order by means of Sylvester's resultant, or eliminant, is well known in the theory of equations, Turnbull [30]; its applications for vibrating systems have been described by Afolabi [29].

The following conditions may be used to test if a given aeroelastic system, whose characteristic polynomial is written in the form of (18), will flutter or not.

$$\text{if } b = 0, \quad \text{flutter occurs when } c_0 = \pm d_0, \quad (22)$$

$$\text{if } d_0 = 0, \quad \text{flutter occurs when } c_0 = \pm b_0, \quad (23)$$

$$\text{if } d_0 = 0, \quad \text{and } b_0 = 0, \quad \text{flutter occurs for all } c_0 \in \mathbb{C}^n, \quad (24)$$

$$\text{if } d_0 \neq 0, \quad \text{and } b_0 \neq 0, \quad \text{flutter occurs when } c_0 = \pm \sqrt{b_0^2 + d_0^2}. \quad (25)$$

The variables $a_0 \cdots d_0$ in the foregoing are functions of λ_0 , and are defined in (16). Although all of the above equations (22)–(25) are theoretically equivalent in that they all give the same flutter boundaries, there are instances when it may be advantageous to use a particular form, rather than another. For example, if $b_0 = 0$ in a model, then it is computationally more efficient to use (22). Similarly, if $d_0 = 0$ in some mathematical model, then flutter boundaries are easier to predict for such a model

using (23). If $b_0 = 0$ and $d_0 = 0$, then flutter *must* occur, as seen from (24). In the most general case, (25) applies and the flutter boundaries may be obtained from

$$c_0^2 = b_0^2 + d_0^2. \quad (26)$$

If it has been definitely determined that flutter will occur, e.g. by using any of (22)–(25), then the flutter frequencies may be computed by means of the formula

$$\lambda_F = \omega_F = a_0, \quad (27)$$

which follows upon substituting (25) in (18). The formula (27) for calculating flutter frequencies is especially efficient because, a_0 is simply the semi-trace of the eigenvalue matrix; the off-diagonal terms in the matrix contribute nothing. Thus, we arrive at the remarkable result:

the off-diagonal terms or, coupling terms, in the eigenmatrix (17) have no influence whatsoever on the flutter frequencies; the flutter speed is determined simply by averaging up the diagonal terms in (17) and equating the sum thus obtained to the eigenvalue parameter, λ as in (27).

The formula (27) is also easy to obtain from the monic form of the characteristic polynomial (18): it is, quite simply, the coefficient of the linear term divided by 2.

6. FLUTTER FREQUENCIES IN STEADY AERODYNAMICS

This flutter problem, for a typical section structural model and a steady flow aerodynamic model, has been treated in several texts; see, for instance, Dowell *et. al.* [22, p.80, §3, eqs. 3.3.48 *et seq.*] from where one gets the lift and moment coefficients

$$L_h = qS \frac{\partial C_L}{\partial \alpha} \alpha, \quad (28)$$

$$M_\alpha = eqS \frac{\partial C_L}{\partial \alpha} \alpha. \quad (29)$$

The equations of motion may be written in the form

$$\begin{bmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_\alpha \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -L_h \\ M_\alpha \end{Bmatrix}. \quad (30)$$

where the lift and moment expressions may be written in the form

$$\begin{Bmatrix} L_h \\ M_\alpha \end{Bmatrix} = \begin{bmatrix} 0 & qS \frac{\partial C_L}{\partial \alpha} \\ 0 & eqS \frac{\partial C_L}{\partial \alpha} \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix}. \quad (31)$$

Expressing the foregoing in matrix form yields $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{L}\mathbf{x}$. The inverse mass matrix is

$$\mathbf{M}^{-1} = \frac{1}{d} \begin{bmatrix} I_\alpha & -S_\alpha \\ -S_\alpha & m \end{bmatrix}, \quad d = mI_\alpha - S_\alpha^2. \quad (32)$$

One may use the above to compute an eigenmatrix, $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$, where $\mathbf{A} = \mathbf{M}^{-1}(\mathbf{K} - \mathbf{L})$, $\lambda = \omega^2$, and hence obtain the characteristic polynomial of the system. The flutter boundaries are then obtained from the conditions giving rise to a loss of transversality.

