Design for Cyclic Loading Endurance of Composites

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SUMMARY

The present paper describes the application of the computer code IPACS (Integrated Probabilistic Assessment of Composite Structures) to aircraft wing type structures. The code performs a complete probabilistic structural analysis for composites taking into account the uncertainties in geometry, boundary conditions, material properties, laminate lay-ups and loads. Results of the analysis are presented in terms of cumulative distribution functions (CDF) and probability density function (PDF) of the fatigue life of a wing type composite structure under different hygrothermal environments subjected to the random pressure. The sensitivity of the fatigue life to a number of critical structural/material variables is also computed from the analysis.

1. INTRODUCTION

The technology of advanced fiber composites has matured to the point where these composites
are prime contenders for various aerospace related applications. Their outstanding mechanical properties are very attractive to the aerospace industry especially the high strength/stiffness to density ratio. They also possess excellent fatigue strength and the ability to resist corrosion and impact. One of the major design considerations for prolonged service of these composites is the fatigue due to cyclic hygral (moisture), thermal, and mechanical (hygrothermomechanical) loading conditions. Furthermore, the mechanical properties of composites depend on a wide variety of variables. The variables are, for example, the fiber and matrix material properties at the constituent level and the fabrication variables at the ply level such as the fiber volume ratio, the void volume ratio, the ply orientation, the ply thickness etc. These primitive variables are statistical in nature, therefore, the mechanical properties of a typical composite can not be reliably quantified using deterministic approaches. Consequently, the behavior of composite structures shows considerable scatter from the mean value. Traditional approaches relied heavily on safety factors to account for the uncertainties/scatter in the response. However, those approaches usually result in an overly conservative design. In addition, they provide no indication of the structural reliability. For example the probability of failure can not be estimated readily.

Recent research activities at NASA Lewis Research Center in the area of Probabilistic Structural Analysis Methods have led to a variety of computer codes which can be used for probabilistic assessment of composite structures. The methodology as shown in Figure 1 is based upon the identification of uncertain variables at every structural level such as at the constituent, ply, or laminate level. The uncertain variables are then filtered through an analyzer which combines the composite mechanics, structural mechanics and probability theory. The output of the analyzer is the desired structural response such as displacement, stress/strain, buckling load, natural frequency, fatigue life, material properties etc. These types of problems can normally be solved by the Monte Carlo simulation method. But, such methods tend to be computationally expensive. In order to save computational time, the newly developed methodology integrates composite mechanics, finite element methods and fast probability integration algorithms to provide efficient and affordable means [1] for the probabilistic assessment of composite structures with inherent uncertainties operating under uncertain environments. The methodology is implemented in the computer code IPACS (Integrated Probabilistic Assessment of Composite Structures). A typical
aircraft wing box structure made of polymer matrix fiber composites is analyzed to demonstrate the code’s capability in the probabilistic fatigue life prediction as well as the identification of the critical variables that have high influence on the fatigue life.

2. FUNDAMENTAL CONSIDERATIONS

Several fundamental aspects underlie the development of a methodology to design cyclic loading endurance of structural components made of polymer matrix composites operating under uncertain loading and environmental conditions. In the present work, the probabilistic structural mechanics methods as applied to composites and simplified math models for predicting the fatigue life are combined sequentially to obtain cumulative distribution functions of the component life. They are briefly described in the following paragraphs.

The fundamental assumptions made in the probabilistic composite mechanics described herein are: (1) the scatter in all the primitive variables, which describe the composite, can be represented by well known probabilistic distributions, so that the values for the primitive variables can be randomly selected from the known distributions for a specific composite, (2) these values can be used in composite mechanics to predict the composite behavior, and (3) the whole process can be repeated many times to obtain sufficient information to develop the distribution of the ply property, the composite property, or the structural response. The primitive variables are identified at micro and macro levels. For example, at the constituent level, the uncertain constituent primitive variables are the fiber and matrix material properties. At the ply level, they are the fiber volume ratio, the void volume ratio, the ply orientation and the ply thickness, etc. At the structural level, uncertain loads, temperatures, geometry or the boundary condition may also be included. All these details are embedded in an integrated code IPACS [2]. The code is developed based on the concept for the probabilistic assessment of composite structures as depicted in Figure 2.

