# Asynchronous Sequential Circuit Design Using Pass Transistor Iterative Logic Arrays 

M. N. Liu, G. K. Maki and S. R. Whitaker<br>NASA Engineering Research Center<br>for VLSI System Design<br>University of Idaho<br>Moscow, Idaho 83843


#### Abstract

The Iterative Logic Array (ILA) is introduced as a new architecture for asynchronous sequential circuits. This is the first ILA architecture for sequential circuits reported in the literature. The ILA architecture produces a very regular circuit structure. Moreover, it is immune to both 1-1 and 0-0 crossovers and is free of hazards. This paper also presents a new critical race free STT state assignment which produces a simple form of design equations that greatly simplifies the ILA realizations.


## 1 Introduction

A major goal of modern Very Large Scale Integrated (VLSI) design is to produce a structure that consists of similar, if not identical, modules. With such a structure only one module needs to be designed and then can be replicated to realize the final circuit. Very few design procedures have been advanced for sequential circuits that produce structures that are easily realized for VLSI implementations.

This paper introduces an Iterative Logic Array (ILA) architecture for the realization of asynchronous sequential circuits. With the ILA architecture, an asynchronous sequential circuit can be built in a very regular form with a single type of ILA module as a building block. Furthermore, the ILA asynchronous circuits have some unique features, such as immunity to both $1-1$ overlapping and $0-0$ crossing, tolerant of function hazards and immunity to input bounce. The fundamental mode operation is still required in the ILA asynchronous sequential circuits.

To further simplify the circuit of ILA module, a new state assignment has been developed to generate a new form of design equations (the simple term equation). In the simple term design equations, each coefficient is simply a state variable or a constant, instead of the random sum-of-product expression found in a traditional design equation. That simplifies the basic ILA module to a simple pass logic multiplexer.

## 2 ILA Architecture

Iterative Logic Arrays (ILA) has been described in the literature for quite some time [1,2]. An ILA circuit consists of an array of identical cells. Generally, as shown in Figure 1, each ILA cell contains two sets of input signals. One set of inputs are applied in parallel, while


Figure 1: A slice of ILA circuit


Figure 2: The overall ILA architecture
the other set of inputs are driven by adjacent cells. Signals normally propagate in only one direction between cells, and outputs are derived only from the serial outputs of the last cell. This paper presents an ILA architecture for sequential circuits in which the next state of each state variable is generated by a slice of concatenated ILA cells. A sequential network is then constructed by placing the ILA slices side by side. The function of a flow table is implemented by interconnecting ILA cells and the input states.

The basic cell of an ILA sequential network consists of a 2 -to- 1 multiplexer (MUX) and a next state forming logic. A MUX cell has a select line $S$, its complement $\bar{S}$ and two data inputs $I_{0}$ and $I_{1}$, such that $Q=S * I_{1}+\bar{S} * I_{0}$.

The simplest way to implement the MUX function is to use a pass transistor circuit. Basically, the pass transistor MUX, excluding level restoration logic, is a module of two pass transistors, which functions as two simple switches. Design considerations, such as level restoration, are assumed to be handled by the output buffers. The circuit design considerations have been discussed in $[3,4, \overline{5}]$.

The overall architecture for the ILA sequential circuit is shown in Figure 2 which implements the next state equation

$$
\begin{equation*}
Y_{i}=f_{i 1} I_{1}+f_{i 2} I_{2}+\cdots+f_{i n} I_{n} \tag{1}
\end{equation*}
$$

Theorem 1 The architecture depicted in Figure 2 is a proper model for an asynchronous sequential circuit.

Proof: It is assumed that an STT assignment is used and the logic equations are defined in Equation 1. Let the present input be $I_{p}$, which means that only MUX cell p passes the next logic state $f_{i p}$ to the buffer. Therefore, $Y_{i}=f_{i p} I_{p}$ for each $y_{i}$. When input $I_{q}$ is present ( $I_{p}=0, I_{q}=1$ ), the only MUX that passes next logic state is cell q . Therefore $Y_{i}=f_{i q} I_{q}$ for each $y_{i}$. Clearly, the architecture realizes Equation 1.

The present state is depicted by the state variables and is fed back to each ILA cell. The logic for each state variable $Y_{i}$ consists of $n$ ILA cells as defined in Figure 2. The set of $n$ cells is described as a slice that realizes a next state variable. Referring to Figure 2, all ILA cells that are driven by the same input state $I_{i}$ belong to the same level. With $n$ input states and $m$ state variables, there are $n$ levels of ILA cells and $m$ slices.

## 3 Design Procedure for Asynchronous ILA Sequential Network

As a general architecture, the ILA architecture can be used to realize the asynchronous design equations of any STT assignment. This section compares the ILA circuits for traditional STT assignments and proposes a new state assignment which minimizes the next state forming logic in the ILA cell.

