S4-34
186466
p. 16

N94-18549

A COUPLED IMPLICIT SOLUTION METHOD FOR TURBULENT SPRAY COMBUSTION IN PROPULSION SYSTEMS

K.-H. CHEN
The University of Toledo, Ohio

and

J.-S. SHUEN
Sverdrup Technology, Inc., Ohio

Aerothermochemistry Branch
Internal Fluid Mechanics Division
NASA Lewis Research Center, Cleveland, Ohio

OBJECTIVES

- Develop an efficient and robust algorithm for multi-phase chemically reacting flows at all speeds, with emphasis on low Mach number flows.
- Calculate turbulent spray combustion flow in a gas turbine combustor.

MOTIVATION

- Many reacting flows in propulsion devices cannot be efficiently calculated by modern compressible flow CFD algorithms, e.g.,
  - rocket motor — wide range of Mach numbers, from near zero velocity at closed end to supersonic at nozzle exit.
  - gas turbine combustor — low subsonic velocity, but large density variation precludes incompressible approach.
- Most low-speed reacting flow codes based on TEACH-type technologies – inefficient and lack of robustness for complex flows.
- Tremendous progress made in high-speed compressible flow CFD in past two decades. Extending application range to low-speed flow regime highly desirable.

OUTLINES

- GOVERNING EQUATIONS
  - Gas-Phase Equations
  - Liquid-Phase Equations
- NUMERICAL ALGORITHM
- NUMERICAL TEST RESULTS
- CONCLUSION
- FUTURE PLAN FOR ALLSPD CODE

GOVERNING EQUATIONS

- **Gas-Phase Equations**

$$\Gamma\frac{\partial\hat{\mathbf{Q}}}{\partial\tau^*}+\frac{\partial\tilde{\mathbf{Q}}}{\partial\tau}+\frac{\partial(\tilde{\mathbf{E}}-\tilde{\mathbf{E}}_v)}{\partial\xi}+\frac{\partial(\tilde{\mathbf{F}}-\tilde{\mathbf{F}}_v)}{\partial\eta}=\tilde{\mathbf{H}}_c+\tilde{\mathbf{H}}_l, \qquad (1)$$

$$\hat{\mathbf{Q}}=\frac{y^\delta}{J}\begin{pmatrix} p_g \\ u \\ v \\ h \\ \kappa \\ \epsilon \\ Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_{N-1} \end{pmatrix}, \quad \Gamma=\begin{bmatrix} 1/\beta & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ u/\beta & \rho & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ v/\beta & 0 & \rho & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ h/\beta-1 & \rho u & \rho v & \rho & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ \kappa/\beta & 0 & 0 & \cdot & \rho & \cdot & \cdot & \cdot & 0 & \cdot & 0 \\ \epsilon/\beta & 0 & 0 & \cdot & \cdot & \rho & \cdot & \cdot & \cdot & 0 & 0 \\ Y_1/\beta & 0 & 0 & 0 & \cdot & \cdot & \rho & \cdot & \cdot & \cdot & 0 \\ Y_2/\beta & 0 & 0 & 0 & 0 & \cdot & \cdot & \rho & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \rho & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \rho & 0 \\ Y_{N-1}/\beta & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \rho \end{bmatrix},$$

$$\mathbf{H}_c=\begin{pmatrix} 0 \\ -\frac{2}{3}\delta\frac{\partial(\mu_e v)}{\partial x} \\ \delta[p-\tau_{\theta\theta}-\frac{2}{3}\frac{\partial(\mu_e v)}{\partial y}] \\ -\frac{2}{3}\delta[\frac{\partial(\mu_e uv)}{\partial x}+\frac{\partial(\mu_e v^2)}{\partial y}] \\ y^\delta(\Psi-\rho\epsilon)-\frac{2}{3}\delta\mu_t v(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}) \\ y^\delta[(c_1f_1\Psi-c_2f_2\rho\kappa)\frac{\epsilon}{\kappa}+\Lambda]-\frac{2}{3}\delta\mu_t v(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})c_1f_1\frac{\epsilon}{\kappa} \\ y^\delta S_1 \\ \cdot \\ \cdot \\ y^\delta S_{N-1} \end{pmatrix},$$

$$\mathbf{H}_l=\begin{pmatrix} \sum_p n_p\dot{m}_p \\ \sum_p n_p\dot{m}_p u_p-\frac{4\pi}{3}\rho_p r_p^3 n_p\frac{du_p}{dt} \\ \sum_p n_p\dot{m}_p v_p-\frac{4\pi}{3}\rho_p r_p^3 n_p\frac{dv_p}{dt} \\ \sum_p n_p\dot{m}_p h_{fs}-4\pi r_p^2 n_p h\Delta T \\ 0 \\ 0 \\ \sum_p n_p\dot{m}_p \\ \cdot \\ \cdot \\ 0 \end{pmatrix},$$

ALLSPD MAIN FEATURES

Present Capabilities

- 2-D and Axisymmetric geometries
- Second-order central difference for both inviscid and viscous terms
- Fully coupled, fully implicit algorithm
- Efficient convergence for wide range of Mach numbers (from $M \leq 10^{-10}$ to supersonic)
- Finite-rate chemistry, realistic thermophysical properties
- Multi-block and body-fitted curvilinear coordinates for complex geometries
- $\kappa - \epsilon$ turbulence model
- Stochastic liquid spray model (dilute spray), vortex model for droplet internal circulation and diffusion
- Still a research code, require experience and knowledge of CFD and flow physics to use.