For an example of how the formulas obtained in the preceding sections by using catastrophe theoretic methods may be implemented, we consider a case previously treated by the classical methods in Dowell *et al.* [22, pp.158-159, eq. 3.8.14 *et seq.*]. The model is for supersonic panel flutter which, according to Dowell *et al.* [22, p. 156], is "somewhat analogous to the typical section model for airfoil flutter". The reason we choose this example is because the lift and moment expressions can be derived in a simple form, which makes it possible to obtain an analytical solution.

The governing equations of motion may be cast in matrix form as

$$ml \begin{bmatrix} \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} + \frac{\rho_\infty U^2}{2M} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0, \quad (33)$$

or, $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{L}\mathbf{q} = 0$. Premultiplying (33) by \mathbf{M}^{-1} leads to the following flutter

determinant, where $\lambda = \omega^2$,

$$|\mathbf{A} - \lambda \mathbf{I}| = \frac{1}{5ml} \begin{vmatrix} 8k + \frac{\rho_\infty U^2}{M} - \lambda_m & -2k + \frac{4\rho_\infty U^2}{M} \\ -2k - \frac{4\rho_\infty U^2}{M} & 8k - \frac{\rho_\infty U^2}{M} - \lambda_m \end{vmatrix} = 0, \quad (34)$$

where $\lambda_m = (5ml)\lambda$. The following characteristic equation is obtained from the above 'flutter determinant', i.e. equation (34),

$$p(\lambda) = a\lambda^2 + b\lambda + c = 0, \quad (35)$$

where $a = 5m^2 l^2$, $b = -16mlk$, $c = 12k^2 + 3\rho_\infty^2 U^4 / M^2$.

In the first instance, one tests if the system will flutter at all. If flutter is to occur, then the discriminant of the characteristic polynomial must vanish at a flutter boundary. The transversality theorem then provides a geometric criterion—i.e. the vanishing of the discriminant of the characteristic polynomial—for locating the flutter boundaries. Computationally, this may be effected by using Sylvester's eliminant if the polynomial is of an arbitrary order (see, for instance, Afolabi [29]).

Since equation (35) is a quadratic, its discriminant is readily computed. If flutter is to occur, this discriminant must vanish. Thus, setting $\Delta = b^2 - 4ac = 0$ in (35) gives the parameter values which guarantee the onset of flutter. When this occurs at a non-transversal intersection of $p(\lambda)$ with the axis of the abscissa, we get the equation of the flutter frequency as

$$\lambda = \omega^2 = -\frac{b}{2a}, \quad (36)$$

or

$$\lambda_F = \frac{8k}{5ml}, \Rightarrow \omega_F = \pm \sqrt{\frac{8k}{5ml}}, \quad (37)$$

which agrees with the result previously given by Dowell *et al.*, [22].

7. CONCLUSIONS

With the aid of the weak transversality theorem from catastrophe theory, simple formulas have been outlined for computing the flutter boundaries of vibrating systems representable as aeroelastic "typical sections", and which are characterized by *asymmetric* system matrices. The procedure developed here provides flutter boundaries much more quickly, and with much less effort, when compared with existing iterative methods. The essence of the procedure is to first compute the characteristic polynomial, and the test for loss of transversality by computing the discriminant of the polynomial. If the polynomial is a quadratic function, then the discriminant $\Delta = b^2 - 4ac$. For a polynomial of arbitrary order, one may compute Δ by means of Sylvester's eliminant. In any case, if $\Delta \neq 0$, then the intersections of the polynomial with the abscissa are always transversal, and coupled mode flutter cannot occur. However, if $\Delta = 0$, then at least one non-transversal intersection exists, and coupled mode flutter may occur.