### 3.1 Simple Term Design Equations

The set of next state equations provides a mathematical model of a sequential circuit. For example, the design equations using Liu's assignment for Table 1 are:

$$
\begin{aligned}
& Y_{1}=y_{1} I_{1}+\left(y_{1} \overline{y_{2}}+y_{2} y_{4}\right) I_{2}+\overline{y_{3}} I_{3} \\
& Y_{2}=y_{1} I_{1}+y_{2} I_{2}+y_{3} I_{3} \\
& Y_{3}=0+\overline{y_{2}} I_{2}+y_{3} I_{3} \\
& Y_{4}=0+y_{2} y_{4} I_{2}+y_{3} I_{3}
\end{aligned}
$$

If an ILA network is utilized to implement each next state equation, then the next state forming logic $f_{i p}$ in Equation 1 would be a random logic function in sum of products form. For example, $f_{12}$ would be $y_{1} \overline{y_{2}}+y_{2} y_{4}$. A circuit to realize the next state forming logic is simply combinational logic and can be formed from NAND gates, NOR gates or from pass transistors.

Some research has been conducted to simplify the next state logic. One such effort was made by Chung-jen Tan [6]. The Tan assignment will produce equations in a form of so called simple product term equation in which each coefficient $f_{i p}$ is simplified to an OR expression, instead of a random sum of products expression.

With traditional state assignments, each $f_{i p}$ term in resulting design equations is a function of the partitioning variables of input $I_{p}[7]$ and the number of partitioning variables

|  | $I_{1}$ | $I_{2}$ | $I_{3}$ | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{A}$ | B | C | 0 | 0 | 0 | 0 |
| B | A | $\mathbf{B}$ | G | 0 | 0 | 1 | 0 |
| C | D | E | $\mathbf{C}$ | 1 | 0 | 0 | 0 |
| D | D | F | C | 1 | 1 | 0 | 0 |
| E | D | $\mathbf{E}$ | G | 1 | 0 | 1 | 0 |
| F | A | $\mathbf{F}$ | C | 0 | 1 | 0 | 0 |
| G | A | H | G | 0 | 1 | 1 | 1 |
| H | D | $\mathbf{H}$ | C | 1 | 1 | 0 | 1 |

Table 1: A flow table and traditional Liu's assignment.
is equal to the number of k -sets in $I_{p}$, the maximum number of inputs to $f_{i p}$ logic is equal to the number of k -sets. Therefore, a completely regular ILA circuit for a traditional assignment would contain a $k$-input next state forming logic for each $f_{i p}$ where $\mathrm{k}=$ "number of k -sets in input $I_{p} "$.

A goal of the new design procedure is to minimize the amount of hardware required by the $f_{i p}$ logic. Physically, the minimal form of $f_{i p}$ logic is a wire. The design equation in which all $f_{i p}$ terms are in minimum logic is called a simple term equation.

Deflnition 1 A simple term next state equation is a design equation in which each $f_{i p}$ coefficient is a single state variable (or its complement) or a constant.

In a simple term equation, each coefficient depends on at most one state variable. This feature has significant impact on the hardware implementation. With a simple term equation, only a wire is required to generate each $f_{i p}$ coefficient. With a procedure discussed in later sections, the simple term equations for Table $\overline{1}$ can be derived as follows,

$$
\begin{aligned}
& Y_{1}=y_{1} I_{1}+y_{3} I_{2}+\overline{y_{2}} I_{3} \\
& Y_{2}=I_{1}+y_{4} I_{2}+y_{2} I_{3} \\
& Y_{3}=I_{1}+y_{3} I_{2}+0 \\
& Y_{4}=\overline{y_{1}} I_{1}+y_{4} I_{2}+\overline{y_{2}} I_{3}
\end{aligned}
$$

As all coefficients are simple terms, no extra logic is required to generate each $f_{i p}$.

### 3.2 New State Assignment

In this research, partition algebra is used to derive simple term equations and synthesize asynchronous circuits. The new state assignment, called $\eta$ assignment, is proposed to produce the simple term design equations. A relationship between $\eta_{i}^{p}$ partitions and $f_{i p}$ coefficients has been presented in the literature [8]. Theorem 2 in [8] shows part of the discovery and has been used to analyze state assignments in predicting hardware.

Theorem 2 (1) If $\eta_{i}^{p}=\tau_{j}$, then $f_{i p}=y_{j}$ or $\overline{y_{j}}$. (2) If all the states of $\eta_{i}^{p}$ are in one block, then $f_{i p}=0$ or 1 .

If all $f_{i p}$ of the design equations meet the conditions of Theorem 2 , then the coefficients will be simple terms. A sufficient condition of $\eta$-partitions for simple term $f_{i p}$ is listed in Corollary 1.

Corollary 1 If all $\eta_{i}^{p}$ partitions satisfy one of the following conditions:

1. $\eta_{i}^{p}=\tau_{j}$
2. $\eta_{i}^{p}=\{\mathcal{S} ; \phi\}$
3. $\eta_{i}^{p}=\{\phi ; \mathcal{S}\}$
then the design will yield simple terms, where $S$ is a set of all flow table states.
Proof: The proof follows directly from satisfying Theorem 2.
Most STT assignments do not meet the conditions of Corollary 1. However, if an assignment can be generated such that for each $\eta$-partition $\eta_{j}^{p}$ under input $I_{p}$ where $\eta_{j}^{p}$ does not satisfy the conditions from Corollary 1, a new $\tau$-partition $\tau_{k}$ will be created where $\tau_{k}=\eta_{j}^{p}$ to allow $f_{j p}=y_{k}$ and produce a simple term equation for $Y_{j}$.

