51-39
187832
p- 28

Adaptive Methods for Nonlinear Structural Dynamics and Crashworthiness Analysis

Ted Belytschko
Northwestern University

ADAPTIVE METHODS FOR NONLINEAR STRUCTURAL DYNAMICS AND CRASHWORTHINESS ANALYSIS

Ted Belytschko
Northwestern University
Evanston, Illinois

ABSTRACT

The objective of this talk is to describe three research thrusts in crashworthiness analysis:

1) adaptivity
2) mixed time integration, or subcycling, in which different timesteps are used for different parts of the mesh in explicit methods
3) methods for contact-impact which are highly vectorizable.

The techniques are being developed to improve the accuracy of calculations, ease-of-use of crashworthiness programs and the speed of calculations. The latter is still of importance because crashworthiness calculations are often made with models of 20,000 to 50,000 elements using explicit time integration and require on the order of 20 to 100 hours on current supercomputers.

The methodologies will be briefly reviewed and then some example calculations employing these methods will be described. The methods are also of value to other nonlinear transient computations.

8

OUTLINE

- Adaptive mesh procedures in nonlinear analysis: why, how, and what is the status
- Subcycling (mixed time integration)
- New highly vectorizable methods for contact impact which are well suited to adaptive methods

Figure 1

PREDICTION

The 1990's will be the decade of *adaptivity*.

adaptive mesh refinement

adaptive targeting

adaptive organization objectives

Figure 2

PREDICTION

There are three types of adaptivity, which are known by the letters r, h, and p. These letters are mnemonic letters and refer to how the refinement is achieved. In r methods the nodes are relocated. In h methods, refinement is achieved by reducing the element size h. In p methods, refinement is achieved by increasing the order p of the element interpolance.

TYPE OF MESH ADAPTIVITY

r — method

relocate nodes

h — method

adapt element size h

p — method

adapt order p of element interpolants

Figure 3

ADAPTIVITY IN NONLINEAR FEM

Adaptive methods are particularly useful in nonlinear problems such as crashworthiness because nonlinear response is often characterized by localization. In the areas of localized response more refinement is needed. When standard method is used, the user of the program must refine the mesh where he anticipates this localized deformation. Therefore, different meshes must be developed for different loadings. For example, in car crash, different meshes must be developed for frontal and rear impact, side impact, and overturning. This can be quite expensive from the viewpoint of manpower.

Why are adaptive methods particularly important in nonlinear problems?

Modes of failure of structures

i. buckling, particularly with formation of hingelines

ii. localization

iii. fracture

All of these involve local phenomena whose location cannot be determined at the outset of a simulation.

Figure 4

COMMENTS ON ADAPTIVITY FOR SHELL AND CRASHWORTHINESS PROBLEMS

In comparing the different types of adaptivity for nonlinear structural dynamics problems such as crashworthiness, the following advantages, which are marked by a plus sign (+), and disadvantages which are marked by a minus sign (-), can be attributed to the various types of methods. From this study we concluded that the h-method was the most suitable method for adaptivity in crashworthiness.

r — method

- \- large elements cannot represent shape of shell
- \+ most accuracy with given NDOF compared to h
- \- history diffusion
- \- elements become distorted - decreases accuracy
- \+ easiest data structure

p — method

- \- awkward in nonlinear dynamics; no good lumped mass
- \+ easy data structure

h — method

- \+ relatively effective
- \+ no distortion of elements
- \- moderately complex data structure

Figure 5

TYPES OF ERROR INDICATORS

Error indicators are an important ingredient in adaptive methods since they are to a guide the refinement of the mesh. Error indicators are classified by Oden in the following classes: residual, interpolation, and post-processing. In the work we are doing, we are using projection error criterions, a post-processing type, because they are very easy to implement and are quite effective for low-order elements.

1. Residual: Compute residual in governing equations and use its norm or use it to drive an element or local enriched solution.

   a) Explicit: Evaluate a norm of the residual.

   b) Implicit: Use residual to drive a local or element error equation.

2. Interpolation Methods: Estimate magnitude of derivatives of higher-order than contained in finite element space.

3. Projection (postprocessing) Methods: Obtain a smoothed solution and compare to finite element solution; sometimes called L2 projection methods.

Figure 6

ADAPTIVE SCHEMES FOR TRANSIENT AND NONLINEAR PROBLEMS

based on constant resource approach

1. Advance the solution n time steps

2. Compute element error indicators θ_e

3. Sort θ_e

4. Fission elements with $\theta_e > tol^{\mathbf{fusion}}$
   Fuse elements with $\theta_e < tol^{\mathbf{fusion}}$

5. Repeat the last n time steps with new mesh (optional)

6. go to 1

Note: If n is too small or $tol^{\mathbf{fission}}$ too close to $tol^{\mathbf{fusion}}$, we encounter "churning" which degrades accuracy. Our recent experience shows 5 is quite important.

Figure 7

REMARKS ON H-ADAPTIVITY

Constraints (or slave nodes in explicit methods) must be introduced at nodes where a large element has two or more neighbors on one side to enforce compatibility; easy in vector methods, awkward in matrix methods.

Usually a group of contiguous elements should be fissioned simultaneously because fissioning a single element does not provide much enrichment; only one new free node.

In wave propagation problems, change in element size can cause spurious reflections.

Usually mesh gradation is limited to 1-irregular meshes: large element cannot have more than 2 small neighbors on any side; see Devloo, Oden and Strouboulis (1987).

Data structure with fission and fusion is complex, particularly for real engineering meshes; see Belytschko, Wong and Plaskacz, *Computers and Structures*, 33(4-5), 1989, pp. 1307-1323.

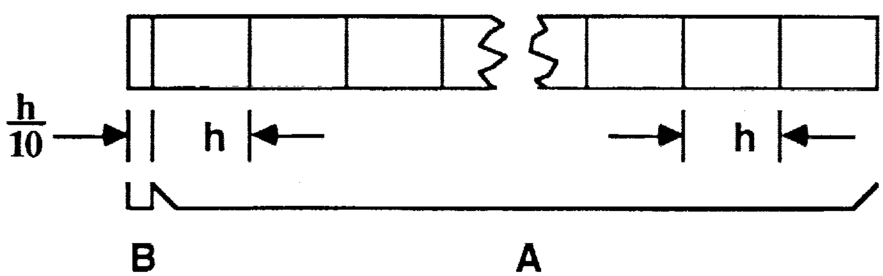
Figure 8

MIXED TIME INTEGRATION

In h-adaptive meshes, large variety of element sizes are found. When explicit methods are used of such meshes, the timestep is reduced dramatically by the presence of small elements. Therefore methods called mixed time integration (or subcycling) are being used.