Future Plans

- Development of a more efficient solver
- Extension to 3-D
- PDF model for turbulence/chemical reaction closure
- Thermal radiation model
- Detailed soot and NOx kinetic models
- Multi-grid and unstructured grid capabilities
- Dense spray and high pressure (near-/super-critical) spray models

Difficulties with Compressible Flow Algorithms at Low Mach Numbers

- Disparities among system's eigenvalues (stiffness), u, $u+c$, $u-c$, resulting in significant slowdown in convergence rate.

- Singular behavior of pressure gradient term in momentum equations as Mach number approaches zero,

$$\rho^* u^{*2} + \frac{p^*}{\gamma M_r^2}$$

As Mach number is decreased, pressure variation ($\Delta p^* \propto M^2$) becomes of similar magnitude as roundoff error of the large pressure gradient term ($p^*/\gamma M_r^2$).

METHOD OF APPROACH

Pressure Singularity Problem

- Pressure decomposed into two parts:

$$p = p_o + p_g$$

p_g replaces p in momentum equations and retains p_g as one of the unknowns.

- Employs conservative form of governing equations, but uses primitive variables

$$(p_g, u, v, h, Y_i)$$

as unknowns. Conservation property preserved and pressure field accurately resolved for all Mach numbers.

Eigenvalue Stiffness Problem

- Pressure rescaled so that all eigenvalues have the same order of magnitude. Physical acoustic waves removed and replaced with pseudo-acoustic waves which travel at speed comparable to fluid convective velocity.

EIGENVALUES RESCALING

$$\lambda = U,\ U,\ \frac{1}{2}\Big[U(1+\frac{\beta}{c^2}) \pm \sqrt{U^2(1-\frac{\beta}{c^2})^2 + 4\beta(\alpha_1^2+\alpha_2^2)}\,\Big],\quad U,\ U\cdots$$

$\alpha_1 = \xi_x, \quad \alpha_2 = \xi_y, \qquad U = \alpha_1 u + \alpha_2 v.$

For well-conditioned eigenvalues, scaling factor β taken to be

$$\beta = u^2 + v^2 \ .$$

LIQUID-PHASE EQUATIONS

- **Droplet Motion Equations**

$$\frac{dx_p}{dt} = u_p, \tag{2}$$

$$\frac{dy_p}{dt} = v_p, \tag{3}$$

$$\frac{du_p}{dt} = \frac{3}{16}\frac{C_D \mu_g Re_p}{\rho_p r_p^2}(u_g - u_p), \tag{4}$$

$$\frac{dv_p}{dt} = \frac{3}{16}\frac{C_D \mu_g Re_p}{\rho_p r_p^2}(v_g - v_p), \tag{5}$$

$$Re_p = 2\frac{r_p \rho_g}{\mu_g}[(u_g - u_p)^2 + (v_g - v_p)^2]^{1/2},$$

$$C_D = \begin{cases} \frac{24}{Re_p}(1 + \frac{Re_p^{2/3}}{6}) & for \quad Re_p < 1000, \\ 0.44 & for \quad Re_p > 1000. \end{cases}$$

- **Droplet Heat and Mass Transfer Equations**

$$\frac{\dot{m}_p'' d_p}{\rho D_f} = 2N_s ln(1 + B), \tag{6}$$

$$\frac{hd_p}{k} = \frac{2N_p ln(1+B)^{Le^{-1}}}{[(1+B)^{Le^{-1}} - 1]} \tag{7}$$

$$N_s = 1 + \frac{0.276 Re_p^{1/2} Pr^{1/3}}{[1 + \frac{1.232}{Re_p Pr^{4/3}}]^{1/2}} \qquad N_p = 1 + \frac{0.276 Re_p^{1/2} Sc^{1/3}}{[1 + \frac{1.232}{Re_p Sc^{4/3}}]^{1/2}},$$

$$B = \frac{Y_{fgp} - \bar{Y}_{fg}}{1 - Y_{fgp}} \qquad Y_{fgp} = \frac{X_{fgp} W_f}{X_{fgp} W_f + (1 - X_{fgp}) W_a}$$

- Droplet Internal Temperature Equations

(Vortex Model)

$$\frac{\partial T_p}{\partial t} = 17\frac{k_l}{C_{pl}\rho_l r_p^2}[\alpha\frac{\partial^2 T_p}{\partial \alpha^2} + (1 + C(t)\alpha)\frac{\partial T_p}{\partial \alpha}] \qquad (8)$$

$$C(t) = \frac{3}{17}(\frac{C_{pl}\rho_l}{k_l})r_p\frac{dr_p}{dt}$$

$$t = t_{inj}, \quad T_p = T_{inj},$$

$$\alpha = 0, \quad \frac{\partial T_p}{\partial \alpha} = \frac{1}{17}(\frac{C_{pl}\rho_l}{k_l})r_p^2\frac{\partial T_P}{\partial t},$$

$$\alpha = 1, \quad \frac{\partial T_p}{\partial \alpha} = \frac{3}{16}r_p\frac{\partial T_P}{\partial r})_s$$