For example, the flow table shown in Table 1 has eight $k$-sets:

$$
\{A B F G\},\{A B\},\{A C D F H\},\{C D E H\},\{C E\},\{D F\},\{B E G\},\{G H\}
$$

A set of $\tau$-partitions can be formed to partition each k -set from the rest of states and the results are $\tau_{1}$ through $\tau_{8}$ in Table 2 (a). Then from the corresponding $\eta$-partitions, it can be found that some $\eta$-partitions, such as $\eta_{1}^{2}, \eta_{3}^{2}, \eta_{4}^{2}$ and $\eta_{7}^{2}$, are not equal to any $\tau$-partitions or a constant. A new $\tau$-partition needs to be formed for each of $\eta$-partitions which do not meet the conditions of Corollary 1 . For each newly created $\tau$-partition, corresponding $\eta$-partitions need to be formed. The new $\tau$-partitions may generate new $\eta$-partitions which do not meet any conditions of Corollary 1. Additional $\tau$-partitions would then be required. The partitioning procedure will continue until the conditions of Corollary 1 are met. In the case of assignment for Table $1, \tau_{9}, \tau_{10}, \tau_{11}, \tau_{12}$ are formed which in turn generate 12 corresponding $\eta$ partitions. The results are shown in Table 2 where all $\eta$-partitions are equal to a $\tau$-partition or a constant.

Simple term equations can be generated once all $\eta$-partitions satisfy Corollary 1. In most cases, however, more $\tau$-partitions (state variables) than necessary have been introduced and can be removed without jeopardizing the simple term feature.

Deflnition $2 A \tau$-partition $\tau_{i}$ is redundant if $\tau_{i}$ and $\eta_{i}^{p}$ for all $I_{p}$ can be eliminated while the resulting next state equations remain the simple term equations and the state assignment remains a critical race free STT assignment.

Theorem 3 In the set of $\tau$-partitions resulting from the $\eta$ assignment, if $a \tau_{i}$, which partitions a k-set $K_{t}$ in $I_{p}$ from other states is not equal to any $\eta_{j}^{p}$ or its logical complement, where $i \neq j$, then $\tau_{i}$ is redundant.

$$
\begin{array}{lll}
\tau_{1}=\{A B F G ; C D E H\} & \tau_{2}=\{A B ; C D E F G H\} & \tau_{3}=\{A C D F H ; B E G\} \\
\tau_{4}=\{C D E H ; A B F G\} & \tau_{5}=\{C E ; A B D F G H\} & \tau_{6}=\{D F ; A B C E G H\} \\
\tau_{7}=\{B E G ; A C D F H\} & \tau_{8}=\{G H ; A B C D E F\} & \tau_{9}=\{A B C E ; D F G H\} \\
\tau_{10}=\{A B D F ; C E G H\} & \tau_{11}=\{C E G H ; A B D F\} & \tau_{12}=\{D F G H ; A B C E\} \\
& (\mathrm{a}) \tau \text {-partitions } & \\
& & \\
\eta_{1}^{1}=\{A B F G ; C D E H\} & \eta_{1}^{2}=\{A B D F ; C E G H\} & \eta_{1}^{3}=\{B E G ; A C D F H\} \\
\eta_{2}^{1}=\{A B F G ; C D E H\} & \eta_{2}^{2}=\{A B ; C D E F G H\} & \eta_{2}^{3}=\{-; A B C D E F G H\} \\
\eta_{3}^{1}=\{A B F G C D E H ;-\} & \eta_{3}^{2}=\{D F G H ; A B C E\} & \eta_{3}^{3}=\{A C D F H ; B E G\} \\
\eta_{4}^{1}=\{C D E H ; A B F G\} & \eta_{4}^{2}=\{C E G H ; A B D F\} & \eta_{4}^{3}=\{A C D F H ; B E G\} \\
\eta_{5}^{1}=\{-; A B F G C D E H\} & \eta_{5}^{2}=\{C E ; A B D F G H\} & \eta_{5}^{3}=\{A C D F H ; B E G\} \\
\eta_{6}^{1}=\{C D E H ; A B F G\} & \eta_{6}^{2}=\{D F ; A B C E G H\} & \eta_{6}^{3}=\{-; A B C D E F G H\} \\
\eta_{7}^{1}=\{-; A B F G C D E H\} & \eta_{7}^{2}=\{A B C E ; D F G H\} & \eta_{7}^{3}=\{B E G ; A C D F H\} \\
\eta_{8}^{1}=\{-; A B F G C D E H\} & \eta_{8}^{2}=\{G H ; A B C D E F\} & \eta_{8}^{3}=\{B E G ; A C D F H\} \\
\eta_{9}^{1}=\{A B F G ; C D E H\} & \eta_{9}^{2}=\{A B C E ; D F G H\} & \eta_{9}^{3}=\{A C D F H ; B E G\} \\
\eta_{10}^{1}=\{A B C D E F G H ;-\} & \eta_{10}^{2}=\{A B D F ; C E G H\} & \eta_{10}^{3}=\{-; A B C D E F G H\} \\
\eta_{11}^{1}=\{-; A B C D E F G H\} & \eta_{11}^{2}=\{C E G H ; A B D F\} & \eta_{11}^{3}=\{A B C D E F G H ;-\} \\
\eta_{12}^{1}=\{C D E H ; A B F G\} & \eta_{12}^{2}=\{D F G H ; A B C E\} & \eta_{12}^{3}=\{B E G ; A C D F H\}
\end{array}
$$