Motivation : in explicit integration with same Δt over entire mesh, stiffest element sets Δt. also called subcycling, explicit-explicit partitions;

example



$\Delta t_{crit} = \min\left(\frac{L}{c}\right)$ c = wave speed

for A $\Delta t_{crit} = \dfrac{h}{c}$

for $A \cup B$ $\Delta t_{crit} = \dfrac{h}{10c}$

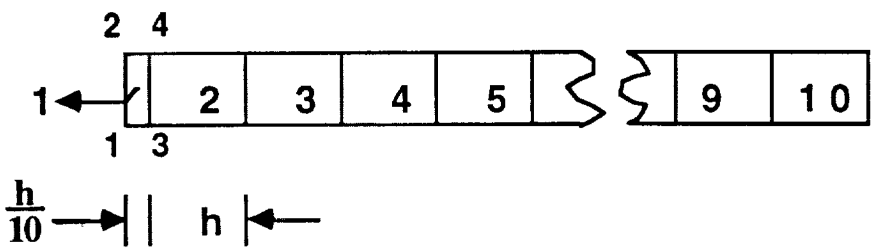
so $A \cup B$ is $10\times$ as expensive as A

Figure 9

Mixed Time Integration

Integrate each element or subdomain with Δt_{crit} using an interface treatment that preserves stability + consistency.

In example



integrate element 1 and nodes 1 to 4 with

$$\Delta t = \frac{h}{10c}$$

elements 2 to 10 and remaining nodes with

$$\Delta t = \frac{h}{c}$$

cost savings: ~ 90%

In adaptive methods, large range of stable time steps is unavoidable, so subcycling is crucial for efficiency.

Figure 10

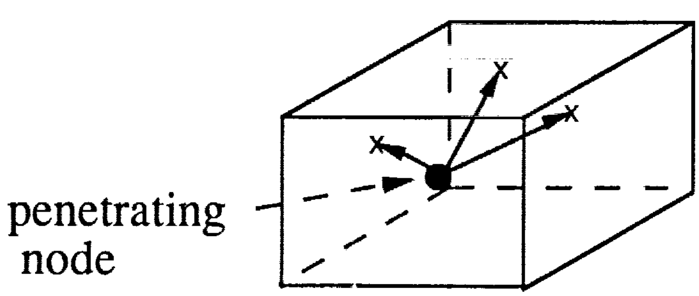
CONTACT-IMPACT

The modeling of contact-impact is very important in the simulation of crashworthiness. However, contact-impact algorithms often require more than fifty percent of the running time of a crashworthiness code because they are not easily vectorized. Therefore we have developed a pinball algorithm which is far more highly vectorizable.

Contact-impact is an important phenomenon in crash analysis, e.g.,

1. engine impact with body, fire wall
2. wheel impact with inner fender
3. contact of collapsing surfaces

Most contact-impact algorithms require many different branches.



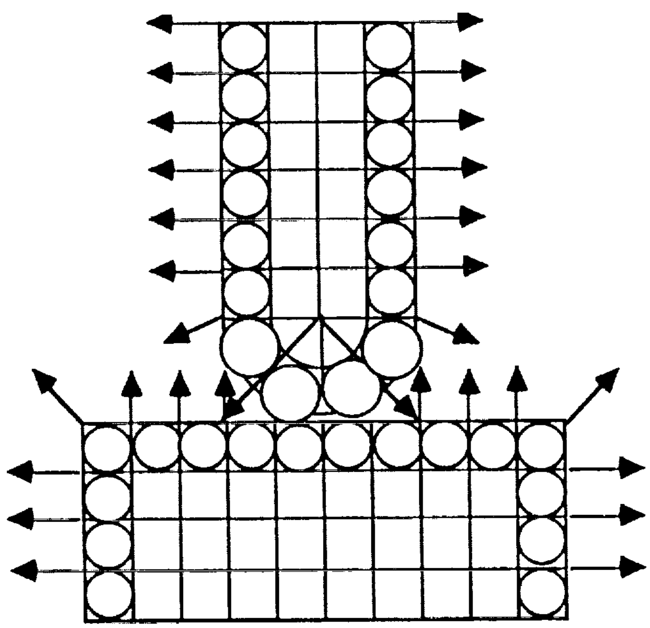
The branch of the algorithm which is activitated depends on which surface is penetrated; there are special branches for edges, etc.

Figure 11

PINBALL PENALTY ALGORITHM

T. Belytschko and M. O. Neal, *International Journal for Numerical Methods in Engineering*, 31, 1991, pp. 547-572.

Interpenetration and interpenetration rate g are computed on pinballs inserted in elements.



Enforces contact-impact conditions on spheres embedded in elements.

As h $\rightarrow$ 0, impenetrability is enforced.

Algorithm is simple and highly vectorizable.

Figure 12

Salient Features of Algorithm

Radius of pinball is determined by equivoluminal expression

$$R^3 = \frac{3V_e}{4\pi}$$

Pinballs are classed by body; for single-surface slideline, smaller R needed.

Interpenetration has occurred when

$$d_{ij} < R_i + R_j$$

$$g = d_{ij}$$

Pinball forces are equally transferred to all nodes of associated element (a surface node option available).

The pinball method automatically places pinballs on outside elements by using assembled surface normal algorithm.

Figure 13

EXAMPLES OF NONLINEAR ADAPTIVE COMPUTATIONS

Nonlinear, transient computations with an explicit nonlinear finite element program WHAMS using h-adaptivity and pinball for contact impact; see Belytschko and Yeh (1992).

An L2 projection on the strain invariants was used to calculate an error estimate.

