N94-17828

IMPLEMENTATION OF MIXED FORMULATION ELEMENTS IN PC/NASTRAN

S1-39
190571
p. 7

By

Harry G. Schaeffer

President
Thoroughbred CAE Software
Louisville, KY 40206

and

Chairman and Professor
Mechanical Engineering Department
University of Louisville
Louisville, KY 40292

SUMMARY

The purpose of this paper is to describe the implementation and use of a consistent family of two and three dimensional elements in NASTRAN. The elements which are based on a mixed formulation include a replacement of the original NASTRAN shear element and the addition of triangular quadrilateral shell elements and tetrahedral, pentahedral and hexahedral solid elements. These elements support all static loads including temperature gradient and pressure load. The mass matrix is also generated to support all dynamic rigid formats.

THEORETICAL CONSIDERATIONS

The principles of virtual and complementary virtual work allow us to formulate the elasticity problem in terms of either displacements or stresses. The formulation presented in (Ref. 1) provides us with the convenience of the displacement approach for statically indeterminate structures and the ease of stress recovery inherent in the stress approach. In the following we briefly outline the procedure for calculating the element stiffness matrix for the mixed formulation.

In order to derive the stiffness matrix we start with the complementary virtual work for the element which can be written as:

$$\delta W_c = \int_V \boldsymbol{\varepsilon}^T d\sigma dV - \int_{S_\sigma} v^T \delta T dS \tag{1}$$

where T is the set of surface tractions on the boundary, S_σ.

The approach taken in the mixed formulation is to assume an equilibrium stress field $\boldsymbol{\sigma}$ within the element described in terms of a set of generalized parameters $\boldsymbol{\beta}$; and to describe the boundary displacements $\mathbf{v}$ in terms of the grid point displacements $\mathbf{u}$. The set of tractions $\mathbf{T}$ on the boundary are related to the stress components $\boldsymbol{\sigma}$ and the geometry of the element boundary so that it can be expressed in terms of the generalized coefficients $\boldsymbol{\beta}$.

The equilibrium stresses are represented in the following form:

$$\boldsymbol{\sigma} = \mathbf{Z}\boldsymbol{\beta} \tag{2}$$

where the stress state does not include rigid body motion. The boundary traction can now be expressed in terms of the stress components and the unit normal to the boundary, which is only a function of geometry. It can thus be represented conceptually in the following form:

$$\mathbf{T} = \mathbf{L}\boldsymbol{\beta} \tag{3}$$

Finally, the displacements along the boundary can be represented in terms of the grid point displacements as:

$$\mathbf{v} = \mathbf{N}\mathbf{u} \tag{4}$$

where $\mathbf{N}$ is a set of assumed shape functions that are appropriate for the order of the polynomial functions $\mathbf{Z}$ chosen to represent the equilibrium stresses.

Using the relationships for $\boldsymbol{\sigma}$, $\mathbf{T}$, and $\mathbf{v}$ and Hooke's law to relate $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ we can now write (1) as:

$$\delta W_c = \boldsymbol{\beta}^T \mathbf{H}\, \delta\boldsymbol{\beta} - \mathbf{u}^T \mathbf{R}\, \delta\boldsymbol{\beta} = 0 \tag{5}$$

where:

$$\mathbf{H} = \int \mathbf{Z}^T \mathbf{E}^{-1} \mathbf{Z}\, dV \tag{6}$$

$$\mathbf{R} = \int \mathbf{N}^T \mathbf{L}\, dS \tag{7}$$

Since $\delta\boldsymbol{\beta}$ is arbitrary it follows that:

$$\boldsymbol{\beta}^T \mathbf{H} - \mathbf{u}^T \mathbf{R} = 0 \tag{8}$$

Solving for $\boldsymbol{\beta}$ gives:

$$\boldsymbol{\beta} = \mathbf{H}^{-1}\mathbf{R}^{T}\mathbf{u} \tag{9}$$

We can now write the internal strain energy in terms of displacements from which it can be seen that the stiffness matrix **k** is:

$$\mathbf{k} = \mathbf{R}^{T}\mathbf{H}^{-1}\mathbf{R} \tag{10}$$

In the next section the set of equilibrium stresses assumed for each of the elements that is included in the PC/NASTRAN element library is described.

Assumed Stress Fields

The assumed stress field used for the three dimensional stress field elements is:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \beta_1 & 0 & \beta_7 & \beta_8 & 0 & \beta_{16} & 0 \\ \beta_2 & \beta_9 & 0 & \beta_{10} & 0 & 0 & \beta_{17} \\ \beta_3 & \beta_{11} & \beta_{12} & 0 & \beta_{18} & 0 & 0 \\ \beta_4 & 0 & 0 & \beta_{13} & 0 & 0 & 0 \\ \beta_5 & \beta_{14} & 0 & 0 & 0 & 0 & 0 \\ \beta_6 & 0 & \beta_{15} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 1 \\ x \\ y \\ z \\ xy \\ yz \\ zx \end{Bmatrix} \tag{11}$$

where only the coefficients terms 1-6 are used for the constant stress tetrahedron which is called the TETRA element in PC/NASTRAN. Similarly the stress field for the two dimensional stress field membrane and bending force and moment resultants are:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \beta_1 & 0 & \beta_4 \\ \beta_2 & \beta_5 & 0 \\ \beta_3 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \tag{12}$$

and

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} \beta_1 & \beta_6 & \beta_4 \\ \beta_2 & \beta_5 & \beta_7 \\ \beta_3 & \beta_9 & \beta_8 \\ \beta_6 - \beta_8 & 0 & \beta_{10} \\ \beta_7 - \beta_9 & \beta_{11} & 0 \end{bmatrix} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \tag{13}$$

respectively. All of the coefficients in equations (12) and (13) are used for the quadrilateral plate element which is called the QUAD4. However only the constant terms 1-3 in equation (12) and 1-7 in equation (13) are used for the triangular element which is called the TRIA3. Shell behavior is represented as the the sum of membrane and bending behavior for both elements.