(b) $\eta$-partitions

Table 2: The $\tau$ and $\eta$ partitions
Proof: If the $k$-set $K_{t}$ in $I_{p}$ is the left block of $\tau_{i}$ and the $\tau_{i}$ is not equal to any $\eta_{j}^{p}$ or its logical complement, for all $i \neq j$, then for every $I_{q}, q \neq p$, there must exist a k-set $K_{k}$ under $I_{p}$ in the right block of $\tau_{i}$ such that the stable state of $K_{t}$ and the stable state of $K_{k}$ are in the same $k$-set in $I_{q}$. Hence the $k$-set $K_{t}$ does not need to be partitioned from $k$-set $K_{k}$ for the input transition $I_{p}$ to $I_{q}$. Moreover, the partitioning variable $y_{i}$ is not needed to generate any simple term equation $Y_{j}$. Therefore, $\tau_{i}$ is redundant.

For example, in Table $2, \tau_{2}, \tau_{5}, \tau_{6}$ and $\tau_{8}$ do not appear in $\eta$-partition set except for $\eta_{2}^{2}, \eta_{5}^{2}, \eta_{6}^{2}$, and $\eta_{8}^{2}$. Those $\tau$-partitions are redundant according to Theorem 3. Another way of looking at this is that $y_{2}$ only appears in the equation for $Y_{2}$ and no other - same for $y_{5}$, $y_{6}$ and $y_{8}$. Therefore, they are redundant.

Theorem 4 If $\tau_{i}$ is a logical complement of $\tau_{j}$, then $y_{i}$ can be replaced by $\overline{y_{j}}$ in all design equations.

Proof: If $\tau_{i}$ is the logical complement of $\tau_{j}$, then $\tau_{i}$ and $\tau_{j}$ partition the same k-sets. Only one of them is needed to meet the partitioning condition for a critical race free STT assignment. Moreover, according to the Rule 2 in Theorem 7, all coefficients where $f_{k p}=y_{i}$ can be replaced by $f_{k p}=\overline{y_{j}}$.

$$
\begin{array}{ll}
\tau_{1}=\{A B F G ; C D E H\} & \tau_{3}=\{A C D F H ; B E G\} \\
\tau_{10}=\{A B D F ; C E G H\} & \tau_{12}=\{D F G H ; A B C E\}
\end{array}
$$

(a) $\tau$-partitions

$$
\begin{array}{lll}
\eta_{1}^{1}=\tau_{1} & \eta_{1}^{2}=\tau_{10} & \eta_{1}^{3}=\tau_{7} \\
\eta_{3}^{1}=1 & \eta_{3}^{2}=\tau_{12} & \eta_{3}^{3}=\tau_{3} \\
\eta_{10}^{1}=1 & \eta_{10}^{2}=\tau_{10} & \eta_{10}^{3}=0 \\
\eta_{12}^{1}=\tau_{4} & \eta_{12}^{2}=\tau_{12} & \eta_{12}^{3}=\tau_{7}
\end{array}
$$

(b) $\eta$-partitions

Table 3: The reduced $\tau$ and $\eta$ partitions

Theorem 4 provides the condition for removing another type of redundancy. For example, $\tau_{4}$ in Table 2 is the logical complement of $\tau_{1}$. Therefore, equation $Y_{4}$ is redundant equation and all coefficients $y_{4}$ in the simple term equations can be replaced by $\overline{y_{1}}$. Similarly, equation $Y_{7}$ is also a redundant equation and all coefficients $y_{7}$ in the simple term equations can be replaced by $\overline{y_{3}}$.

Theorem 4 does not specify which $\tau$-partition should be removed if $\tau_{i}$ is a logical complement of $\tau_{j}$. The final result may be quite different if $\tau_{i}$ or $\tau_{j}$ is removed because removing such a $\tau$-partition may make more $\tau$-partitions become redundant with the condition of Theorem 3. In order to obtain a better result, the redundancy specified by Theorem 3 first needs to be removed. That will allow the designer to make a better choice by checking fewer $\tau$ and $\eta$-partitions. For example, $\tau_{9}$ and $\tau_{11}$ become redundant partitions after removing $\tau_{4}, \tau_{7}$ and corresponding $\eta$-partitions $\eta_{4}^{p}, \eta_{7}^{p}, \forall p=1,2,3$.

By eliminating redundant $\tau$-partitions and $\eta$-partitions, the number of partitions is significantly reduced. In the example above, 8 out of $12 \tau$ partitions are removed. Table 3 (a) and (b) show the results.

