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Butler Analyses

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INTRODUCTION

There is a constant need to be able to solve for enforced motion of structures. Spacecraft need to be qualified for acceleration inputs. Truck cargoes need to be safeguarded from road mishaps. Office buildings need to withstand earthquake shocks. Marine machinery needs to be able to withstand hull shocks. All of these kinds of enforced motions are being grouped together under the heading of seismic inputs.

Attempts have been made to cope with this problem over the years and they usually have ended up with some limiting or compromise conditions. The crudest approach was to limit the problem to acceleration occurring only at a base of a structure, constrained to be rigid. The analyst would assign arbitrarily outsized masses to base points. He would then calculate the magnitude of force to apply to the base mass (or masses) in order to produce the specified acceleration. He would of necessity have to sacrifice the determination of stresses in the vicinity of the base, because of the artificial nature of the input forces.

The author followed the lead of John M. Biggs[1] by using relative coordinates for a rigid base in a 1975 paper[2], and

1. "Introduction to Structural Dynamics" by John M. Biggs, McGraw Hill 1964, Sec 6.2.
2. "Fidelity in Shaker Simulation Analysis with NASTRAN", T. G. Butler, January 1975, Orally presented at the First MSC NASTRAN Colloquium.

again in a 1981 paper[3]. This method of relative coordinates was extended and made operational as DMAP ALTER packets to rigid formats 9, 10, 11, & 12 under contract N60921-82-C-0128. This method was presented at the twelfth NASTRAN Colloquium.[4] Another analyst in the field, Gary L. Fox, develped a method [5] that computed the forces from enforced motion then applied them as a forcing to the remaining unknowns after the knowns were partitioned off. The method was translated into DMAP ALTER's, but was never made operational. All of this activity jelled into the current effort. Much thought was invested in working out ways to unshakle the analysis of enforced motions from the limitations that persisted. In the following theoretical development the avenue to complete generality is charted. The method is in the process of being coded and will be implemented as four new rigid formats.

THEORY

Seismic analysis in the displacement method becomes especially challenging, because forces are required in NASTRAN to provide loading for the dynamic solutions. The attempt here is to admit displacement histories as acceptable loadings by converting them into equivalent force loadings. The development of this theory will start with a statement of the general dynamic equation based upon all freedoms being present before any constraints or reductions are applied; this is known as the P-set

3. "Dynamic Structural Responses to Base Acceleration", Thomas G. Butler, Proceedings of the Conference on Finite Element Method & Technology, March 1981; Paper No. 8.
4. "Seismic Analysis Capability in NASTRAN", Thomas G. Butler and Robert F. Strang; Proceedings of the 12th NASTRAN Colloquium, May 7-11, 1984, pp 92 - 131.
5. "Solution of Enforced Boundary Motion in Direct Transient and Harmonic Problems", Gary L. Fox, Proceedings of the Ninth NASTRAN Users Colloquium, Oct 22-23, 1080, pp 96 - 105.

(set of all freedoms obtained from all points, grid and extra) in NASTRAN.

$$\left[\left[M_{PP}\right]p^2 + \left[B_{PP}\right]p + \left[K_{PP}\right]\right]\left\{u_P(t)\right\} = \left\{P_P(t)\right\}, \qquad (1)$$

where lower case p stands for the differential operator d/dt. Freedoms which are directly exposed to seismic forcings (accelerations, velocities, & displacements) will be given the designation "C" (standing for contact freedoms) and the complement of this set with respect to the P-set will be designated "J". The P-set of Equation (1) will be partitioned between J & C to get

$$\left[\left[\frac{M_{CC} \mid M_{CJ}}{M_{JC} \mid M_{JJ}}\right]p^2 + \left[\frac{B_{CC} \mid B_{CJ}}{B_{JC} \mid B_{CC}}\right]p + \left[\frac{K_{CC} \mid K_{CJ}}{K_{JC} \mid K_{JJ}}\right]\right]\left\{\frac{u_C(t)}{u_J(t)}\right\} = \left\{\frac{P_C(t)}{P_J(t)}\right\}. \qquad (2)$$

Points will be allowed to be loaded with both displacement and force histories. This will provide for such cases as a space craft being tested in a centrifuge with a shaker on board. In such a case there will be body forces being applied by the centrifuge on all points including contact points, $P_C(t)$, and complement points, $P_J(t)$; and displacement histories being applied by the shaker, $u_C(t)$. Single point constraints (SPC's) can be applied only to J dof's, but multipoint constraints (MPC's) can exist between C & J dof's, however the C freedoms must be chosen as independent when defining the constraint. Thus the known quantities in equation (2) are the forces on the complement set P_J, the forces on the contact set P_C, and the displacement histories at the contact set u_C, pu_C, and p^2u_C.