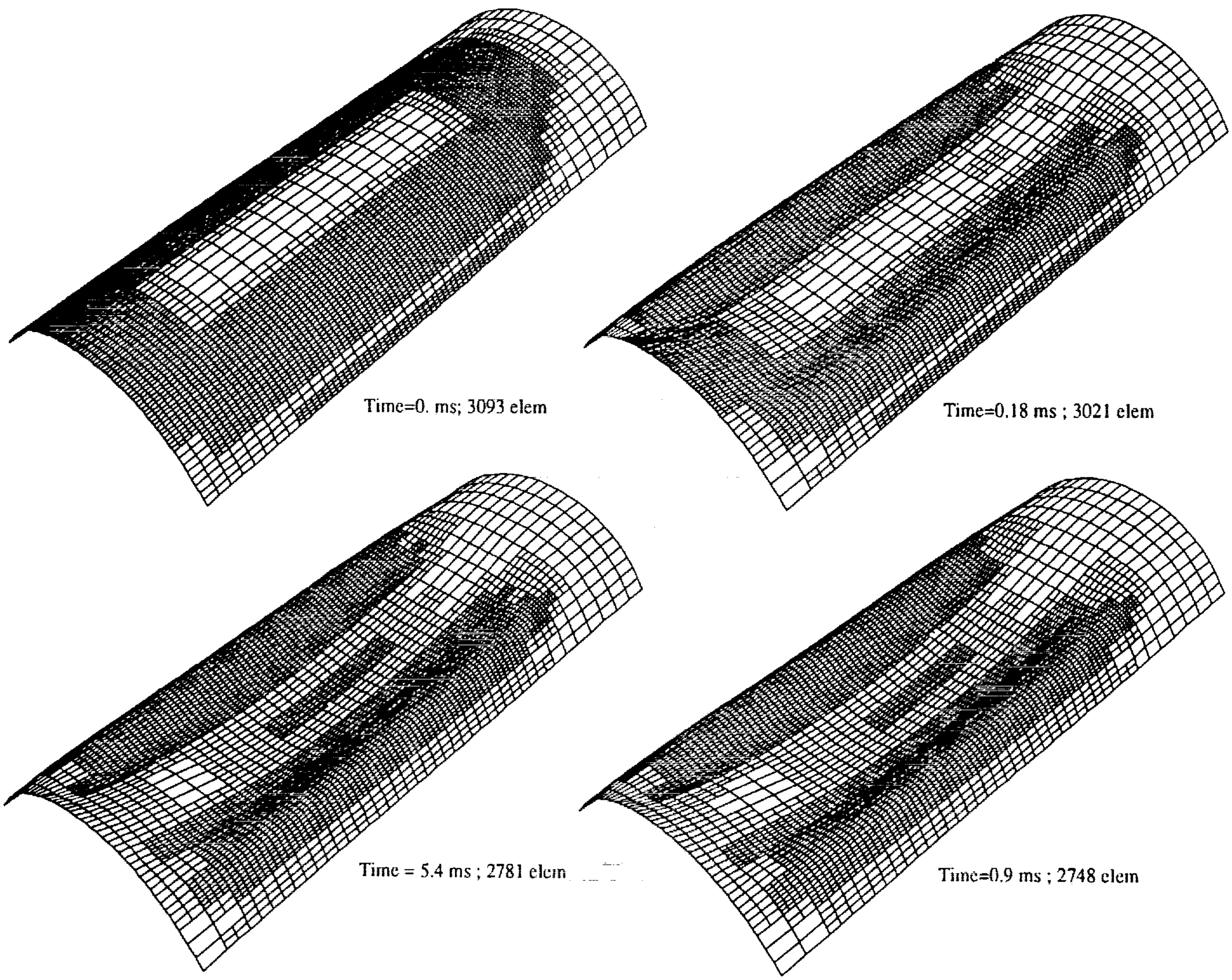
A commercial version of this program is available from:

KBS2, Inc.
455 Frontage Road
Burr Ridge, IL 60521
(708) 850-9444
Fax (708) 850-9455

Figure 14

TWO-LEVEL ADAPTIVE MESH OF CYLINDRICAL PANEL

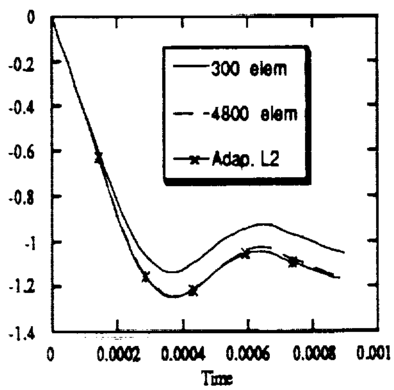
This shows an h-adaptive solution of a cylindrical panel which is impulsively loaded. Notice that the elements are refined along the side and at the support, where there is severe plastic bending deformation, and hinge lines consequently form.



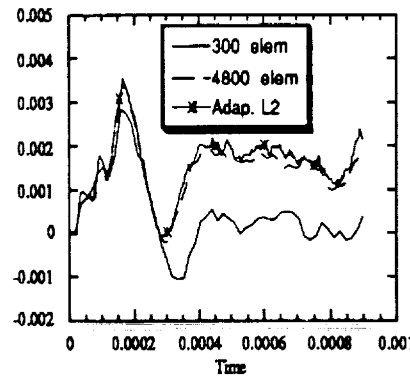
Two-level adaptive mesh of cylindrical panel.

Figure 15

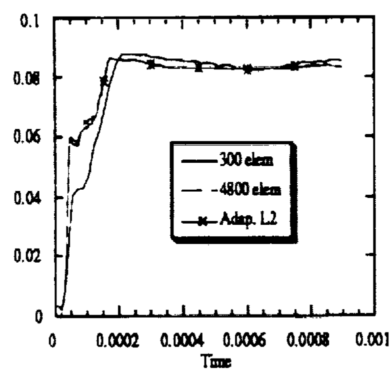
This figure shows a comparison between solutions obtained by h-adaptivity and those obtained using a very fine mesh and a coarse mesh. As can be seen, the adaptive solution compares well to the fine mesh solution. The differences in the displacements obtained by the coarse mesh and the fine mesh are not large, but for some of the stresses and strains, significant improvement is obtained by the use of adaptivity.