NUMERICAL ALGORITHM

- Gas-Phase - ALLSPD code
- Liquid-Phase
  - Droplet motion equations (ODE) - Runge-Kutta method.
  - Droplet internal equations (PDE) - implicit method (Thomas algorithm).
  - Determination of spray time step for integration.
  - Stochastic separate flow model.
- Interaction Between Two Phases

SPRAY TIME STEP

- **Droplet Velocity Relaxation time (t_r)**

$$t_r = \frac{16}{3}(\frac{\rho_l}{\rho_g})(\frac{r_p^2}{\nu})(C_D Re_p)^{-1}.$$

- **Droplet Life Time (t_l)**

$$t_l = \frac{r_p}{3\dot{m}_p'' \rho_l}.$$

- **Droplet Surface Temperature Constraint Time (t_s)**

$$t_s = ln(\frac{1}{1 - \frac{\Delta T_p}{A}})/A'$$

$$\frac{dT_p}{dt} = \frac{6}{\rho_l C_v d_p}[h(\bar{T} - T_p) - \dot{m}_p'' h_{fg}]$$

- **Local Grid Time Scale (t_g)**
- **Turbulent Eddy-Droplet Interaction Time (t_i)**

$$t_i = t_e, \quad if \quad L_e > \tau|\vec{u}'' - \vec{u}_p''|$$

$$t_i = min(t_e, t_t), \quad if \quad L_e < \tau|\vec{u}'' - \vec{u}_p''|$$

where

$$L_e = C_\mu^{3/4}\kappa^{3/2}/\epsilon$$

$$t_e = L_e/(2\kappa/3)^{1/2}.$$

$$t_t = -\tau ln[1 - L_e/(\tau|\vec{u}'' - \vec{u}_p''|)]$$

- **Spray Time step - ΔT_{spr}**

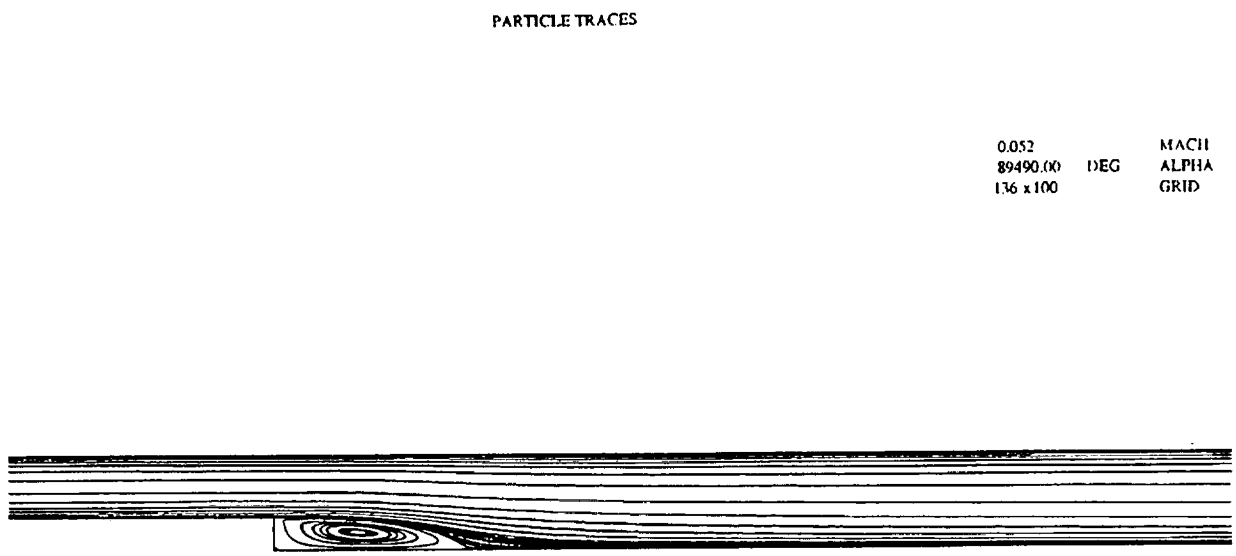
$$\Delta t_{spr} = \alpha \text{ min } (t_r, t_l, t_s, t_g, t_i)$$