Since the set of u_C are known, the terms involving them can be expanded from equation (2). Take the known terms in the upper partition first:

$$\left[\left[M_{CC}\right]p^2 + \left[B_{CC}\right]p + \left[K_{CC}\right]\right]\left\{u_C\right\} . \qquad (3)$$

The dimension of each of these 3 terms is force. Designate the set of terms in expression (3) as P_C^C; i.e. the forces from displacement histories on the contact freedoms. Next the known terms in the lower partition expands into:

$$\left[\left[M_{JC}\right]p^2 + \left[B_{JC}\right]p + \left[K_{JC}\right]\right]\left\{u_C\right\} \tag{4}$$

Designate the set of terms in expression (4) as P_J^C; i.e. the forces on those complement freedoms, J, from displacement histories due to their being coupled to the contact freedoms, C.

The first term of expression (3) $\left[M_{CC}\right]p^2\left\{u_C\right\}$ constitutes forces that develop from the accelerations of masses at the contact surface. The first term of expression (4) $\left[M_{JC}\right]p^2\left\{u_C\right\}$ constitutes forces that develop in the "complement" set from the accelerations of interior masses due to their couplings with the contact set. The second term of expression (3) $\left[B_{CC}\right]p\left\{u_C\right\}$ constitutes forces from the speeding of dampers that are connected between members of the contact set. The second term of expression (4) $\left[B_{JC}\right]p\left\{u_C\right\}$ constitutes forces that develop in the "complement" set from the speeding of dampers that are connected between the interior and the contact set. The third term of expression (3) $\left[K_{CC}\right]\left\{u_C\right\}$ constitutes forces that develop from the deformation of elastic elements that are connected between members of the contact set. The third term of expression (4) $\left[K_{JC}\right]\left\{u_C\right\}$ constitutes forces that develop in the "complement" set from the deformation of elastic elements that are connected between the interior and contact set. The portrayal of the forces on the interior dof's must be extracted from the J partitioning of the P-set, otherwise an incorrect distribution would result from the increased coupling if they were extracted from a reduced order such as N-set or A-set.

The scheme here is to treat the excitation histories as known *only* for the purpose of computing forces that develop from displacements on contact points. Once the forces from displacement histories are defined they will be added to boundary force histories to give an array of excitations expressed entirely of forces in spite of the fact that part develop from displacement histories. After the forces from displacement histories are fully defined, the contact freedoms $u_C(t)$ will henceforth be treated as *unknown*. In effect the scheme is to re-solve for displacement histories that are already known. This can be characterized with the following example. Put simply; if one were to look at a single dof system dynamic equation

$$mp^2x(t) + bpx(t) + kx(t) = P(t) \tag{5}$$

one could compute the value of the external forcing $P(t)$ if all three of the displacement histories were known. For the opposite case, one could treat $P(t)$ as known in equation (5), and integrate it to find the acceleration, velocity and displacement at any time. The result would be to recover the values that were originally known (assuming perfect differentiation and integration routines). This is not an unreasonable approach in view of the power in today's computers.

With the displacements on contact points being treated as unknowns, the forces in equation (2) can now be augmented with the forces from displacement histories as follows:

$$\left[\begin{bmatrix} M_{CC} & M_{CJ} \\ M_{JC} & M_{JJ} \end{bmatrix}p^2 + \begin{bmatrix} B_{CC} & B_{CJ} \\ B_{JC} & B_{CC} \end{bmatrix}p + \begin{bmatrix} K_{CC} & K_{CJ} \\ K_{JC} & K_{JJ} \end{bmatrix}\right]\left\{\begin{matrix} u_C(t) \\ u_J(t) \end{matrix}\right\} = \left\{\begin{matrix} P_C(t) + P_C^C(t) \\ P_J(t) + P_J^C(t) \end{matrix}\right\} \tag{6}$$

$u_C(t)$ would be recovered if $P_C(t)$ & $P_J(t)$ were null.