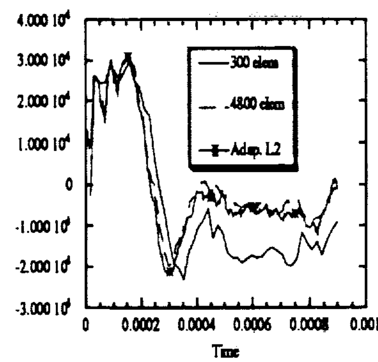
Displacement of point A
in cylindrical panel (MAXLEV=2)



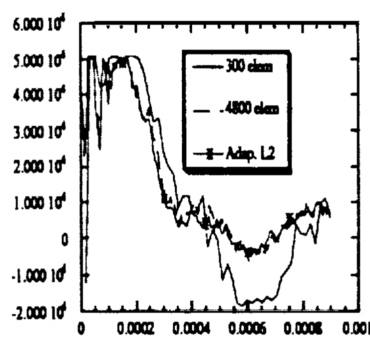
Strain Exx of element B
in cylindrical panel (MAXLEV=2)



Strain Eyy of element B
in cylindrical panel (MAXLEV=2).



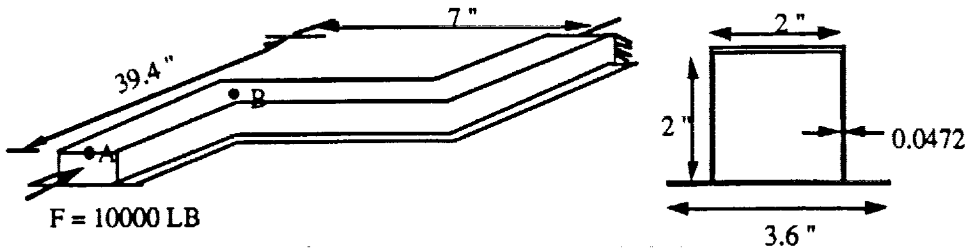
Stress Sxx of element B
in cylindrical panel (MAXLEV=2)



Stress Syy of element B in cylindrical panel (MAXLEV=2).

Figure 16

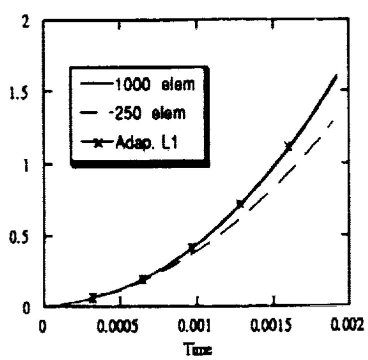
This shows results for an S-beam which is impulsively loaded. Again, significant differences occur in some of the strains for a coarse mesh solution as compared to an adaptive or fine mesh solution.



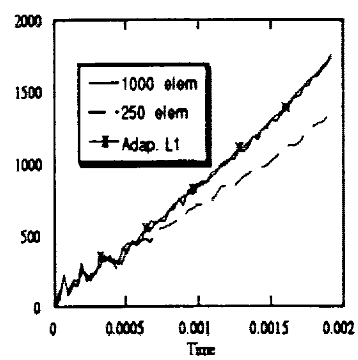
T - shape cross section

Material number 2 (Table 6)

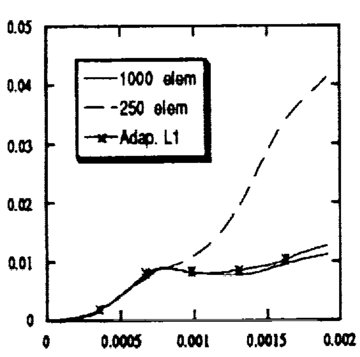
Geometry of T-shape cross section beam.



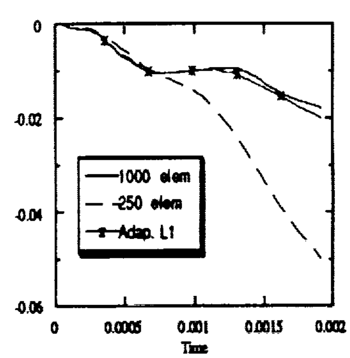
Axial displacement Dz of point A
in T-beam (MAXLEV=1).



Axial velocity Vz of point A
in T-beam (MAXLEV=1)



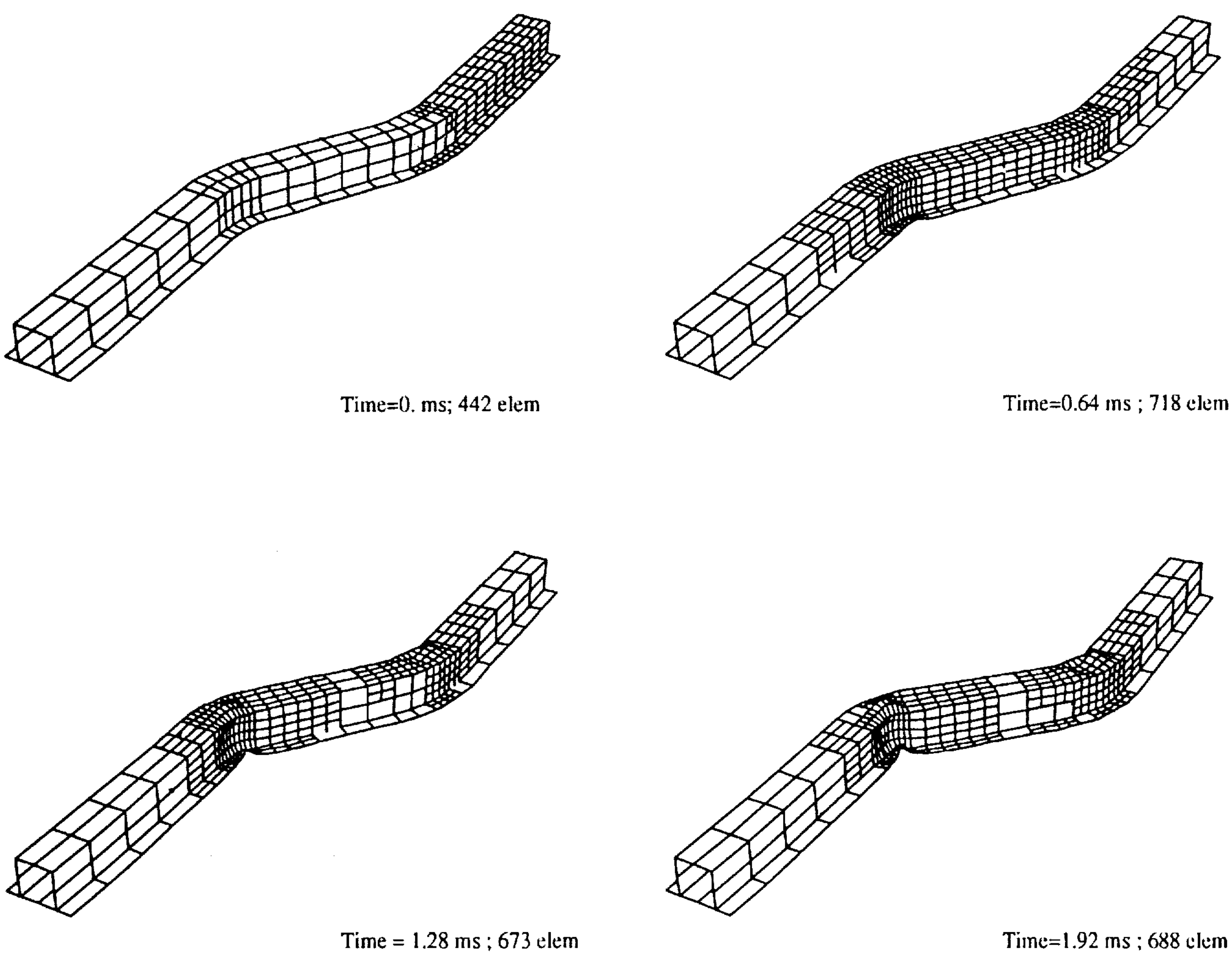
Strain Exx of element B
in T-beam (MAXLEV=1).