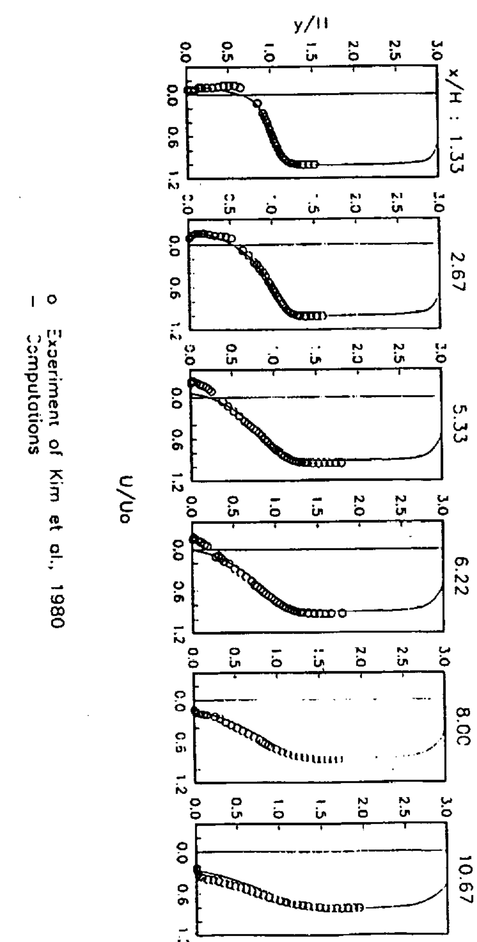
INTERACTION BETWEEN TWO PHASES

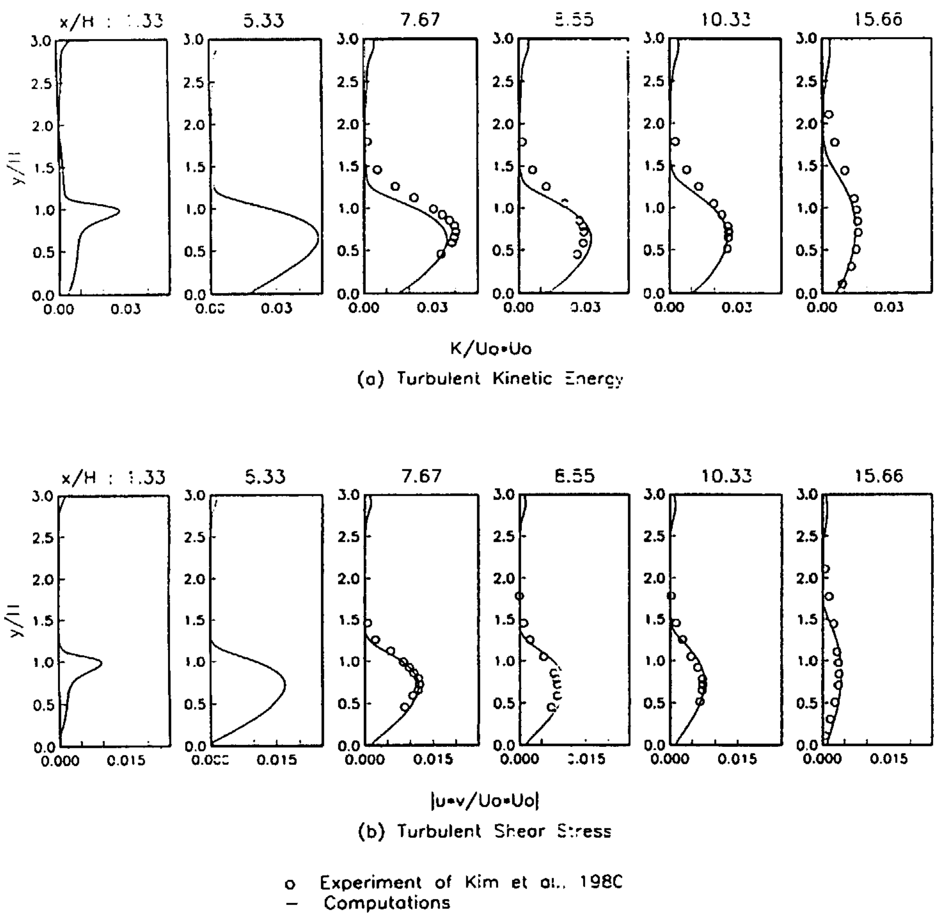
1. Initialize gas and liquid phase variables.
2. Solve liquid-phase equations.
3. Evaluate spray source term, H_l.
4. Solve gas-phase equations and update gas-phase variables.
5. Update spray source term, H_l ?
   No, go to step 4.
   Yes, go to step 2.