By using Theorem 3, two stable states under the same input state may have an identical state assignment if they have the same next states under all inputs. To have identical assignment for two stable states is equivalent to merge two corresponding rows in the flow table. However, if the outputs associated with these two stable states are not compatible, such a merging is invalid. A unique state has to be assigned to each stable state so that two outputs can be distinguished.

A partitioning chart is introduced to facilitate state assignment reduction and avoid such invalid merging. The chart has intersection for each pair of $k$-sets in an input. For each $\tau$-partition $\tau_{i}$ introduced by the state assignment, $\tau_{i}$ will be placed in the intersections where a pair of $k$-sets are partitioned by $\tau_{i}$. When a $\tau_{j}$ is removed by the state assignment reduction procedure in accordance with Theorem $3, \tau_{j}$ must be removed from all intersections in the partitioning chart.

If a blank intersection in a partitioning chart was left once $\tau_{j}$ was removed, then the compatibility of two corresponding rows in the flow table would have to be checked. If


Figure 3: The partitioning chart for the final $\tau$ partitions
two outputs are not compatible, $\tau_{j}$ should not be removed even though $\tau_{j}$ is redundant according to Theorem 3. The partitioning chart for the final $\eta$ assignment is shown in Figure 3. There is at least one $\tau$-partition at each intersection.

The following procedure formalizes the $\bar{\eta}$ assignment assignment.

## Procedure $1 \eta$ assignment generation:

Step 1. Select an initial set of $\tau$-partition $\tau_{i}$ that partition each $k$-set $K_{i}$ from the rest of states. Create a partitioning chart for $k$-sets.
Step 2. Generate all $\eta$-partitions $\eta_{i}^{p}$ for each input $I_{p}$.
Step 3. For any $\eta_{i}^{p}$ that do not meet the conditions of Corollary 1, generate a new $\tau$ partition $\tau_{k}$ such that $\tau_{k}=\eta_{i}^{p}$. Add $\tau_{k}$ to the partitioning chart. Return to step 2. The state assignment process is complete when every $\eta_{i}^{p}$ satisfies the conditions of Corollary 1.
Step 4. Remove each $\tau$-partition $\tau_{i}$ that meets the conditions of Theorem 3 if there are no merging problem in the partitioning chart by removing $\tau_{i}$. Also, for each $\tau_{i}$ removed, remove $\eta_{i}^{p}$ under all $I_{p}$. Repeat Step 4 until all such $\tau_{i}$ have been eliminated.
Step 5. For each pair of $\tau$-partitions $\tau_{i}$ and $\tau_{j}$ that are logical complements, remove $\tau_{i}$ and $\eta$-partitions $\eta_{i}^{p}$ under all $I_{p}$. Once such $a \tau_{i}$ is removed, return to Step 4.

Since Procedure 1 involves iterations of step 2 and step 3 , it is useful to know if the nümber of $\tau$-partitions is finite. The following theorem shows closure of Procedure 1.

Theorem 5 The number of $r$-paritions neded to generale any arbitrary $\eta$ assigntinent is finite.

|  | $I_{1}$ | $I_{2}$ | $I_{3}$ | $y_{1}$ | $y_{3}$ | $y_{10}$ | $y_{12}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{A}$ | B | C | 1 | 1 | 1 | 0 |
| B | A | $\mathbf{B}$ | G | 1 | 0 | 1 | 0 |
| C | D | E | $\mathbf{C}$ | 0 | 1 | 0 | 0 |
| D | D | F | C | 0 | 1 | 1 | 1 |
| E | D | E | G | 0 | 0 | 0 | 0 |
| F | A | F | C | 1 | 1 | 1 | 1 |
| G | A | H | G | 1 | 0 | 0 | 1 |
| H | D | H | C | 0 | 1 | 0 | 1 |

Table 4: The result of $\eta$ assignment

Proof: If there are $\mathrm{k} k$-sets in a column of $I_{p}$, the maximum number of $\tau$-partitions that could be generated is

$$
\binom{k}{0}+\binom{k}{1}+\binom{k}{2}+\cdots+\binom{k}{k}=2^{k} .
$$

This is finite.

Corollary 2 A simple term $\eta$ assignment exists.
Proof: A flow table must have at least one input state with more than one $k$-set, otherwise the circuit is purely combinational. With $k$ k-set ( $k \geq 2$ ), Theorem 5 shows that the maximum of $\tau$-partitions is $2^{k}$. Therefore, an assignment exists because a set of $\tau$-partitions can be constructed.

Theorem 6 The $\eta$ assignment is a valid $S T T$ assignment.
Proof: A Liu assignment will produce $\tau$-partitions which partition the k -sets. The $\tau$ partitions in this assignment consists of two sets of partitions. The first consists of the initial set of $\tau$-partitions that partitions individual $k$-sets. The second set consists of the $\tau$-partitions that are created from $\eta$-partitions that do not meet conditions of Corollary 1. Hence, both type of $\tau$-partitions are elements of a valid Liu assignment and therefore the overall assignment meets the conditions of an STT assignment.