The simplified math models for predicting the fatigue life are embedded in the in house code ICAN (Integrated Composite Analyzer) [3] for polymer matrix composite. The complete details of these models are outlined in the references [4,5]. The underlying assumptions in the
development of these models are the following:

1. Holes, slits, and impact damage (defects) induce similar strength degradation in fiber-composite laminates where the characteristic dimensions of these defects are negligible compared to the planform dimensions of the laminate. If this is not the case, the effects of the defects must be evaluated using appropriate structural analyses.

2. Fatigue degrades all ply strengths at approximately the same rate \([6,7]\).

3. All types of fatigue degrade laminate strength linearly on a semilog plot including: (a) mechanical (tension, compression, shear, and bending); (b) thermal (elevated and cryogenic temperature); (c) hygral (moisture); and (d) combinations (mechanical, thermal, hygral, and reverse-tension compression). Experimental data for compression fatigue can be found in reference \([8]\).

4. Laminated composites generally exhibit linear behavior to initial damage under uniaxial or combined loading including hygrothermal effects.

5. All ply stresses (mechanical, thermal, and hygral) are predictable by using linear laminate theory.

6. Stress concentration factors for circular holes are available. They can be obtained from the literature \([9]\) or can be predicted by using finite element analysis.

3. PROBABILISTIC COMPUTER CODE IPACS

IPACS is a computer code for the probabilistic analysis of composite structures. It integrates several NASA in-house computer programs developed in recent years such as COBSTRAN \([10]\), PICAN \([11]\) and PROCSAN \([12]\). COBSTRAN (Composite Blade Structural Analyzer) is a dedicated finite element model generator for structural analyses of composite structures. PICAN (Probabilistic Integrated Composite Analyzer) enables the computation of the perturbed and probabilistic composite material properties at the ply and the laminate level. PROCSAN (PRObabilistic Composite Structural ANAlysis) determines the perturbed and probabilistic structural response at global, laminate and ply levels. PICAN and PROCSAN share the FPI (Fast Probability Integrator) module \([13]\) for the application of fast probability integration algorithm to obtain cumulative distribution functions of the material properties and the structural...
The Fast Probability Integration (FPI) is an approximate technique for the probabilistic analysis of the structural performance. The major advantage of fast probability integration is the speed. FPI techniques are an order of magnitude or greater more efficient than Monte Carlo simulation methods. This is especially true in the tails of the distribution, i.e., very high or low probabilities since the FPI solution time is independent of the probability level; whereas, in simulation methods the computational time increases with very high or low probability levels. Also, FPI will evaluate information that describes the relative importance of each random variable. These "sensitivity factors" can be a valuable aid to optimizing a design.

In IPACS the probabilistic assessment of composite structures starts with the identification of uncertain primitive variables at constituent and ply levels. These variables are then selectively perturbed several times in order to create a database for the determination of the relationship between the desired structural response (or the desired material property) and the primitive variables. For every given perturbed primitive variable, micromechanics is applied to determine the corresponding perturbed mechanical properties at the ply and laminate level. Laminate theory is then used to determine the perturbed resultant force/moment-strain/curvature relationship. With this relationship at the laminate level, a finite element perturbation analysis is performed to determine the perturbed structural responses corresponding to the selectively perturbed primitive variables. This process is repeated until enough data is generated and the proper relationship between structural responses and primitive variables can be determined through a numerical procedure.

With the known probabilistic distributions of the primitive variables and a numerically determined relationship between the structural response and the primitive variables, fast probability integration is applied. For every discrete response value, a corresponding cumulative probability can be computed quickly by FPI. This process is repeated until the cumulative distribution function can be appropriately represented. The probabilistic material properties at ply and laminate levels are also computed in the same way as for the structural responses. The output information from FPI for a given structural response includes its discrete CDF values,
the coefficients for a special type of probability distribution function, and the sensitivity factors of the primitive variables to the structural response.

4. DETERMINATION OF THE FATIGUE LIFE

The most critical areas (hot spots) of the structural components are identified first based on the finite element analysis. The force and the moment resultants at the critical section are saved for the computation of the fatigue life using ICAN's durability analysis module. The cyclic life is determined based on the first ply failure criteria. According to this criteria the number of cycles of any given set of loading combinations that give rise to failure in one of the plies at the critical section is assumed to give a conservative estimate of the cyclic life. The laminate, by virtue of the unfailed remaining plies, may be able to survive additional cycles of load. The procedure for the determination of the fatigue life is shown in Figure 3.