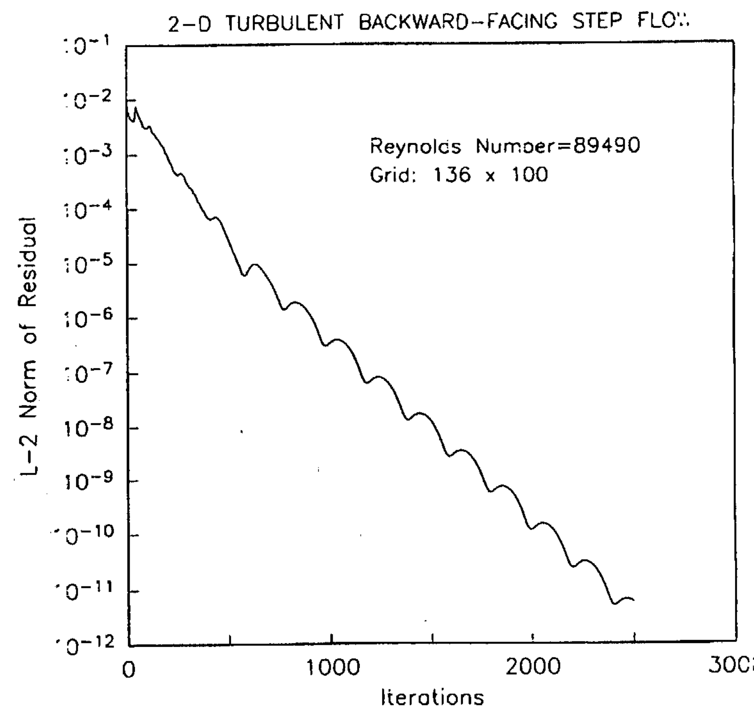
NUMERICAL TEST RESULTS

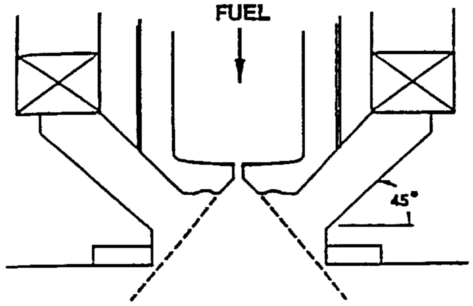
- Turbulent Backward-Facing Step Flow - non-reacting.
- Evaporating Turbulent Spray Flow.
- Gas Turbine Spray Combustion Flow.


PARTICLE TRACES
0.052 MACH
89490.00 DEG ALPHA
136 x 100 GRID


y/H
x/H : 1.33
2.67
5.33
6.22
8.00
10.67
U/Uo
o Experiment of Kim et al., 1980
— Computations