The result of $\eta$ assignment for flow table Table 1 is given in Table 4.

### 3.3 Design Equation Generation

With the $\eta$ assignment, the next design step is to associate a unique state variable $y_{i}$ with each $\tau$-partition $\tau_{i}$ and determine each $f_{i p}$ term.

Theorem 7 The next state equations can be derived from the $\eta$-partitions with the following roles:

1. If $\eta_{i}^{p}=\tau_{j}$, then $f_{i p}=y_{j}$.
2. If the states in the left block of $\eta_{i}^{p}$ are the same as states in the right block of $\tau_{j}$ and vice versa, then $f_{i p}=\overline{y_{j}}$.
3. If all the states of $\eta_{i}^{p}$ are in the left block, then $f_{i p}=1$.
4. If all the states of $\eta_{i}^{p}$ are in the right block, then $f_{i p}=0$.

Proof: The proof follows directly from Theorem 2 since the $\eta$ assignment assigns state variable $y_{j}=1$ to states in the left block and $y_{j}=0$ to states in the right block of $\tau$ partition $\boldsymbol{\tau}_{\boldsymbol{j}}$.

The following example gives the next state equations by applying Theorem 7 to the result of $\eta$ assignment in Table 3.

Example 1 Applying the revised conditions of Theorem 7 to each $\eta$-partition $\eta_{i}^{p}$ in Table 9, the next state equations can be derived as follows:

$$
\begin{aligned}
& Y_{1}=y_{1} I_{1}+y_{10} I_{2}+\overline{y_{3}} I_{3} \\
& Y_{3}=I_{1}+y_{12} I_{2}+y_{3} I_{3} \\
& Y_{10}=I_{1}+y_{10} I_{2}+0 \\
& Y_{12}=\overline{y_{1}} I_{1}+y_{12} I_{2}+\overline{y_{3}} I_{3}
\end{aligned}
$$

The $\eta$ assignment guarantees to produce the simple term equations in which all coefficients are single variables or constants. The general form for the next state equation is

$$
Y_{i}=\sum_{p=1}^{n} f_{i p} I_{p}
$$

It has been shown in [10] that this equation can be represented as a pass logic expression. Since only one $I_{p}=1$ at a time and the case when all $I_{i}=0$ is an undefined situation in an asynchronous flow table, an ILA network will let $y_{i}$ be passed to $Y_{i}$ if the term $\overline{I_{n}}\left(y_{i}\right)$ is added. The equation of $Y_{i}$ then becomes

$$
\begin{equation*}
Y_{i}=I_{1}\left(f_{i 1}\right)+\overline{I_{1}}\left(\cdots I_{p}\left(f_{i p}\right)+\overline{I_{p}}\left(\cdots\left(I_{n}\left(f_{i n}\right)+\overline{I_{n}}\left(y_{i}\right) \cdots\right) .\right.\right. \tag{2}
\end{equation*}
$$

The advantage of this equation is to allow $Y_{i}$ to maintain the same state when all $I_{i}=0$, which can happen during an input transition. For example, the simple term equation for $Y_{1}$ in Example 1 is

$$
Y_{1}=y_{1} I_{1}+y_{10} I_{2}+\overline{y_{3}} I_{3}
$$

Putting this into the form of Equation 2, the expression can then be converted into

$$
Y_{1}=I_{1}\left(y_{1}\right)+\overline{I_{1}}\left(I_{2}\left(y_{10}\right)+\overline{I_{2}}\left(I_{3}\left(\overline{y_{3}}\right)+\overline{I_{3}}\left(y_{1}\right)\right)\right)
$$

Similarly, all other equations can be converted to the pass logic expressions in the same way. The results are as follows:


Figure 4: ILA realization of state variable $Y_{1}$

$$
\begin{aligned}
& Y_{1}=I_{1}\left(y_{1}\right)+\overline{I_{1}}\left(I_{2}\left(y_{10}\right)+\overline{I_{2}}\left(I_{3}\left(\overline{y_{3}}\right)+\overline{I_{3}}\left(y_{1}\right)\right)\right) \\
& Y_{3}=I_{1}(1)+\overline{I_{1}}\left(I_{2}\left(y_{12}\right)+\overline{I_{2}}\left(I_{3}\left(y_{3}\right)+\overline{I_{3}}\left(y_{3}\right)\right)\right) \\
& Y_{10}=I_{1}(1)+\overline{I_{1}}\left(I_{2}\left(y_{10}\right)+\overline{I_{2}}\left(I_{3}(0)+\overline{I_{3}}\left(y_{10}\right)\right)\right) \\
& Y_{12}=I_{1}\left(\overline{y_{1}}\right)+\overline{I_{1}}\left(I_{2}\left(y_{12}\right)+\overline{I_{2}}\left(I_{3}\left(\overline{y_{3}}\right)+\overline{I_{3}}\left(y_{12}\right)\right)\right) .
\end{aligned}
$$

Obviously, the next state logic $f_{i p}$ is minimized to a wire if the set of next state equations are all simple term equations. It is straightforward then to map the design equations to an ILA network. Figure 4 shows an ILA realization of state variable $Y_{1}$.