5. NUMERICAL EXAMPLES AND DISCUSSIONS

The probabilistic fatigue life of a composite wing type structure was computationally simulated with the computer code IPACS. The wing consists of the skin, spars and ribs made of graphite/epoxy composite material. The laminate configurations for the skin, spars and ribs are [0₂/45₂/-45₂/90₂/0₂]s, [0₈] and [0₆] respectively. The geometry of the structure and the finite element model are shown in Figure 4. The model consists of 840 nodes and 908 4-node shell elements (6 dof per node).

Uncertainties in material properties are identified at all composite levels. At the constituent level, ten material properties for the fiber and seven material properties for the matrix are modelled as uncertain variables. Their respective probability distribution type and associated parameters are listed in Table 1. At the ply level, the fabrication variables (the fiber volume ratio, the void volume ratio, the ply orientation, the ply thickness) are also treated as random variables. Their statistics are listed in Table 2. The random loading is due to the fluctuating random pressure which is assumed to be a narrowband Gaussian random process with mean equal to 4 psi and the coefficient of variation equal 0.03. The variation of the mean pressure
across the wing surface is shown in Figure 5. The uncertain load variable $F_t$ for the fatigue life analysis is modelled by equation (1).

$$ F_t = D F_p $$

where $F_p$ is a random variable with Rayleigh distribution for the peak value of a narrowband Gaussian process. $D$ represents the loading effect which takes into account the dynamic loading aspect [14]. In this paper, $D$ is set to be 1 for a quasi static analysis. The wing was analyzed under six different hygrothermal environments. The material properties at a given hygrothermal condition is shown in equation (2) [6].

$$ M_p = \left[ \frac{T_{GW} - T_T}{T_G - T_0} \right]^{1/2} M_{p0} $$

where

- $M_p$ = the material property at a given hygrothermal condition
- $T_{GW}$ = glass transition temperature of the resin at that moisture content.
- $T_T$ = test or use temperature at which $M_p$ is to be predicted.
- $T_{T0}$ = test temperature at which $M_{p0}$ was measured.
- $M_{p0}$ = reference material property

Results from these six cases were summarized in Table 3. The cumulative distribution functions (CDF) and the probability density functions (PDF) of the fatigue life of the upper skin (under compression) for these six cases were simulated and plotted in Figures 6 to 11.

In case 1, the temperature on the structure is 70 °F without moisture. The critical fatigue cycle, at which the cumulative probability is equal to 0.001, is found to be $10^{10.76}$. In second case, the moisture content of the wing increases to 1 % and the temperature remains the same. The composite material properties were changed by the moisture and moisture induced forces were also included in the finite element analysis. The critical fatigue life was reduced from $10^{10.76}$ in case 1 to $10^{10.60}$ by the moisture effect alone.
In case 3, thermal loads and material degradation due to temperature were included by raising the temperature to 200 °F and keeping moisture content to zero. In this case, the critical fatigue life was reduced to $10^{8.81}$. In fourth case, thermal loads were excluded and the temperature degradation on the composite material properties was considered. The critical fatigue life was reduced to $10^{8.18}$. By comparing with the case 3, it was found that the temperature degradation on material properties results in a fatigue life reduction, however, the thermal loads increase the compressive fatigue life.

In case 5, both thermal and moisture induced loads as well as the material property degradation under hygrothermal environments were considered. The critical fatigue life was reduced to $10^{8.09}$. In case 6, the thermal loads were not used in the analysis and a critical compressive fatigue life reduction was again observed.

The important observations from the study of these six cases are: (1) The fatigue life of complex composite structures in adverse environments can be evaluated probabilistically using IPACS; and (2) The hygrothermal environment has severe degradation effects on the compressive fatigue life.

Sensitivity analysis was performed for each hygrothermal environment. This analysis identifies those uncertain variables which have a major influence in the probabilistic fatigue life. The eight most important random variables for the compressive fatigue life of the composite wing under complete hygrothermal consideration (case 5, at 0.001 cumulative probability level) are shown in Figure 12. These sensitivity factors indicate that uncertainties in: (1) the fiber modulus, the fiber volume ratio and the matrix shear strength of the skin, (2) the fiber modulus and the fiber volume ratio of the frame, and (3) the cyclic load have the most significant (about 40%) influence on the composite wing compressive fatigue life. The matrix tensile strength and the void volume ratio of the skin also have a significant influence (about 20%). The important observation is that the local stiffness and the local strength (matrix dominated) are important in the composite fatigue life based on the first ply failure criteria after the uncertainty in the cyclic load.
6. CONCLUSIONS

The results of an investigation to probabilistically evaluate the composite structures for the cyclic load endurance are as follows:

(1) A formal methodology and a computer code IPACS to probabilistically quantify the uncertainties in composite structures has been developed and demonstrated. This methodology is computationally efficient and accurate for probabilistic assessment of composite structures.