IMPLEMENTATION

An early decision was made to replace the original two and three dimensional elements with a consistent family of elements rather than to add to the existing family. PC/NASTRAN thus no longer includes the TRIMEM, QDMEM, HEXA1, HEXA2 etc. These have been replaced with

TRIA3	A triangular shell element with three vertex grid points
QUAD4	A quadrilateral shell element with four vertex grid points
TETRA	A tetrahedral solid element with four vertex grid points
PENTA	A pentahedral solid element with six vertex grid points
HEXA	A hexahedral solid element with eight vertex grid points

Element Matrix Generation

The element subroutines for the generation of element stiffness, mass and stress matrices are called by EMGPRO in the EMG module. The stiffness and mass matrices together with their directory entries are written using EMGOUT for later use by the Element Matrix Assembler (EMA). In addition the stress matrices and their directory is written out for subsequent use in generating thermal loads and in recovering element stresses.

Element Load Generation

The calculation of element-dependent loads including thermal loading which is specified by the standard NASTRAN thermal load Bulk Data and the grid point forces due to pressure load requires access to the element stress matrix and element geometry, respectively. Existing routines were modified to include the new elements and a new capability for generating grid point forces from surface pressure data was implemented. The associative Bulk Data is called the PLOAD4 which allows the user to define a surface traction with respect to either element of the global set of coordinates.

Elements Stresses and Forces

The SDR2 module was modified to accept the stress matrix and directory files produced in EMG. The stress recovery subroutines were written to interface with subroutine SDR2E.

Output Routines

The OFP module was modified to print the element stresses and forces for the new elements. Since the stress output is easily calculated at any point in the domain of the element, the stress and element forces are printed at the element centroid and at each vertex point.

In addition to the standard Output File Processor, separate binary files for each behavioral variable selected in Case Control can be created as a user option. The data structure of each binary file closely follows that of the associated file that is created for the OFP. The benefit in having the binary files is they can be read directly in binary format rather than parsing the ASCII output print file as many post processor programs do, thereby leading to a great speed increase especially for large print files. Another benefit is a reduction in the computer disk storage resources required to store the output.

Other Modifications

Several additional modifications were made to PC/NASTRAN to improve the user friendliness and efficiency of the analysis program. These are:

1. Grid Point Resequencing

   Grid point resequencing is automatically executed as a default but may be bypassed at user option. The resequencing strategies available to the user include Reverse Cuthill-McKee and Gibbs-Poole-Stockmeyer.

2. Automatic Constraint Generation

   In order to remove unconnected degrees of freedom a procedure is introduced to determine whether a singularity at the grid point level exists in the assembled stiffness matrix. If one does exist the automatic constraint generator determines whether a single point constraint or multiple point constraint equation is required to remove the constraint. The USET is updated accordingly and if the constraint is an MPC the associated data are written to a file and added to any MPC constraints selected by Case Control and those defined by rigid elements.

   The automatic MPC capability means that grid point singularities which do not align with displacement coordinate degrees of freedom are handled correctly. The improvement can be demonstrated easily using a single rod element whose axis is not aligned with the displacement coordinate system as described in (Ref. 2).

3. Modified Givens Procedure

As new users of NASTRAN can attest, Fatal Error 3053 – MAA is singular is rather esoteric to the uninitiated. For the initiates it means that Givens Method for eigenvalue extraction has been selected. The associated transformation of the eigenvalue problem to standard form requires that the mass matrix be non-singular.

It can be time consuming to determine the set of massless degrees of freedom which must be removed by static condensation prior to using Givens method. An alternative is to reformulate the eigenvalue problem using a shift point so that the matrix is to be decomposed is always nonsingular. This method is called Modified Givens.

Dynamic Solutions

The solution sequences for normal modes, transient dynamic response and frequency response have been modified as required for the new elements. The eigenvalue solution options have been verified by solving for the modes and frequencies of several test models. In general, the results for the eigenvalues are identical for Givens, Modified Givens and the inverse power methods. Testing also shows that Givens and modified Givens will handled approximately 250 degrees of freedom before spilling.

The transient response and frequency response algorithms for both the modal and direct formulations produce results that agree well with those obtained from other NASTRAN implementations. At this time the random response capability has not been implemented.

RESULTS AND CONCLUSIONS

The implementation of mixed formulation elements in PC/NASTRAN has shown that:

1. **NASTRAN is a powerful test bed for the development of computational structural mechanics algorithms.**
2. **PC/NASTRAN provides a low-cost powerful computational environment on Personal Computers.**
3. **The mixed formulation elements generally equal the performance of displacement-based elements with the same number of vertex grid points.**

REFERENCES

1. Lee, S. W. and Pian, T. H. H., '*Improvement of plate and shell finite element by fixed formulation*', AIAA Journal, 16,29-34(1978).
2. Schaeffer, H. G., *A Guide to Finite Element Analysis Using PC/NASTRAN*, Thoroughbred CAE Software, Louisville, KY 1992.