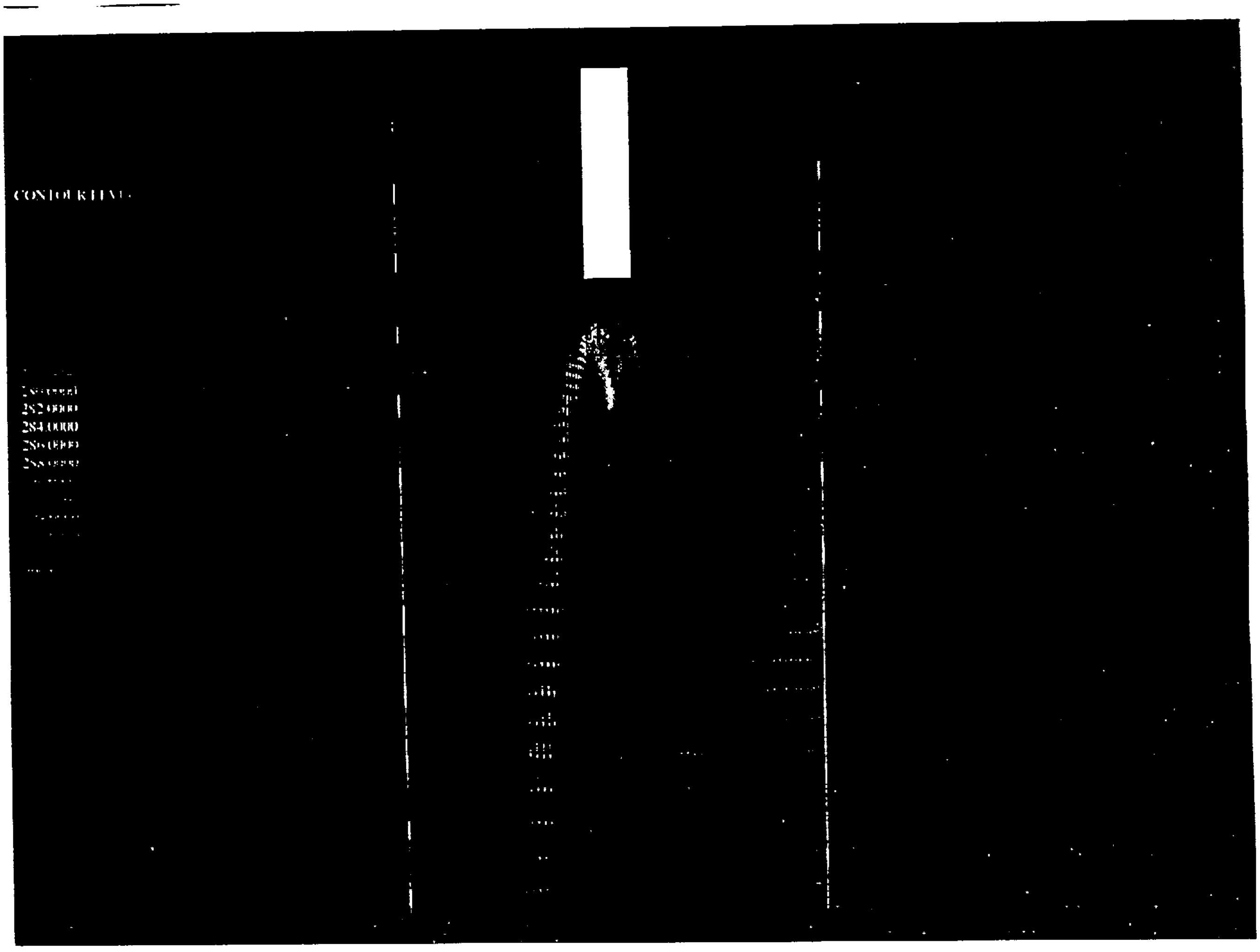

x/H : 1.33
5.33
7.67
8.55
10.33
15.66
y/H
K/Uo•Uo
(a) Turbulent Kinetic Energy
|u•v/Uo•Uo|
(b) Turbulent Shear Stress
o Experiment of Kim et al., 1980
– Computations

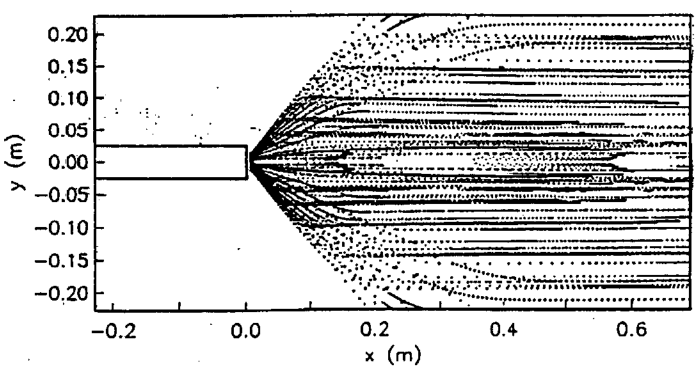

2-D TURBULENT BACKWARD-FACING STEP FLOW
Reynolds Number=89490
Grid: 136 x 100
L-2 Norm of Residual
Iterations