This lays the groundwork for implementation. Provision must be made for admitting displacement history specifications as bulk data; i.e. $p^2\{u(t)\}$, $p\{u(t)\}$, and $u(t)$. Next, the computation of $P_C^C(t)$ and $P_J^C(t)$ must be provided for. Different parts of a structure can have certain portions involved in a given displacment excitation while other portions could be subject to distinctly different excitations. Thus a framework is needed for the spatial specification of each distinct excitation. There can also be spatially distinct time delays associated with individual excitations. But a mechanism already exists in NASTRAN for such specifications: i.e. DAREA for spatial specification of magnitudes, TABLEDi for time varying amplifications, and DELAY for spatial specifications of time delays. All of these can be used with impunity and without confusion with respect to the normal input of dynamic data by requring unique set ID numbers and by having a seismic assembler of enforced loadings. A new case control command called SEISLOAD and a new bulk data card called SEISLOAD will be put into service. Bulk SEISLOAD will act much like TLOADi and RLOAD cards in organizing the spatial, temporal, and phase aspects of displacement excitations. It will incorporate one additional BCD field to specify the type of displacement being input; DISP, or VELO, or ACCE. SEISLOAD case control command will activate the bulk SEISLOAD card much like the DLOAD case contol command that activates the bulk DLOAD card. The Input File Processor (IFP) will assemble the seismic bulk data into the initial data block called DYNAMICS. Case control will direct the data from its SEISLOAD card to read the data from the DYNAMICS data block with a new functional module SPD (seismic pool distributor) whose function would be similar to the DPD (dynamics pool distributor) to prepare SEISLT (seismic load table) and SEISRL (seismic response list) similar to the DLT & TRL. Now comes the actual work of processing these tables and

lists into actual force histories. SEISLT & SEISRL would be input to a second new module SEISLG (seismic load generator) that would treat each distinct displacement excitation as an individual case. That is, SEISLG would form the partitioning vector of the P-set between the C & J sets for one distinct loading. It would compute the equivalent set of three force loadings and ready it for combining with loads from Load generator modules; then turn to the next distinct case and build another partitoning vector for this succeeding case and proceed as before in computing the equivalent set of three loadings. A record should probably be kept for purposes of checking and in setting up output sets for recovery of proof of re-solving for the input specifications.

There are several situations that must be anticipated. First an important premise must be stated. REGARDLESS OF WHAT COMPONENTS OF SEISMIC EXCIATION ARE SPECIFIED (p^2U, pU, OR U), ALL THREE COMPONENTS EXIST AS A CONSEQUENCE OF THE EXISTENCE OF ANY ONE OF THEM. For example, if a seismic acceleration were given as a specification for excitation, the associated velocity and displacment histories can be derived by integration. All 3 components of a seismic disturbance can produce excitation in a structure provided that the structure contains appropriate elements that are coupled to the contact points. Therefore if only one or two out of the three components are specified, the analysis must be equipped to derive the missing component(s). This means that seismic specifications must be differentiated and/or integrated to complete the description of the excitation. Modules will need to be written to perform both integration and differentiation of these displacement histories. The options would be these when all three components are needed:

(a) Only DISP is specified on the SEISLOAD card.
     Consequence: Differentiate twice to obtain seismic velocity
     and seismic acceleration.

(b) Only VELO is specified on the SEISLOAD card.
     Consequence: Differentiate once to get seismic acceleration.
     Integrate once to get seismic displacement.

(c) Only ACCE is specified on the SEISLOAD card.
     Consequence: Integrate twice to get seismic velocity and
     seismic displacement.

Once the three components of seismic excitation are fully enunciated for one case they will be ready for delivery to SEISLG for computation of forces. Each such triplet of histories must be identified with its associated spatial companion. Some connection must be made with Case Control so as to keep these various combinations of load separated for purposes of managing the solution and data recovery operations.

SEISLG must operate similar to TRLG in that it should produce P-set forces, and D-set forces, and S-set forces. It will do this for the C-set based on the SEISLOAD data. It will also have to determine which of the J-set are loaded and to what extent, due to their individual coupling and prepare these additional loadings. After the dynamic load generator has done its work on normal forcing, the forces due to displacements should be added into the three different partitions of load vectors such as the P_P vector.

$$\{P_P\} = \left\{P_P + P_P^C\right\} = \left\{\frac{P_C}{P_J}\right\} + \sum_{i=1}^{k} \left\{\frac{P_C^C}{P_J^C}\right\}_i , \qquad (7)$$

where i represents a distinct contact set.

For each C dof there exists a distinct set of coupling to the J dof's for mass and for elasticity, and for damping. Therefore, for each C dof for each C point there will be a distinct C-J partitioning vector. For example, if there are 2 C-points and if each point were being excited in 2 translational dof's, there are 4 possible couplings for mass, 4 possible couplings for damping, and 4 possible couplings for stiffness. Thus there would be 3 x 4 = 12 distinct J-C vectors, 12 distinct DAREA patterns, 12 distinct TLOAD1 combinations, 2 x 2 X 3 = 12 distinct TABLED1 histories, 3 x 4 = 12 DELAY spatial distributions, and 1 SEISLOAD assemblage.