Strain Eyy of element
in T-beam (MAXLEV=1)

Figure 17

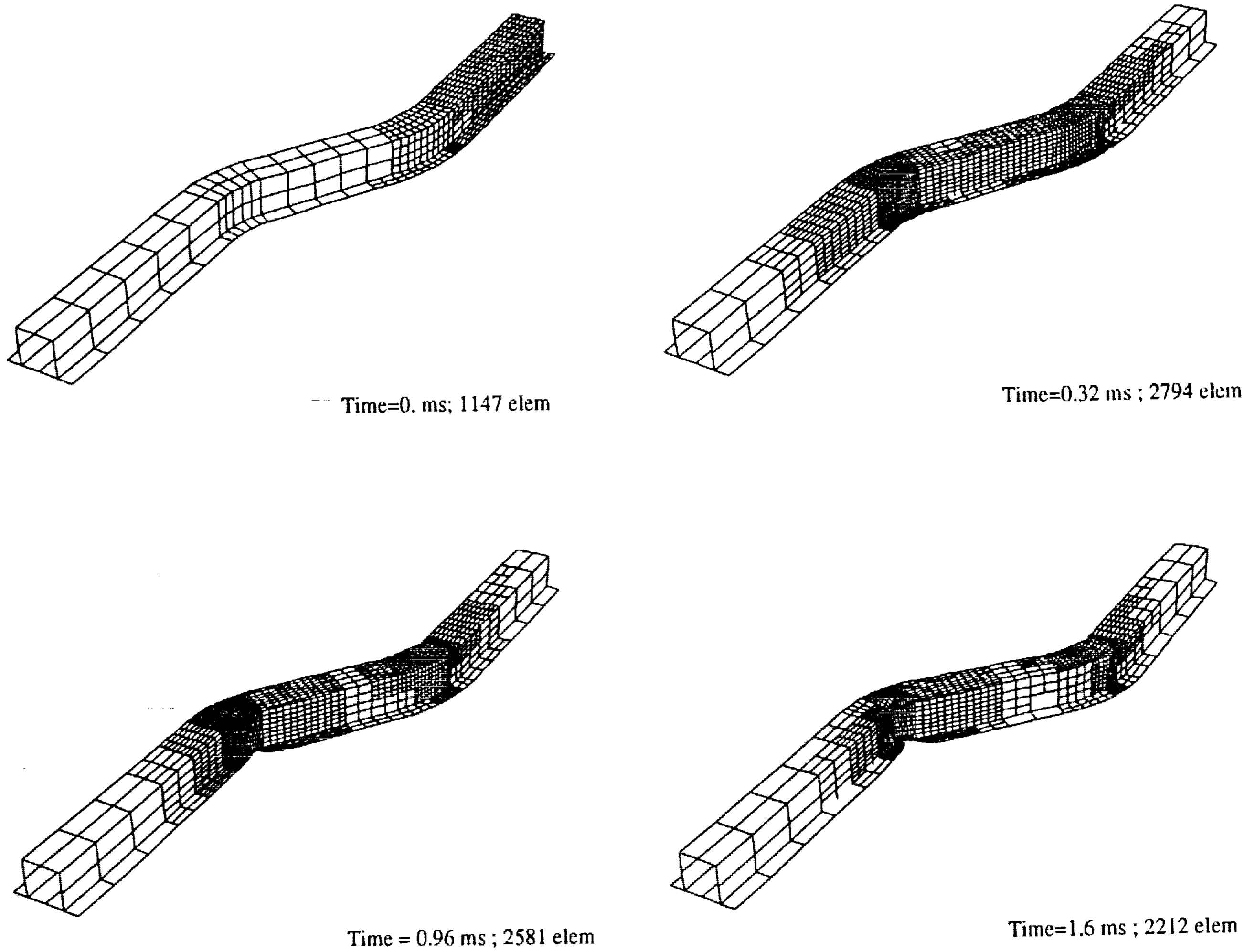
This shows the evolution of the mesh for the S-beam; note that the h-refinement occurs at a corner where local buckling takes place.



One-level adaptive mesh of T-beam.

Figure 18

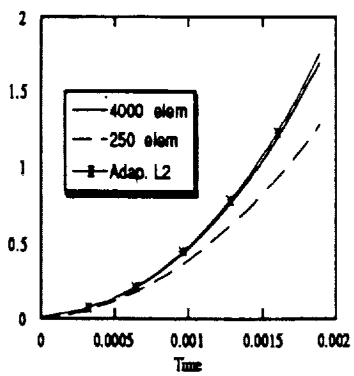
This is the same problem with a higher level of adaptivity. Far more elements are placed in the region of local buckling.



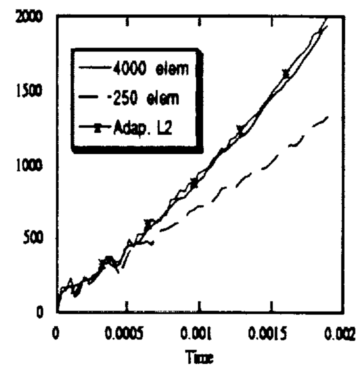
Two-level adaptive mesh of T-beam.