## 4 Input and Hazard Characteristics

In addition to the regularity of the ILA network with the $\eta$ assignment, an ILA realization has other features such as (1) immunity to input $1-1$ overlapping and $0-0$ crossing, (2) immunity to input bounce while 1-1 overlapping is not present, and (3) free of transition path hazards and input state transition hazards.

Potential conflicts arise in asynchronous sequential networks when more than one input state is present at a time (1-1 overlapping), or when none of the input states are active ( $0-0$ crossing). The overlapping and crossing situations occur because two input states rarely switch at exactly the same time due to differences in delay through forming logic. Most design procedures avoid such uncertainties by setting a constraint either to forbid 1-1 overlapping or to forbid $0-0$ crossing of input states. Another form of hazard on an input signal which may cause the circuit to malfunction is the dynamic hazard when input transitions.

Theorem 8 The ILA architecture tolerates $1-1$ input overlapping.
Proof: In Equation 2, assume the input state $I_{p}$ have higher priority relative to input state $I_{q}$. In the other words, $I_{p}$ is the control variable of an ILA cell which is closer to the output than the ILA cell with the control variable $I_{q}$. If both $I_{p}$ and $I_{q}$ are asserted 1, then $Y_{i}$ will assume value specified by $f_{i p}$ rather than $f_{i q}$.

In the case where the input switches from $I_{p}$ to $I_{q}$, when $I_{p}$ is 1 , it passes $f_{i p}$ to $Y_{i}$ and meanwhile cuts off the path of $f_{i q}$ to $Y_{i}$, no matter if $I_{q}$ is set to 1 or 0 . With such an
architecture, the 1-1 overlapping of input $I_{p}$ and $I_{q}$ has the same effect on the output $Y_{i}$ as $I_{p}=1, I_{q}=0$.

In the case where the input switches from $I_{q}$ to $I_{p}$, when $I_{q}=1$, all $Y_{i}$ are determined by the $f_{i q}$ values which are propagated through the ILA. Once $I_{p}=1$, the $f_{i p}$ values are passed through the ILA independent of whether $I_{q}$ is 0 or 1 . Hence the circuit assumes the proper next state value.

For example, in Figure 4, if $I_{1}$ and $I_{2}$ are active, the circuit for all next state variables will be under the control of $I_{1}$ only, and $Y_{1}$ will assume the value of $y_{1} . y_{10}$ can be passed to $Y_{1}$ only if $I_{1}=0$ while $I_{2}$ is active. Once $I_{1}$ is set to 1 again, the value of $y_{10}$ at $Y_{1}$ will become $y_{1}$ immediately (assuming $\overline{I_{1}}$ is set to 0 simultaneously).

The $0-0$ crossing happens when all input states are 0 . Let the input change from $I_{p}$ to $I_{q}$. If input $I_{p}$ goes to 0 before $I_{q}$ goes to 1 , there will be a period that all input lines are 0 . In traditional design, the outputs of the design equations could assume an undeterminant state when all inputs are 0 . The ILA architecture solves the problem by providing a path for each state variable to pass $y_{i}$ to $Y_{i}$. When all inputs are 0 , it allows $Y_{i}$ to maintain its current value $y_{i}$.

Again, Figure 4 can be used to show the feature of $0-0$ crossing tolerance. For example, assuming the input transition is from $I_{2}$ to $I_{3}$, when $I_{2}=1, y_{10}$ is passed to $Y_{1}$, and $y_{1}$ is fed back into the last ILA cell under $\overline{I_{3}}$. Once the network is stable, the level of $y_{10}$ is passed to $Y_{3}$. When $I_{2}$ is set to 0 , the path provided by $\overline{I_{1}}, \overline{I_{2}}$ and $\overline{I_{3}}$ will still maintain the level of $Y_{1}$ at the current value of $y_{1}$. The output of the network remains unchanged during the period when all inputs are 0 until $I_{3}$ is set to 1 . Then a new level of $y_{7}$ will be passed to $Y_{1}$ and the network assumes a new state.

An input bounce can be considered a dynamic hazard during the transition of input states. With the ability to tolerate input bounce, the ILA network allows extra input transitions to occur before the circuit is stablized. However, the input bounce can be tolerated only if it is not necessary to also tolerate 1 -1 overlapping. If an input $I_{q}$ bounce occurs when two input states $I_{p}$ and $I_{q}$ are overlapping, an ambiguity is created regarding the interpreration of the transition; it could be one transition from $I_{p}$ to $I_{q}$ or three transitions from $I_{p}$ to $I_{q}$ then to $I_{p}$ back to $I_{q}$. In order to avoid the ambiguity, it is assumed that during the input state transition from $I_{p}$ to $I_{q}$ the circuit tolerates either the bounce condition or the 1-1 overlapping, but not both.

Theorem 9 For the simple term equations derived from an $\eta$ assignment, all partitioning variables under input $I_{p}$ do not change as the circuit transitions from one state to another under $I_{p}$.