Translated into a specific example, if the two C-points were numbered 50 and 60 and the excitations were in axial (x=1) and transverse (y=2) directions, there will be 4 distinct acceleration histories: 50(x) and 50(y) plus 60(x) and 60(y). The mass coupling between 50(x) and its J neighbors would probably have a different pattern than that of the mass coupling between 50(y), 60(x) and 60(y) and their respective J neigbors. So the DAREA content for the spatial loading from the acceleration excitation at 50(x) will have to be derived from the mass coupling to 50(x). Fortunately the DELAY content for the spatial time lapse of the acceleration history at 50(x) will be the same as the DAREA content for 50(x). Similarly, the DAREA & DELAY distributions for 50(y), 60(x), and 60(y) will have to be derived from the mass couplings between their J neighbors and at the respective points 50(y), 60(x), and 60(y).

This same pattern of reasoning applies to the formation of loadings for displacement histories stemming from stiffness coupling between the C dof's and their J neighbors. And again this same reasoning applies to the formation of loadings for the

velocity histories stemming from damping coupling from the C dof's and their J neighbors. TLOAD1's and SEISLOAD for the 12 loadings can be described thusly:

ACCE @ 50(x) TLOAD1

| 1 | DAREA from mass coupling to 50(x) | TABLED1 from acce history at 50(x) | DELAY from mass coupling to 50(x) |

VELO @ 50(x) TLOAD1

| 2 | DAREA from damp coupling to 50(x) | TABLED1 from velo history at 50(x) | DELAY from damp coupling to 50(x) |

DISP @ 50(x) TLOAD1

| 3 | DAREA from stiff coupling to 50(x) | TABLED1 from disp history at 50(x) | DELAY from stiff coupling to 50(x) |

ACCE @ 50(y) TLOAD1

| 4 | DAREA from mass coupling to 50(y) | TABLED1 from acce history at 50(y) | DELAY from mass coupling to 50(y) |

VELO @ 50(y) TLOAD1

| 5 | DAREA from damp coupling to 50(y) | TABLED1 from velo history at 50(y) | DELAY from damp coupling to 50(y) |

DISP @ 50(y) TLOAD1

	DAREA from	TABLED1 from	DELAY from
6	stiff coupling	disp history	stiff coupling
	to 50(y)	at 50(y)	to 50(y)

ACCE @ 60(x) TLOAD1

	DAREA from	TABLED1 from	DELAY from
7	mass coupling	acce history	mass coupling
	to 60(x)	at 60(x)	to 60(x)

VELO @ 60(x) TLOAD1

	DAREA from	TABLED1 from	DELAY from
8	damp coupling	velo history	damp coupling
	to 60(x)	at 60(x)	to 60(x)

DISP @ 60(x) TLOAD1

	DAREA from	TABLED1 from	DELAY from
9	stiff coupling	disp history	stiff coupling
	to 60(x)	at 60(x)	to 60(x)

ACCE @ 60(y) TLOAD1

	DAREA from	TABLED1 from	DELAY from
10	mass coupling	acce history	mass coupling
	to 60(y)	at 60(y)	to 60(y)

VELO @ 60(y) TLOAD1

	DAREA from	TABLED1 from	DELAY from
11	damp coupling	velo history	damp coupling
	to 60(y)	at 60(y)	to 60(y)

DISP @ 60(y) TLOAD1

	12	DAREA from	TABLED1 from	DELAY from
		stiff coupling	disp history	stiff coupling
		to 60(y)	at 60(y)	to 60(y)

COMBINED SEISLOAD

| 13	1.0	1.0 ACC @ 50(X)	1.0 VEL @ 50(X)	1.0 DIS @ 50(X)
		1.0 ACC @ 50(Y)	1.0 VEL @ 50(Y)	1.0 DIS @ 50(Y)
		1.0 ACC @ 60(X)	1.0 VEL @ 60(X)	1.0 DIS @ 60(X)
		1.0 ACC @ 60(Y)	1.0 VEL @ 60(Y)	1.0 DIS @ 60(Y)

Now all bookkeeping is in the hands of Case Control and the loads are all in terms of force, so the dynamic solution can proceed as it normally does, including the recovery of data. The output should provide bookkeeping for the several C sets that were fed to the SPD (Seismic Pool Distributor module) so that a separate reporting of these dynamic displacements can be assembled for comparison with the specified seismic histories and/or a differencing should take place to give a measure of the effectiveness in re-solving for the specified seismic inputs.