(2) The fatigue life of the complex composite structures in adverse environments can be probabilistically assessed using IPACS.

(3) For the specific cases studied in this paper, it is found that based on the first ply failure criteria the fatigue life of the composite structure is reduced significantly due to the hygrothermal effect.

(4) The uncertainties in the cyclic load, the local stiffness and the local strength (matrix dominated) influence the compressive fatigue life significantly compared to uncertainties from other factors.

7. REFERENCES


8. SYMBOLS

\[ E_{f1} \] : fiber modulus in longitudinal direction

\[ E_{r2} \] : fiber modulus in transverse direction

\[ G_{f2} \] : in-plane fiber shear modulus

\[ G_{r3} \] : out-of-plane fiber shear modulus
\( \nu_{n2} \) : in-plane fiber Poisson’s ratio  
\( \nu_{r23} \) : out-of-plane fiber Poisson’s ratio  
\( \alpha_{n1} \) : fiber thermal expansion coefficient in longitudinal direction  
\( \alpha_{r22} \) : fiber thermal expansion coefficient in transverse direction  
\( S_{ft} \) : fiber tensile strength  
\( S_{fc} \) : fiber compressive strength  
\( E_m \) : matrix elastic modulus  
\( G_m \) : matrix shear modulus  
\( \nu_m \) : matrix Poisson’s ratio  
\( \alpha_m \) : matrix thermal expansion coefficient  
\( S_{mt} \) : matrix tensile strength  
\( S_{mc} \) : matrix compressive strength  
\( S_{ms} \) : matrix shear strength  
\( \text{stdv} \) : standard deviation  
\( \text{cov} \) : coefficient of variation  
\( \text{fvr} \) : fiber volume ratio  
\( \text{vvr} \) : void volume ratio  
\( \theta_p \) : ply misalignment  
\( t_{ps} \) : ply thickness of skin  
\( t_{pt} \) : ply thickness of frames
Table 1. Material Properties at the Constituent Level for the Skin and Frames

<table>
<thead>
<tr>
<th>Unit</th>
<th>Distribution Type</th>
<th>Mean</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_{11}</td>
<td>Msi</td>
<td>Normal</td>
<td>31.0</td>
</tr>
<tr>
<td>E_{22}</td>
<td>Msi</td>
<td>Normal</td>
<td>2.0</td>
</tr>
<tr>
<td>G_{12}</td>
<td>Msi</td>
<td>Normal</td>
<td>2.0</td>
</tr>
<tr>
<td>G_{23}</td>
<td>Msi</td>
<td>Normal</td>
<td>1.0</td>
</tr>
<tr>
<td>ν_{12}</td>
<td>---</td>
<td>Normal</td>
<td>0.2</td>
</tr>
<tr>
<td>ν_{23}</td>
<td>---</td>
<td>Normal</td>
<td>0.25</td>
</tr>
<tr>
<td>α_11</td>
<td>ppm/°F</td>
<td>Normal</td>
<td>-0.55</td>
</tr>
<tr>
<td>α_{22}</td>
<td>ppm/°F</td>
<td>Normal</td>
<td>5.6</td>
</tr>
<tr>
<td>S_{11}</td>
<td>Ksi</td>
<td>Weibull</td>
<td>400</td>
</tr>
<tr>
<td>S_{12}</td>
<td>Ksi</td>
<td>Weibull</td>
<td>400</td>
</tr>
<tr>
<td>E_m</td>
<td>Msi</td>
<td>Normal</td>
<td>0.5</td>
</tr>
<tr>
<td>G_m</td>
<td>Msi</td>
<td>Normal</td>
<td>0.185</td>
</tr>
<tr>
<td>ν_m</td>
<td>---</td>
<td>Normal</td>
<td>0.35</td>
</tr>
<tr>
<td>α_{m}</td>
<td>ppm/°F</td>
<td>Normal</td>
<td>42.8</td>
</tr>
<tr>
<td>S_{m1}</td>
<td>Ksi</td>
<td>Weibull</td>
<td>15</td>
</tr>
<tr>
<td>S_{m2}</td>
<td>Ksi</td>
<td>Weibull</td>
<td>35</td>
</tr>
<tr>
<td>S_{m3}</td>
<td>Ksi</td>
<td>Weibull</td>
<td>13</td>
</tr>
</tbody>
</table>
Table 2. The Fabrication Variables