Proof: The $\eta$ assignment produces the simple term equations of the form

$$
Y_{i}=\cdots+y_{i} I_{p}+\cdots
$$

where $y_{i}$ is the partitioning variable of $I_{p} . \quad y_{i}$ is the only partitioning variable for all transition paths in the $\eta$ assignment. According to Theorem 2 in [7], $y_{i}$ will not change in


Figure 5: A traditional realization of a partial flow table
any transition under $I_{p}$. Hence, the partitioning variables of the simple term equations do not change during the input transitions.

From Theorem 9, the partitioning variables do not change when an input signal $I_{p}$ undergoes a bounce. In the case that a dynamic hazard presents when $I_{p}$ transitions from 0 to 1 , the circuit begins to transition to the new state when $I_{p}$ goes to 1 . Meanwhile, the partitioning variables of $I_{p}$ remain stable. Since the partitioning variables determine all $f_{i p}$ values [7] and since the partitioning variables are uneffected by the input bounce, the circuit will assume the proper next state value when $I_{p}$ is stablized.

In the case that a dynamic hazard presents when $I_{p}$ transitions from 1 to 0 , the present state is simply passed to the next state once $I_{p}$ goes to 0 , as this represents a $0-0$ crossover condition. The circuit does not transition. When $I_{p}$ returns to 1 on the bounce condition, again from Theorem 9, there will be no transition in partitioning next state variables. Since the circuit is a function only of the partitioning variables [7], output of the circuit remains unchanged.

A critical race free asynchronous sequential network may still malfunction due to unwanted switching transients in the combinational circuit. A transition path hazard is one that is present within the states of a transition path. The simple term equations for the next state variables have the form

$$
Y_{i}=\cdots+y_{i} I_{p}+\cdots
$$

where $y_{i}$ is the partitioning variable of $I_{p}$. From Theorem 9 , partition variable $y_{i}$ does not change as the circuit transitions from one state to another in $I_{p}$. In general, since the partitioning variables of the simple term equations do not change during the transition, it is impossible for a hazard of any kind to occur. Therefore, there cannot be any static, dynamic and function hazards that occur during the transition between unstable and stable states.

A circuit free of static and dynamic hazard may have a hazard problem caused by the change of more than one variable in the design equation. One type of such multi-variable change hazard is a function hazard. The problem can be illustrated by the partial flow
table in Figure 5 where design equations for $Y_{i}, Y_{j}$ and a schematic for $Y_{i}$ are shown. A hazard exists when an input changes from $I_{2}$ to $I_{1}$. If the delay of AND gate- 1 is longer than the total delays of AND gate-2, the OR gate and the buffer, then the output of gate-1 will remain 0 after the input changes to $I_{1}=1$ and $I_{2}=0$. That will in turn cause $y_{i}$ to remain at 0 and lock the output of gate- 1 to 0 . The result is that state variables $y_{i} y_{j}$ are set to 00 instead of intended value 10 . Moreover, the circuit is locked up in the wrong state until a reset line is provided.

The a problem occurs in a traditional design where a slow gate has the same effect as a $0-0$ crossover. Input $I_{p}=0$ will set the product term to 0 . The ILA circuit solves the problem by maintaining the same state when all inputs go to 0 . Also, the $1-1$ overlapping problem which may arise in the traditional design due to a slow gate can be tolerated by ILA circuit. The 1-1 overlapping and 0-0 crossover properties of the ILA architecture prevents the circuit from malfunction due to such input state transition hazards.

## References

[1] C. Roth, Fundamentals of Logic Design, 3rd Ed,. St. Paul, Minn., West Publishing, 1985.
[2] D. Givone, Introduction to Switching Circuit Theory, McGraw-Hill, Inc., 1970.
[3] S. Whitaker, "Design of Asynchronous Sequential Circuits Using Pass transistors," Ph.D Dissertation, Úniversity of Idaho, Feb. 1988.
[4] S. K. Gopalakrishnan and G. K. Maki, "VLSI Asynchronous Sequential Circuit Design", ICCD, Sept, 1990, pp. 238-242.
[5] S. Whitaker and G. Maki, "Pass-Transistor Asynchronous Sequential Circuits", IEEE JSSC, Vol.24, No.1, Feb. 1989, pp. 71-78.
[6] C. Tan, "State Assignments for Asynchronous Sequential Machines", IEEE Transactions on Computers, Vol. C-20, No. 4, April 1971, pp. 382-391.
[7] S. Gopalakrishnan, G. Kim and G. Maki, "Implications of Tracey's Theorem to Asynchronous Sequential Circuit Design", The 2nd NASA SERC Symposium on VLSI Design, November, 1990, pp. 9.1.1-9.1.11.
[8] G. Maki, D.Sawin and B. Jeng, "Improved State Assignment Selection Tests", IEEE Transactions on Computer, Dec. 1972, pp.1443-1449.
[9] J. Tracey, "Internal State Assignment for Asynchronous Sequential Machines", IEEE Transactions on Electronic Computers, Vol. EC-15, Aug. 1966, pp. 551-560.
[10] S. K. Gopalakrishnan, "Design of VLSI Asynchronous Sequential Machines," Ph.D Dissertation, University of Idaho, December, 1989.