Figure 19

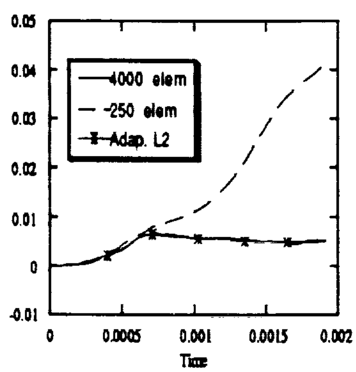
This again compares displacement in strains for coarse mesh, fine mesh, and adaptive solutions. Again the adaptive solutions agree very closely with the fine mesh solution.



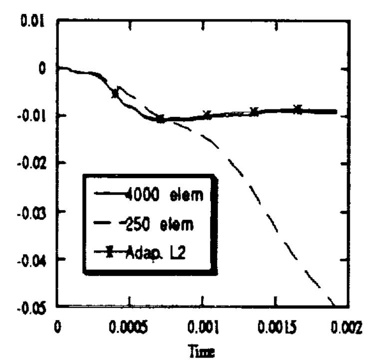
Axial displacement Dx of point A
in T-beam (MAXLEV=2).



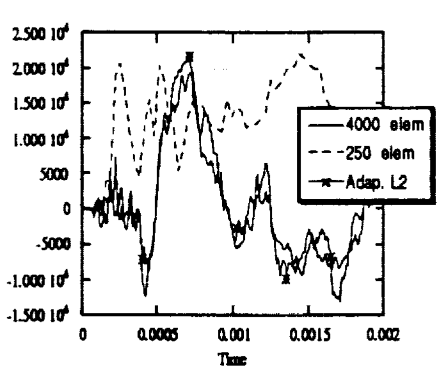
Axial velocity Vz of point A
in T-beam (MAXLEV=2)



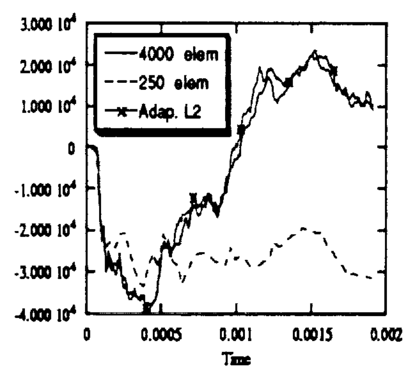
Strain Exx of element B
in T-beam (MAXLEV=2)



Strain Eyy of element B
in T-beam (MAXLEV=2)



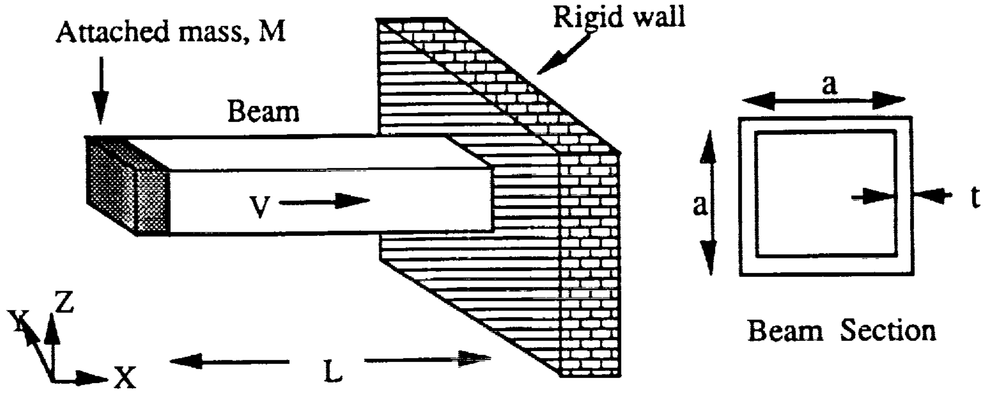
Stress Sxx of element B
in T-beam (MAXLEV=2).



Stress Syy of element B
in T-beam (MAXLEV=2).

Figure 20

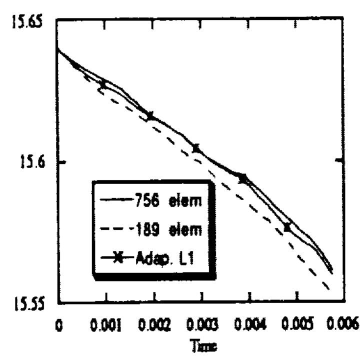
This is a solution of a box beam which has an initial velocity as shown with an attached mass at the back. This problem is often considered a model for crash analysis. The solutions for fine mesh, coarse mesh and adaptive meshes are shown; the adaptive solution agrees well with the fine mesh.



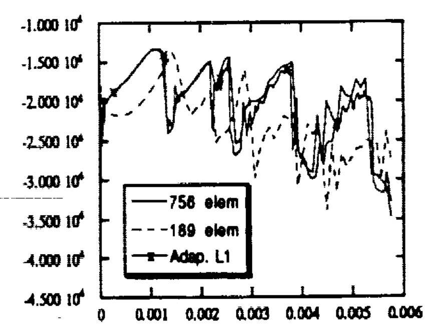
Geometry: L = 0.1500 m
a = 0.0300 m
t = 0.0015 m

Initial condition: V=15.64 m/sec
Material number 3 (Table 6)

Box beam problem.



Velocity of point A in the box-beam (MAXLEV=1).



Reaction force of box-beam (MAXLEV=1).

Figure 21

This shows a timing for a full car model which is shown on the next page. It is solved with full contact-impact and subcycling. The important thing to notice is that subcycling gives a speedup of 1.7 and that the effective element cycle time on a CRAY-YMP here is 12 microseconds.