<table>
<thead>
<tr>
<th>Unit</th>
<th>Distribution Type</th>
<th>Mean</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>fvr</td>
<td>--- Normal</td>
<td>0.60</td>
<td>0.02</td>
</tr>
<tr>
<td>vvr</td>
<td>--- Normal</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>( \theta_p )</td>
<td>degree Normal</td>
<td>0.00</td>
<td>0.50 (stdv)</td>
</tr>
<tr>
<td>( t_{pk} )</td>
<td>in Normal</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>( t_{ps} )</td>
<td>in Normal</td>
<td>0.08</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 3. Summary for the Probabilistic Compressive Fatigue Life under Different Hygrothermal Environments

<table>
<thead>
<tr>
<th>CASE</th>
<th>REFERENCE TEMPERATURE E</th>
<th>OPERATING TEMPERATURE E</th>
<th>MOISTURE CONTENT</th>
<th>( \log_{10}(\text{CYCLES}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70 °F</td>
<td>70 °F</td>
<td>0 %</td>
<td>10.76</td>
</tr>
<tr>
<td>2</td>
<td>70 °F</td>
<td>70 °F</td>
<td>1 %</td>
<td>10.60</td>
</tr>
<tr>
<td>3</td>
<td>70 °F</td>
<td>200 °F</td>
<td>0 %</td>
<td>8.81</td>
</tr>
<tr>
<td>4</td>
<td>200 °F</td>
<td>200 °F</td>
<td>0 %</td>
<td>8.18</td>
</tr>
<tr>
<td>5</td>
<td>70 °F</td>
<td>200 °F</td>
<td>1 %</td>
<td>8.09</td>
</tr>
<tr>
<td>6</td>
<td>200 °F</td>
<td>200 °F</td>
<td>1 %</td>
<td>7.37</td>
</tr>
</tbody>
</table>

* Probability (Fatigue life < Critical Fatigue life) = 0.001
Figure 1. Probabilistic Assessment of Composite Structures.
Figure 2. Concept of Probabilistic Assessment of Composite Structures
INPUT: temperature, moisture, static loads, cyclic loads, cyclic degradation coefficient $\beta^*$

Number of cycles $N$

Calculate static stress $\sigma_{st}$, cyclic stress $\sigma_{cyc}$, static strength $S_{st}$, cyclic strength $S_{cyc} = S_{st} (1 - \beta \log_{10} N)$, for ply $i$ ($i=1,NL$ where $NL$: total number of plies)

Calculate safety margins: $R$

$$R(i,j) = 1 - \left( \frac{\sigma_{st}}{S_{st}} + \frac{\sigma_{cyc}}{S_{cyc}} \right)_j$$

j: longitudinal, transverse, shear

no

i=NL

Select a smaller N

< 0

yes

Select a larger N

> 0

$R(i,j) = 0$

Fatigue life = N cycles

- Coefficient to be determined for the specific composite system

Figure 3: The Procedure for the Determination of the Fatigue Life
Figure 4. Geometry and Finite Element Model of a Composite Wing

Figure 5. Mean Pressure Variation on a Composite Wing
Figure 6. Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the Compressive Fatigue Life of the Composite Wing for Case 1 (70°F)

Figure 7. Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the Compressive Fatigue Life of the Composite Wing for Case 2 (70°F, 1% Moisture)

Figure 8. Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the Compressive Fatigue Life of the Composite Wing for Case 3 (200°F)
Figure 9. Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the Compressive Fatigue Life of the Composite Wing for Case 4 (200°F, without Thermal Loads)

![Figure 9](image_url_9)

Figure 10. Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the Compressive Fatigue Life of the Composite Wing for Case 5 (200°F, 1% Moisture)

![Figure 10](image_url_10)

Figure 11. Cumulative Distribution Function (CDF) and Probability Density Function (PDF) of the Compressive Fatigue Life of the Composite Wing for Case 6 (200°F, 1% Moisture, without Thermal Loads)

![Figure 11](image_url_11)
Figure 12. Sensitivity Factors of Probabilistic Compressive Fatigue Life of the Composite Wing at 0.001 Cumulative Probability for Case with 200° F Temperature and 1% Moisture
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