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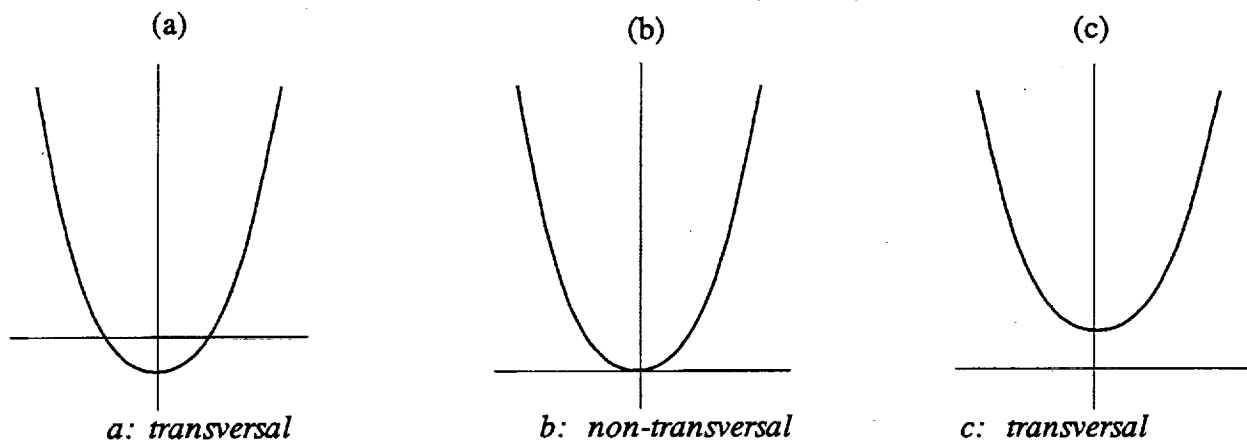


Fig. 1: Transversal and non-transversal intersections

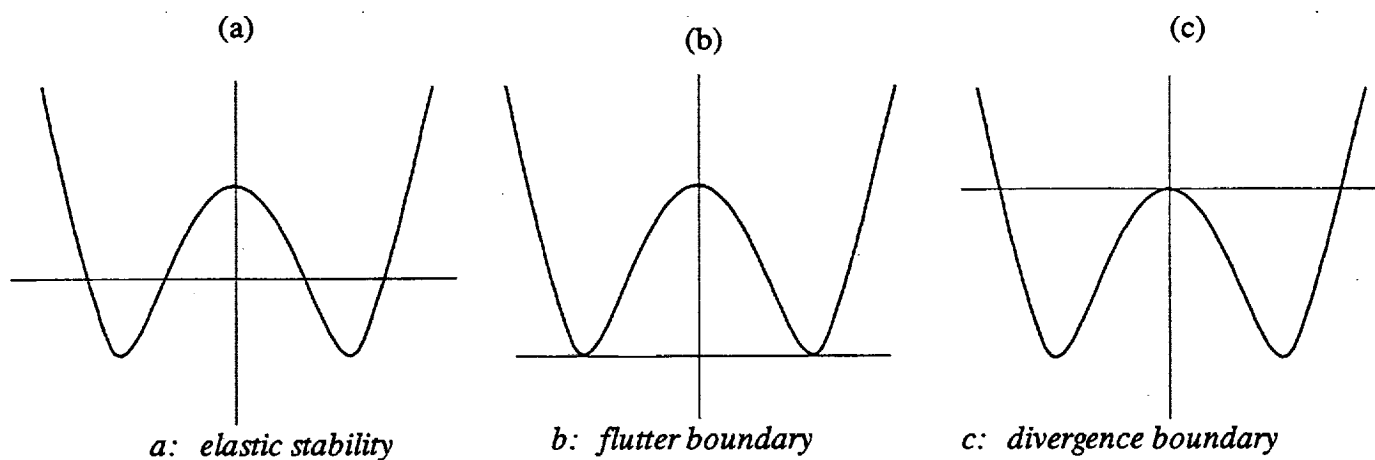


Fig. 2: Symmetric unfolding of the cusp catastrophe germ showing transversal and non-transversal intersections of the characteristic polynomial of a vibrating system with two degrees of freedom.

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