Timing

FULL-CAR MODEL

Elements:	17,297
Mass (kg):	1,880
Time steps:	78,274
80 msec simulation	

CRAY-YMP

Without subcycling: 128 elements/block	7.63 hrs
Element cycle time:	20 μsec
With subcycling: 64 elements/block	4.39 hrs
Effective element cycle time:	12 μsec
Speedup:	1.7

Figure 22

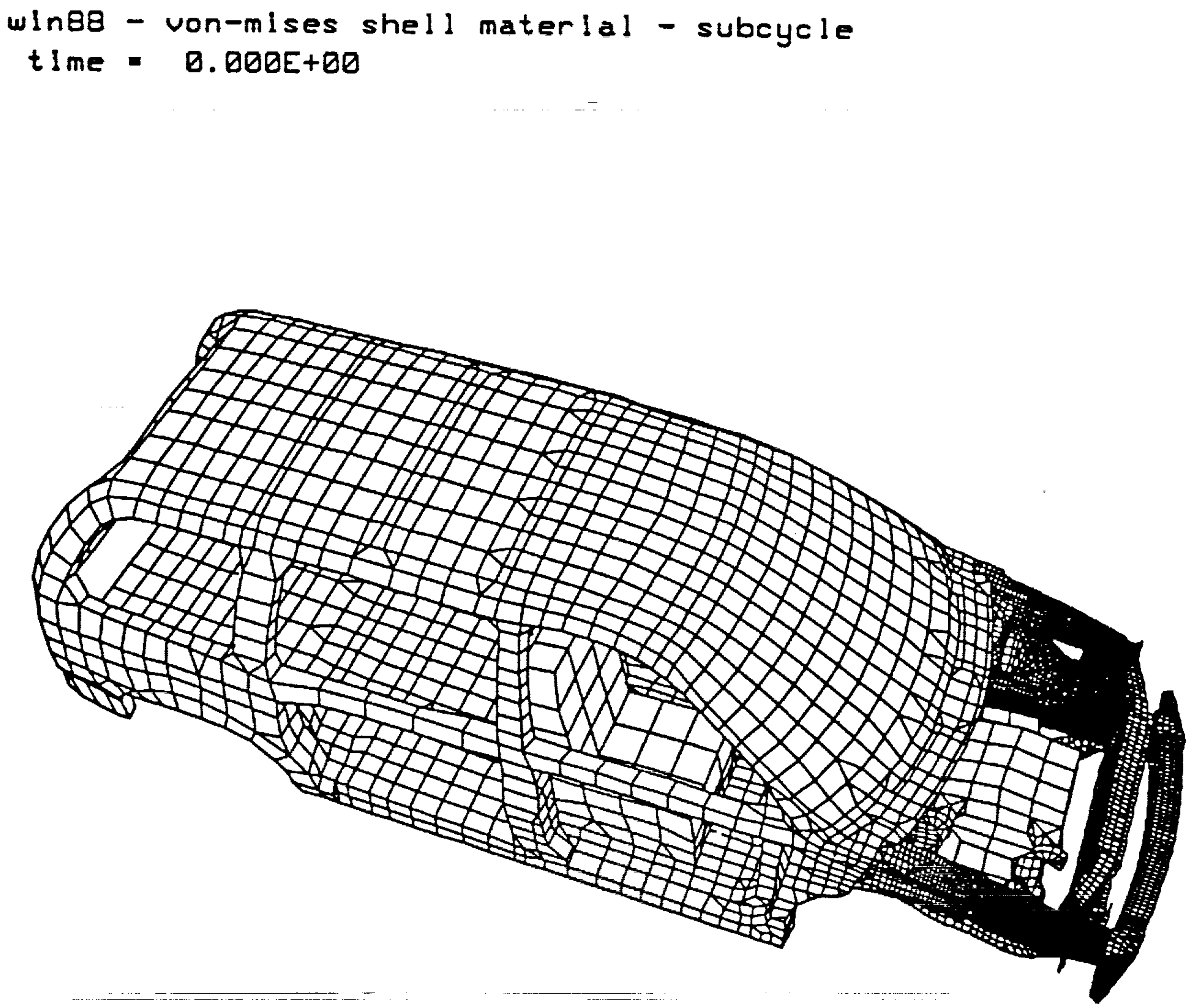


Figure 23

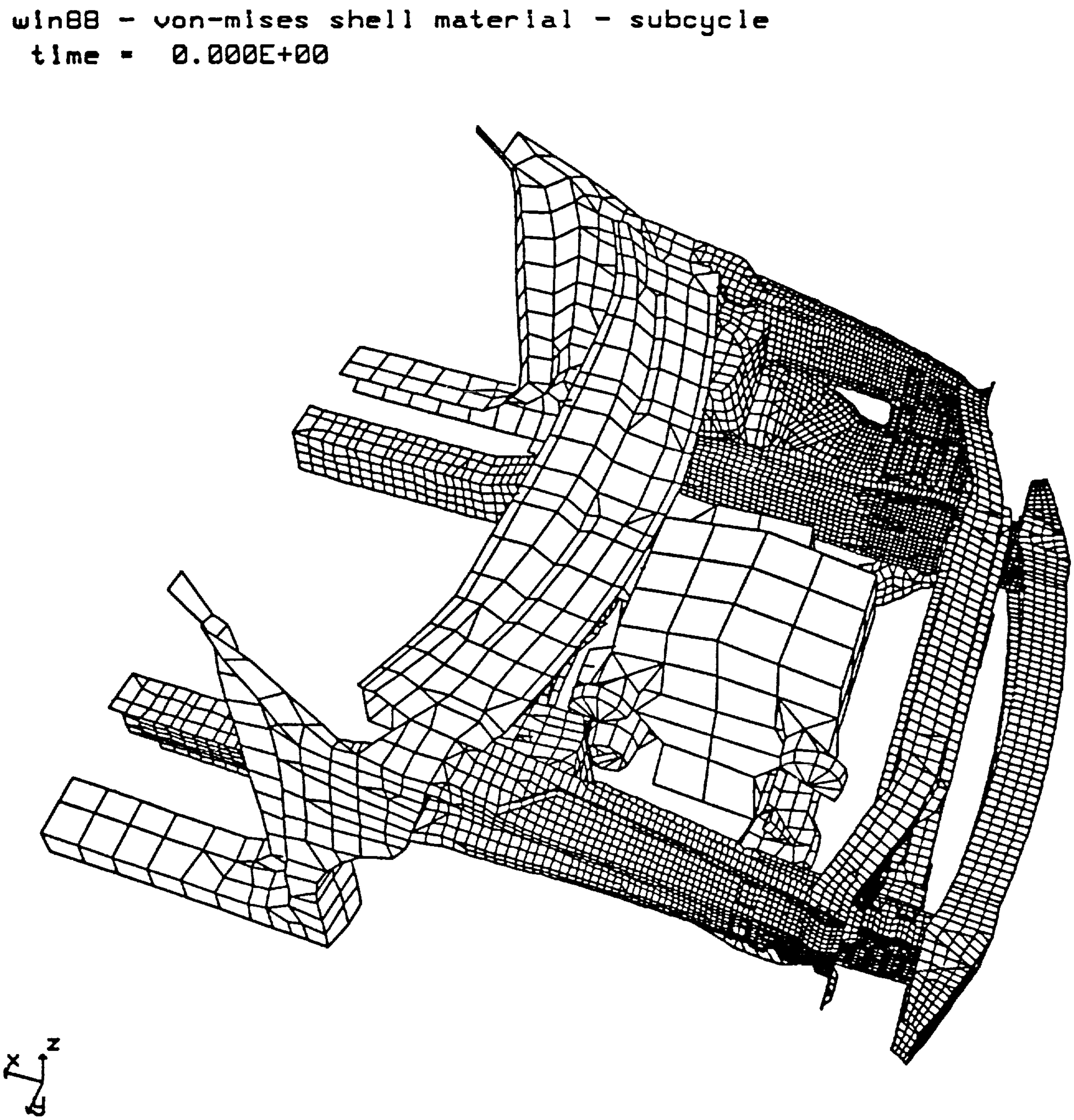


Figure 24

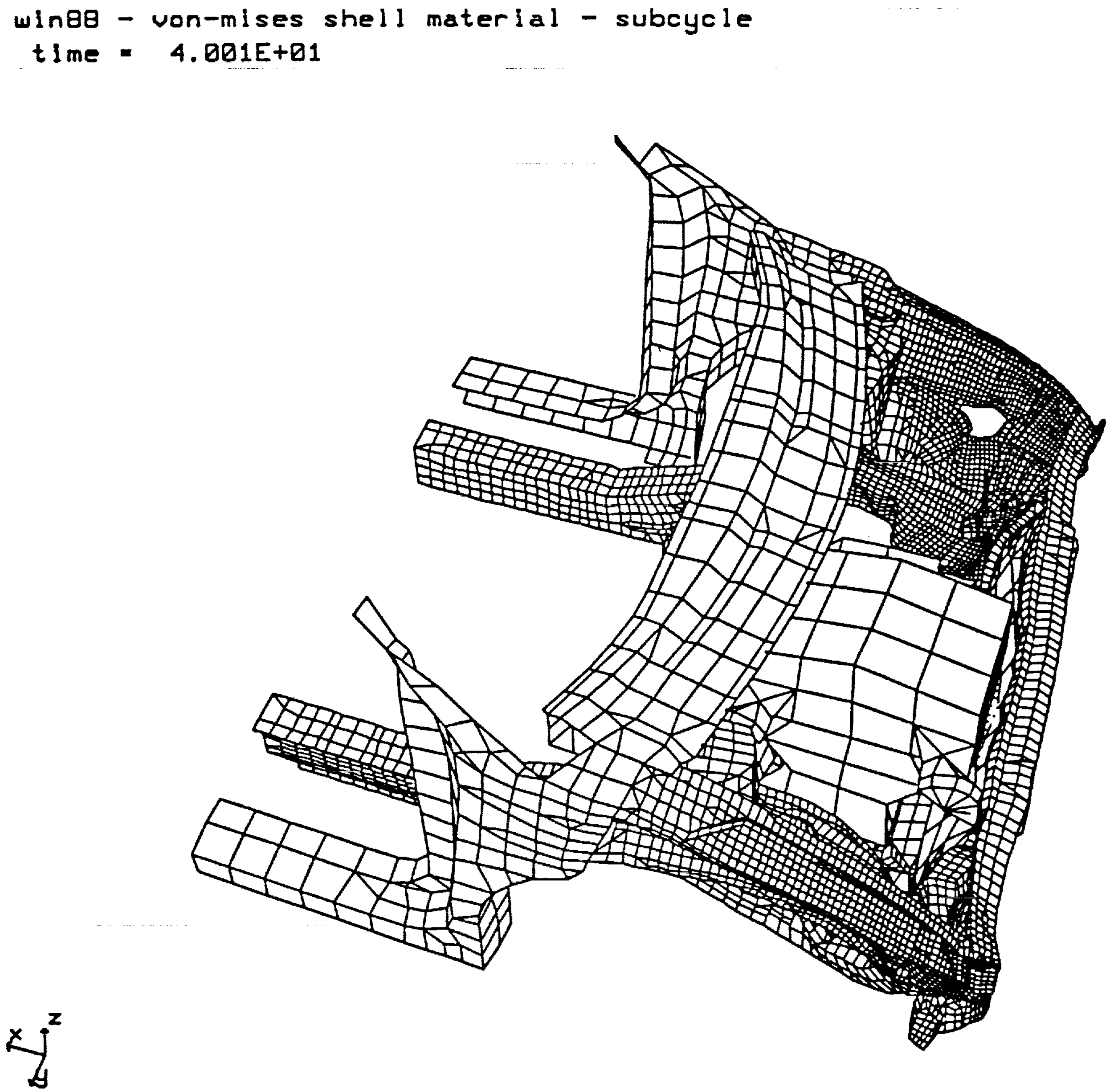


Figure 25

Implementation and Stability

Time steps are assigned to nodes and element blocks automatically.

Elements are sorted by Δt_e^{crit}

$$\Delta t_e^{crit} \leq \Delta t_2^{crit} \leq ..... \leq \Delta t_n^{crit}$$

Elements are arranged in blocks so that time steps of adjacent elements have integer ratios.*

Blocking of elements is necessary to take advantage of vectorization.

For analysis of stability, see Belytschko and Lu, ASME publication edited by G. Hulbert, et al, 1992.

*A new algorithm which does not require integer ratios has recently been developed (Belytschko and Neal, *Computer Methods in Applied Mechanics and Engineering*, 31, 1989, pp. 547-570).

Figure 26

REMARKS AND CONCLUSIONS

- H-adaptivity is a promising technique for simulating nonlinear structural response and structural failure.
  - Improves accuracy.
  - Simplifies model preparation.
- Subcycling and advanced contact-impact methods such as the pinball method can improve efficiency of explicit dynamic codes and is essential with h-adaptivity.
- Improved error criteria are needed for adaptive methods for nonlinear solid mechanics.

Figure 27