APPLICATION

This theory has been implemented in DMAP form for Direct Transients. Although the problems were small pilot examples they included extra points and DMIG matrices and involved excitations from mass coupling, damping coupling and stiffness coupling. The theory has been thoroughly certified. The pilot problem, shown

in the plot, represents a simple truss bridge on three foundations with a seismic wave travelling in the positive x direction and disrupting these foundataions.
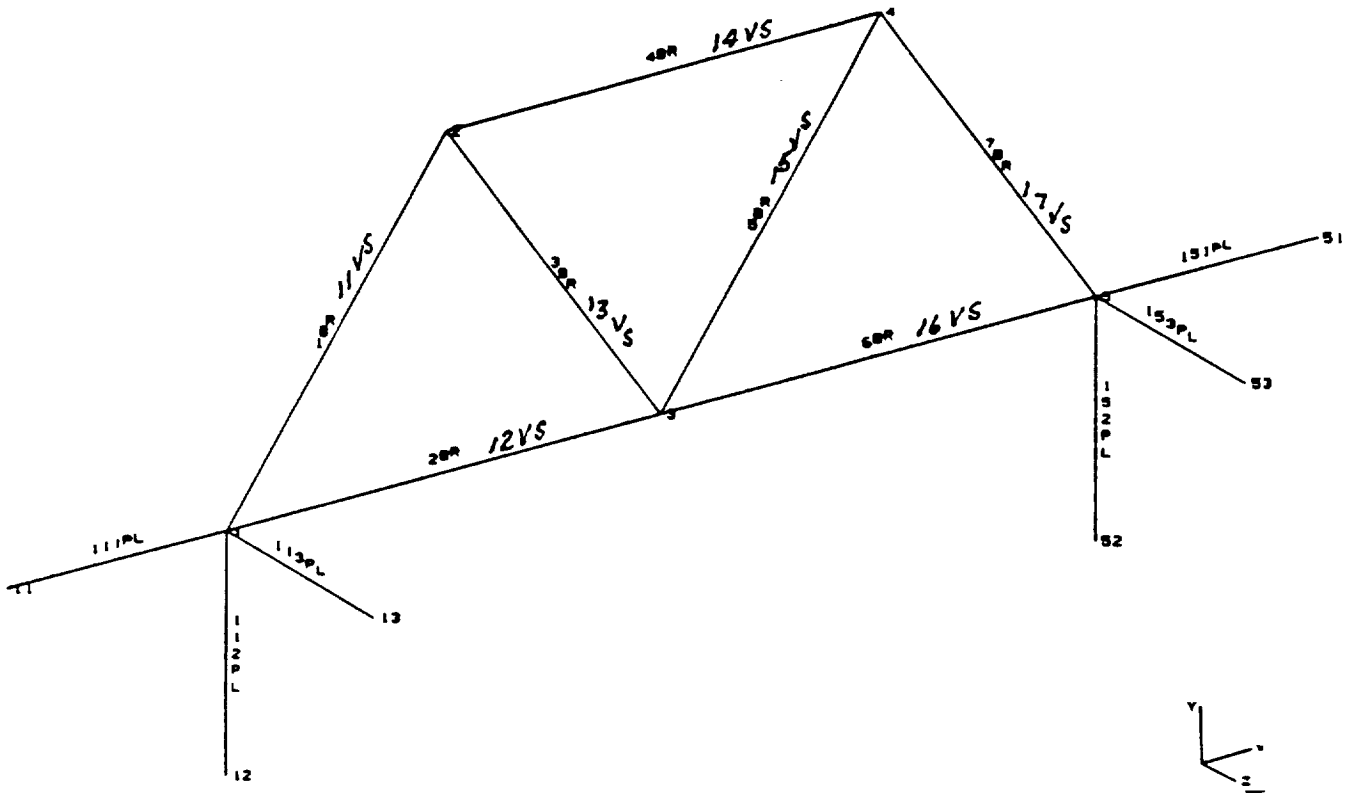
CONCLUSION

Here at last is an automatic method for handling enforced motion that is completely general. The method has been shown to be operational in a DMAP mode. There is no special burden on the analyst except to provide the usual engineering information giving the particulars of his problem. The coding will be completed by the summer of 1993 and will be available in the 1994 release of NASTRAN.