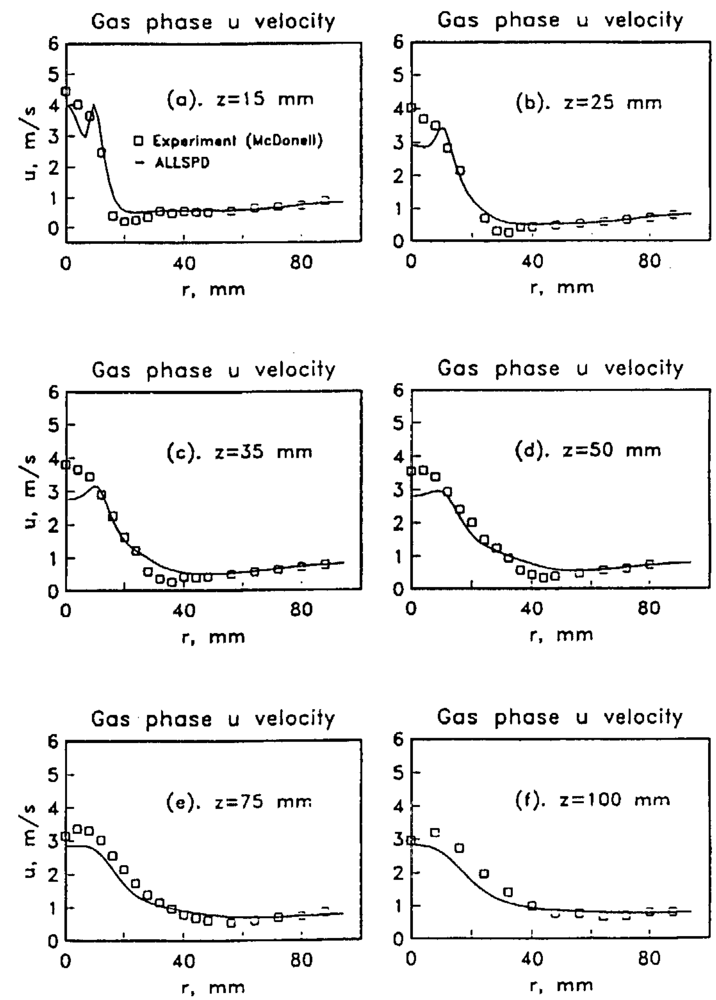
OPERATING MODE 1
NO ATOMIZING AIR

LIQUID FLOW RATE : 1.26 grams / sec
AIR FLOW RATE : 0.00 grams / sec

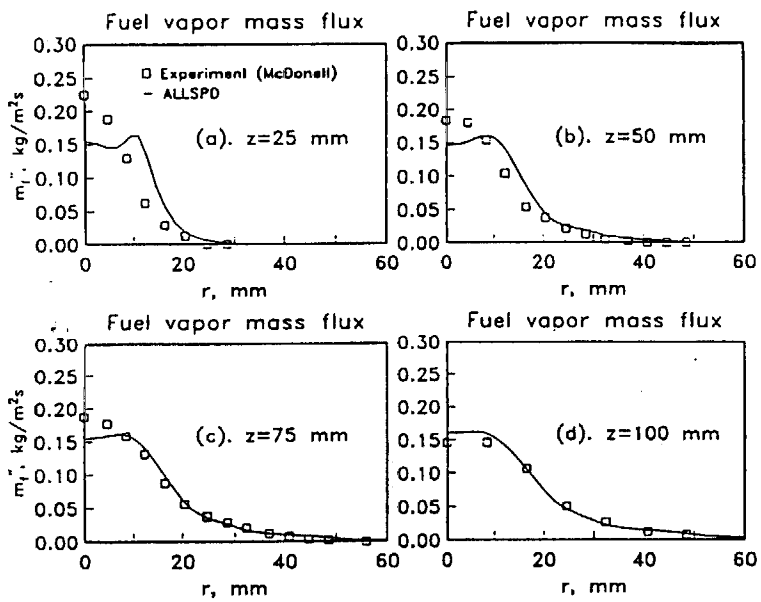


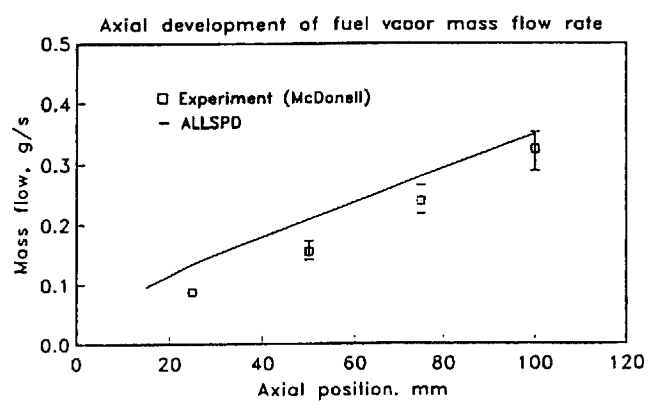


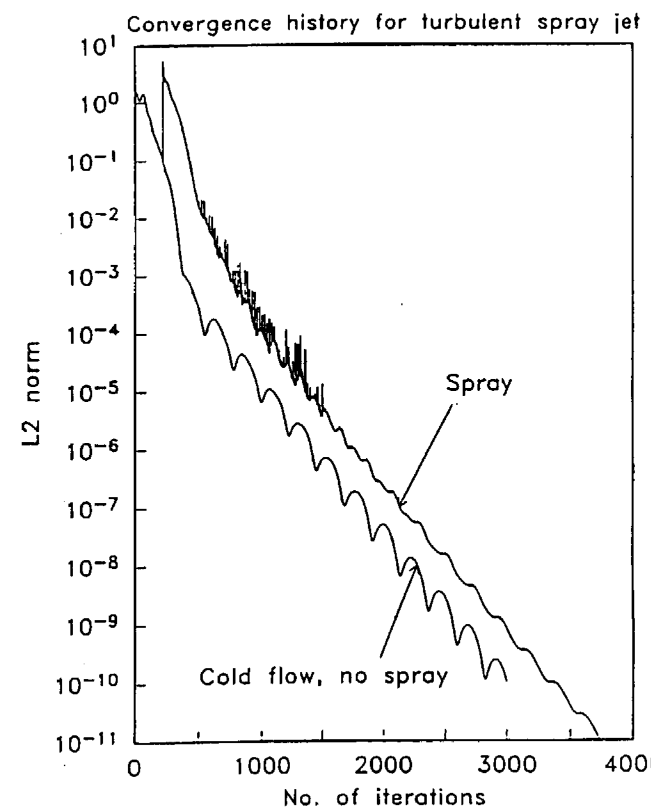
y (m)
x (m)



Gas phase u velocity
(a). z=15 mm
□ Experiment (McDonell)
– ALLSPD
(b). z=25 mm
(c). z=35 mm
(d). z=50 mm
(e). z=75 mm
(f). z=100 mm
u, m/s
r, mm


Fuel vapor mass flux
Experiment (McDonell)
ALLSPD
(a). z=25 mm
(b). z=50 mm
(c). z=75 mm
(d). z=100 mm
m_f'', kg/m^2s
r, mm


Axial development of fuel vaoor mass flow rate
Experiment (McDonell)
ALLSPD
Mass flow, g/s
Axial position, mm


Convergence history for turbulent spray jet
L2 norm
Spray
Cold flow, no spray
No. of iterations

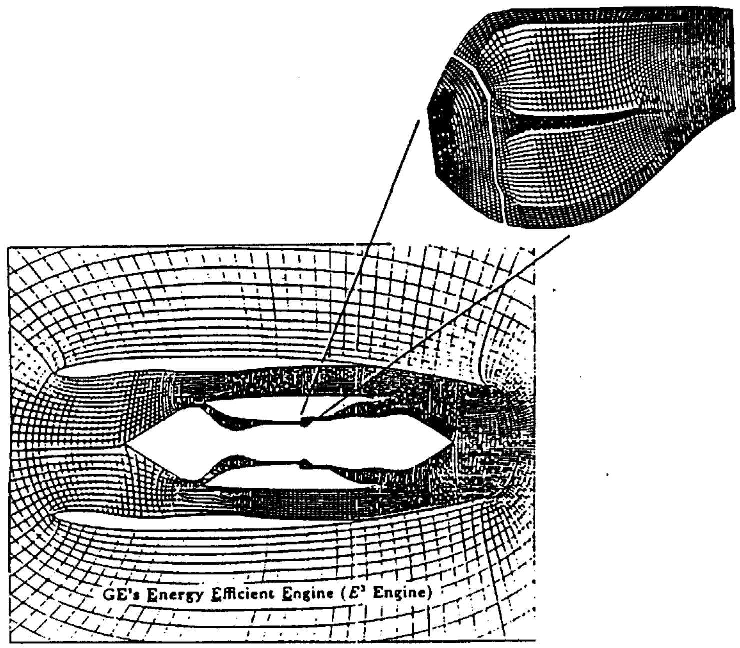
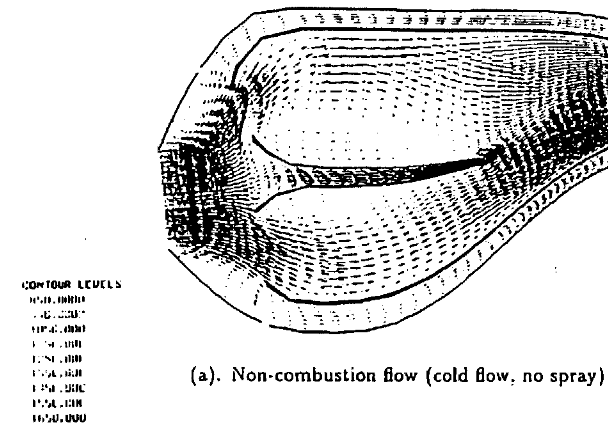
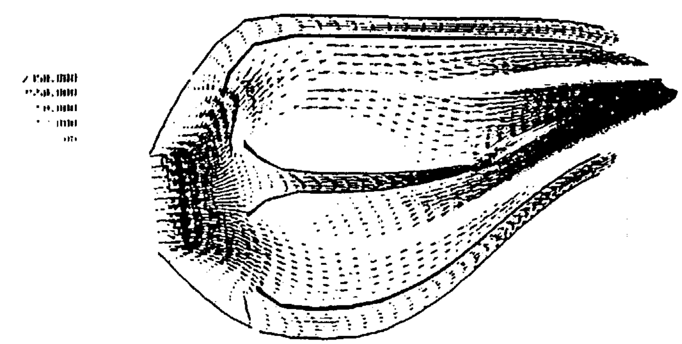


Fig. 9 Center-plane cut for GE's EEE gas turbine engine and the grid for the combustor.



(a). Non-combustion flow (cold flow, no spray)



(b). Spray combustion flow

Fig. 11 Velocity vectors (colored by temperature) for (a) non-combustion and (b) spray combustion cases for the gas turbine spray combustion flow.

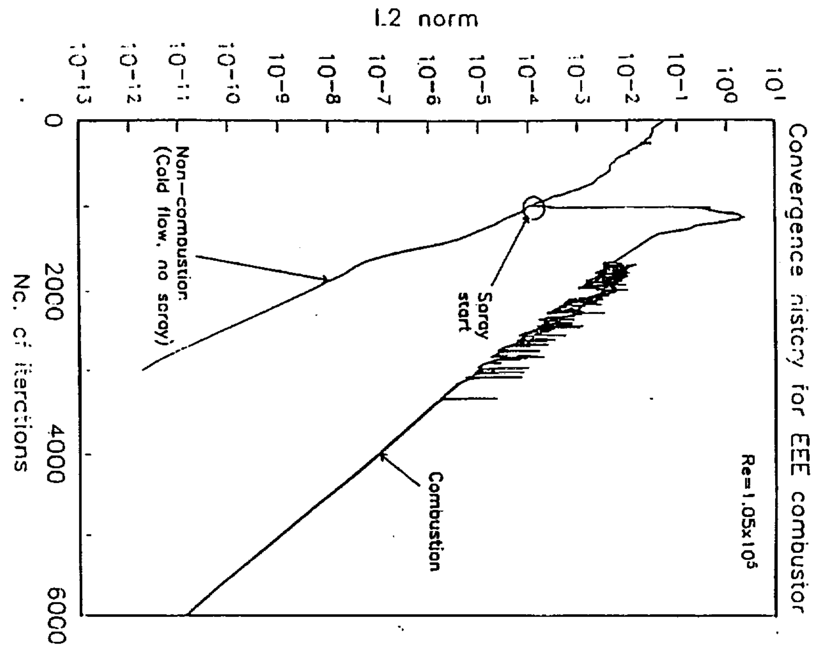


Fig. 13 Convergence history for both non-combustion and spray combustion cases for the gas turbine spray combustion flow.

CONCLUSION

- ALLSPD code efficiently coupled with the SSF spray model.
- Satisfactory convergence property for flows with and without spray.